# Introduction to Particle Physics
## - Chapter 1 -

# A collection of “known” items

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# Chapter summary:

- Cross section
- Life time
- Resonances
- S matrix and transition probabilities
- Fermi Golden rule
- QED and Feynman diagrams

# What we measure: cross-section

$a + b \rightarrow$ anything

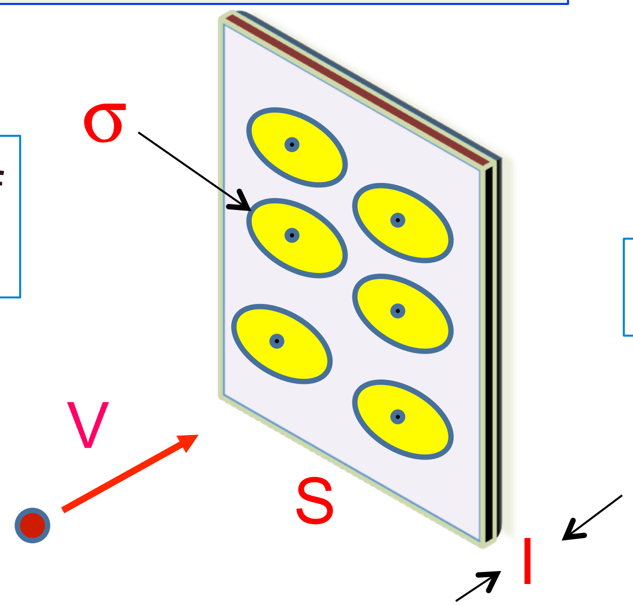
Bullet (beam)

target

**The cross section is proportional to the probability of a given process**

geometrical definition

σ: effective area of a target particle




Thin target approximation:
l << attenuation lenght λ

$N_t$: number of target particles

S: total target area

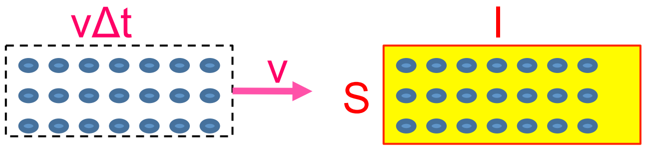
Probability that the bullet hit a target particle

$$p = \frac{\text{effective area}}{\text{total area}} = \frac{N_t \cdot \sigma}{S}$$

# Cross-section

**a + b $\longrightarrow$ anything** ....... **fixed target** .....

$N_b$: number of bullet particles




$N_t$: number of target particles

$$\boxed{N_b = n_b \cdot S \cdot v\Delta t}$$

$\left[ n_x = \text{ particle density} \right]$

$$\boxed{N_t = n_t \cdot S \cdot \ell}$$

$$\boxed{\Phi = n_b \cdot v = \text{ particles flux}}$$

$$\boxed{\Rightarrow \; N_b = \Phi \cdot S \cdot \Delta t}$$

$$\left( N.B. \quad \lambda = \frac{1}{n_t \cdot \sigma} \right)$$

**Let's compute the number of interactions (N$_{\text{events}}$)**

$$N_{events} = N_b \cdot p = N_b \cdot \frac{N_t \cdot \sigma}{S} = \Phi \cdot S \cdot \Delta t \cdot \frac{N_t \cdot \sigma}{S} = \Phi \cdot N_t \cdot \sigma \cdot \Delta t$$

$$\Longrightarrow \quad \sigma = \frac{N_{events}}{\Delta t} \cdot \frac{1}{\Phi} \cdot \frac{1}{N_t}$$

**Number of interactions in the time interval Δt** $\quad \dfrac{N_{eventi}}{\Delta t} = \sigma \cdot \Phi \cdot N_t$

**Interaction probability (transition probability ) per unit of time, unit of area and only one target particle:**

$$\boxed{W = \sigma \cdot \Phi}$$

# What we measure: life time

$a \quad \longrightarrow \text{anything}$ ← Unstable particle decay

$$dN = -\Gamma_{tot} \cdot N \cdot dt$$

The total number of particle decays is proportional to the total number of particles in the sample (N) and to the time interval dt. The decay probability (Γtot) is independent from the "past history".

$$N(t) = N(0) \cdot e^{-\Gamma_{tot} \cdot t} \quad \text{(number of particles at time t)}$$

$\Gamma_{tot}$ = total width (transition probability W)

$$\tau = \frac{1}{\Gamma_{tot}} \quad \text{(life time)}$$

A particle may decay in several final states. At every state is associated a given transition probability (partial width)

$$\Gamma_{tot} = \sum_{i=1}^{n} \Gamma_i \quad ; \quad \Gamma_i = \text{partial width.}$$

Branching Ratio B.R.

$$\text{B.R.} = \frac{\Gamma_i}{\Gamma_{tot}} \quad \left[ = \frac{N_i}{N_{tot}} \right]$$

# "Formation" Resonance

$$a + b \rightarrow R \rightarrow a + b$$

**Elastic Cross-Section**

$$a + b \rightarrow R \rightarrow X$$

**Total Cross-Section**

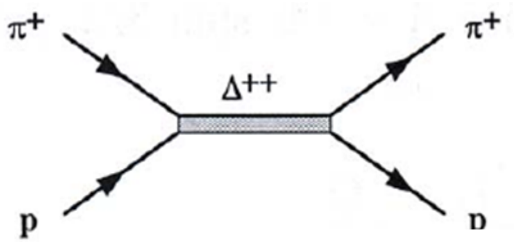
- The scattering process happens through the "formation" of an intermediate resonant state R;
- The resonance can decay in:
  - same particles of the initial state (elastic scattering)
  - other particles (anelastic scattering)
- The resonance is described by the Breit-Wigner formula:

$$\sigma(E) = \frac{4\pi \hbar^2}{p_{cm}^2} \frac{2J+1}{(2S_a+1)\cdot(2S_b+1)} \left[ \frac{\Gamma_{in} \cdot \Gamma_{fin}}{(E-M_R)^2 + \Gamma^2/4} \right]$$

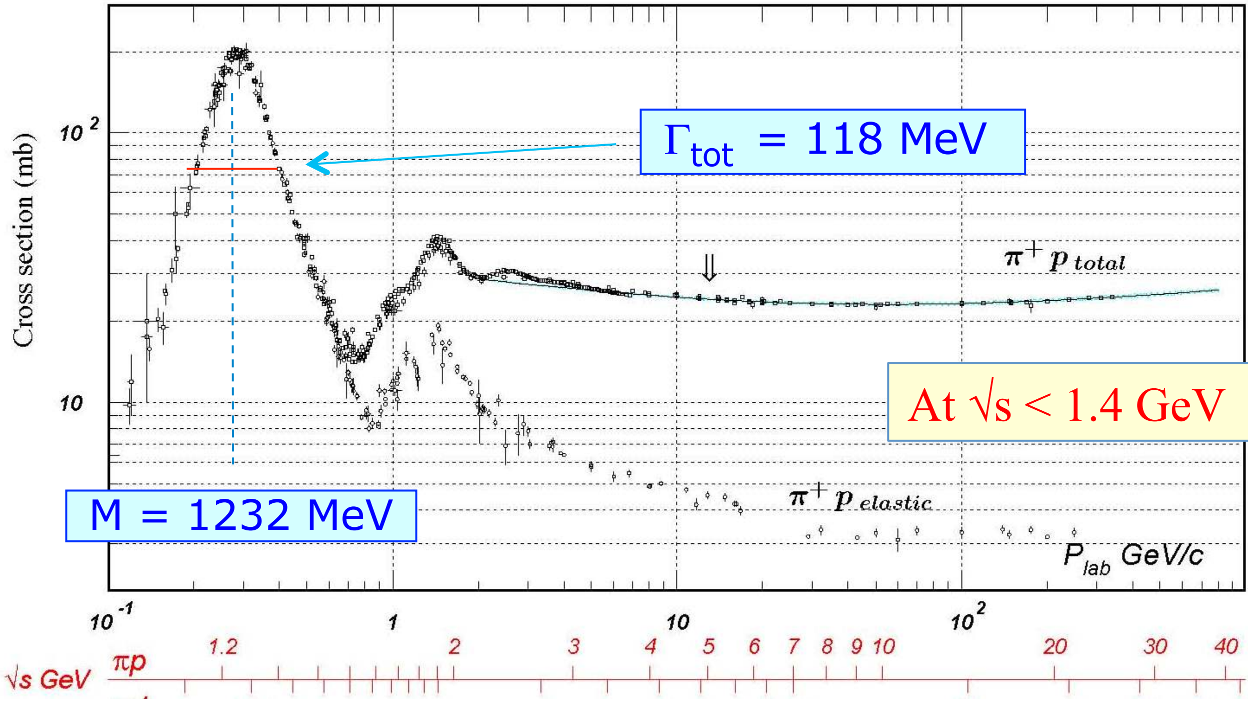
- $P_{cm}$: beam momentum in the center of mass reference frame
- E: center of mass energy ($\sqrt{s}$)
- $M_R$: resonance mass

- $S_a$, $S_b$ :initial state spins
- J : resonance spin
- $\Gamma$, $\Gamma_{in}$, $\Gamma_{fin}$ : resonance total and partial widths

# The resonance Δ




Peak in the elastic cross section $\pi^+ p$




$\sigma_{peak}$ = 195 mb

$\Gamma_{tot}$ = 118 MeV

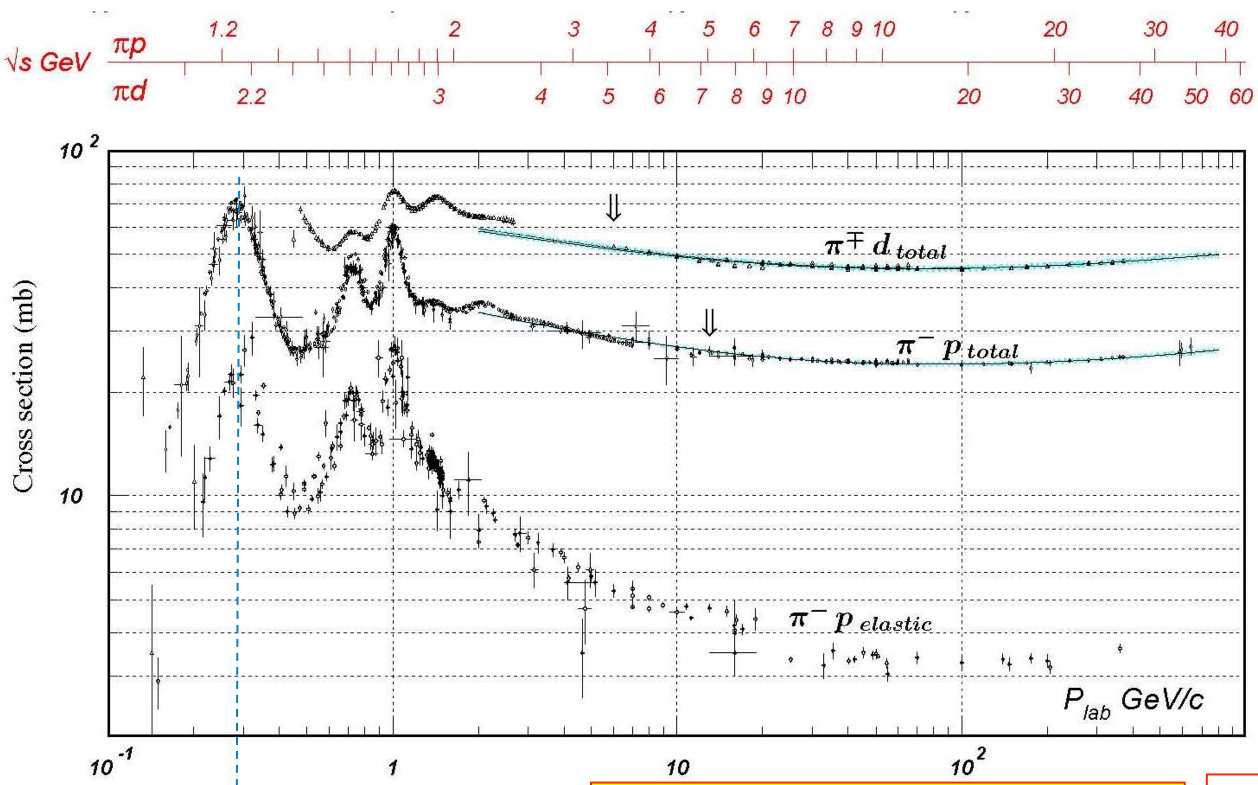
At $\sqrt{s} < 1.4$ GeV $\sigma_{elast} = \sigma_{total}$

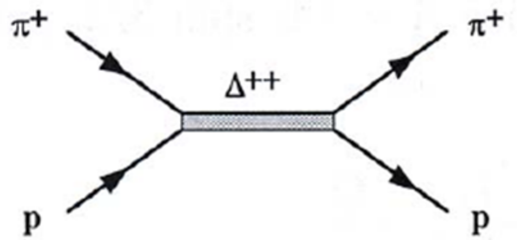
M = 1232 MeV

$$\tau = \frac{h}{\Gamma_{tot}} = \frac{6.58 \cdot 10^{-16}\ eV \cdot s}{118 \cdot 10^{6}\ eV} = 5.6 \cdot 10^{-24}\ s$$

From angular distribution of the decay products we derive that the spin of the Δ is 3/2

# The resonance Δ: Cross section $\pi^- p$, $\pi^- n$, $\pi^+ p$, $\pi^+ n$




$\pi^+ p \rightarrow \Delta^{++}$

$\pi^+ n \rightarrow \Delta^{+}$

$\pi^- p \rightarrow \Delta^{0}$

$\pi^- n \rightarrow \Delta^{-}$

M = 1232 MeV ⇒ peak position is the same

… and $\Gamma_{tot}$ is the same too

N.B. in the $\pi^- p$ channel $\sigma_{elastic}$ and $\sigma_{total}$ are different ⇒

$\sigma_{picco}$ $(\pi^- p \rightarrow \pi^- p)$ = 22 mb

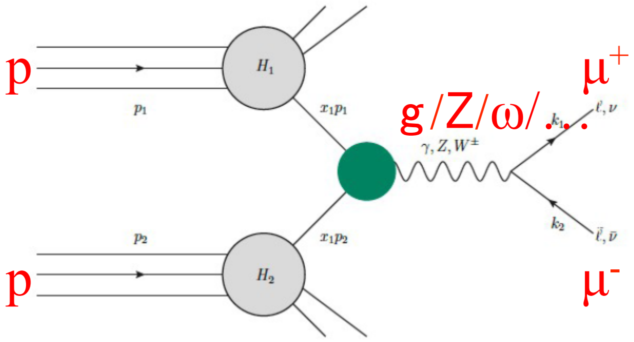
$\sigma_{picco}$ $(\pi^- p \rightarrow \pi^0 n)$ = 45 mb

Question: why $\sigma_{elastic}$ in the channels $\pi^- p$ e $\pi^+ p$ are different?
The answer is in the Δ isospin.

# Production Resonance: an example

$$p+p \rightarrow \mu^+ + \mu^- + X$$

production resonance



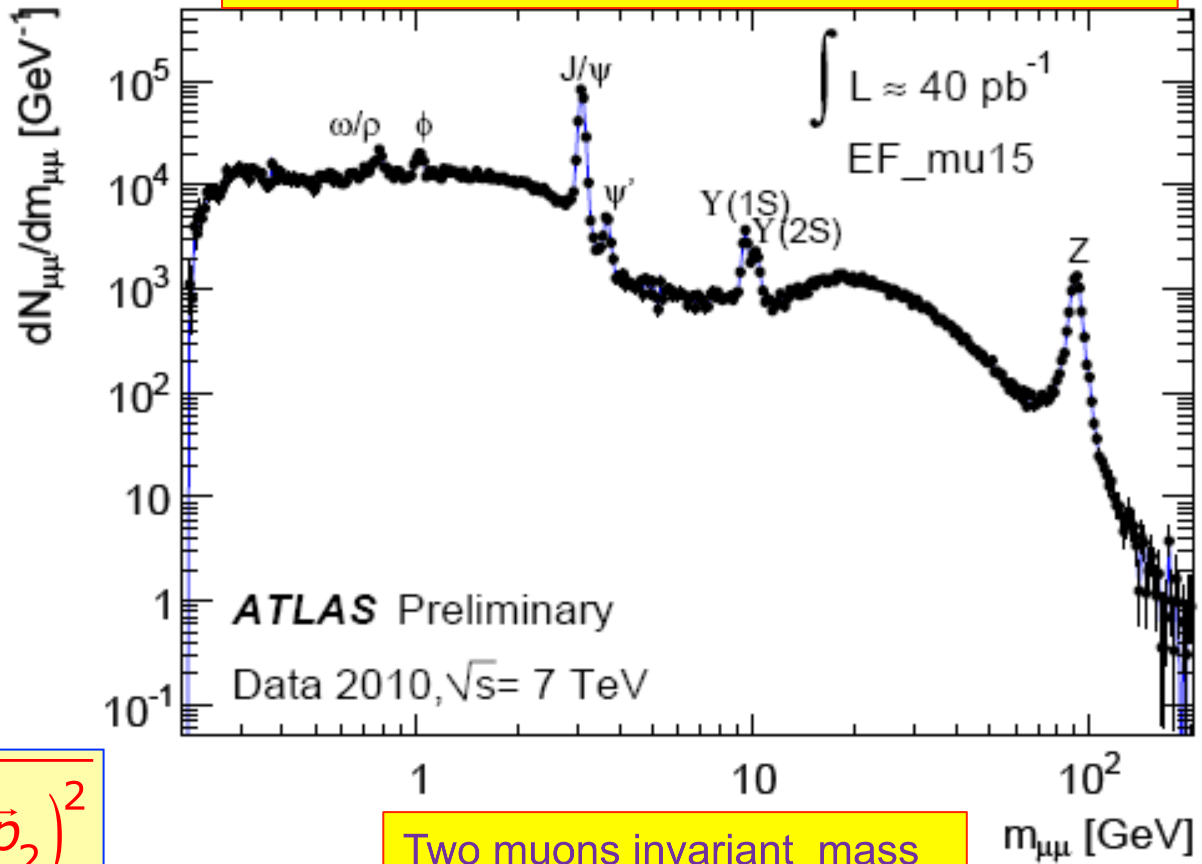

Drell-Yan process

$$m_{\mu\mu} = \sqrt{\left(E_1 + E_2\right)^2 - \left(\vec{p}_1 + \vec{p}_2\right)^2}$$

It is a relativistic invariant

ATLAS: 50 years of history in one slide

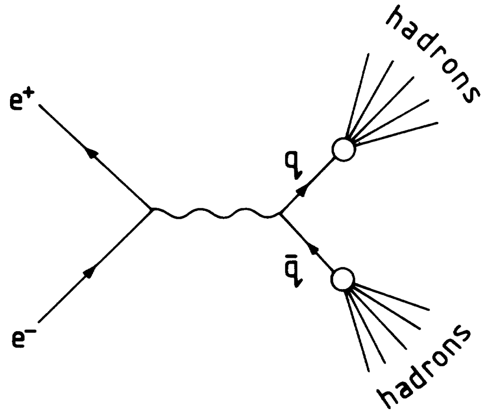
Spin 1 mesons with different quark compositions




Two muons invariant mass

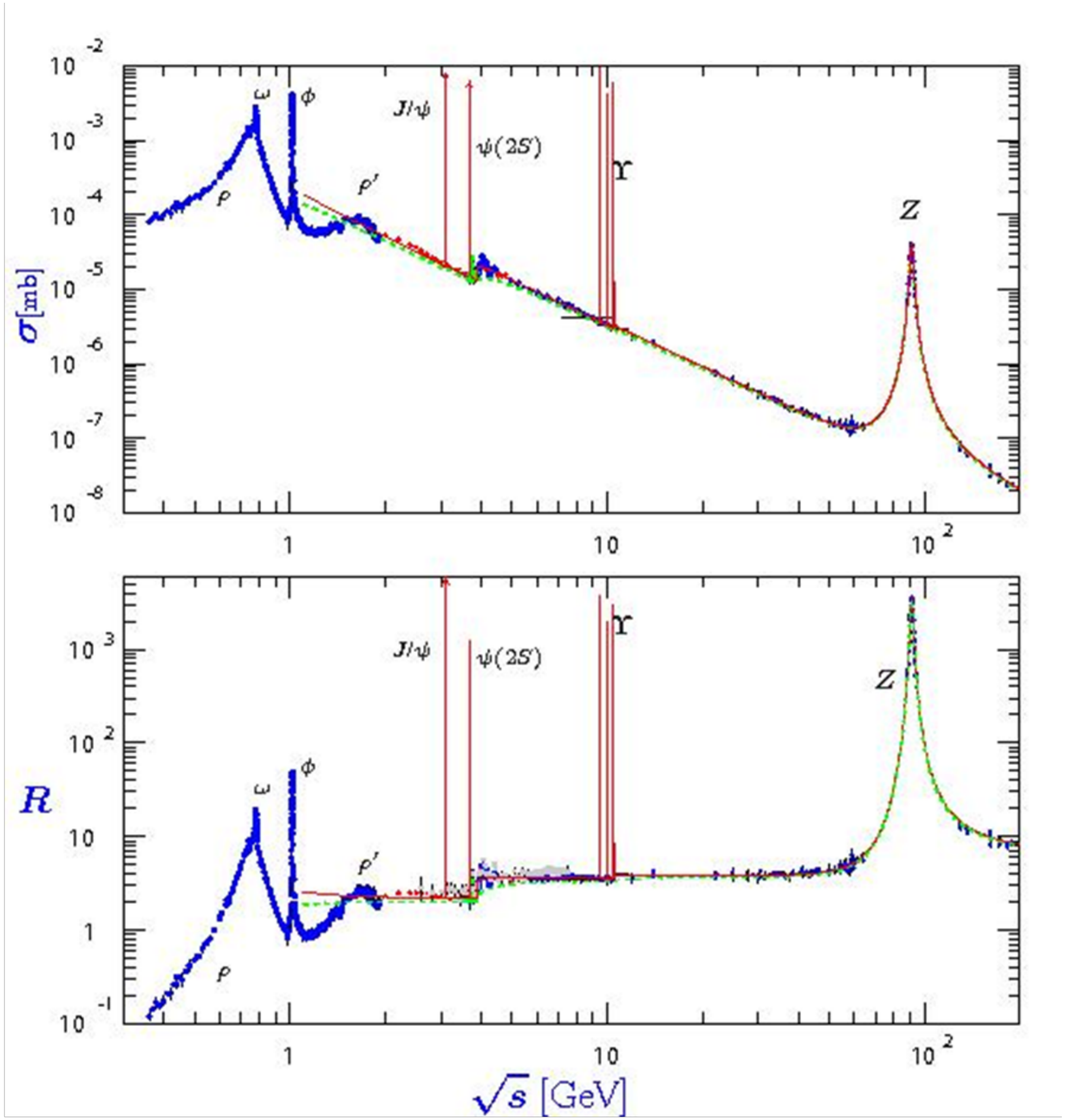
# Cross Section $e^+e^- \to$ hadrons

$$\sigma\left(e^+e^- \to hadrons\right)$$




$$R = \frac{\sigma\left(e^+e^- \to hadrons\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}$$

N.B.: the resonances are much narrower than the case of $\mu^+\mu^-$ invariant mass



# Exercise

In an experiment at a proton-proton collider have been identified, among the several particles produced in the final states, two muons back to back (that is collinear) of opposite charge. One has a momentum of 47 MeV/c and the other one of 31 GeV/c (this kinematic configuration is very unlikely and it is very difficult to measure a momentum as low as 47 MeV/c, but it has the merit to simplify the computation). Assuming that the two particles are the daughters of a mother particle, find out the mass of the mother and, assuming a 5% error on the mass, guess which is the particle.

$$P_1 = -47\ \text{MeV} \longleftarrow \bullet \longrightarrow P_2 = 31\ \text{GeV}$$

The mother's mass can be inferred from the quadrimomentum of the two muons:

$$E_1 = \sqrt{m_\mu^2 + p_1^2} = \sqrt{105^2 + 47^2} = 115\ \text{MeV} \qquad E_2 = \sqrt{m_\mu^2 + p_2^2} = \sqrt{0.105^2 + 31^2} \simeq 31\ \text{GeV}$$

$$E_f = E_1 + E_2 = 0.115 + 31 = 31.1\ \text{GeV} \qquad \vec{p}_f = \vec{p}_1 + \vec{p}_2 = -0.047 + 31 = 30.95\ \text{GeV/c}$$

The square of the quadrimomentum is a relativistic invariant and it is equal to the mass squared of the mother (in the mother rest frame its energy is equal to its mass and its momentum is zero):

$$m = \sqrt{E_f^2 - \vec{p}_f^{\,2}} = \sqrt{31.1^2 - 30.95^2} = 3.05\ \text{GeV/c}^2$$

A 5% error on the mass value gives an uncertainty on the mass of 0.15 GeV/c$^2$. We should check all neutral particles in the mass range 2.90 – 3.20 GeV/c$^2$. A good candidate is the $J/\Psi$ whose mass is 3.096 GeV/c$^2$.

# Exercise

The neutral pion has been discovered in the photoproduction on the proton at rest ($\gamma+p\rightarrow\pi^0+p$). Compute the minimal photon energy in the laboratory frame to achieve this reaction. ($m\_\pi^0$= 135 MeV/$c^2$ ; m_p=938 MeV/$c^2$)

In order to compute the threshold energy we have to assume that the particles in the final state are produced at rest in the center of mass reference system. We also remind you that the square of the total quadri-momentum is a relativistic invariant.

Let's call M the proton mass and m the $\pi^0$ mass:

$$P_\gamma=(E_\gamma,\vec{P}_\gamma)\ ;\ \ E_p=(M,0)$$

$$\Rightarrow\ P^{Lab.}_{iniz.}=(E_\gamma+M,\vec{P}_\gamma)\ ;\ \ P^{C.M.}_{fin.}=(m+M,0)$$

$$\left(P^{Lab.}_{iniz.}\right)^2=\left(P^{C.M.}_{fin.}\right)^2\ \Rightarrow\ \left(E_\gamma+M\right)^2-\vec{p}^{\,2}_\gamma=\left(m+M\right)^2$$

$$\Rightarrow\ E_\gamma=m\left(1+\frac{m}{2M}\right)=135\left(1+\frac{135}{2\cdot 938}\right)=145\ \text{MeV}$$

# Usefull tips

- In one gram of matter there are about $N_A$ nucleons (the atomic weight of a protone/neutron is 1)
- A few quantities given in a more suitable units

$$\hbar = 6.58 \cdot 10^{-16}\ eV \cdot s$$

$$c = 3 \cdot 10^{23}\ fm \cdot s^{-1} = 30\ \text{cm} \cdot \text{ns}^{-1}$$

$$\hbar c = 197\ \text{MeV} \cdot fm$$

- Conversion factor in the natural units system ($\hbar=c=1$):

$$1\ \text{MeV} = 1.52 \cdot 10^{21}\ s^{-1} \quad ; \quad 1\ \text{MeV}^{-1} = 197\ fm$$

$$1\ \text{s} = 3 \cdot 10^{23}\ \text{fm} \quad ; \quad 1\ \text{s}^{-1} = 6.5 \cdot 10^{-16}\ eV$$

$$1\ \text{m} = 5.07 \cdot 10^{6}\ \text{eV}^{-1} \quad ; \quad 1\ \text{m}^{-1} = 1.97 \cdot 10^{-7}\ eV$$

# A bit of theory: the S matrix

- We have an initial state |i> that evolves in the final state |f> due to an interaction;
- We work in the Dirac representation (interaction representation);
- $H = H_0 + V_I$, where $H_0$ is the free Hamiltonian and $V_I$ is the interaction Hamiltonian;
- The S matrix (function of $V_I$) drives the state evolution from time $t_0$ until time t;

$$\boxed{|\Psi_I(t)\rangle = S(t_0,t)|\Psi_I(t_0)\rangle}$$

- where

$$\boxed{S(t,t_0)=\exp\left[-\frac{i}{h}\int_{t_0}^{t} V_I(t')dt'\right]}$$

We have a conceptual problem to solve the integral because at different time t' the $V_I$ are not granted that commute with each other. We introduce a procedure of time ordering (Time order product) that lead to the concept of "propagator".

# A bit of theory: the S matrix

- We change the differential equation into an integral equation:

$$S(t,t_0)=1-\frac{i}{h}\int_{t_0}^{t} V_I(t')\cdot S(t',t_0)dt'$$

- it can be solved by successive iterations. Hence we have first order term, second order term and so on and so forth. We apply the Time order product: the bigger t (that comes afterward) to the left and the time smaller (that comes before) to the right:

$$S^0(t,t_0) = 1 \;;$$

$$S^1(t,t_0) = 1-\frac{i}{h}\int_{t_0}^{t} dt_1 V_I(t_1)\cdot 1 \;;$$

$$S^2(t,t_0) = 1-\frac{i}{h}\int_{t_0}^{t} dt_1 V_I(t_1)\cdot 1 + \left(-\frac{i}{h}\right)^2 \int_{t_0}^{t} dt_1 V_I(t_1)\int_{t_0}^{t_1} dt_2 V_I(t_2);$$

- by appropriate symmetrization of the integral on the integration domain we have:

$$S(t,t_0)=\sum_{n=0}^{\infty}\frac{1}{n!}\left(-\frac{i}{h}\right)^n\int_{t_0}^{t} dt_1 \cdots \int_{t_0}^{t} dt_n T\left[V_1(t_1)\cdots V_n(t_n)\right]$$

- N.B. Wick's theorem transforms the Time order product into Normal order product (ordering of the creation and annihilation operators) plus the field contractions (propagators).

# A bit of theory: the S matrix

- We want to evaluate the S Matrix between the time $-\infty$ and $+\infty$; that is we have a free state |i> and we would like to know how it evolves after the interaction:

$$\boxed{|\Psi(\infty)\rangle = S(-\infty,\infty)|\mathrm{i}\rangle}$$

- the amplitude probability to find a particular final state |f> is:

$$\boxed{\langle \mathrm{f}|\Psi(\infty)\rangle = \langle f \mid S(-\infty,\infty)|\mathrm{i}\rangle = \langle f \mid S|\mathrm{i}\rangle = S_{fi}}$$

- expansion of |Ψ(∞)> in a complete set of eigenstates:

$$|\Psi(\infty)\rangle = \sum_f | f\rangle\langle f |\Psi(\infty)\rangle = \sum_f | f\rangle S_{fi}$$

- Transition probability from the state |i> to the state |f>:

$$\boxed{\left|\langle \mathrm{f}|\Psi(\infty)\rangle\right|^2 = \mathrm{S}_{\mathrm{fi}}^2 \quad \text{(eigenstates normalized to 1)}}$$

- Unitarity of the S Matrix (probability conservation):

$$\boxed{\sum_f S_{fi}^2 = 1}$$

It can not be violated in any case and in any way!

# The T matrix

- Let's introduce the transition matrix **T** by factorizing the identity matrix I: **S = I + T**; then:

$$S_{fi} = \langle f|S|i\rangle = \delta_{fi} + i(2\pi)^4\,\delta^4(p_f - p_i)\langle f|T|i\rangle$$

- In order to have the transition probability we need to take the square of the second term, therefore we will have a factor like this:

$$(2\pi)^8\left|\delta^4(p_f - p_i)\right|^2 = (2\pi)^4\,\delta^4(p_f - p_i)\cdot(2\pi)^4\,\delta(0)$$

When $V \to \infty$ and $T \to \infty$ we have $(2\pi)^4\,\delta(0) \to VT$

$$\Rightarrow \quad W_{fi} = (2\pi)^4\,\delta^4(p_f - p_i)\left|\langle f|T|i\rangle\right|^2 \cdot VT$$

- To get rid of V and T, we consider the transition probability per unit of time and normalized to the volume V, but we have to pay attention to the normalization chosen (one particle per unit of volume, two particles, etc…)

- With the help of Feynman diagrams and with the proper normalization to get rid of V and T, one can compute the element of the matrix $\mathbf{T_{fi}}$ or, with another terminology, the element $\mathbf{M_{fi}}$.

# Fermi's golden rule

- It can be deduced with the time dependent perturbation theory of non relativistic quantum mechanics, but it is also valid in this context:

Transition probability per unit of time

$$\boxed{W_{fi} = \frac{2\pi}{h}\left|M_{fi}\right|^2 \cdot \rho(E)}$$

Phase space

Matrix element

- The amplitude **$M_{fi}$** contains the dynamical information of the process

- The phace space contains only the kinematical information of the process. It depends on the masses, energies and momenta of the particles involved in the process. It is more likely a process to occur if there is "more room to manoeuver".

  - For istance a particle does not decay in two particles whose masses are bigger than the initial mass because the phase space is zero.

N.B. **$W_{fi}$** is a relativistic invariant. **$M_{fi}$** and **$\rho(E)$** could be both invariant or just as a product

# An example: QED

- QED handles the interaction of electrons/positrons with an electromagnetic field e.m.

$$\boxed{L = L_0 + L_I}$$

$$L_0 = N\left[\bar{\psi}(x)\left(i\gamma^\mu\delta_\mu - m\right)\psi(x) - \frac{1}{2}\left(\delta_\nu A_\mu(x)\delta^\nu A^\mu(x)\right)\right]$$

$$L_I = N\left[-J^\mu(x)A_\mu(x)\right] = N\left[e\cdot\bar{\psi}(x)\gamma^\mu A_\mu(x)\psi(x)\right]$$

Normal order product: creation operators on the left and annihilation operators on the right

- $$H_I = -L_I = -eN\left[\bar{\psi}(x)\not{A}(x)\psi(x)\right] = -eN\left[\left(\bar{\psi}^+ + \bar{\psi}^-\right)\left(\not{A}^+ + \not{A}^-\right)\left(\psi^+ + \psi^-\right)\right]$$

Annihil. posit.

Create elec.

Annihil. photon

Create photon

Annihil. elect.

Create posit.

- Example: Compton scattering

$$\underbrace{e^- + \gamma}_{\text{Destroyed}} \rightarrow \underbrace{e^- + \gamma}_{\text{Created}}$$

- The Normal order product, at the lowest order, must contain the operators:

$$\bar{\psi}^- \not{A}^- \psi^+ \not{A}^+$$

Create elect.

Create photon

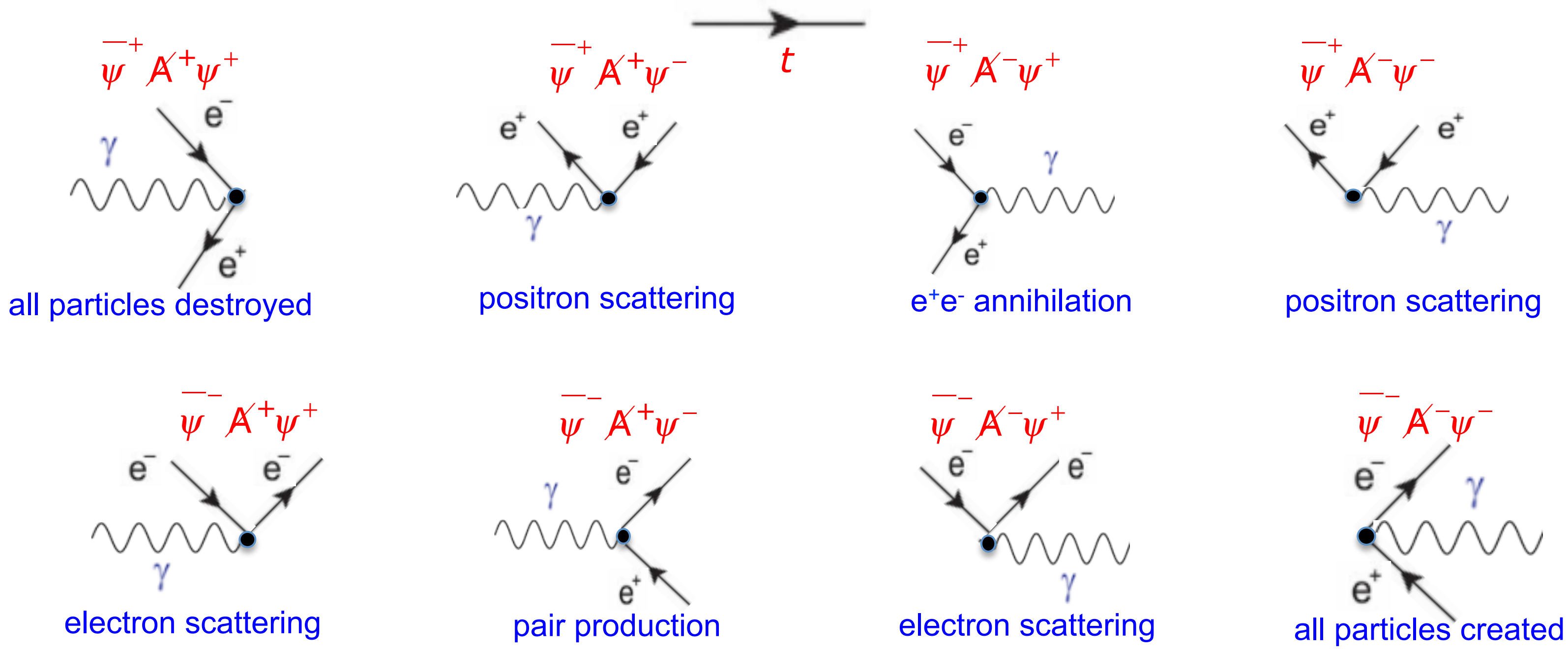
Annihilate photon

Annihilate elect.

# QED: Feynman's diagrams

$$H_I = -eN\left[\left(\overline{\psi}^{+} + \overline{\psi}^{-}\right)\left(\not{A}^{+} + \not{A}^{-}\right)\left(\psi^{+} + \psi^{-}\right)\right]$$

- If we multiply the operators among themselves we have eight processes:

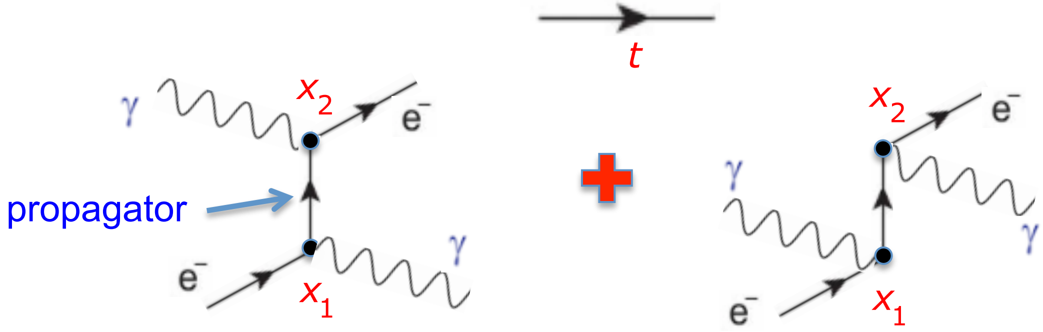



- All these processes do not conserve the quadrimomentum at the vertex. In order to have real processes we must combine two diagrams, that is we have to go to the secord order of the S matrix expansion.

$$\langle f \mid S^{(1)} | i \rangle = 0 \quad \Rightarrow \quad \langle f \mid S^{(2)} | i \rangle$$

# Compton scattering

$$e^- + \gamma \rightarrow e^- + \gamma$$

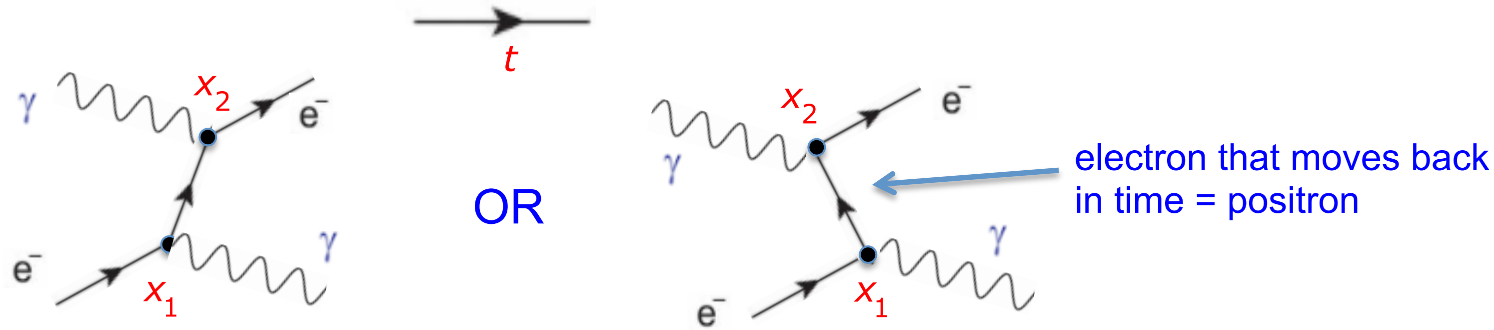



- The photon can be destroyed in $X_2$ and created in $X_1$ or the contrary;
- The propagator connects two vertexes. It is a virtual particle for which hold the relation:

$$E^2 - p^2 \neq m^2$$

- In case of fermionic propagator, if $t_1 < t_2$ we can think of it as a virtual electron that goes from $X_1$ to $X_2$, otherwise it is a virtual positron. The Time order product takes care of this.

For example:



# Radiative Correction

- Let's take, for instance, the  fundamental vertex of QED and let's see how it is modified by the four contributions of the second order terms of the S matrix expansion:

- If we apply the Feynman rules to compute the second order terms, we find that these are divergent, that is they give as a result infinite (this divergence is also present in the classical electrodynamics, let's take for instance the self-energy of the electron: $U=e^2/r$).

- The "solution" of the problem is complex but just to make it simple we could say:
  - an electron not interacting has a bare mass and a bare charge that are infinite as well;
  - the interaction with the field changes these infinite values toward the values measured experimentally:

$$\boxed{\infty - \infty = \text{ finite value}}$$

- IL MOMENTO MAGNETICO DELL'ELETTRONE È PROPORZIONALE ALLO SPIN

$$\vec{\mu} = -g \frac{e}{2m} \vec{S}$$

- $g$ È IL RAPPORTO GIROMAGNETICO. NELLA TEORIA DI DIRAC $g$ È UGUALE A 2 (VALORE MISURATO), MENTRE CLASSICAMENTE SAREBBE DOVUTO ESSERE UGUALE A 1

- MISURE PIÙ PRECISE DANNO PER $g$ UN VALORE LEGGERMENTE MAGGIORE DI 2 (ANOMALIA)

- DALLA QED, FACENDO LO SVILUPPO FINO AL TERZO ORDINE, RISULTA:

$$\left(\frac{g-2}{2}\right)^{\text{elettrone}} = 0.5\left(\frac{\alpha}{\pi}\right) - 0.32848\left(\frac{\alpha}{\pi}\right)^2 + 1.19\left(\frac{\alpha}{\pi}\right)^3 + \dots$$

$$\left(\frac{g-2}{2}\right)^{\text{muone}} = 0.5\left(\frac{\alpha}{\pi}\right) + 0.76578\left(\frac{\alpha}{\pi}\right)^2 + 24.45\left(\frac{\alpha}{\pi}\right)^3 + \dots$$

RISULTATI ~ 2000

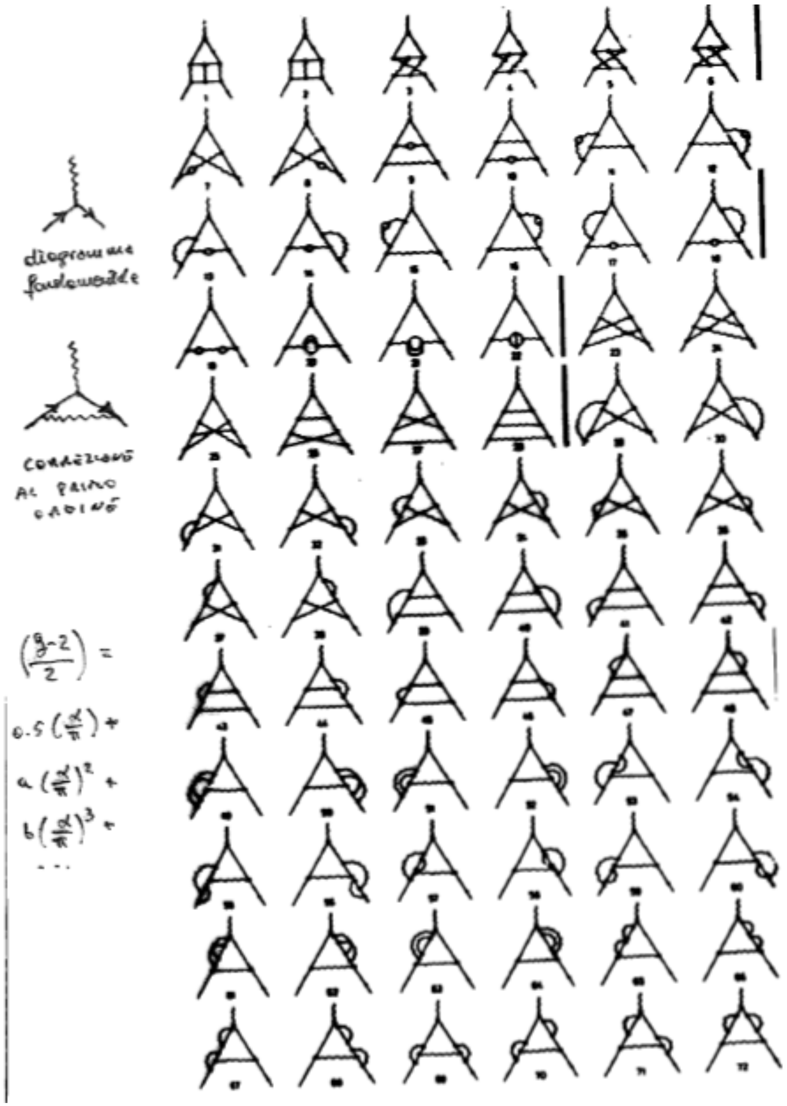
| | ELETTRONE | MUONE |
|---|---|---|
| PREDETTO | $11\,596\,524 \pm 4 \cdot 10^{-10}$ | $11\,659\,180 \pm 100 \cdot 10^{-10}$ |
| MISURATO | $11\,596\,521.9 \pm 0.1 \cdot 10^{-10}$ | $11\,659\,230 \pm 80 \cdot 10^{-10}$ |

N.B. SI PUÒ RISCRIVERE LA CORREZIONE INTRODUCENDO UNA COSTANTE DI ACCOPPIAMENTO $\alpha$ EFFICACE

$$\frac{g-2}{2} = \frac{0.5}{\pi}\,\alpha_{\text{eff.}}$$

$$\alpha_{\text{eff}}^{\text{muone}} > \alpha_{\text{eff.}}^{\text{elettrone}}$$

# Anomalous magnetic moment of the electron

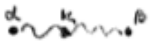



Correzioni al vertice del terzo ordine

1) PER CIASCUN VERTICE SCRIVERE UN FATTORE $ie\gamma^{\alpha}$ $[e \propto \sqrt{4\pi\alpha}]$

2) PER CIASCUNA LINEA FOTONICA INTERNA (PROPAGATORE), LABELLATA DAL MOMENTO K, SCRIVERE:

$$i\,\frac{-g_{\alpha\beta}}{k^2+i\varepsilon}$$




3) PER CIASCUNA LINEA FERMIONICA INTERNA (PROPAGATORE), LABELLATA DAL MOMENTO P, SCRIVERE:

$$i\,\frac{1}{\not p - m + i\varepsilon}$$ [su un altro testo: $i\,\frac{\gamma^{\mu}p_{\mu}+m}{p^2-m^2}$ ]

4) PER CIASCUNA LINEA ESTERNA, SCRIVERE UNO DEI SEGUENTI:

a) ELETTRONE INIZIALE: $u_r(p)$
b) ELETTRONE FINALE: $\bar{u}_r(p)$
c) POSITRONE INIZIALE: $\bar{v}_r(p)$
d) POSITRONE INIZIALE: $v_r(p)$
e) FOTONE INIZIALE: $\varepsilon_{r\alpha}(k)$
b) FOTONE FINALE: $\varepsilon_{r\alpha}(k)$

5) METTERE IN ORDINE I FATTORI SPINORIALI DA DESTRA A SINISTRA IN MODO DA SEGUIRE IL VERSO DELLE FRECCE SULLE LINEE FERMIONICHE

6) PER OGNI LOOP FERMIONICO, PRENDERE LA TRACCIA E MOLTIPLICARE PER $-1$

7) AD OGNI VERTICE SI DEVE CONSERVARE IL QUADRIMPULSO. PER OGNI QUADRIMOMENTO NON DETERMINATO DALLA REGOLA DI CONSERVAZIONE FARE L'INTEGRAZIONE $(2\pi)^{-4}\int d^4q$

8) MOLTIPLICARE PER UN FATTORE DI FASE $\delta_p = \pm 1$ IN BASE AL NUMERO DI SCAMBI FERMIONICI PARI O DISPARI NECESSARI PER IL NORMAL ORDER PRODUCT

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# End of chapter 1