Introduction to Particle Physics
- Chapter 5 -

Colour and QCD

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Chapter summary:

- Symmetry of the Δ^{++} wave function
- The colour
- Experimental evidences of the colour
- Gluons and the QCD
- Asymptotic freedom
- hadronization
- Experimental cross-checks of the QCD predictions

Δ^{++} symmetry

- The Δ^{++} quark composition is:

$$\Delta^{++}: \ J^P = \frac{3}{2}^+ \Rightarrow u\uparrow u\uparrow u\uparrow$$

- We can factorize the wave function as:

$$\Psi = \psi(\text{spatial}) \cdot \chi(\text{spin}) \cdot \phi(\text{flavour})$$

- L=0 (baryon 3/2 of lowest mass). χ and ϕ are also symmetric, therefore the total wave function is symmetric, in contradiction with the Pauli's principle.

- In 1964 Greenberg (then Han and Nambu) introduced a new internal degree of freedom that he called colour, therefore:

$$\Psi = \psi(\text{spatial}) \cdot \chi(\text{spin}) \cdot \phi(\text{flavour}) \cdot \xi(\text{colour})$$

- ξ(colour) is antisymmetric and restores the right connection spin-statistic of the wave function.

The colour

- The base assumption of the theory is that every quark can exist in three different states of colour:

$$\xi^c = r, g, b \quad (red, green, blue)$$

- The three states can be represented as colour spinors:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad ; \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

and we can repeat whatever we said before about SU(3), but now we are talking about colour SU(3):

1. we have 8 generators of the symmetry group;

2. we can pick out 2 diagonal generators that have the colour spinors as eigenstates.

The Gell-mann matrices

$$U = e^{-\frac{1}{2} i \vartheta \hat{n} \cdot \vec{\lambda}}$$

Rotation in the space of colour

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$I_3 = \frac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

I_3 = colour isospin

$$Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Y = colour hypercharge

The colour quantum numbers

- The diagonal operators are λ_3 and λ_8, where:

$$\lambda_3|r\rangle = \frac{1}{2}|r\rangle \;;\; \lambda_8|r\rangle = \frac{1}{\sqrt{3}}|r\rangle \quad ; \quad \text{etc...}$$

- Let's introduce a colour I_3^c and a colour hypercharge Y^c, defined as:

$$I_3^c \equiv \lambda_3 = \;;\quad Y^c = \frac{1}{\sqrt{3}}\lambda_8$$

- With this definition we have the following quantum numbers:

	quark			antiquark	
	I_3^c	Y^c		I_3^c	Y^c
r:	$\frac{1}{2}$	$\frac{1}{3}$	$\bar{r}$:	$-\frac{1}{2}$	$-\frac{1}{3}$
g:	$-\frac{1}{2}$	$\frac{1}{3}$	$\bar{g}$:	$\frac{1}{2}$	$-\frac{1}{3}$
b:	0	$-\frac{2}{3}$	$\bar{b}$:	0	$\frac{2}{3}$

The values I_3^c and Y^c are additive quantum numbers.

Colour confinement

- With the introduction of the colour hypothesis the number of quarks are multiplied by three, therefore we should have a proliferation of particles; for instance the proton could exist as:

$$p = u_r u_b d_g \; ; \; u_r u_g d_b \; ; \; u_b u_g d_b \; ; \; \text{etc....}$$

but they have never been seen any colored hadrons.

- so we postulate that all free hadrons observed in nature must have:

$$I_3^c = 0 \; ; \; Y^c = 0$$

that is they must be colour singlets.

This the hypothesis of colour confinement

N.B. The charge conjugation transforms a red quark in an antired antiquark with opposite values of I_3^c e Y^c

The hadrons

- The only possibilities to have a "white" hadron are:
  - combining three quarks together;
  - combining a quark and an antiquark;

baryons = qqq
mesons = $q\bar{q}$

$p = u_r u_g d_b$; $\bar{p} = \bar{u}_r \bar{u}_g \bar{d}_b$; etc....
π = rr + gg + bb

but we must not have combinations like these ones:

qq ; $qq\bar{q}$; qqqq ; etc ....

- In principle we could have these combinations:

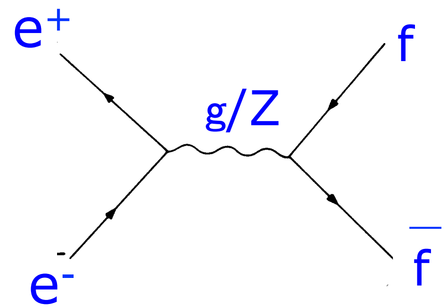
$q\bar{q}q\bar{q}$; $qqqq\bar{q}$

(recently they have been found resonances that can be interpreted as 4 quarks states)

The SU(3) symmetry of colour is an exact symmetry, contrary to the SU(3) symmetry of flavour that is an approximate symmetry.

Hence quarks with different colour have exactly the same mass.

Collider e^+e^-

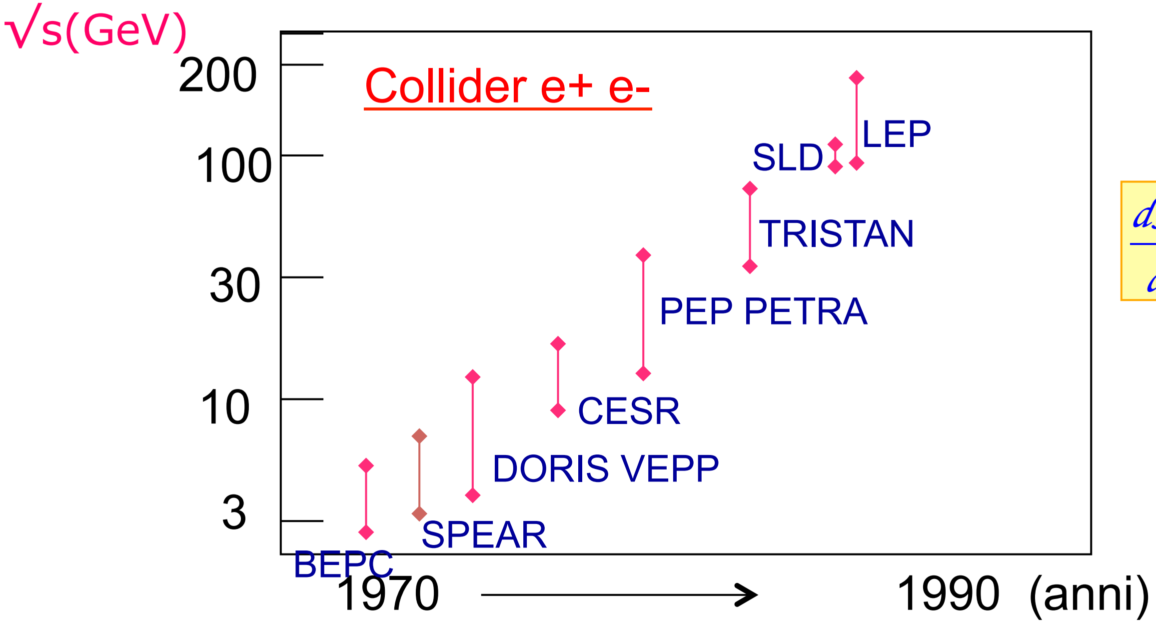


Pros of an e^+e^- collider:

- initial state completely defined (pointlike particle);
- $\sqrt{s}$ defined = $2xE_{beam}$

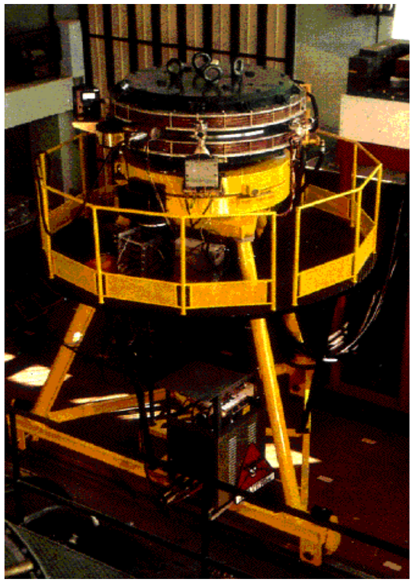
Cons of an e^+e^- collider:

- It is more difficult to reach high energies (with respect to a proton collider) because of the synchrotron radiation;
- Lower luminosity with respect to a fixed target experiment.



$$\frac{d\mathcal{N}}{dt} = \mathcal{L} \cdot \sigma$$

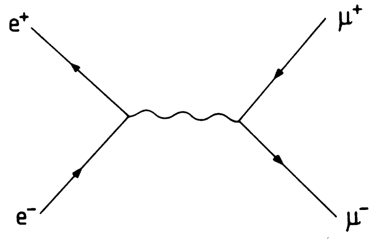
ADA – 1961 - LNF

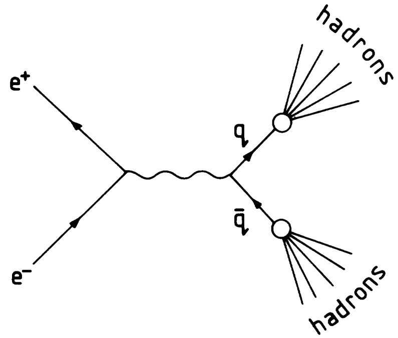


$\sqrt{s}$ = 500 MeV

Experimental evidence of the colour

- One of the most convincing evidence of the existence of the colour comes from the comparison of the cross-sections of the two following processes:





$e^+e^- \to \mu^+\mu^-$ $\qquad$ $e^+e^- \to q\bar{q}$

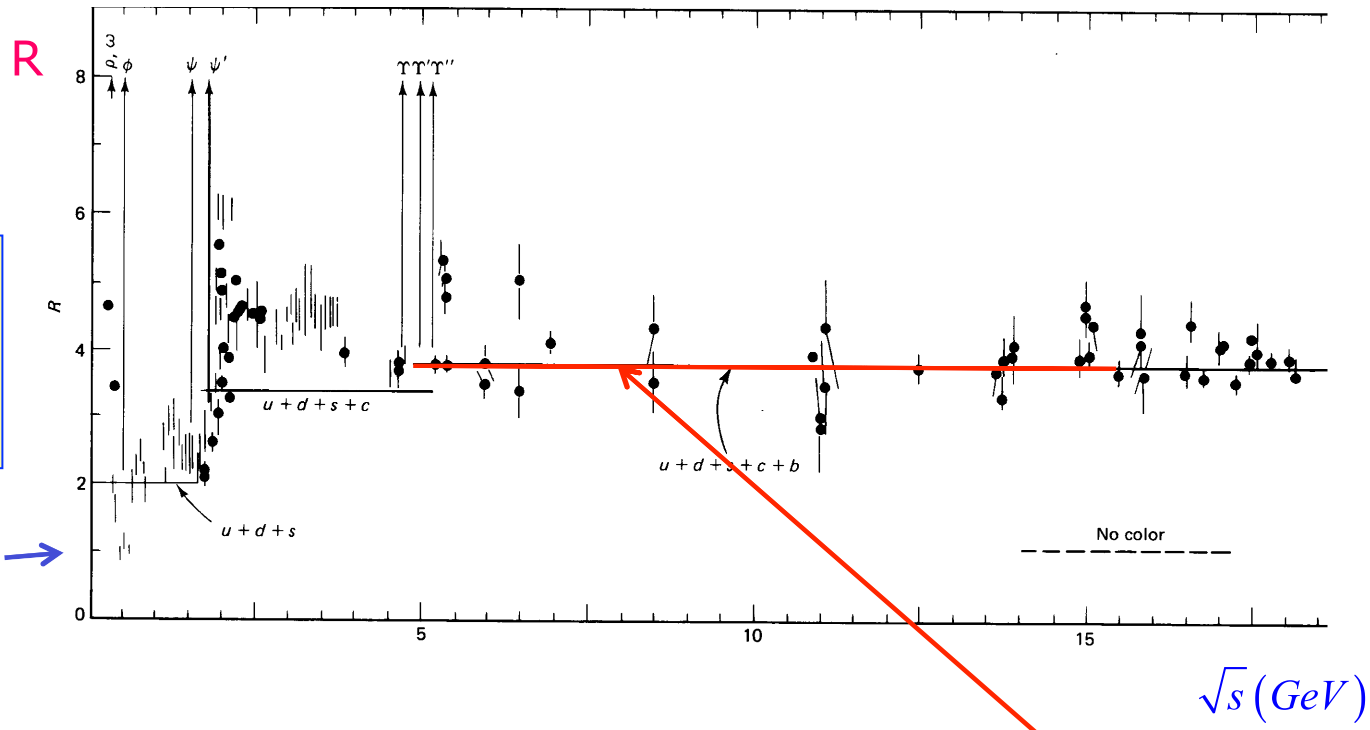
- If we do not take into account the quark hadronization, and assuming that √s is bigger than the fermion masses, the amplitudes of the two graphs differ only because of the electrical charge of the fermions in the final state.

- Without the existence of the colour: $R = \dfrac{\sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = \sum_{i=1}^{n} Q_i^2$

Experimental evidence of the colour

If the colour exist, R must be multiplied by 3 because the number of colours of the quarks is 3

R is a function ofi √s. Above the threshold of b production we have:

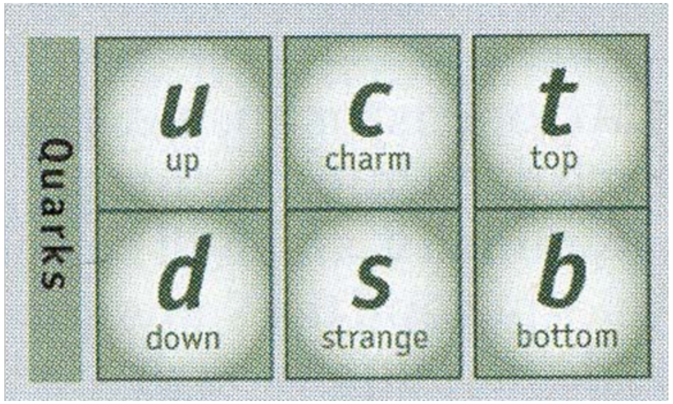


Without colour: $R = \frac{11}{9}$

With the colour: $R = 3 \cdot [(-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2] = \frac{33}{9} = 3.67$

Mass of the quarks (PDG 2010)

Quark	Mass	Q	I_3	s	c	b	t
u	1.7 – 3.3 MeV	2/3	1/2	0	0	0	0
d	4.1 – 5.8 MeV	-1/3	-1/2	0	0	0	0
s	80 – 130 MeV	-1/3	0	-1	0	0	0
c	1.18 – 1.34 GeV	2/3	0	0	+1	0	0
b	4.13 – 4.37 GeV	-1/3	0	0	0	-1	0
t	172 ± 1.6 GeV	2/3	0	0	0	0	+1



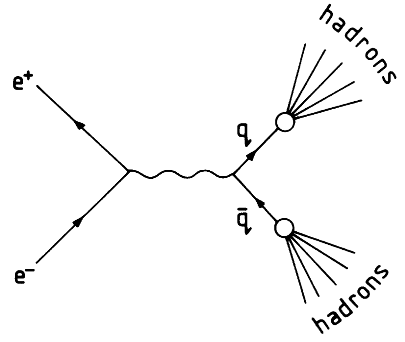
$$Q = \frac{2}{3}e$$

$$Q = -\frac{1}{3}e$$

Exercise

Question: what is the value of R in an e^+e^- collider having $\sqrt{s}$ = 6 GeV?

- N.B.: the collider always produce $q\overline{q}$ pair;



- They are produced only the pairs such that:

$$\boxed{\sqrt{s} > 2m_q}$$

- Then if the collider has a center of mass energy of 6 GeV, they are produced pairs of quark uu, dd, ss, cc but NOT bb or tt

$$\Rightarrow \quad R = 3\sum_{i=1}^{n} Q_i^2 = 3\cdot[(-\frac{1}{3})^2 + (\frac{2}{3})^2 + (-\frac{1}{3})^2 + (\frac{2}{3})^2] = \frac{30}{9} = 3.33$$

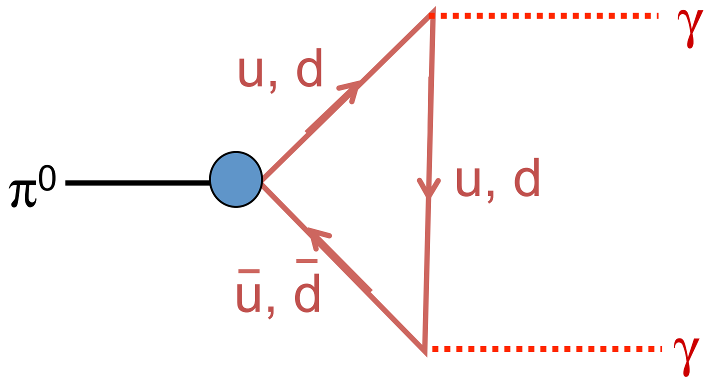
π^0 decay

1949-50 The decay $\pi^0 \rightarrow \gamma\gamma$ calculated and measured by Steinberger.

1967 Veltman calculates the π^0 decay rate using modern field theory and finds that the π^0 does not decay!

1968-70 Adler, Bell and Jackiw "fix" field theory and now π^0 decays but decay rate is off by factor of 9.

1973-4 Gell-Mann and Fritzsch (+others) use QCD with 3 colors and calculate the correct π^0 decay rate.



Triangle Diagram

Each color contributes one amplitude.

Three Colors change the decay rate by 9.

$$\Gamma\left(\pi^0 \rightarrow \gamma\gamma\right) = 7.73\left(\frac{N_c}{3}\right)^2 \text{ eV} \quad \left[\textit{predicted}\right]$$

$$\Gamma\left(\pi^0 \rightarrow \gamma\gamma\right) = 7.7 \pm 0.6 \text{ eV} \quad \left[\textit{measured}\right]$$

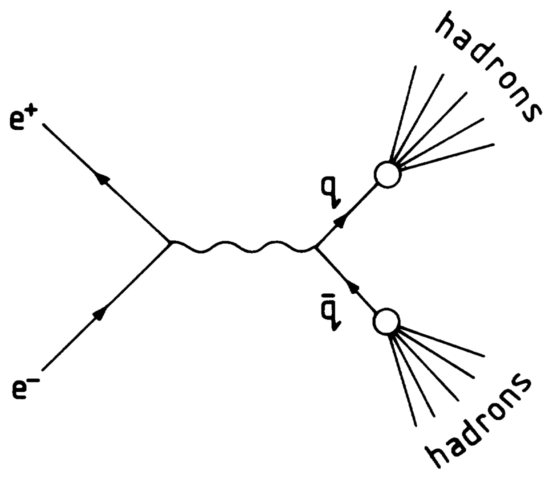
$$\Rightarrow \quad N_c = 2.99 \pm 0.12$$

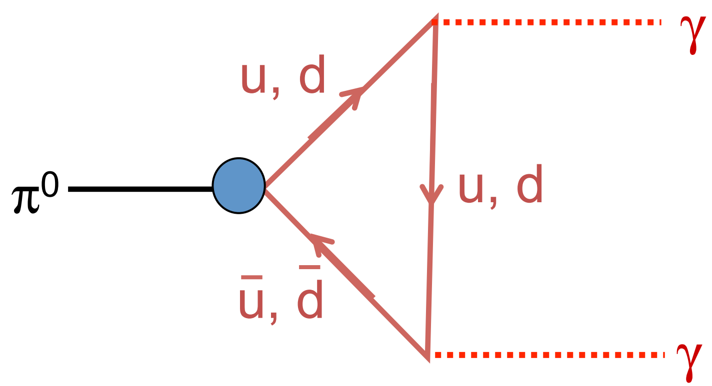
question

- Why in the ratio R

$$R = \frac{\sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

we need to multiply R by 3 in order to take into account the number of colours, while in the π^0 decay we have to multiply the probability of the decay (Γ_{to}t) by a factor 9?





Gluons and QCD

- The theory that describes the strong interactions in the Standard Model is called Quantum ChromoDynamics (QCD).

- The QCD is similar to the QED, its properties can be derived from the gauge invariance of the transformations in the SU(3) space of colour.

- From this invariance derives that the strong interactions are mediated by 8 neutral bosons of spin 1 and mass zero that are called gluons. The gluons are related to the 8 generators of the rotations in the SU(3) space.

- The QED is invariant for a gauge transformation U(1). This is an abelian symmetry, therefore the photon, that couples to the electrical charge, does not carry an electrical charge (it is not charged) so that the photons do not interact with each other.

- SU(3) is a non abelian symmetry, therefore the gluons, that couple to the colour charges, do carry a colour charge (actually a colour and an anticolour), therefore they can couple among themselves. This feature gives rise to verteces with three or four gluons. These verteces have as a consequence the colour confinement and the asyntotic freedom.

- Gluons do not distinguish the quark flavours, hence the strong interactions are invariant for rotations in the SU(3) space of flavour.

Colour of the gluons

- In analogy with the mesons octet, we can represent the eight gluons in the following way (n.b. this only one of the many possible representations):

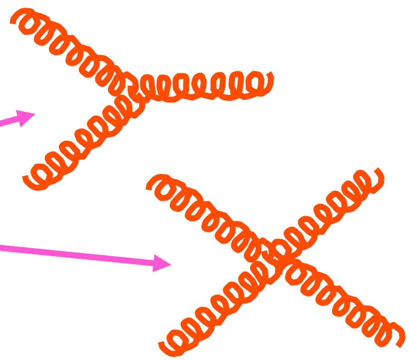
$$g_1=R\bar{G},\ \ g_2=R\bar{B},\ \ g_3=G\bar{R},\ \ g_4=G\bar{B},\ \ g_5=B\bar{R},\ \ g_6=B\bar{G},$$

$$g_7=\sqrt{\frac{1}{2}}\left(R\bar{R}-G\bar{G}\right),\ \ g_8=\sqrt{\frac{1}{6}}\left(R\bar{R}+G\bar{G}-2B\bar{B}\right)$$

- N.B. g_7 e g_8 are "colorless" but they are not colour singlets.
- La nineth combination (3x3) is a colour singlet and it can not be a mediator between "colored" quarks.
  If it exists we would have strong interaction between colorless hadron with infinite range.

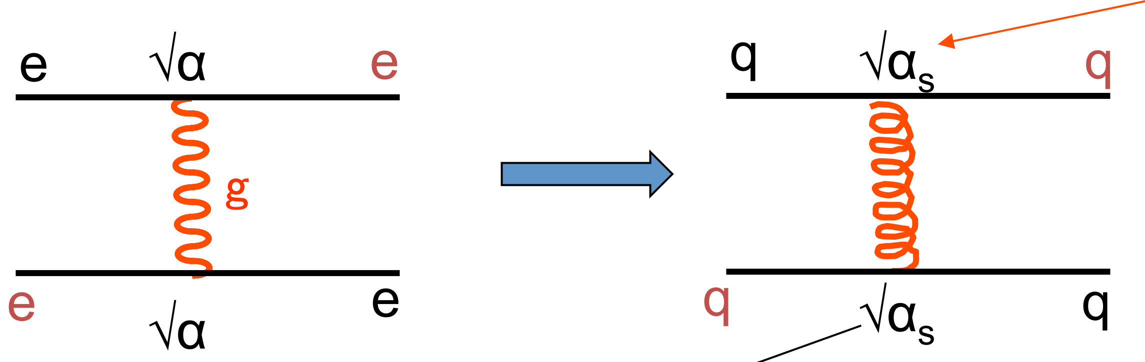
$$\sqrt{\frac{1}{3}}\left(R\bar{R}+G\bar{G}+B\bar{B}\right)$$

Remember, the gluons too are "colored" and the can interact among themselves.

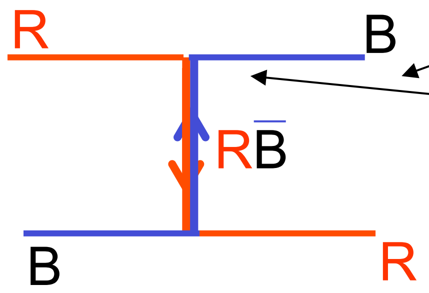


Colour of the gluons

- electrical charge → colour charge
- 3^2-1=8 matrices/operators → 8 gluons
- quarks have colours RGB
- The colour is exchanged by 8 « bicoloured » gluons



α_s = coupling constant of the strong interactions.



In this vertex the gluon $\bar{B}$R transforms the quark R→B (by removing the quark R and leaving the quark B, which is balanced by the component $\bar{B}$ of the gluon)

<u>In every vertex we must conserv the colour.</u>

Questions

- Are the quarks “real”? What is their spin?

- Why do not observe free quarks?

- Do they exist the gluons?

- What is a jet of particles?

- What computations we can do with QCD?

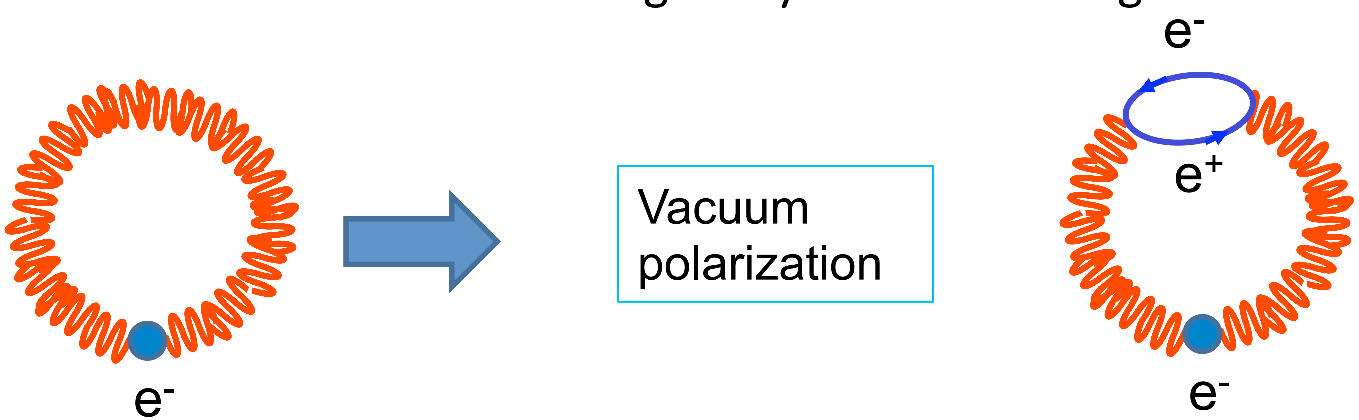
running of α_{QED}

- In classical electromagnetism the potential energy of an electron in the field generated by the same electron (self-energy) is equal to:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

The potential energy goes to infinity since r goes to zero.

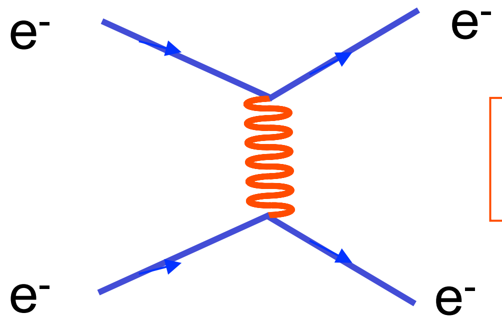
- The self-interaction in the field theory is described as photons that are emitted and then are absorbed again by the same charge:



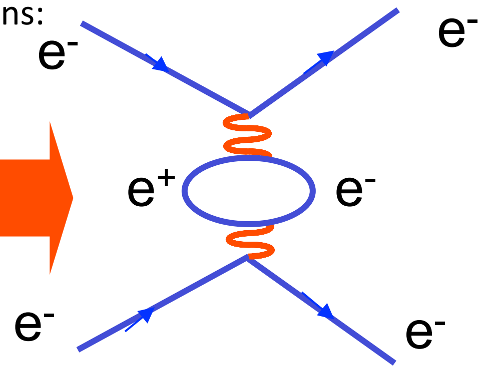
- The positron is “attracted” by the electron and it will “screen” the charge of the electron in a such a way that its effective value diminishes.
- The more you go into the positron “cloud” the lesser will be the shielding effect, so the electron effective charge increases.

running of α_{QED}

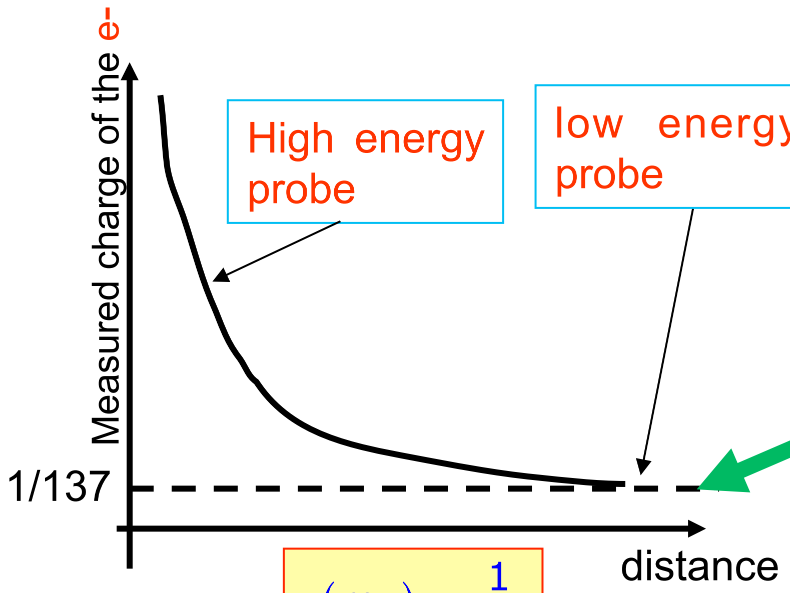
- Let's consider the interaction between two electrons:

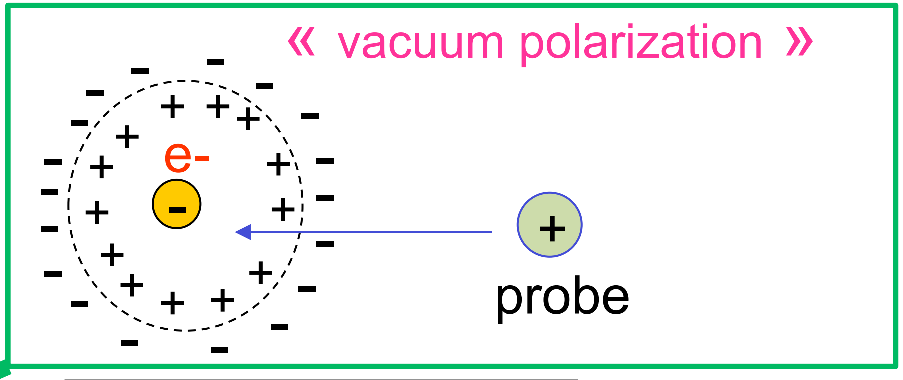


The interaction is modified by the vacuum polarization



- A consequence of the vacuum polarization is that the charge of the electron becomes a function of the energy of the "probe" (that is of the other electron). The positrons "screen" the charge e^-; the nearer we get to the charge the lesser the "screening" is and the effective charge "increase".





$$\alpha(m_Z) = \frac{1}{128}$$

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\frac{Q^2}{\mu^2}}$$

μ: scale factor

discovery of the asymptotic freedom in QCD: 1973

The Nobel Prize in Physics 2004

"for the discovery of asymptotic freedom in the theory of the strong interaction"

David J. Gross

1/3 of the prize

USA

Kavli Institute for Theoretical Physics, University of California
Santa Barbara, CA, USA

b. 1941

H. David Politzer

1/3 of the prize

USA

California Institute of Technology
Pasadena, CA, USA

b. 1949

Frank Wilczek

1/3 of the prize

USA

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

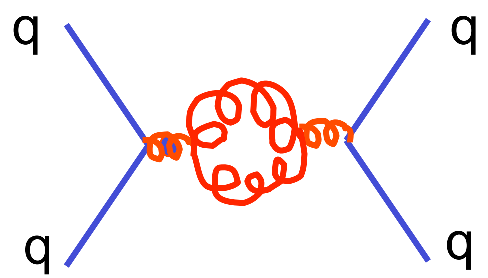
b. 1951

running of α_s

- Let's consider the strong interaction between two quarks:

The production of virtual qq pair in the gluon propagator produces the same screening effect of the colour charge as in QED, hence the charge should diminish at the increase of the distance (that is at low momentum transfer).

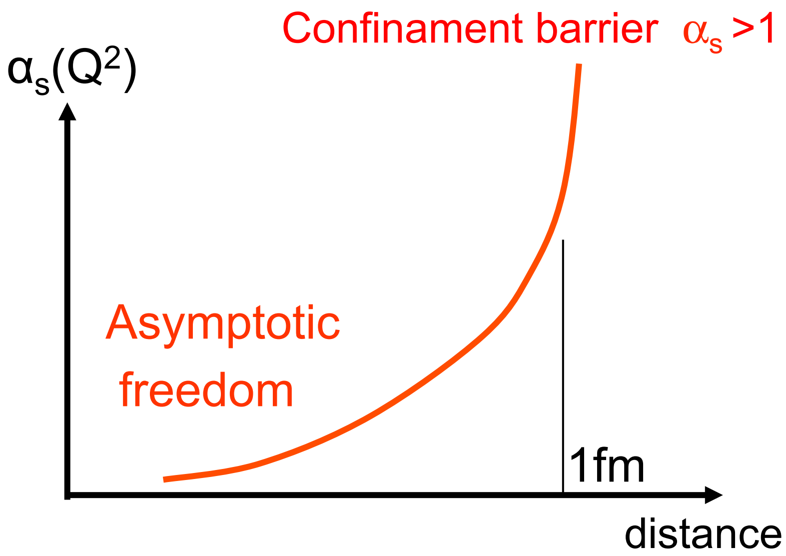
But since the gluons are "coloured" they exist also diagrams like this one that modify the interaction and produce an effect of "antiscreening"



[a fermions loop has opposite sign with respect to a bosons loop]

running of α_s

- The effect of the gluon self-interaction is such that:



$$\alpha_s\left(Q^2\right) = \frac{12\pi}{(33-2f)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

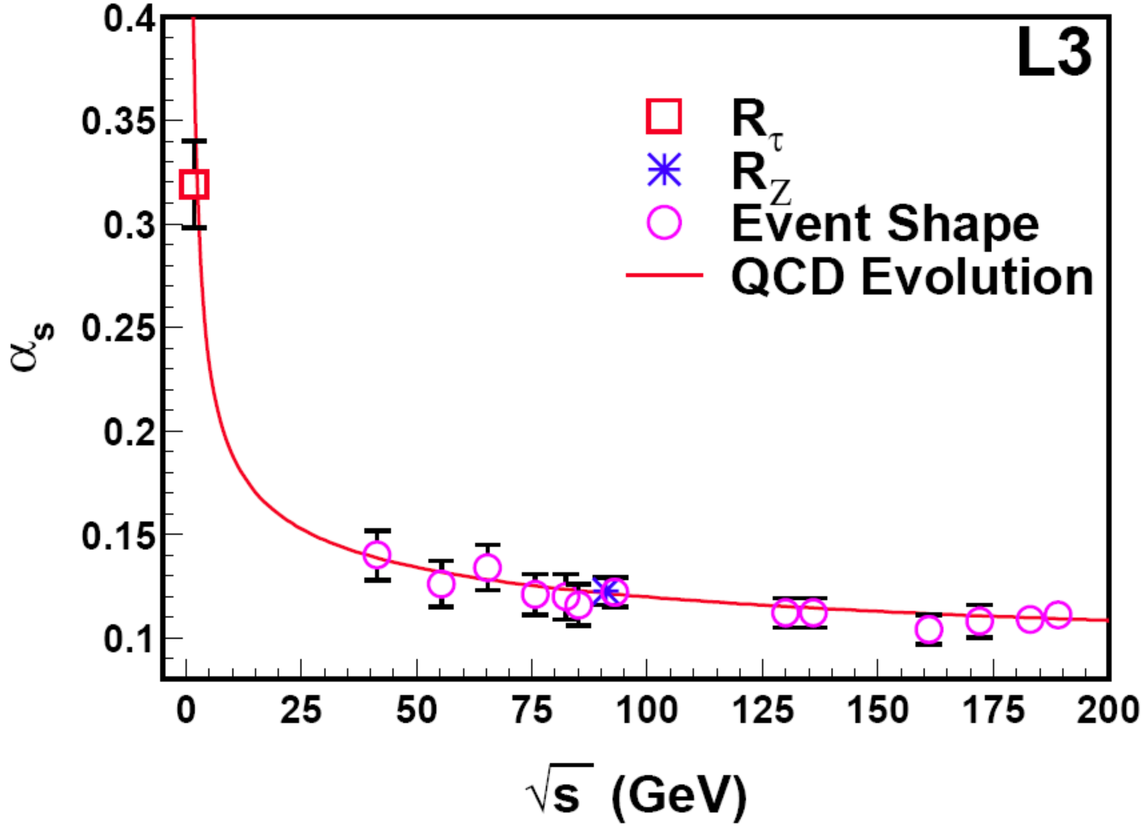
f = number of quarks with $4m^2<Q^2$
Λ= scale (~200 MeV with f=4)

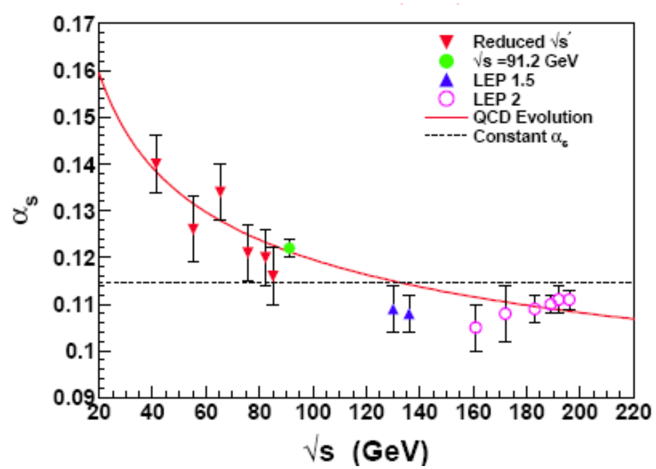
$Q^2 \sim \Lambda^2$ strong coupling → ~~perturbation approach~~
$Q^2 \gg \Lambda^2$ weak coupling → perturbation approach

At high momentum transfer (that is at small distances) α_s is small and we can do QCD calculation with the perturbative method. At low momentum transfer the constant is big and we can not use the perturbative method.

Measurement of the running of α_s

- The coupling constant α_s has been measured by several experiments at various range of Q^2. For instance at LEP has been found:



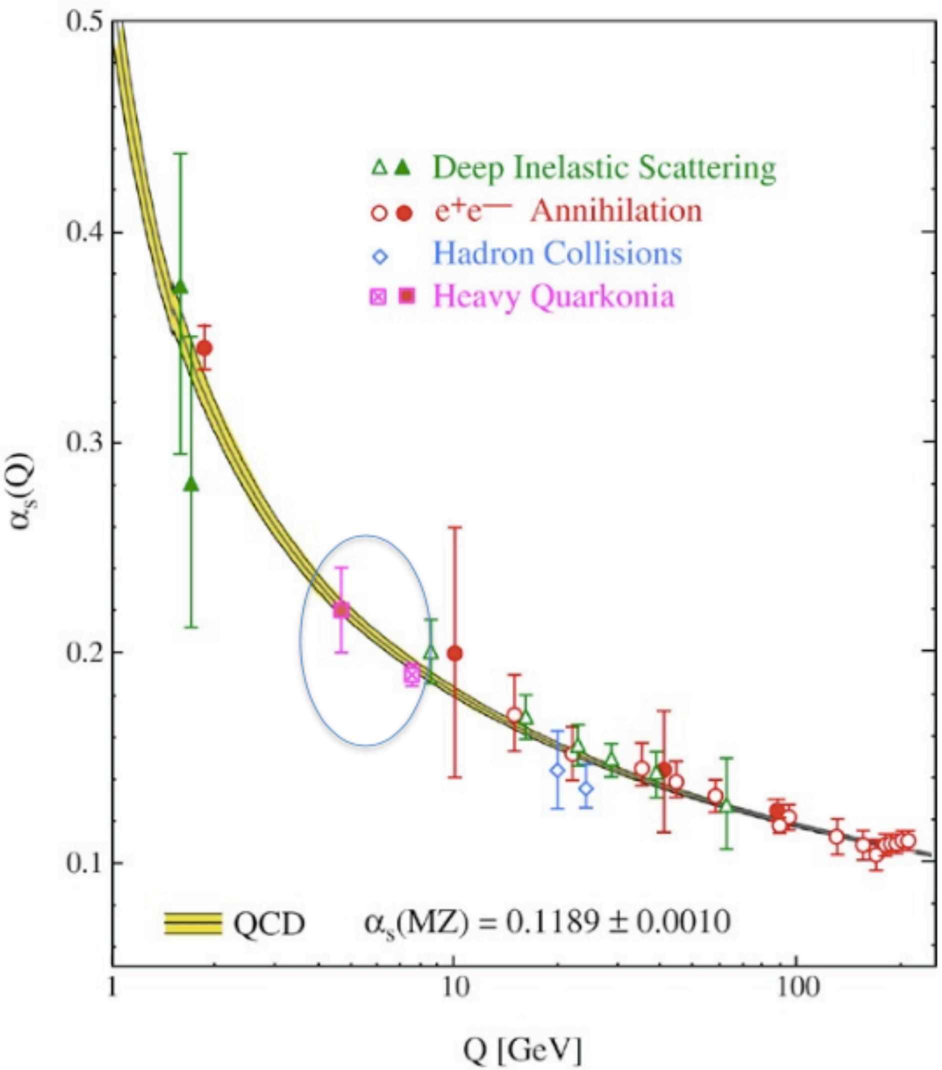


The data confirm the running of a_s

N.B. We are at energies much higher than Λ_{QCD}

Measurement of the running of α_s

- Other measurements of α_s:



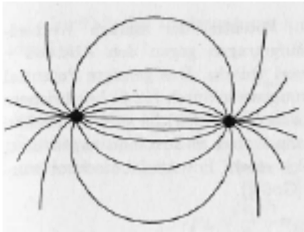
$$\alpha_s\left(Q^2\right) = \frac{12\pi}{\left(33-2f\right)\log\left(\frac{Q^2}{\Lambda^2}\right)}$$

QCD potential

- The potential energy of the static interaction between a quark-antiquark pair (as in a colour singlet like a meson) can be summarized in the following way:

  - At short distance between the quarks (< 0.1 fm) is dominant the exchange of a single gluon (because α_s is "small") and the potential of the interaction is similar to the QED potential due to a photon exchange:

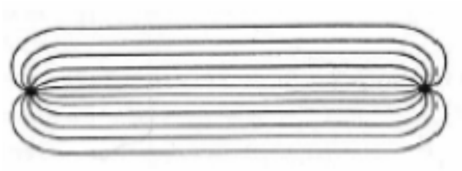
$$\boxed{V(r) = -\frac{4}{3}\frac{\alpha_s}{r}}$$



  - With the increase of the distance also α_s increase and we do not have any longer the exchange of a single gluon but we enter in a non perturbative regime, where we can only do numerical calculations. These are consistent with a potential at large distances that can be parametrized in the following way:

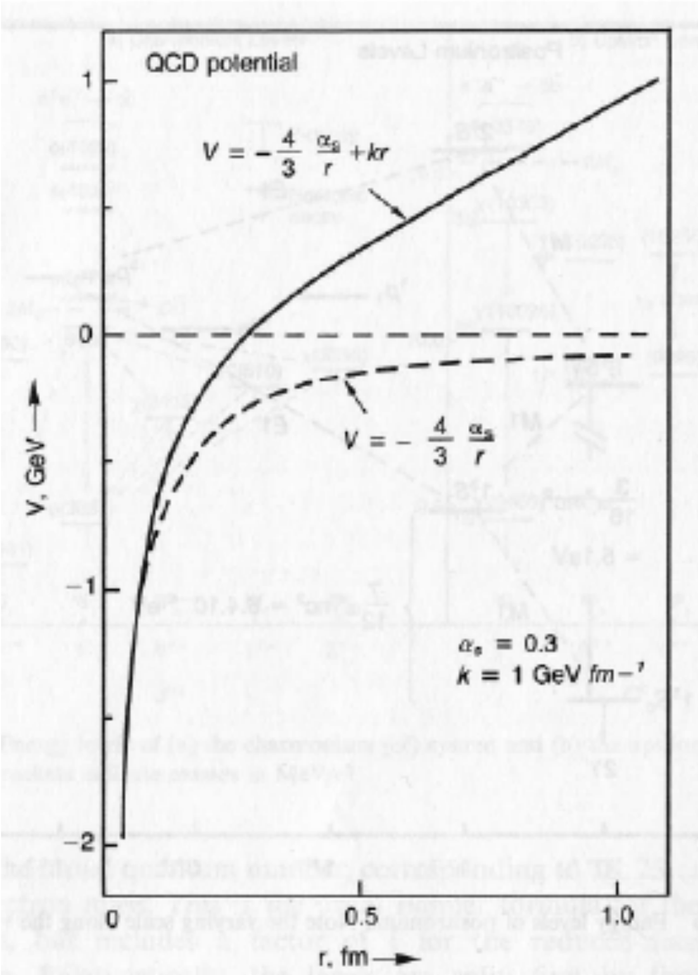
$$\boxed{V(r) = kr}$$

$$\boxed{k \approx 1\ \mathrm{GeV \cdot fm^{-1}}}$$



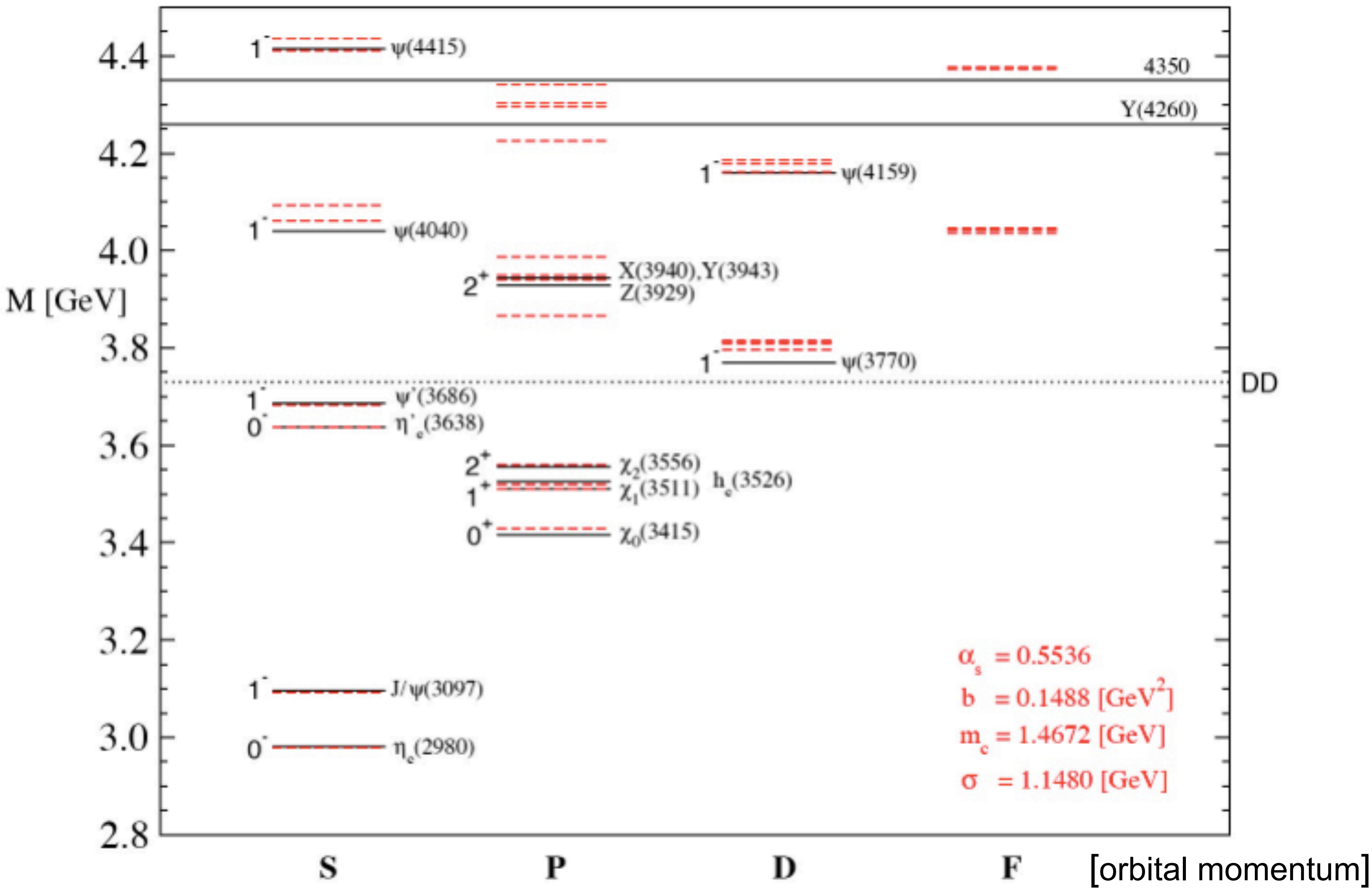
QCD potential

- When the two quarks recede from each other, the linear term of the potential becomes dominant "trapping" the energy inside the system (as it happens when you stretch a spring).

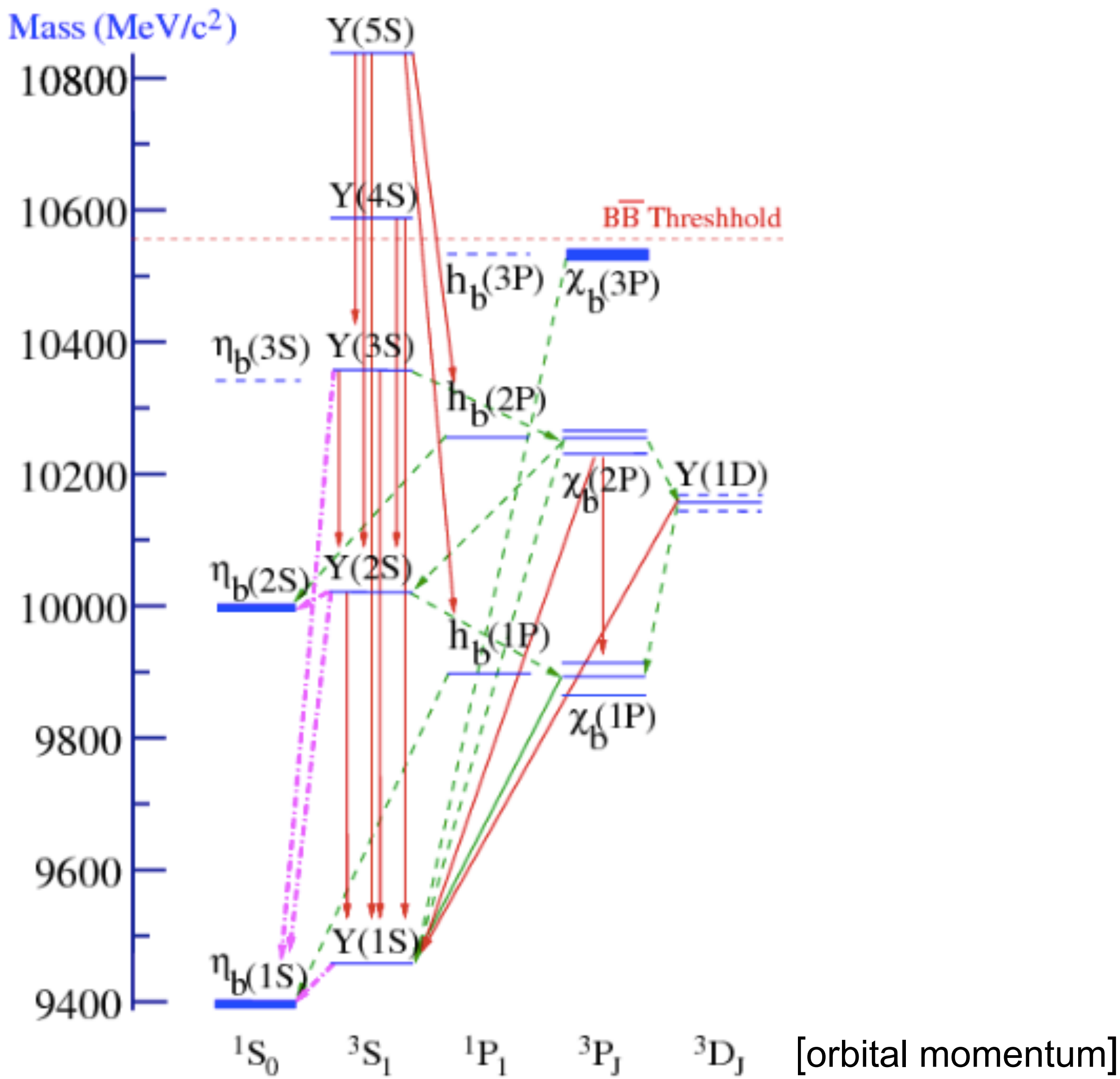


This kind of potential allows to explain the excited states of a heavy quark-antiquark pair, like the charmonium ($c\bar{c}$ bound states) or the bottomonium ($b\bar{b}$ bound states)

charmonium spectra

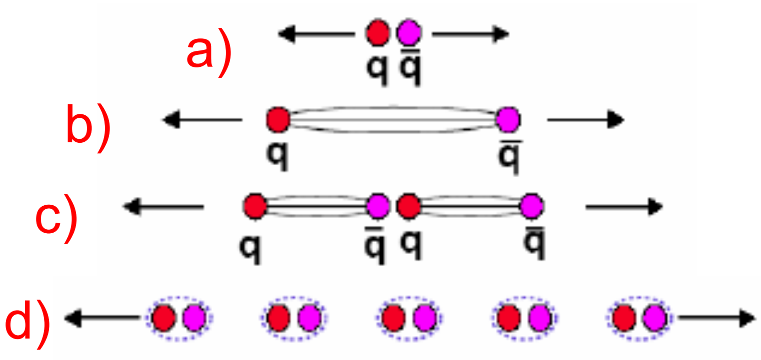


bottomonium spectra

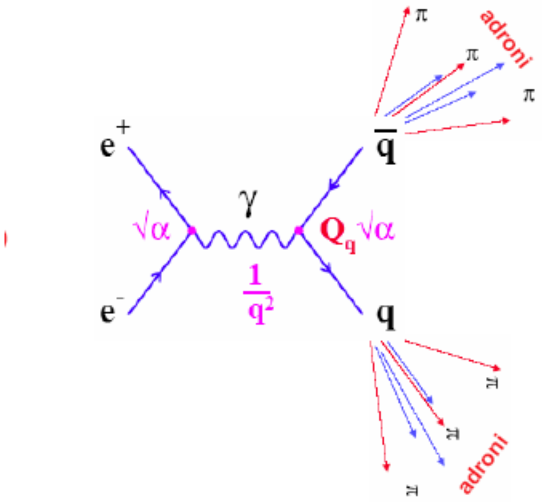


hadronization

A consequence of the asymptotic freedom is the quark confinement within a hadron.



a) The quarks are close and their interaction energy is small.
b) The quarks move apart and their interaction energy increases (α_s increases with the distance).
c) At a certain moment, for distances of the order of one fm, it becomes energetically convenient to break the "tube", producing a new quark-antiquark pair at the two new ends. We have now a second meson, which is colour neutral.
d) The process (a non perturbative one) called hadronization continues untill we have all "white" hadrons in the final state.

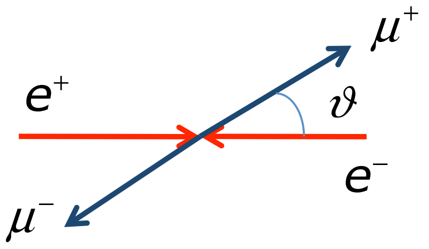


The hadrons in the final states keep "memory" of the initial momentum of the quark and give origin to a jet of particles

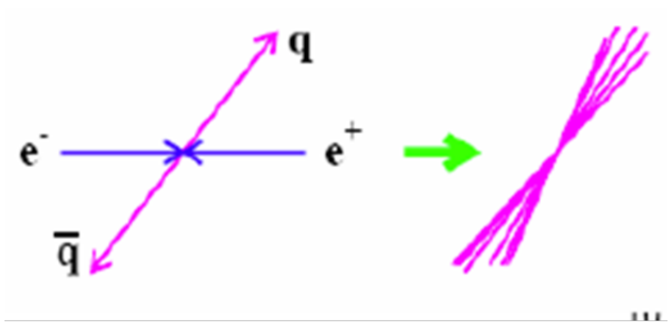
spin of the quark

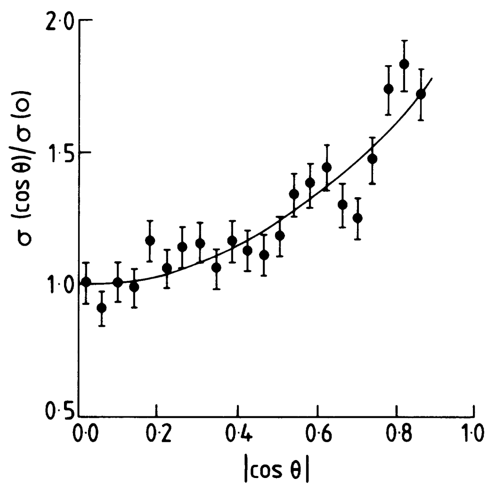
- We can measure the spin of the quarks from the jet angular distributions.
- The differential cross-section of the process $e^+e^- \to \mu^+\mu^-$ is:

$$\boxed{\frac{d\sigma}{d\cos\theta} = A(1+\cos^2\theta)}$$



- If the quarks are fermions of spin ½ , the differential cross-section of the process $e^+e^- \to q\bar{q}$ must have the same behaviour.

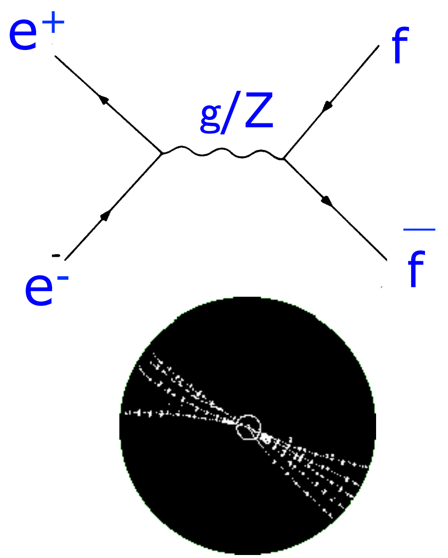


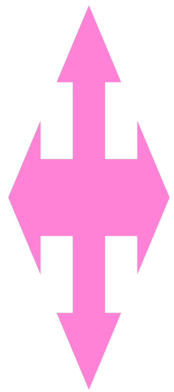


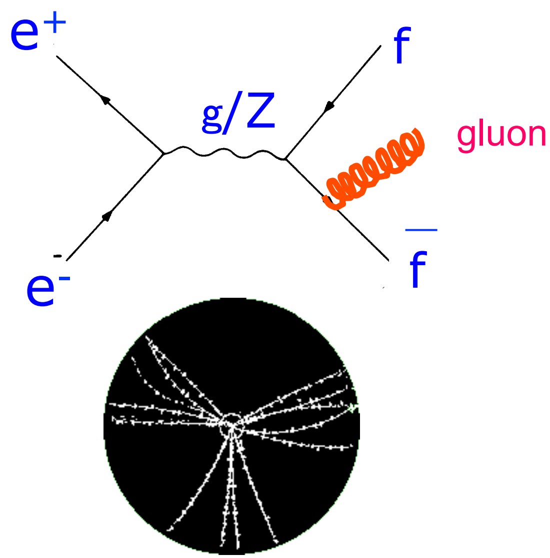
quarks have spin ½

discovery of the gluon

- The gluon was discovered in 1979 at Petra, an e^+e^- collider of $\sqrt{s}$=27 GeV at Desy (Hamburg)





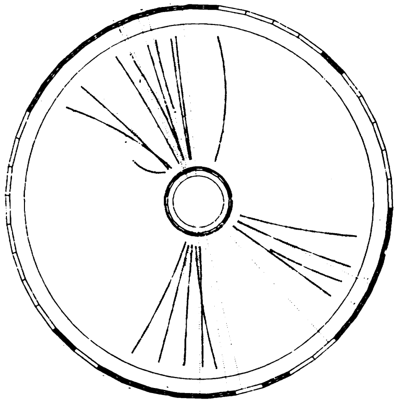


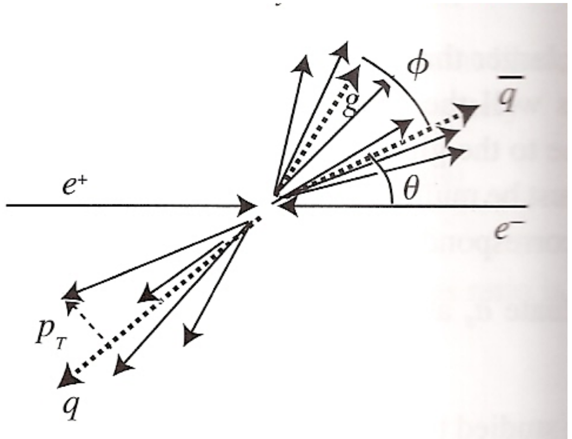
- From the quarks in the final state can be radiated a gluon. Whenever this gluon has enough energy, it can hadronize in its own way independently from the quarks, and it gives rise to a third jet of particles.
- People were looking for events with 3 jets in the transverse plane with respect to the beam axis, where the sum of the quadrimomentum must be zero.

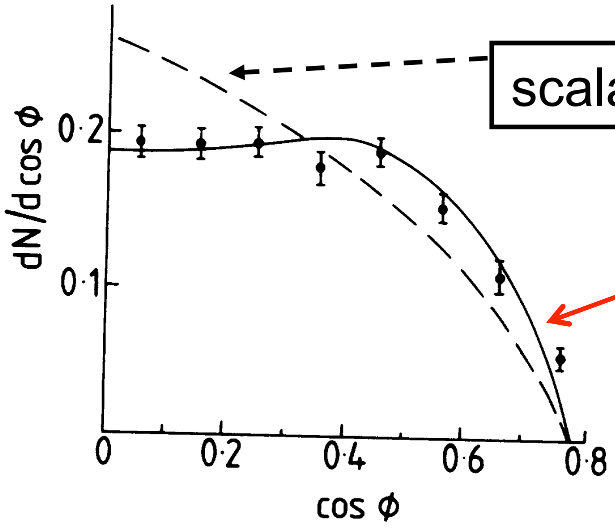
(A jet is made by a group of hadrons whose total quadrimomentum is equal to the quark quadrimomentum that gave origin to the jet).

spin of the gluon

- The spin of the gluon can be deduced from the differential cross-section with respect to the angle φ between the gluon and the axis of the other two jets, in the center of mass frame of the two jets.







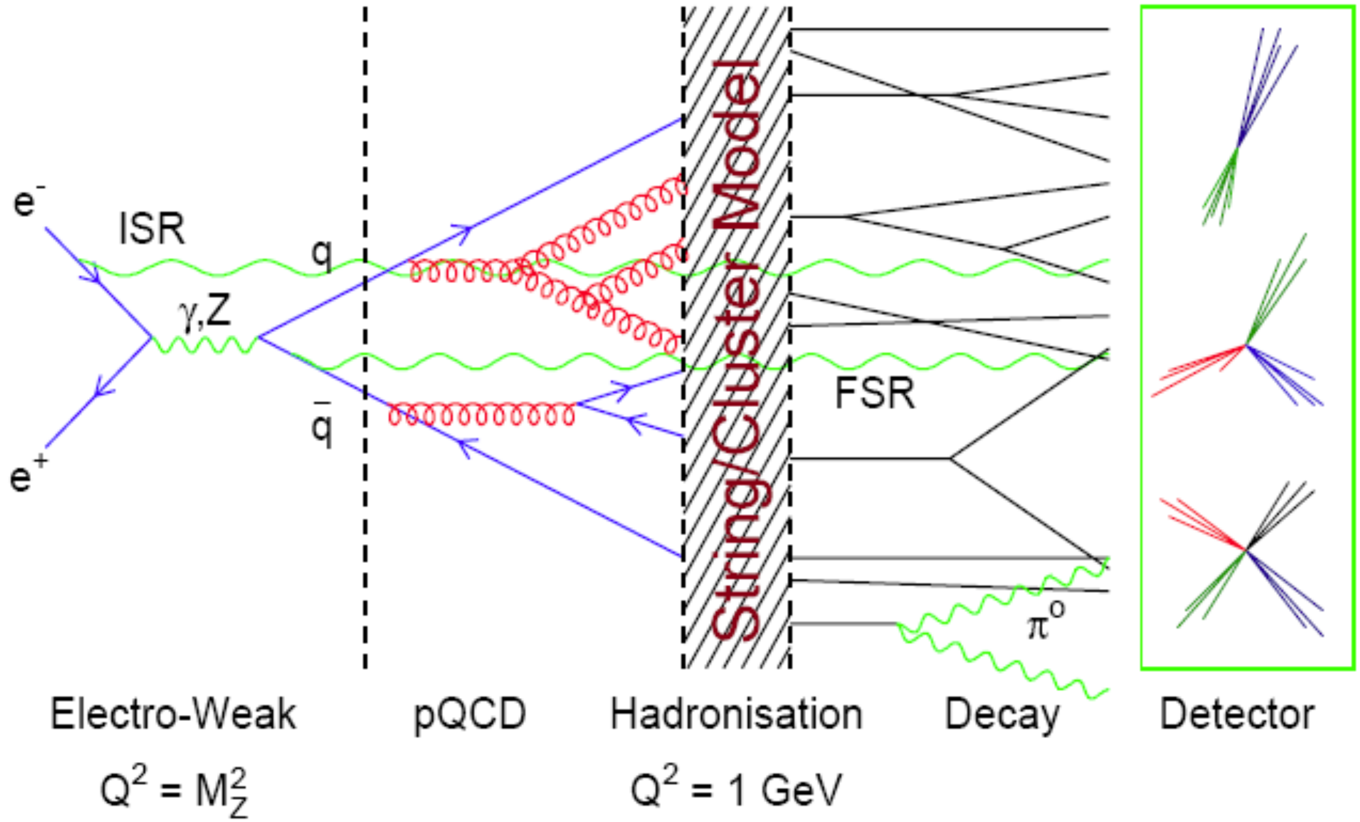
scalar gluons

QCD, vector gluons

gluons have spin 1

hadronization

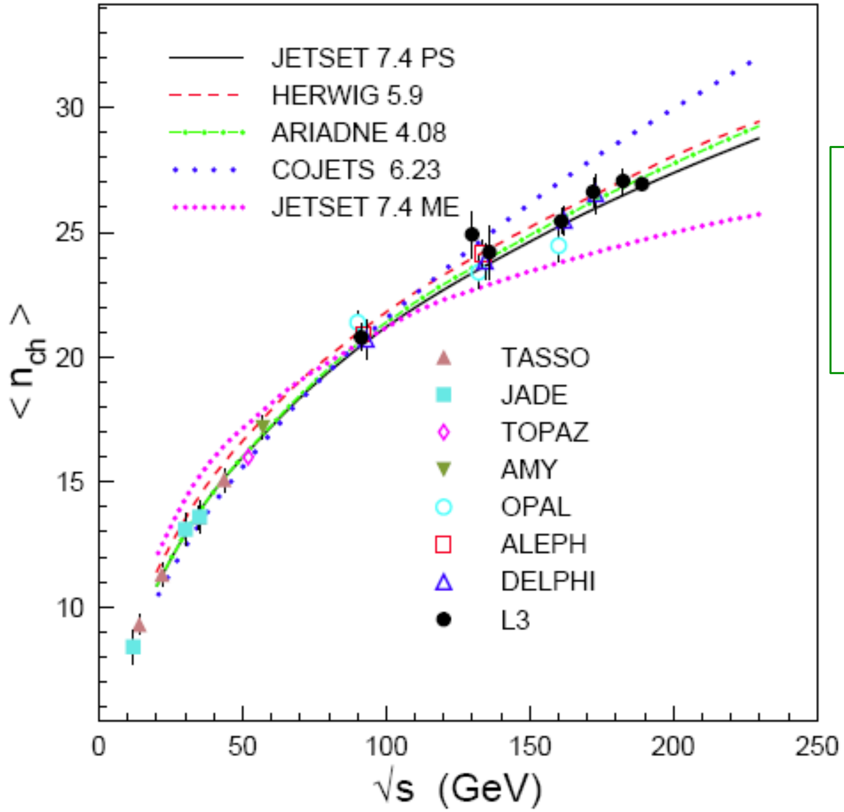
- The initial quarks are coloured. The final hadrons are white.
- The formation process of the hadrons is called hadronization. It happens for energies "around" 1 GeV and the process is not perturbative, so it can be described only by phenomenological models.



Comparison between hadronization models

- The degree of "goodness" of the various hadronization models can be deduced from the comparison of Montecarlo predictions with experimental data for several quantities that characterize a hadronic event. For instance:

Average number of charged particle in a jet as a function of the center of mass energy of the system e^+e^-.

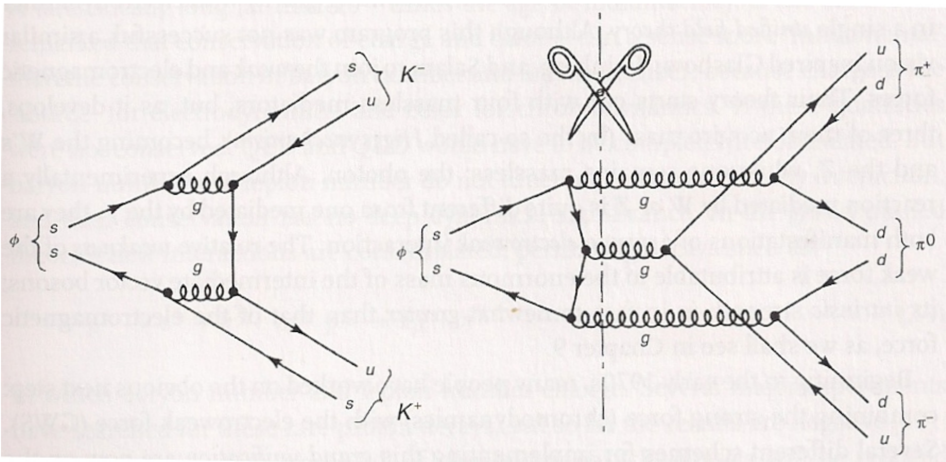


For instance the MC Jetset 7.4 ME and Cojets 6.23 do not reproduce the data well enough at high energy.

OZI rule and the QCD

1/2

- By using the asymptotic freedom we can explain the OZI rule:

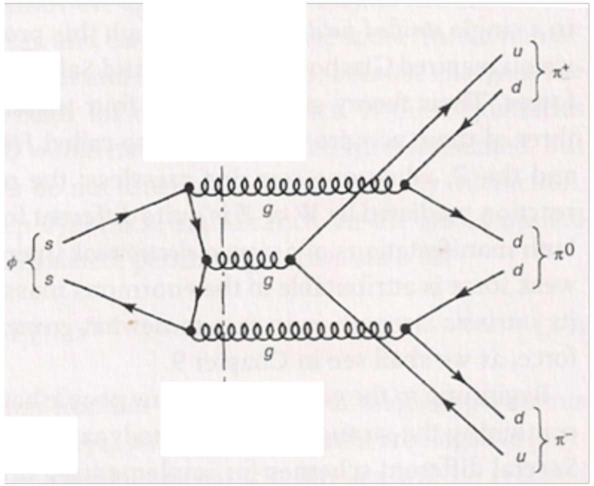


a) $\Phi \to K^+K^-$ b) $\Phi \to \pi^+\pi^-\pi^0$

OZI rule: if the diagram can be divided in two pieces cutting only gluon lines (without cutting the externa lines), the process is suppressed.

- In the diagram a) the emitted gluons are “soft”, that is they carry a small momentum, therefore their coupling constant is big.
- On the contrary in the process b) the gluons are “hard” because they carry the momentum necessary to form the hadrons in the final state, hence their coupling constant is small. Therefore the amplitude of this process is reduced with respect to the previous case.

OZI rule: why three gluons?



- The initial state and the final state are colour singlet, then the two states must be connected by a gluon combination that is a colour singlet.
  The minimum number of gluons to do it is two.

- The ϕ, as well as the J/ψ, has the same quantum numbers of the photon, including C=-1. The gluon too has charge conjugation equal to =-1, then the number of exchanged gluons has to be odd (multiplicative quantum number).
  Therefore the minimum number of gluons exchanged is three.

Exercise

- In an asymmetric e^+e^- collider, the electron beam has energy 4.5 GeV and the positron beam has 2.0 GeV.
  a) Compute the center of mass energy;
  b) determine which quark pairs can be produced in the e+e- annihilation:

$$e^- = 4.5 \text{ GeV} \longrightarrow \quad \longleftarrow e^+ = 2 \text{ GeV}$$

The center of mass energy is equal to √s.

$$s = \left(p(e^+) + p(e^-)\right)^2 \qquad \begin{cases} p(e^+) = \left(E^+;0,0,-E^+\right) \\ p(e^-) = \left(E^-;0,0,E^-\right) \end{cases}$$

$$\Rightarrow s = 4 \cdot E^+E^- \quad \Rightarrow \quad \sqrt{s} = 2\sqrt{E^+E^-} = 2\sqrt{4.5 \cdot 2} = 6 \text{ GeV}$$

In this collider we can produce $d\bar{d}$, $u\bar{u}$, $s\bar{s}$ and $c\bar{c}$ pairs, but not $b\bar{b}$ because the mass of the quark b is about 4.5 GeV

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End of chapter 5