Introduction to Particle Physics
- Chapter 7 -

Weak Interactions

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Chapter summary:

- Beta decay.
- Fermi’s theory of Beta decay
- Sargent’s rule
- Parity violation in the weak interactions.
- Two components theory of the neutrino.
- Goldhaber’s experiment.
- V-A interaction.
- Helicity eigenstates and chiral eigenstates.
- The W boson.
- The Weinberg’s angle.
- Charged pion decay.
- Charged K decay in the muon channel.
- The Cabibbo’s angle.
- Weak isospin doublets.
- GIM effect.
- The quark charm.
- CKM matrix.

Introduction to weak interactions

- Let's recall the life time of a few decays:

$\Delta^{++} \rightarrow p\pi$	$\sim 10^{-23}$ s	strong int.
$\Sigma^0 \rightarrow \Lambda\gamma$	$\sim 6 \cdot 10^{-20}$ s	1 γ, e.m. int.
$\pi^0 \rightarrow \gamma\gamma$	$\sim 10^{-16}$	2 γ, e.m. int.
$\Sigma \rightarrow n\pi$	$\sim 10^{-10}$ s	weak int.
$\pi^- \rightarrow \mu^- \nu_\mu$	$\sim 10^{-8}$ s	
$\mu^- \rightarrow e^- \nu_e \nu_\mu$	$\sim 10^{-6}$ s	
$n \rightarrow p\, e^- \nu_e$	~ 15 min	

N.B. we observe the weak interactions only when the strong and e.m. interactions are forbidden.

- We need to explain the enourmos range in the life time going from 10^{-12} s untill 15 min.
- The weak interactions are also characterized by cross-sections extremely small (~10^{-39} cm^2=1 fb)

$$\sigma(\nu_\mu + N \rightarrow N + \pi + \mu) = 10^{-38}\ \text{cm}^2\ (10\ \text{fb})\ \text{a 1 GeV}$$

$$\sigma(\pi + N \rightarrow N + \pi\ ) = 10^{-26}\ \text{cm}^2\ (10\ \text{mb})\ \text{a 1 GeV}$$

- The weak interactios violate many conservation rules (parity, charge conjugation, strangeness, etc...)
- Because of their "weakness", the weak interactions can be observed in the "standard" matter only in the beta decay, because they do not give origin to any bound states. However they are the base fuel for the stars functioning , therefore without the weak interactions we coud not exist

$$p+p \rightarrow d+e^+ + \nu_e$$

Beta decay

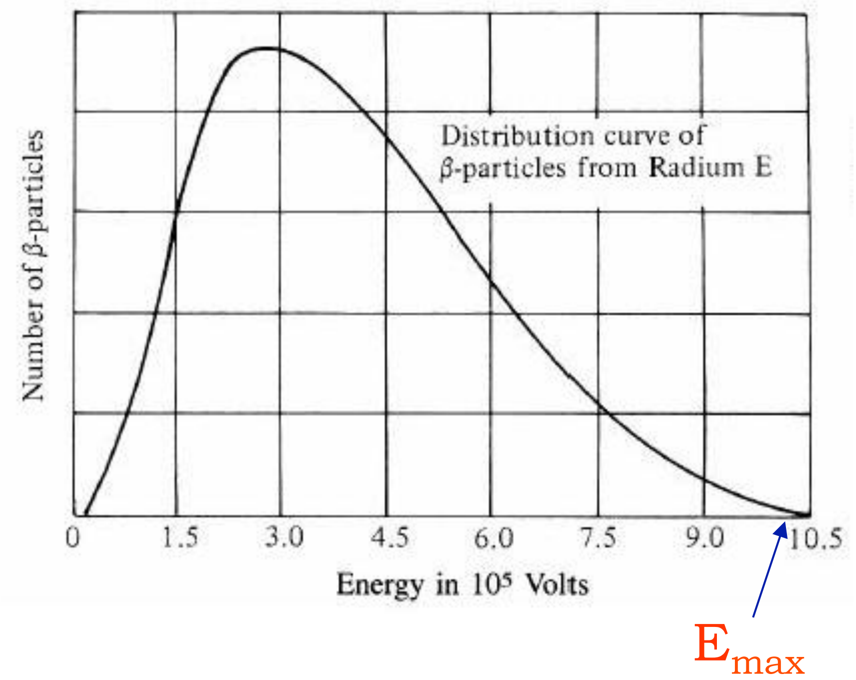
Most of our knowledge about the base principles of beta decay is based upon nuclei beta decays.

$n \rightarrow p+e^{-}+\bar{\nu}_e \quad \Rightarrow \quad (A,Z) \rightarrow (A,Z+1)+e^{-}+\bar{\nu}_e$

$p \rightarrow n+e^{+}+\nu_e \quad \Rightarrow \quad (A,Z) \rightarrow (A,Z-1)+e^{+}+\nu_e$

$e^{-}+p \rightarrow n+\nu_e \quad \Rightarrow \quad (A,Z)+e^{-} \rightarrow (A,Z-1)+\nu_e$

- The existence of the β^+ decay was established in 1934 by Curie and Joliot.
- In 1919 Chadwick discovered that the electron in the β decay had a continuos spectrum.



- The maximum energy of the spectrum corresponds fairly well to the Q of the reaction (Q=M(A,Z)-M(A,Z+1)), while for the rest of the spectrum there is a violation of the energy conservation rule.
- Moreover there is also a violation of the momentum and angular momentum conservation rules (without the introduction of the neutrino).

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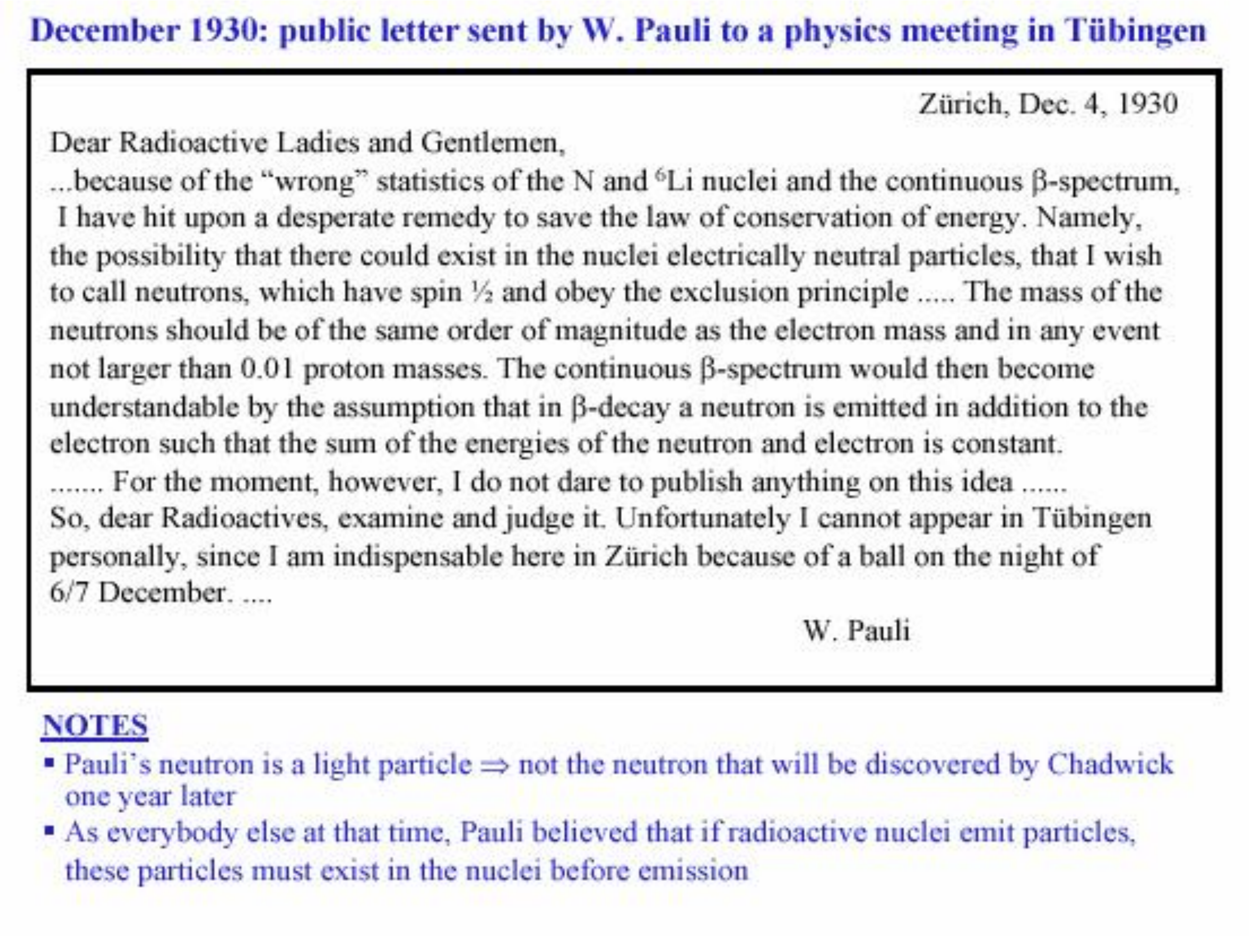
Neutrino ”creation”

- To re-establish the various conservation laws, in 1930 Pauli made the hypothesis of the existence of a very small neutral particle: the neutron (later renamed neutrino by Fermi).



December 1930: public letter sent by W. Pauli to a physics meeting in Tübingen

Zürich, Dec. 4, 1930

Dear Radioactive Ladies and Gentlemen,

...because of the “wrong” statistics of the N and ^{6}Li nuclei and the continuous β-spectrum, I have hit upon a desperate remedy to save the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin ½ and obey the exclusion principle ..... The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous β-spectrum would then become understandable by the assumption that in β-decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and electron is constant.

.... For the moment, however, I do not dare to publish anything on this idea ......

So, dear Radioactives, examine and judge it. Unfortunately I cannot appear in Tübingen personally, since I am indispensable here in Zürich because of a ball on the night of 6/7 December. ....

W. Pauli

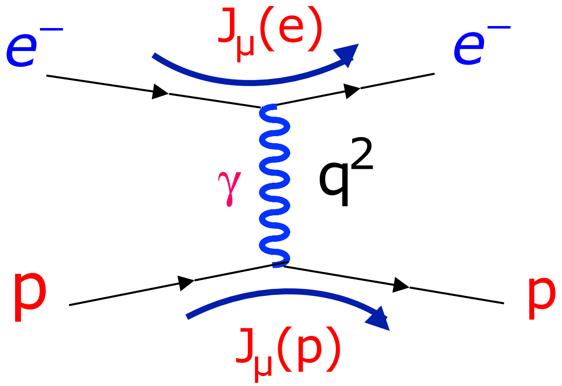
NOTES

- Pauli’s neutron is a light particle ⇒ not the neutron that will be discovered by Chadwick one year later
- As everybody else at that time, Pauli believed that if radioactive nuclei emit particles, these particles must exist in the nuclei before emission

- This letter is very important for Physics … but it is also interesting from a sociological point of view ☺
- The first theory of β decay was done by Fermi in 1934.
- The neutrino was discovered by Reines e Cowan “only” in 1958
- We have three kind of neutrinos; recently have been established the flavour neutrino oscillations that imply that neutrinos have masses different from zero, although they are very small and not yet measured.

Fermi's theory of β decay

- In 1934 Fermi did the first theory of β decay; he took as a model the QED description of the electron-proton scattering:

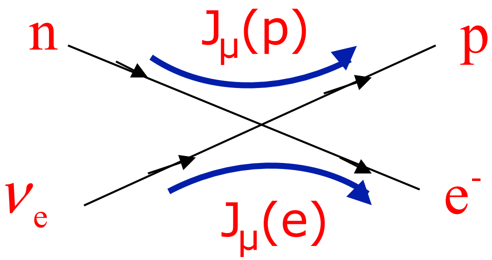


The matrix element is proportional to:

$$\boxed{M_{fi} \approx -\frac{1}{q^2} J_\mu(e) J^\mu(p)}$$

$$M_{fi} \approx \left(\bar{u}_e \sqrt{\alpha}\gamma^\mu u_e\right)\frac{g_{\mu\nu}}{q^2}\left(\bar{u}_p \sqrt{\alpha}\gamma^\nu u_p\right)$$

- Fermi made the hypothesis of a pointlike interaction like: $n+\nu \rightarrow p+e^-$ (that is like $n \rightarrow \bar{\nu}+p+e^-$)



→ <u>There is no propagator</u>

$$\boxed{M_{fi} \approx G\left(\bar{u}_p \gamma^\mu u_n\right)\left(\bar{u}_e \gamma_\mu u_\nu\right)}$$

vector-vector interaction

The G constant is known as Fermi's constant and it is related to the square of the "weak charge".

interaction between two (charged) currents: hadronic and leptonic currents.

$\bar{u}_p$ creates a proton (or destroys an antiproton)
u_n destroys a neutron (or creates an antineutron)
$\bar{u}_e$ creates an electron (or destroys a positron)
u_ν destroys a neutrino (or creates an antineutrino)

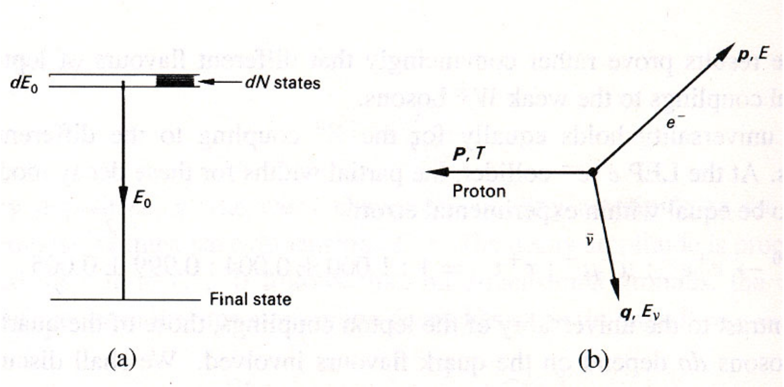
Nuclear β decay

- The transition probability (the decay rate per unit of time) can be found by using the Fermi's golden rule:

$$W=\frac{2\pi}{\hbar}G^2|M|^2\frac{dN}{dE_0}$$

$$\frac{dN}{dE_0}$$: phase space

$|M|^2$ is the matrix element squared. It is computed by integrating over all angles of the final particles, by summing over the final spin states and by averaging on the initial spin states. It is a constant of order one.



Fermi decays: $J_{leptoni}=0 \Rightarrow |M|^2 \approx 1$

Gamow-Teller decays: $J_{leptoni}=1 \Rightarrow |M|^2 \approx 3$

- E_0 is the energy available in the final state (it is equal to the Q of the reaction). The energy spread dE_0 is present because the energy of the initial state is not precisely known due to the finite lifetime (Heisemberg's principle).

$$\vec{P}+\vec{q}+\vec{p}=0 \qquad T+E_\nu+E=E_0$$

- In the nuclear β decays E_0 is of the order of 1 MeV. The proton kinetic energy is of the order of 10^{-3} MeV and can be neglected. The proton is there just to ensure the momentum conservation.

$q_\nu=E_0-E_e$ The energy is shared between the electron and the neutrino

The phase space

- The number of available states for an electron with momentum between p and p+dp, confined in the volume V, within the solid angle dΩ, is:

$$dN = \frac{Vd\Omega}{(2\pi)^3\hbar^3}p^2dp$$

- We normalize the wave function to V=1, we sum over the entire solid angle and we ignore the effect of the spin on the angular distribution. We get the following phase space for the electron and neutrino:

$$dN_e = \frac{4\pi p^2 dp}{(2\pi)^3\hbar^3} \quad ; \quad dN_\nu = \frac{4\pi q_\nu^2 dq_\nu}{(2\pi)^3\hbar^3}$$

- The two phase space factors are independent because there is no correlation between q and p, since it is a three bodies decay the proton will absorb the remaining momentum difference. The proton momentum is fixed (given q and p) so there is no proton phase space factor.

- The number of final states is: $dN = dN_e \cdot dN_\nu = \frac{(4\pi)^2}{(2\pi)^6\hbar^6}p^2q_\nu^2dpdq_\nu$

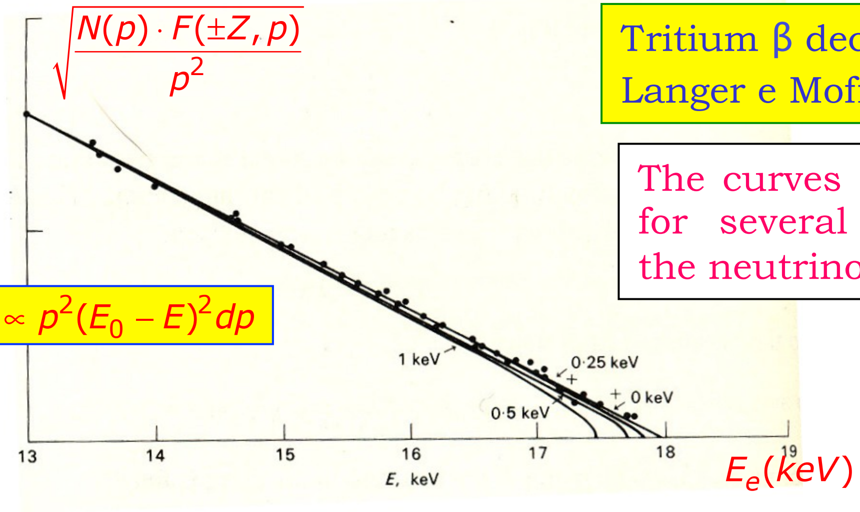
- For a given value of the electron energy E, the neutrino energy E_ν is fixed as well as its momentum:

$$q_\nu \equiv E_\nu = (E_0 - E) \quad ; \quad \Rightarrow dq_\nu = dE_0 \quad \Rightarrow \frac{dN}{dE_0} = \frac{dN}{dq_\nu} = \frac{1}{4\pi^4\hbar^6}p^2(E_0 - E)^2dp$$

- Once we have integrated the transition probability W over the entire solid angle, M^2 is equal to a constant, therefore the electron energy spectrum is entirely due to the phace space form:

$$\boxed{N(p)dp \propto p^2(E_0 - E)^2dp}$$

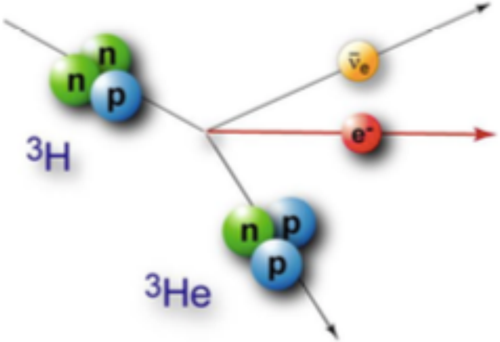
Kurie plot



$$N(p)dp \propto p^2(E_0-E)^2dp$$

Tritium β decay.
Langer e Moffatt (1952)

The curves are plotted for several values of the neutrino mass.



$$E_0 = 18.6\ keV \qquad T_{1/2} = 12.3 \text{ years}$$

- If we plot $(N(p)/p^2)^{1/2}$ versus the electron energy, we get a straight line that crosses the x-axis at $E=E_0$. This graph used to study β decay was developed by Franz N.D. Kurie.
- Experimentally we need to include a correction factor F(Z,p) to take into account the Coulomb interaction between the electron and the nucleus.
- If the neutrino has a mass, its effect would be to modify the distribution in the following way:

$$N(p)dp \propto p^2(E_0-E)^2\sqrt{1-\left(\frac{m_\nu}{E_0-E}\right)^2}\,dp$$

- The Kurie plot is modified in a way that the curve crosses the x-axis at $E=E_0-m_\nu$. This is how we try to measure the neutrino-e mass. Unfortunately in this region there are very few events and it is very difficult to perform the experiment. At the moment we have only an upper limit.

$$m_{\nu_e} \leq 2.2 \text{ eV}$$

Mainz exp. ; 2000

The Sargent's rule

- The total decay rate is obtained by integrating the spectrum N(p)dp. It can be done analitically; however in the cases where the electron is relativistic we can use the approximation p≈E and we get a very simple formula:

$$\boxed{N \propto \int_0^{E_0} E^2 (E_0 - E)^2 dE \propto E_0^5}$$

- The decay rate is proportional to the fifth power of the energy available in the process. This is the **Sargent's rule.**

- If we consider all the numerical factors in the process, we get: $W = \dfrac{G^2 |M|^2 E_0^5}{60\pi^3 (hc)^6 h}$ (per $E_0 >> m_e$)

- The Fermi's constant G can be found, as we will see later, from the life time measurements of a few β decays (and with some theoretical speculations, see Cabibbo's angle) or in a more precise way from the muon life time.

- From the PDG we get: $\boxed{\dfrac{G}{(hc)^3} = 1.16637(1) \cdot 10^{-5}\ \mathrm{GeV}^{-2}}$

- Replacing the numerical values in the formula we get: $\dfrac{1}{\tau} = W = \dfrac{1.11}{10^4} |M|^2 E_0^5\ \mathrm{s}^{-1}$ (E_0 in MeV)

- For instance, if $E_0 \approx 100$ MeV like in the muon decay and $M^2=1$, we get: $\tau_\mu = \dfrac{1}{W} \approx 10^{-6}\ \mathrm{s}$ ($\tau_\mu = 2.2\ \mu\mathrm{s}$)

N.B. it is the E_0^5 dependence that explains the huge range in the life time of the decays mediated by the weak interactions.

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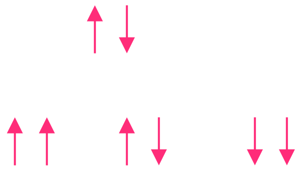
The nuclear β decay

- The nuclear β decays can be discriminated as allowed transitions and forbidden transitions.
- The allowed ones are the most common and are characterized by the fact that the electron and neutrino emitted DOES NOT carry any spatial angular momentum, that is they are in the S-state (L=0). This is justified by the fact that the two leptons have energies of the order a 1 MeV.
- The transitions with L=1 are called first forbidden, the ones with L=2 second forbidden and so on. They have a lifetime considerably longer that the allowed transitions.
- Since e and ν have spin ½ , the nucleus spin change can be either 0 or 1. The transitions with ΔJ=0 are called Fermi transitions while the ones with ΔJ=1 are called Gamow-Teller transitions.

transitions	ΔJ nucleus	Leptonic state
Fermi	ΔJ=0	singlet
Gamow-Teller	ΔJ=1 $\Delta J_z=0,\pm1$	triplet



- Since e-ν have L=0, there is no change in the spatial angular momentum of the nucleus, therefore its parity will not change. The nuclues undergoes a spin flip for the G.T. transitions.

Fermi: $0^+ \to 0^+, \Delta\vec{J}=0$	**G.-T. :** $1^+ \to 0^+, \Delta\vec{J}=1$	**Mixed:** $\frac{1}{2}^+ \to \frac{1}{2}^+, \Delta\vec{J}=0,1$
$^{10}C \to {}^{10}B^* e^- \bar{\nu}_e$ $^{14}O \to {}^{14}N^* e^+ \nu_e$	$^{12}B \to {}^{12}C e^- \bar{\nu}_e$	$n \to p e^- \bar{\nu}_e$

Measurement of the Fermi constant

- The decay rate can be written in a different way with respect to the Sargent rule. We write explicitly the proton mass m, then we include the phase space factors and the Coulombian correction F(±Z,p) in a dimensionless function f(±Z,E_0/m_e) that can be computed analitically.

$$\frac{1}{\tau} = W = \frac{(mc^2)^5}{2\pi^3 h(hc)^6} G^2 |M|^2 f(\pm Z, E_0)$$

$$\Downarrow$$

$$G^2 |M|^2 = \frac{\text{constant}}{f \cdot \tau} \qquad (\text{constant} = \frac{2\pi^3}{m^5})$$

			E_0 (MeV)			$\text{MeV}^2 \cdot \text{fm}^6$
decadimento	*transizione*	τ (s)	W	p_e^{max}	$f\ \tau$	$g^2\ \lvert M_{if}\rvert^2$
$n \to p\ e^-\ \bar{\nu}$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	890	1.29	1.18	1.61 10^3	4.25 10^{-8}
${}^3_1H \to {}^3_2He\ e^-\ \bar{\nu}$	$\frac{1}{2}^+ \to \frac{1}{2}^+$	5.60 10^8	0.53	0.14	1.63 10^3	4.20 10^{-8}
${}^{14}_8O \to {}^{14}_7N^*\ e^+\ \nu$	$0^+ \to 0^+$	102	2.32	2.26	4.51 10^3	1.52 10^{-8}
${}^{34}_{17}Cl \to {}^{34}_{16}S\ e^+\ \nu$	$0^+ \to 0^+$	2.21	4.97	4.94	4.54 10^3	1.51 10^{-8}
${}^6_2He \to {}^6_3Li\ e^-\ \bar{\nu}$	$0^+ \to 1^+$	1.15	4.02	3.99	1.17 10^3	5.85 10^{-8}
${}^{13}_5B \to {}^{13}_6C\ e^-\ \bar{\nu}$	$\frac{3}{2}^- \to \frac{1}{2}^-$	2.51 10^{-3}	13.4	13.4	1.11 10^3	6.17 10^{-8}

- In spite of the big variations of lifetime due to the strong dependency of the function f from p_e^{max}, the product G^2M^2 is about the same in all decays.
- However we observe a small difference due to the type of nuclear transition: Fermi, Gamow-Teller or mixed transitions.
- If we consider a pure Fermi transition, we get: $\frac{G}{(\hbar c)^3} = 1.140(2) \cdot 10^{-5}\ \text{GeV}^{-2}$

that is slightly different from the one quoted by the PDG taken from the muon decay. We will see later the reason of such a discrepancy (Cabibbo's angle).

Generalization of the Fermi Lagrangian

- There is no a priori reasons that justify a vectorial weak current in the Fermi's Lagrangian.
- The simplest Lorentz invariant Lagrangian is:

$$L_i = \sum_r C_r(\bar{\psi}_p O_r \psi_n)(\bar{\psi}_e O_r \psi_\nu) + \sum_r C_r(\bar{\psi}_n O_r^\dagger \psi_p)(\bar{\psi}_\nu O_r^\dagger \psi_e) \longleftarrow \text{Hermitian conjugate}$$

- C_r are constants that determine the intensity of the the interaction. The operators O_r are:

$O_S = 1$ scalar ; $O_A = \gamma^\mu \gamma_5$ axial vector

$O_V = \gamma^\mu$ vector ; $O_P = i\gamma_5$ pseudoscalar

$O_T = \sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ antisymmetric tensor

- Since the weak interactions do not conserve the parity, we will show later how to introduce this feature in the Lagrangian. In what follows we assume that the parity is conserved.

- In the pseudoscalar term the matrix element is multiplied by the β (=v/c) of the nucleon, therefore this term can be neglected in the Lagrangian, since β is of the order 10^{-3}.

- Moreover, by examining the electron and neutrino spin correlations, it turns out that only the vector or scalar terms can contributes to the pure Fermi decays (ΔJ=0), while only the axial or tensorial terms can contribute to the Gamow-Teller decays.

Determination of the coefficients C_r

- If we consider only the allowed β decays (ΔL=0) where there is no parity change, we can write the Lagrangian in the following way:

$$\boxed{L_i = \sum_{i=S,V} C_i(\bar{\psi}_p O_i \psi_n)(\bar{\psi}_e O_i \psi_\nu) + \sum_{j=T,A} C_j(\bar{\psi}_p O_j \psi_n)(\bar{\psi}_e O_j \psi_\nu)}$$

Fermi Gamow-Teller

- The electron or positron energy spectrum in the β decays can be written as:

$$\frac{dn_{\mp}}{dE_e} = \frac{P_e E_e}{2\pi^3}(E_0 - E)^2 \left[\ |M_F|^2 (C_S^2 + C_V^2) + |M_{GT}|^2 (C_T^2 + C_A^2) \pm \frac{2me}{Ee}(|M_F|^2 C_s C_V + |M_{GT}|^2 C_T C_A) \ \right]$$

M_F and M_{GT} are the Fermi and Gamow-Teller matrix elements; E_0 is the maximum electron energy.

- From this expression we see that there is no interference between Fermi and Gamow-Teller transitions, while there is interference between the terms S and V and the terms A and T.

- Since we have observed pure Fermi transitions or pure Gamow-Teller transitions, we can NOT have:

$$C_S = C_V = 0 \qquad \text{or} \qquad C_A = C_T = 0$$

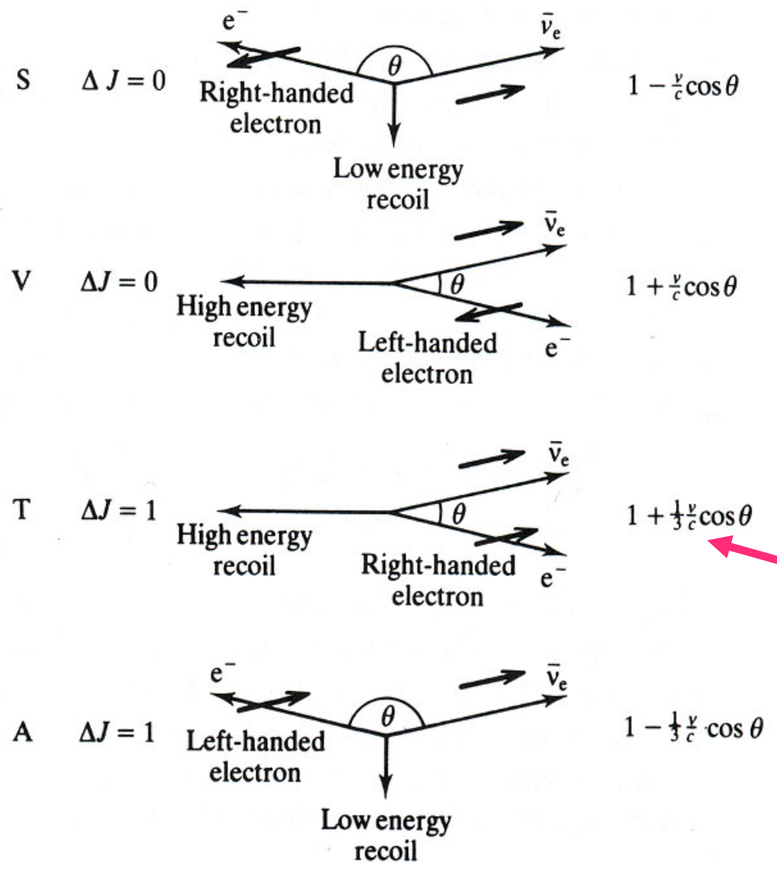
Determination of the coefficients C_r

- From the Kurie plot of the Fermi or Gamow-Teller transitions, we determine the ratio:

$$\frac{C_S \cdot C_V}{C_S^2 + C_V^2} = 0.00 \pm 0.15 \quad ; \quad \frac{C_T \cdot C_A}{C_T^2 + C_A^2} = 0.00 \pm 0.02$$

therefore the data suggest (Michel,1957) that C_s or C_V are zero and C_T or C_A are zero.

To determine which term is zero, we examine the correlation between the electron and neutrino directions.



Angular distribution

N.B. we measure the angle between the electron and the nucleus "recoil".

Data are "peaked" at ϑ=0, so C_s is zero.

The electron is "lefthanded"

Data are "peaked" at ϑ=180, so C_T is zero.

The electron is "lefthanded"

The factor 3 comes from the three possible total spin orientations.

N.B. the antineutrino is always righthanded

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Measurement of C_V and C_A

- We can write down the electron spectrum as: $\frac{dn}{dE_e} = \frac{P_e E_e}{2\pi^3}(E_0 - E)^2\left[C_V^2 |M_F|^2 + C_A^2 |M_{GT}|^2\right]$
- If we integrate over the electron spectrum we get the number of counting per unit of time:

$$n = \frac{1}{\tau} = \frac{1}{2\pi^3}\left[C_V^2 |M_F|^2 + C_A^2 |M_{GT}|^2\right] \underbrace{\int_{m_e}^{E_0} P_e E_e (E_0 - E)^2 dE_e}_{\boxed{m^5 \cdot f}} \qquad |M_F|^2 \approx 1 \quad ; \quad |M_{GT}|^2 \approx 3$$

N.B. In order to have f dimensionless , we normalize the energy with the mass m, that can be the electron mass or the proton mass. N.N.B. f takes into account also the Coulomb correction, that it is different for elec. and positr.

- The parameters C_V and C_A are inferred from the lifetime of some nuclear β decays. Actually what is measured is the half-life that is related to the mean lifetime in the following way:

$$N(t_2) = \frac{1}{2}N_0 = N_0 \cdot e^{-\frac{t_2}{\tau}} \quad \Rightarrow \quad t_2 = \tau \cdot \ln 2$$

- From the pure Fermi decays (for instance $^{14}O \rightarrow ^{14}N^*$): $C_V = \frac{G}{(hc)^3} = 1.140(2) \cdot 10^{-5}\ \text{GeV}^{-2} = \frac{10^{-5}}{M_p^2} \quad (h = c = 1)$
- From the neutron decay (mixed decay): $\frac{1}{f \cdot t} = \left[C_V^2 + 3 \cdot C_A^2\right]\frac{m^5}{2\pi^3 \ln 2} = (1080 \pm 16)^{-1} s^{-1}$
- Comparing with the O^{14} decay (2 proton decays): $\frac{(ft)^{14}O}{(ft)n} = \frac{C_V^2 + 3 \cdot C_A^2}{2C_V^2} = \frac{3100 \pm 20}{1080 \pm 16} \Rightarrow \boxed{\left|\frac{C_A}{C_V}\right| = 1.25 \pm 0.2}$

From polarized neutrons decay we deduce that the sign of C_A is negative

The τ-θ puzzle

- There were two strange particles, with the same mass and lifetime, that decayed in two final states with opposite parity:

$$\theta \rightarrow \pi\ \pi \qquad ; \qquad \tau \rightarrow \pi\ \pi\ \pi$$

- Parity of the meson θ (K → π π):

  Pions have spin zero, therefore due to the conservation of the total angular momentum, the K spin must be equal to the relative orbital angular momentum of the two pions system.

  Hence the parity of the system is equal to: $\eta = (-1)^{L}$

  $$\Rightarrow \quad J^{P} = 0^{+}, 1^{-}, 2^{+}, 3^{-} \cdots$$ (natural spin parity)

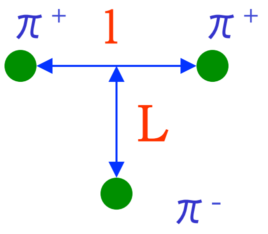
  If we consider the neutral K decaying into two π^0 that are two identical bosons, the wave function must be symmetric, so they are allowe only the even L values:

  $$\Rightarrow \quad J^{P} = 0^{+}, 2^{+} \cdots$$ ⇒ even K spin and positive parity

- Parity of the meson τ (K → π π π):

  We can handle the system as a di-pion (for instance two pions with the same charge plus the third one).

  Let's call l the relative orbital angular momentum of the two pions and let's call L the angular momentum of the third pion with respect to the pion pair.

  The parity of the three pions system is:

  $$\eta = (-1)^{3} \cdot (-1)^{l} \cdot (-1)^{L} = -(-1)^{L}$$

N.B. l must be even because the two pions have the same charge, therefore are two identical particles.

Moreover remember that the pion has negative intrinsic parity.

The τ-θ puzzle

- The total spin J of the three pions system must lie in the interval: $|L-l| \le J \le |L+l|$

  therefore we have the following combinations:

l	L	J^p
0	0	0^-
0	1	1^+
0	2	2^-
2	0	2^-
2	1	$1^+ . 2^+ , 3^+$
2	2	$0^-, 1^-, 2^-, 3^-, 4^-$

$$\eta = -(-1)^L$$

To determine which is the right spin assignment we need to study the angular distributions of the decay products as a function of the various J combinations (partial waves expansion and Dalitz plot)

- From these studies we deduce that the combination must be:

$$J^P = 0^- \quad \text{or} \quad 1^+ \quad \text{but not} \quad 1^-$$

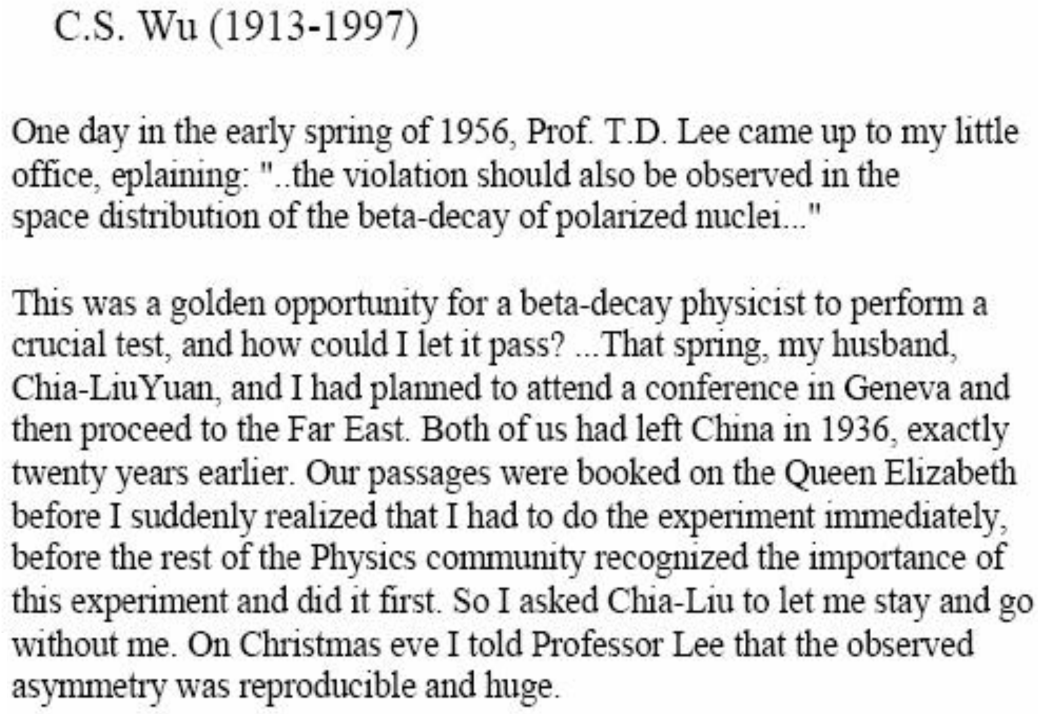
If we include also the effects of the phase space, we have:

$$\boxed{J^P = 0^-}$$

(N.B. the K has spin 0)

- Therefore the τ had negative parity while the θ had positive parity, hence the so-called τ-θ puzzle. T.D.Lee and C.N.Yang made the hypothesis that the weak interactions violate the parity conservation and they suggest a few experimental checks to verify it.

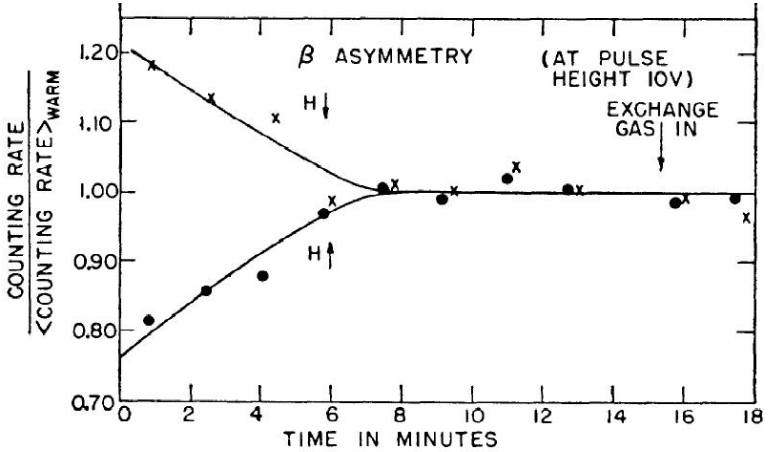
Madame Wu’s experiment



C.S. Wu (1913-1997)

One day in the early spring of 1956, Prof. T.D. Lee came up to my little office, eplaining: "..the violation should also be observed in the space distribution of the beta-decay of polarized nuclei..."

This was a golden opportunity for a beta-decay physicist to perform a crucial test, and how could I let it pass? ...That spring, my husband, Chia-LiuYuan, and I had planned to attend a conference in Geneva and then proceed to the Far East. Both of us had left China in 1936, exactly twenty years earlier. Our passages were booked on the Queen Elizabeth before I suddenly realized that I had to do the experiment immediately, before the rest of the Physics community recognized the importance of this experiment and did it first. So I asked Chia-Liu to let me stay and go without me. On Christmas eve I told Professor Lee that the observed asymmetry was reproducible and huge.



Phys. Rev., 105, 1413 (1957)

- Also this letter is a fondamental one for Physics … and to understand the sociology of the physicists and the scientists in general!

- R.L.Garwin,L.M.Lederman,M.Weinrich

  Phys. Rev., 105, 1415 (1957)
- J.I.Friedman,V.L.Telegdi

  Phys. Rev., 106, 1290 (1957)

(parity violation in the pion decay)

1/2

Weyl equation: two components neutrino theory

- In 1929, just after the publication of the Dirac equation, Weyl published a very simple and elegant theory about massless particles of spin ½ for which the helicity is a good quantum number.
- At the time of the pubblication the theory didn't have very much success since there were no known massless particles of spin ½
- However, even after the introduction of the neutrino by Pauli, Pauli himself disregarded the Weyl's theory because it violated the parity.
- Only after 1957 the Weyl's theory received the deserved credit.

- Let's start from the Dirac equation in the momentum space:

$$\left(\gamma^\mu p_\mu - m\right)\psi(p_\mu) = 0$$

if we set m=0 and we remember that $\gamma^0=\beta$ and $\gamma^i = \beta\alpha^i$, we have:

$$\left(\gamma^0 E - \vec{\gamma}\cdot\vec{p}\right)\psi(p_\mu) = 0 \quad \Rightarrow \quad \left(\beta E - \beta\vec{\alpha}\cdot\vec{p}\right)\psi(p_\mu) \Rightarrow H\psi(p) \equiv \vec{\alpha}\cdot\vec{p}\psi(p) = E\psi(p)$$

- To study the Weil equation is preferable to use the Weil representation (or chiral representation), where γ^5 is diagonal, instead of the Dirac-Pauli representation where γ^0 is diagonal.

$$\vec{\alpha} = \begin{pmatrix} -\vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \quad ; \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad \gamma_0 = \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad ; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad ; \quad \gamma_5 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

2/2

Weyl equation

$$\boxed{\vec{\alpha}\cdot\vec{p}\psi = E\psi}$$

- We can write down the 4-compenents spinor ψ as: $\psi = \begin{pmatrix}\chi \\ \varphi\end{pmatrix}$ χ and φ are 2 components spinors
- The Weyl equation can be written as:

$$\begin{pmatrix} -\vec{\sigma}\cdot\vec{p} & 0 \\ 0 & \vec{\sigma}\cdot\vec{p} \end{pmatrix}\begin{pmatrix}\chi \\ \varphi\end{pmatrix} = \mathrm{E}\cdot\begin{pmatrix}\chi \\ \varphi\end{pmatrix} \Rightarrow \begin{cases} -\vec{\sigma}\cdot\vec{p}\chi = E\chi \\ \vec{\sigma}\cdot\vec{p}\varphi = E\varphi \end{cases}$$

We have two decoupled equations!

- Since the neutrino is massless, we have $E^2=P^2$. For each equations we have two solutions, one with positive energy and another one with negative energy.
- The solutions with positive energy correspond to neutrinos while the ones with negative energy correspond to antineutrinos.

- positive energy solutions: $E = |\vec{p}| \Rightarrow \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\chi = -\chi \quad ; \quad \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\varphi = \varphi$

(lefthanded neutrino ; righthanded neutrino)

$\left(\frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\right.$ is the helicity projector$\left.\right)$

- negative energy solutions: $E = -|\vec{p}| \Rightarrow \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\chi = \chi \quad ; \quad \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}\varphi = -\varphi$

(righthanded antineutrino ; lefthanded antineutrino)

N.B. The Weyl equation violates the parity because the lefthanded neutrino and the righthanded neutrino are described by two different spinors (χ and ϕ) that are decoupled.

helicity of the neutrino

- According to the Weyl's theory, the neutrino can exist only in one state of definite helicity, either positive or negative. This is the maximum possible parity violation.

- It was necessary an experiment to prove that the neutrino has only one helicity state and to discover if it is a positive or negative helicity state. In this case the antineutrino will have a helicity opposed to the neutrino one.

- The neutrino interacts only through weak interactions and it is very difficult to detect it, and pratically impossible to measure its helicity.

- However in 1958 Goldhaber, Grodzins and Sunyar designed and realized a very ingenious experiment to measure the neutrino helicity: they will measure the helicity of a photon that has the same helicity as the neutrino one.

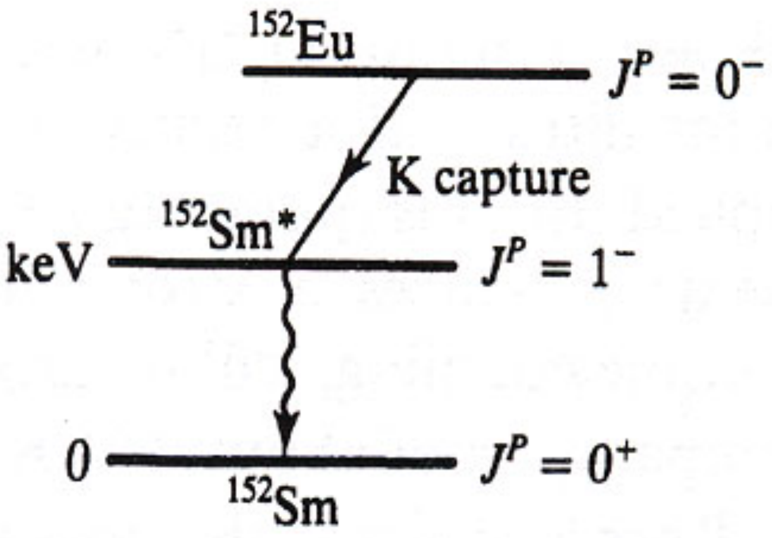
- The result of the experiment is that the neutrino is lefthanded and the antineutrino is righthanded. So the "true" wavefunction is described by the spinor χ.

N.B. Since the charge conjugation transform a neutrino in an antineutrino, but without changing the helicity, so also the charge conjugation is violated by the weak interaction.

Measurement of the neutrino helicity

- Goldhaber et al. found that the ^{152}Eu had the right features needed for the experiment: measure the photon helicity and deduce the neutrino helicity.

- A given metastabile state of the ^{152}Eu, through a K electron capture , decays in 24% of the cases in an excited state of ^{152}Sm*, which in turn decays to the ground state by emitting a photon of 961 keV.



a) Electron capture:

$$^{152}Eu + e^- \to {}^{152}Sm^* + \nu_e$$

– it is a two body final state
– neutrino energy: E_ν = 840 keV

b) Radiative decay:

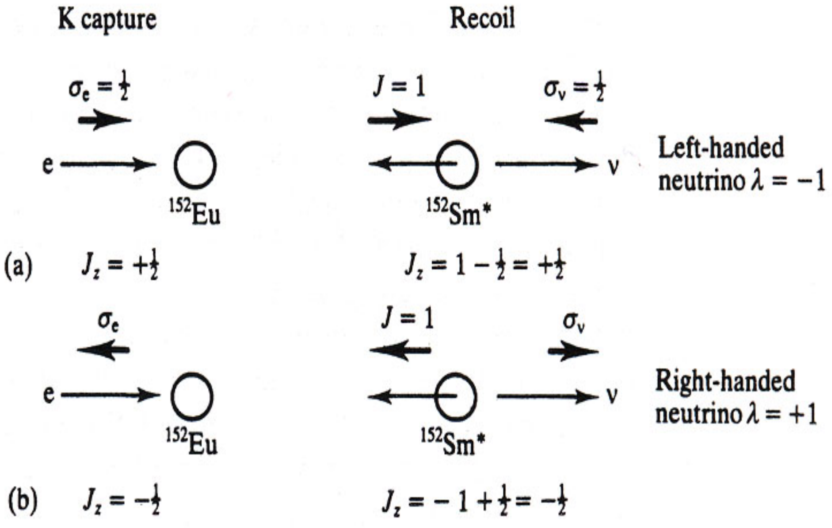
$$^{152}Sm^* \to {}^{152}Sm + \gamma$$

– The Samarium decays in flight
– photon energy: E_γ = 961 keV

To be noticed:

1) The spin of the Europium is zero;
2) The excited Samarium decays in flight;
3) The neutrino and the photon have almost the same energy.

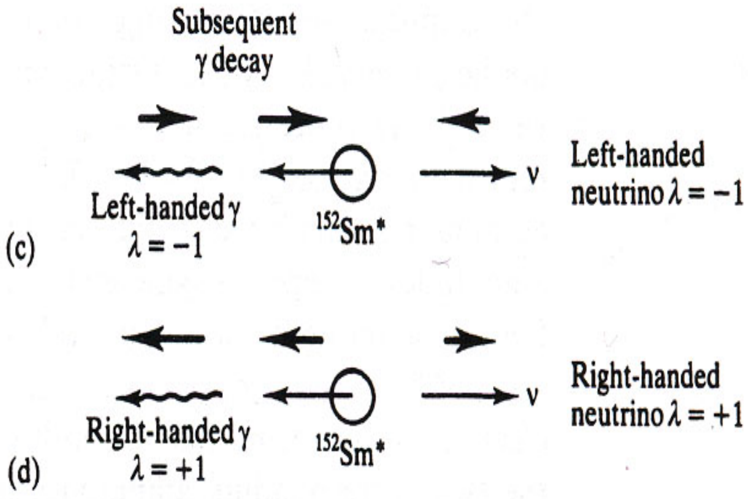
neutrino and photon helicity



$$^{152}Eu + e^- \rightarrow {}^{152}Sm^* + \nu_e$$

1) Due to the conservation of the total angular momentum, the neutrino has the spin opposed to the one of the electron captured (that we don’t know what it is).

2) The excited Samarium and the neutrino have the same helicity since it is a two body final state (The Europium and the captured electron can be considered at rest).



N.B. ONLY the photons going along the ^{152}Sm* line of flight have the same helicity of the neutrino.

$$^{152}Sm^* \rightarrow {}^{152}Sm + \gamma$$

1) The lifetime of the ^{152}Sm* is about 10^{-14} s, so it decays before coming to rest.

2) Since the Samarium has spin zero, the photon “carries” away the spin of the excited Samarium.

3) only the photons decaying along the direction of the ^{152}Sm* line of flight have the same helicity of the excited Samarium and therefore the same helicity of the neutrino.

Two experimental problems

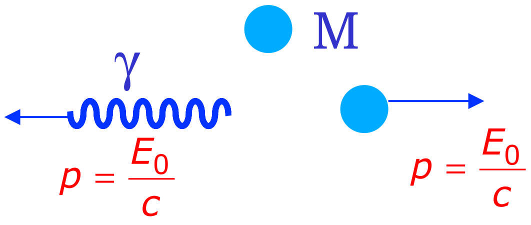
1) **How to measure the photon helicity?**

it can done by examining the transmission of the photons through magnetized iron. The dominant interaction with matter of photon of energy 961 keV is the Compton effect and the method relies on the fact that **the cross-section for Compton scattering is spin dependent**. The transmission is greatest when the photon spin is parallel to the electron spin.

2) **How to select the photons decaying along the ^{152}Sm* line of flight?**

This can be done using **the method of resonant scattering** devised by Goldhaber et al.

- In the emission of a photon from an excited state with energy of excitation E_0, a momentum E_0/c must be imparted to the emitting nucleus and consequently the energy of the photon is reduced by an amount $E_0^2/2Mc^2$ where M is the mass of the nucleus (the nucleus is not relativistic).
- Similarly, on absorption, an extra energy $E_0^2/2Mc^2$ must be supplied to counteract the nuclear recoil.
- This energy, $\Delta E=E_0^2/Mc^2$, lost by recoil in emission and absorption, is in general much greater than the level width so that resonant absorption will take place only if extra energy, equal to the energy lost, is supplied to the emitted photon.
- In this experiment it is precisely those photons emitted in the direction of recoil of the ^{152}Sm* which have the correct energy to undergo resonant absorption. The recoiling ^{152}Sm* has a velocity of E_ν/Mc^2 that by Doppler effect will increase the energy of the emitted photon. The resonant condition require that $E_\nu\cos\theta \approx E_0$, so it is important that $E_\nu \approx E_0$; thermal motion will supply the small amount of energy still missing that permit that the resonance condition can be met in practice.



$$p = \frac{E_0}{c} \qquad p = \frac{E_0}{c}$$

$$\Rightarrow K = \frac{p^2}{2m} = \frac{E_0^2}{2Mc^2}$$

$$E_\gamma = E_0 - \frac{E_0^2}{2Mc^2}$$

emission

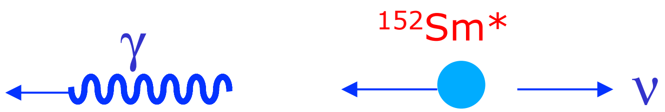
$$E_\gamma = E_0 + \frac{E_0^2}{2Mc^2}$$

absorption

$$E_\nu \cos\theta \approx E_0$$

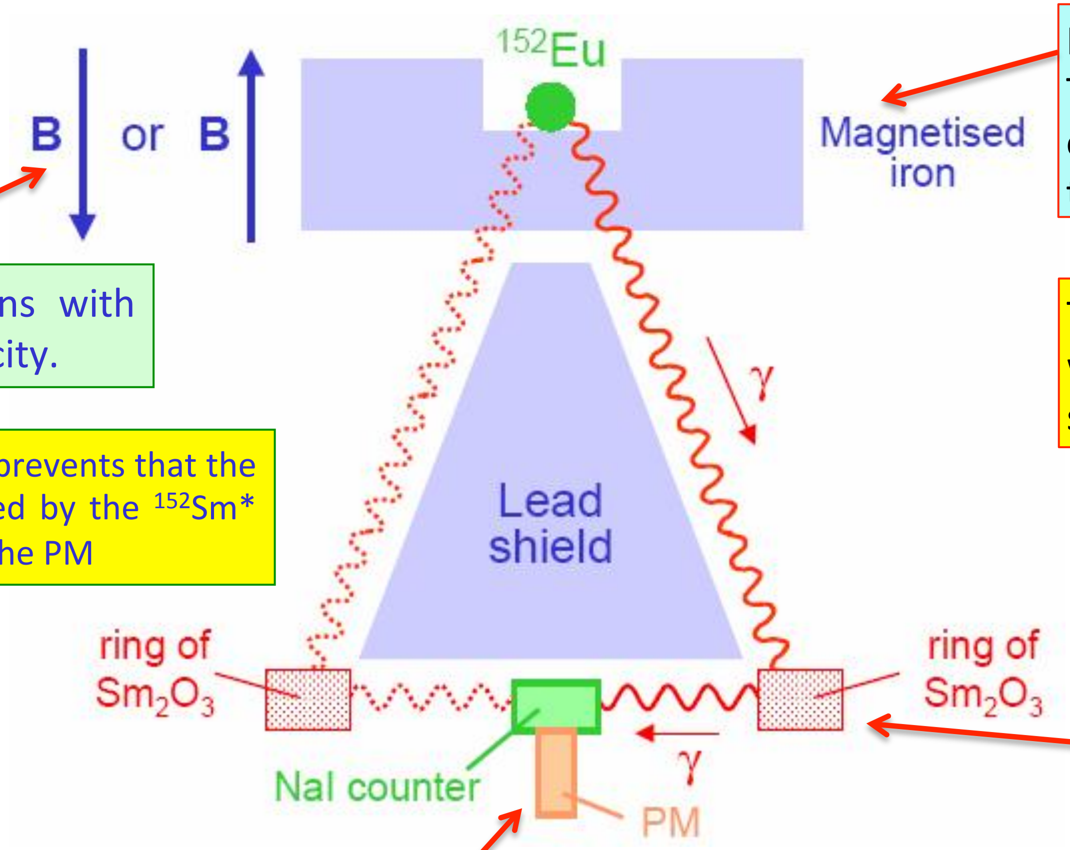
resonant condition

The Goldhaber's experiment



$$p_{Sm^*} = \frac{E_\nu}{c} \approx \frac{E_0}{c}$$

The experiment measures only the photons emitted along the $^{152}Sm^*$ line of flight



It will orient the electron spin. The spins are directed in the opposite direction with respect to the B field.

Select photons with negative helicity.

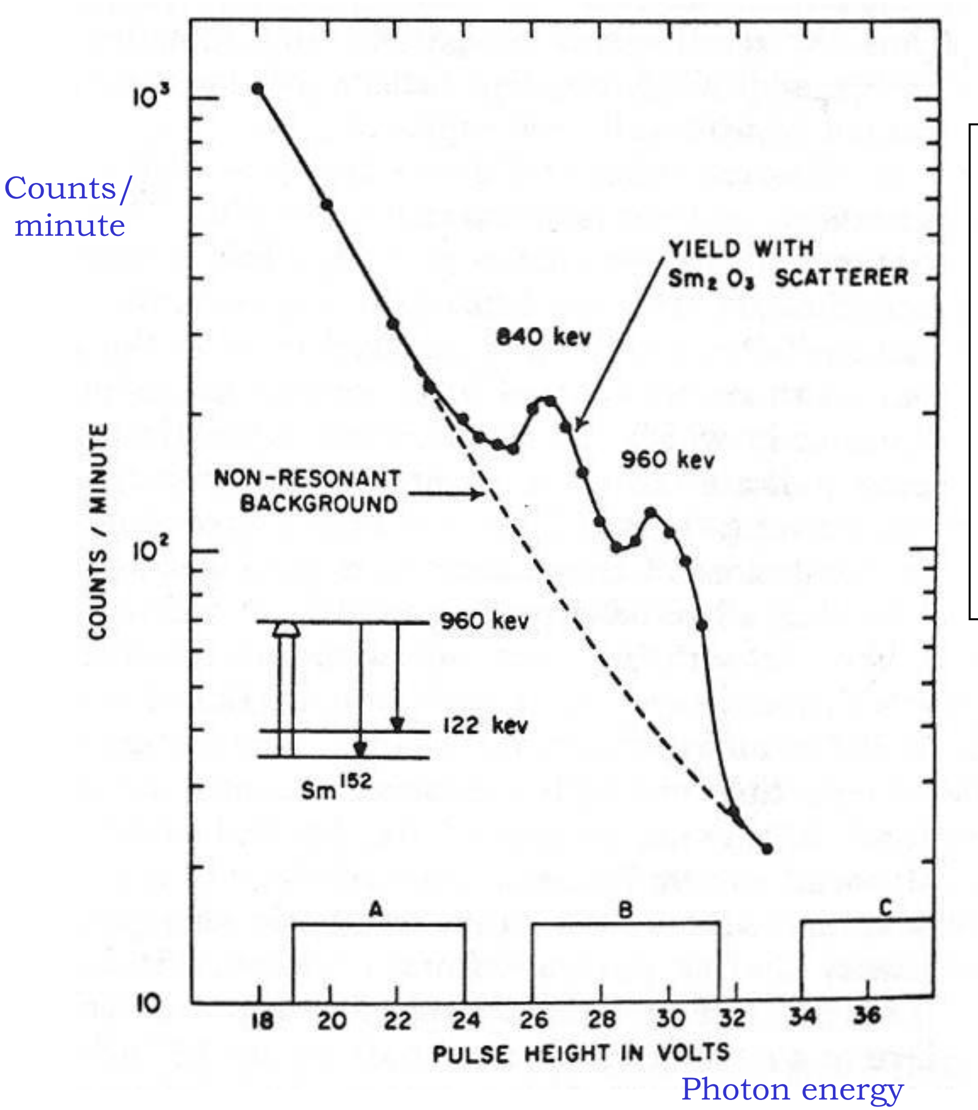
The transmission is maximal when the photon and electron spins are parallel.

Lead shield. It prevents that the photons emitted by the $^{152}Sm^*$ reach directly the PM

Oxide of Samarium.
It absorbs the photons and can re-emit only those that meet the resonant condition.

PhotoMultiplier. It detects the photons emitted by the scintillator NaI

The Goldhaber's experiment: results



- The resonant peak (actually two peaks) is obtained only for a given B-field configuration selecting lefthanded photons, therefore also the neutrinos are lefthanded.

- The helicity of the antineutrino has been measured by studying the decay of polarized neutrons and it turns out that the antineutrinos are righthanded.

The neutrino is lefthanded

The antineutrino is righthanded

V-A interaction

- Let’s summarize what we have experimentally verified so far about weak interactions:

1. In the Fermi interactions we have only the vectorial term ($O_i=\gamma^\mu$) while Gamow-Teller interactions we have only the axial term ($O_j=i\gamma^\mu\gamma^5$);
2. The neutrino has negative helicity;
3. The weak interactions violate the parity. In order to take into account also this feature we have to introduce in the Lagrangian a pseudo scalar term next to the scalar one. This is done with the substitution:

$$\boxed{C_i \to \left(C_i + C_i^{'}\gamma^5\right)\frac{1}{\sqrt{2}}}$$

The factor 1/√2 is needed to retain the original value of $G\cdot C_V$ (Fermi constant)

- The Fermi Lagrangian, with the parity violation, becomes: $L_i = \sum\limits_{i=V,A}\frac{1}{\sqrt{2}}(\bar{\psi}_p O_i \psi_n)\left[\bar{\psi}_e O_i (Ci + C_i^{'}\gamma^5)\psi_\nu\right]$
- From the helicity neutrino experiments we know that: $C_i = 1 \quad ; \quad C_i^{'} = -1$
- Let’s write down explicitily the Fermi constant:

$$\Rightarrow L_i = \frac{G}{\sqrt{2}}\left\{C_V(\bar{\psi}_p\gamma^\mu\psi_n)\left[\bar{\psi}_e\gamma_\mu(1-\gamma^5)\psi_\nu\right] + C_A(\bar{\psi}_p i\gamma^\mu\gamma^5\psi_n)\left[\bar{\psi}_e i\gamma_\mu\gamma_5(1-\gamma^5)\psi_\nu\right]\right\}$$

- By using the properties of the γ matrices we have: $\boxed{\left(\gamma^5\right)^2 = 1 \quad ; \; \gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu}$

$$\Rightarrow L_i = \frac{G}{\sqrt{2}}\left[\bar{\psi}_p\gamma^\mu(C_V + C_A\gamma^5)\psi_n\right]\left[\bar{\psi}_e\gamma_\mu(1-\gamma^5)\psi_\nu\right]$$

V-A interaction

- Let's recall that in a pure Fermi transition we measure the product G·C_V. If we compare this number with the value of G measured in a pure leptonic decay, like for instance the muon decay where we do not have the term C_V, we find that the two values are in good agreement, therefore we deduce that:

$$\boxed{C_V = 1 \qquad \Rightarrow \qquad C_A = -1.26 \pm 0.02}$$

- C_A is not equal to 1 because the strong interactions modify the hadron axial current while the vector current remains unchanged. If we take other hadronic weak decays, besides the neutron, we have:

$$\Lambda \to p + e^- + \bar{\nu}_e \Rightarrow \frac{C_A}{C_V} = -0.72 \quad ; \quad \Sigma^- \to n + e^- + \bar{\nu}_e \quad \Rightarrow \quad \frac{C_A}{C_V} = +0.34$$

- However if we ignore the strong interactions corrections on the axial current, we can put:

$$\boxed{C_A = -C_V = -1}$$

(this setting has been validated in the weak interactions between neutrinos and quarks)

- Therefore we can write again the Lagrangian in the following way:

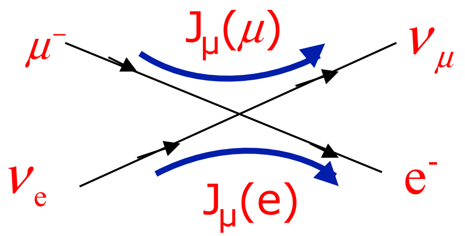
$$\boxed{L_i = \frac{G}{\sqrt{2}}\left[\bar{\psi}_p \gamma^\mu (1-\gamma^5)\psi_n\right]\left[\bar{\psi}_e \gamma_\mu (1-\gamma^5)\psi_\nu\right]}$$

- This is the so-called V-A interaction. Besides the factor (1-γ^5) is the same Lagrangian originally proposed by Fermi.
- The factor (1-γ^5) is very important because because, as we will see later, selects only one defined helicity (actually chirality) of the fermions that participate in the weak interactions.

The universal Fermi interaction

- Let's consider the muon decay:

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$



- The Lagrangian can be written as: $L_i = \frac{G}{\sqrt{2}}\left[\bar{\psi}_{\nu_\mu}\gamma^\rho(1-\gamma^5)\psi_\mu\right]\left[\bar{\psi}_e\gamma_\rho(1-\gamma^5)\psi_{\nu_e}\right]$
- It is a pure V-A interaction: the vector and axial currents have the same intensity and opposed sign.
- The muon lifetime, taking into account the phase space factor, can be written as:

$$\boxed{\frac{1}{\tau_\mu} = W = \frac{G^2 m_\mu^5}{192\pi^3}}$$

$m_\mu = 105.658369\ (9)\ \text{MeV}$

$\tau_\mu = (2.19703\ (4)) \cdot 10^{-6}\ \text{s}$

- From the muon mass and lifetime we get: $G = (1.4358\ (1)) \cdot 10^{-62}\ \text{J} \cdot \text{m}^3$
- From the pure Fermi β decay (0 → 0) we measure: $G \cdot C_V = (1.4116\ (8)) \cdot 10^{-62}\ \text{J} \cdot \text{m}^3$

  by comparing the two values we get C_V=0.98 (see Cabibbo's angle)

- The near equality of the Fermi constant obtained from an analysis of nuclear β decay, involving hadrons as well as leptons, and that derived from muon decay involving only leptons suggests a universality of the weak charge; the value of the weak charge is the same for all particles which posses it (it like the electrical charge e). The so-called universal Fermi interaction assigns a single global constant G for the coupling between any four fermion fields.

N.B. the enormous span in lifetime is a kinematical effect

The current-current hypothesis

- The neutron decay is described by the product of two currents:

$J_n^\mu = \bar{\psi}_p \gamma^\mu (1-\gamma^5)\psi_n$ (Neutron current if C_A=-1) **x** $J_e^\mu = \bar{\psi}_e \gamma^\mu (1-\gamma^5)\psi_{\nu_e}$ (Electron current)

- The muon decay is described by the product of two lepton currents, the electron and the muon ones:

$$J_\mu^\rho = \bar{\psi}_\mu \gamma^\rho (1-\gamma^5)\psi_{\nu_\mu}$$ (muon current)

- These are charged currents, because there is a change between the initial and the final particle charge present in the current.

- This description was generalized by Feynman and Gell-Mann to include all weak processes (actually only the charged current processes because at that time the neutral currents were not yet known).

- We define a leptonic weak current that is the sum of all lepton currrents:

$J_l^\mu = \bar{\psi}_e \gamma^\mu (1-\gamma^5)\psi_{\nu_e} +$ the current for the other leptons, with equal amplitude due to leptonic universality.

and one hadronic current: $J_h^\mu = \bar{\psi}_p \gamma^\mu (1-\gamma^5)\psi_n +$ similar terms for strange particles

- therefore all amplitudes of the weak processes can be written as:

$$\boxed{\mathcal{M} = \frac{G}{\sqrt{2}} J^\mu \cdot J_\mu^\dagger}$$

due to electric charge conservation, in the amplitude must appears a raising charge current and a lowering charge current.

- N.B. In the modern formalism, we prefere to define the current with the factor ½ $(1-\gamma^5)$ instead of the old $(1-\gamma^5)$, therefore:

$$\boxed{\mathcal{M} = 4\frac{G}{\sqrt{2}} J^\mu \cdot J_\mu^\dagger}$$

Reminder: Dirac equation

- Let's recall the Dirac-Pauli representation of the γ matrices, where the β matrix is diagonal:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \quad ; \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma_0 = \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad ; \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad ; \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

- original formulation of the Dirac equation:

$$Hu \equiv (\vec{\alpha}\cdot\vec{p} + \beta m)u = Eu \quad \Rightarrow \quad Hu \equiv \begin{pmatrix} m & \vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -m \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = E\begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

two components spinors

$$\Rightarrow \begin{cases} \vec{\sigma}\cdot\vec{p}u_B = (E-m)u_A & \text{E} < 0 \\ \vec{\sigma}\cdot\vec{p}u_A = (E+m)u_B & \text{E} > 0 \end{cases}$$

We make use of the spinor χ
$\chi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $\chi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- Solution with positive energy (E > 0):

$$u_A^{(s)} = \chi^{(s)} \quad \text{(s=1,2)}$$

$$\Rightarrow u_B^{(s)} = \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \quad \Rightarrow u^{(s)} = N\begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(s)} \end{pmatrix}$$

normalization factor

- Solution with negative energy (E < 0): $u_B^{(s)} = \chi^{(s)}$

$$\Rightarrow u_A^{(s)} = \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\chi^{(s)} = -\frac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi^{(s)}$$

$$\Rightarrow u^{(s+2)} = N\begin{pmatrix} \frac{-\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi^{(s)} \\ \chi^{(s)} \end{pmatrix}$$

$$u^{(3,4)}(-p) \equiv v^{(2,1)}(p)$$

- The solutions u(1,2) with positive energy describe the electrons while the u(3,4) with negative energy describe the positrons.

$$\left[\text{N.B. } \mathit{Parity}: \ \psi(\text{x}) \rightarrow \psi'(\text{-x}) = \gamma^0\psi\right]$$

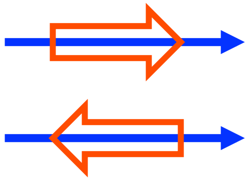
Helicity operator

- The eigenstates of the Dirac equation with a definite energy are doubly degenerate (there are two states with the same energy), therefore it must exist another observable that commutes with the Hamiltonian (that is with the momentum operator since we are dealing with a free particle) that permits to distinguish the two states.
- The following operator fulfil this requirement:

$$\vec{\Sigma}\cdot\hat{p} \equiv \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix} \quad ; \quad \vec{\Sigma} \equiv \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \text{Spin operator ;} \quad \hat{p} = \frac{\vec{p}}{|\vec{p}|} \quad \text{Momentum unit vector}$$

- $\vec{\Sigma}\cdot\hat{p}$ is the spin projection along the momentum direction; it is a good quantum number to distinguish the two solutions.
- This quantum number is called **helicity**. Its two eigenvalues are:

$$h = \begin{cases} +1 \\ -1 \end{cases}$$



- If we choose the z-axis along the momentum direction, we have $\vec{p}=(0,0,p)$, therefore:

$$\vec{\sigma}\cdot\hat{p}\chi^{(s)} = \sigma_3\chi^{(s)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\chi^{(s)} = h\chi^{(s)} \qquad \text{where } h = \pm 1$$

The spinor $\chi^{(s)}$ is eigenstate of the helicity with eigenvalue ±1 (but only with this choise of the reference system). (N.B. sometimes we introduce the factor ½ in the helicity definition.)

Relation between helicity and γ^5

$$\gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad u^{(s)} = N\begin{pmatrix}\chi^{(s)} \\ \frac{\vec\sigma\cdot\vec p}{E+m}\chi^{(s)}\end{pmatrix} \qquad u^{(s+2)} = N\begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E-m}\chi^{(s)} \\ \chi^{(s)}\end{pmatrix}$$

$$\boxed{\chi^{(1)} = \begin{pmatrix}1\\0\end{pmatrix} \; ; \; \chi^{(2)} = \begin{pmatrix}0\\1\end{pmatrix}}$$

- Let's apply the matrix γ^5 to a Dirac spinor (we ignore the normalization factor N):

$$\gamma_5 u^{(1,2)} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix}\chi \\ \frac{\vec\sigma\cdot\vec p}{E+m}\chi\end{pmatrix} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m}\chi \\ \chi\end{pmatrix} \quad ; \quad \gamma_5 u^{(3,4)} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}\begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E-m}\chi \\ \chi\end{pmatrix} = \begin{pmatrix}\chi \\ \frac{\vec\sigma\cdot\vec p}{E-m}\chi\end{pmatrix}$$

- We make use of the following property: $\frac{\vec\sigma\cdot\vec p}{E+m}\cdot\frac{\vec\sigma\cdot\vec p}{E-m} = \frac{\vec\sigma\cdot\hat p p}{E+m}\cdot\frac{\vec\sigma\cdot\hat p p}{E-m} = (\vec\sigma\cdot\hat p)^2\frac{p^2}{E^2-m^2} = 1$ $\qquad \boxed{(\sigma_i)^2 = 1}$

$$\gamma_5 u^{(1,2)} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m}\chi \\ \frac{\vec\sigma\cdot\vec p}{E+m}\frac{\vec\sigma\cdot\vec p}{E-m}\chi\end{pmatrix} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{E-m}\end{pmatrix}\begin{pmatrix}\chi \\ \frac{\vec\sigma\cdot\vec p}{E+m}\chi\end{pmatrix} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{E-m}\end{pmatrix}u^{(1,2)}$$

$$\gamma_5 u^{(3,4)} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m}\frac{\vec\sigma\cdot\vec p}{E-m}\chi \\ \frac{\vec\sigma\cdot\vec p}{E-m}\chi\end{pmatrix} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{E-m}\end{pmatrix}\begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E-m}\chi \\ \chi\end{pmatrix} = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{E-m}\end{pmatrix}u^{(3,4)}$$

$$\boxed{\gamma^5 u(p) = \begin{pmatrix}\frac{\vec\sigma\cdot\vec p}{E+m} & 0 \\ 0 & \frac{\vec\sigma\cdot\vec p}{E-m}\end{pmatrix}u(p)}$$

- If the particle has m=0 (massless particle) or E >> m, we have E=p, therefore :

$$\boxed{\gamma^5 u(p) = (\vec\Sigma\cdot\hat p)\; u(p)}$$

N.B. for a massless antiparticle: $\gamma^5 v(p) = -(\vec\Sigma\cdot\hat p)\; v(p)$

γ^5 corresponds to the helicity operator for massless particles

Helicity projector and chiral eigenstates

- We can verify that the operator ½ (1-γ^5) behaves as a helicity projector:

$$\frac{1}{2}(1-\gamma^5)u(p) = \begin{cases} 0 & \text{if u(p) has helicity } +1 \\ u(p) & \text{if u(p) has helicity } -1 \end{cases} \qquad \textbf{(for m=0)}$$

- Let's recall the form of the weak current:

$$J_l^\mu = \bar{\psi}_e \gamma^\mu \frac{1}{2}(1-\gamma^5)\psi_{\nu_e} \quad ; \quad J_l^{\mu\dagger} = \bar{\psi}_{\nu_e}\gamma^\mu \frac{1}{2}(1-\gamma^5)\psi_e$$

- Therefore in the w.i. contribute only the state with a definite helicity; in particular only lefthanded neutrinos and, as we will see, righthanded antineutrinos. In the limit of high energy (E >> m) also for the massive fermions only the lefthanded state intervenes in the weak interactions.

- We can define now the **chiral eigenstates** (from the greek word chiros, hand; they are the states that distinguish the left hand from the right hand). These states coincide with the helicity eigenstates only for massless particles. This happens because the helicity is a good quantum number only for massless particles that move at the speed of light, while for massive particles it is always possible to find another reference system where the helicity flips the sign.
- The chiral eigenstates are called lefthanded or righthanded states; they have helicity equal to ±1 only for massless particles or, with a good approximation, for particles with E >> m.

Definition:

$$u_L(p) \equiv \frac{1-\gamma^5}{2}u(p) \quad ; \quad v_L(p) \equiv \frac{1+\gamma^5}{2}v(p)$$

$$u_R(p) \equiv \frac{1+\gamma^5}{2}u(p) \quad ; \quad v_R(p) \equiv \frac{1-\gamma^5}{2}v(p)$$

antiparticles

Weak vector current

$$u_L(p) \equiv \frac{1-\gamma^5}{2}u(p) \quad ; \quad v_L(p) \equiv \frac{1+\gamma^5}{2}v(p)$$

$$u_R(p) \equiv \frac{1+\gamma^5}{2}u(p) \quad ; \quad v_R(p) \equiv \frac{1-\gamma^5}{2}v(p)$$

- Let's see the projector for the adjoint spinor. Let's recall that γ^5 is hermitian ($\gamma^5 = \gamma^{5\dagger}$) and it anticommute with the other γ matrices ($\gamma^\mu \gamma^5 = - \gamma^5 \gamma^\mu$), therefore:

$$\bar{u}_L = u_L^\dagger \gamma^0 = u^\dagger \frac{1-\gamma^5}{2}\gamma^0 = u^\dagger \gamma^0 \frac{1+\gamma^5}{2} = \bar{u}\frac{1+\gamma^5}{2}$$

$$\bar{v}_L(p) \equiv \bar{v}(p)\frac{1-\gamma^5}{2} \quad ; \quad \bar{u}_R(p) \equiv u(p)\frac{1-\gamma^5}{2} \quad ; \quad \bar{v}_R(p) \equiv \bar{v}(p)\frac{1+\gamma^5}{2}$$

- A few properties of the projector:

$$\left(\frac{1-\gamma^5}{2}\right)^2 = \frac{1}{4}\left[1-2\gamma^5+(\gamma^5)^2\right] = \frac{1}{4}\left[1-2\gamma^5+1\right] = \frac{1-\gamma^5}{2}$$

A projector applied twice gives the same result

$$\gamma^\mu \frac{1-\gamma^5}{2} = \frac{1+\gamma^5}{2}\gamma^\mu \quad \Rightarrow \gamma^\mu \frac{1-\gamma^5}{2} = \gamma^\mu \frac{1-\gamma^5}{2}\frac{1-\gamma^5}{2} = \frac{1+\gamma^5}{2}\gamma^\mu \frac{1-\gamma^5}{2}$$

- Let's recall one example of weak current (vertex W-e-ν):

$$J_\mu^- = \bar{\nu}\gamma_\mu \frac{(1-\gamma^5)}{2}e$$

(destroy an electron and create a neutrino)

$$J_\mu^- = \bar{\nu}\gamma_\mu \frac{(1-\gamma^5)}{2}e = \bar{\nu}\frac{(1+\gamma^5)}{2}\gamma_\mu \frac{(1-\gamma^5)}{2}e = \bar{\nu}_L \cdot \gamma_\mu \cdot e_L$$

We got a pure vector current between two lefthanded particles (eventually Fermi was right ☺)

Chiral simmetry

- As we have seen the (charged) weak current couples only lefthanded electrons with lefthanded neutrinos (this is the parity violation of the w.i.), while the electromagnetic current does not distinguish the **chirality** of the particles involved in the interaction.
- However also for QED we can make use of the chiral eigenstates:

$$u = \frac{1-\gamma^5}{2}u + \frac{1+\gamma^5}{2}u = u_L + u_R \quad (\text{also } \bar{u} = \bar{u}_L + \bar{u}_R)$$

$$\Rightarrow \ J_\mu^{em} = -\bar{e}\gamma_\mu e = -(\bar{e}_L + \bar{e}_R)\gamma_\mu(e_L + e_R) = \ -\bar{e}_L\gamma_\mu e_L - \bar{e}_R\gamma_\mu e_R$$

- this is true because the “crossed” terms are not present:

$$\bar{e}_L\gamma_\mu e_R = \bar{e}\frac{1+\gamma^5}{2}\gamma_\mu\frac{1+\gamma^5}{2}e = \bar{e}\gamma_\mu\frac{1-\gamma^5}{2}\frac{1+\gamma^5}{2}e = 0 \qquad \textit{because}: \left(1-\gamma^5\right)\left(1+\gamma^5\right) = 1 - \left(\gamma^5\right)^2 = 1 - 1 = 0$$

- therefore the e.m. interactions conserve the chirality of the fermions involved. This is true because it is a vector current. It can be proved that also an axial current conserve the chirality.

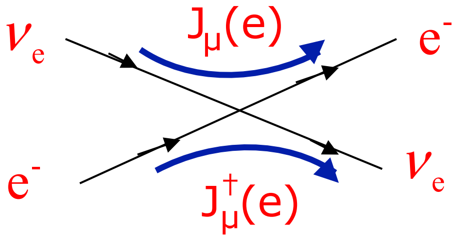
- Let’s see what happens to a scalar term, like the mass term appearing in the Dirac Lagrangian:

$$m\bar{e}e = m\bar{e}\left[\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2}\right]e = m\left[\bar{e}\left(\frac{1-\gamma^5}{2}\right)^2 e + \bar{e}\left(\frac{1+\gamma^5}{2}\right)^2 e\right] = m(\bar{e}_R e_L + \bar{e}_L e_R)$$

- The mass terms mix states with different chirality, therefore **they break the chiral symmetry**. This caused a lot of problems to the first version of the electroweak theory of Glashow, where all fermions had to be massless. The problem was solved by Weimberg e Salam by intoducing in the theory the Higgs mechanism of a spontaneous breaking of a local gauge symmetry.

Unitarity violation

- Let's look at this process $\nu_e + e^- \rightarrow \nu_e + e^-$; in the Fermi theory can be written as:



$$\mathscr{M} = \frac{4G}{\sqrt{2}}\left[\bar{u}_e\gamma^\mu \frac{1-\gamma^5}{2} u_{\nu_e}\right]\left[\bar{u}_{\nu_e}\gamma^\mu \frac{1-\gamma^5}{2} u_e\right]$$

- With this amplitude, and neglecting the electron mass, we get the following cross-section:

$$\sigma(\nu_e + e^- \rightarrow \nu_e + e^-) = \frac{G^2}{\pi} s$$

s is the center of mass energy squared

- From the formalism of the partial waves expansion we know that we have a maximum value for the elastic scattering cross-section, compatible with the conservation of unitarity:
- If we ignore the spin, the maximum cross-sections is:

$$\sigma_{el}^{max} = \frac{4\pi h^2}{p_{cm}^2}(2l+1) = \frac{4\pi h^2}{p_{cm}^2} = \frac{4\pi}{p_{cm}^2} \quad (h=1)$$

Scattering in S wave for pointlike particles

at high energy ν and e are lefthanded → J=0 (S wave)

- Therefore: $\boxed{\frac{G^2}{\pi} s \leq \frac{4\pi}{p_{cm}^2}}$; let's recall that $s = (p_\nu + p_e)^2$

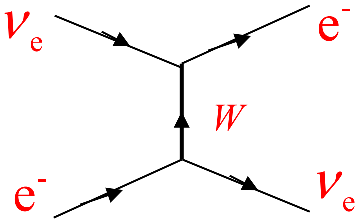
- In the lab. s=2m$_e$E$_0$, while in the center of mass frame: $s = (2p_{cm})^2 \Rightarrow p_{cm}^2 = \frac{s}{4}$

$$\Rightarrow \frac{G^2}{\pi} s \leq \frac{16\pi}{s} \Rightarrow \sqrt{s} \leq 2\sqrt{\frac{\pi}{G}} = 2\sqrt{\frac{\pi}{1.67 \cdot 10^{-5}}} \approx 870 \text{ GeV}$$

including the spin, the Fermi cross-section violates the unitarity at √s≈ √G ≈300 GeV

Intermediate Vector Boson

- The divergent behaviour of the cross-section can be avoided if, in analogy with QED, we introduce an intermediate vector boson as a propagator of the weak interactions.
- The diagram of the scattering process becomes:



The propagator of a massive boson of spin 1 is: →

$$\frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}}{M_W^2 - q^2}$$

- The matrix element can be written as: $\mathcal{M} = \left[\frac{g}{\sqrt{2}}\bar{u}_e\gamma^\mu\frac{1-\gamma^5}{2}u_{\nu_e}\right]\frac{1}{M_W^2 - q^2}\left[\frac{g}{\sqrt{2}}\bar{u}_{\nu_e}\gamma_\mu\frac{1-\gamma^5}{2}u_e\right]$

- g is a dimensionless coupling constant;
- the factors √2 and ½ are introduced to get the conventional definition of g;
- Since the range of the weak interactions is extremely short (of the order 10^{-3} fm), then the mass of the intermediate vector boson must be very big;
- for weak processes where q^2 transferred is small, like the β decays or the muon decay, we have $q^2 << M_w^2$, therefore we can neglect q^2 with respect the W mass in the propagator expression.

- If we compare the Fermi matrix element: $\mathcal{M} = \frac{G}{\sqrt{2}}\left[\bar{u}_p\gamma^\mu(1-\gamma^5)u_n\right]\left[\bar{u}_e\gamma_\mu(1-\gamma^5)u_\nu\right]$

with the one with the W boson, we get:

$$\boxed{\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}}$$

W boson mass

- From the previous relation we get the W boson mass: $\boxed{\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}} \Longrightarrow M_W = \sqrt{\frac{g^2\sqrt{2}}{8G}}$
- if we do the hypothesis that g ≈ e, we have: $\frac{e^2}{4\pi} = \alpha = \frac{1}{137} \Rightarrow g^2 \approx e^2 = \frac{4\pi}{137}$; $G = \frac{10^{-5}}{M_p^2}$
- Putting all together we get: $M_W = \sqrt{\frac{\frac{4\pi}{137}\sqrt{2}}{8 \cdot 10^{-5}}} \cdot M_p \approx 37.4 \text{ GeV}$
- actually: $e = g\sin(\theta_W) \Rightarrow \text{Mw} \approx \frac{37.4}{\sin(\theta_W)} = 80.425 \pm 0.038 \text{ GeV}$

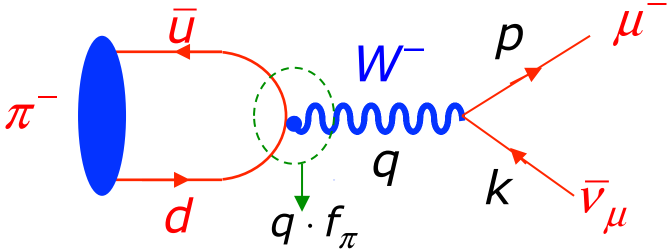
θ_W is the weak angle, known as Weinberg angle

The weak interactions are "weak" not because of the "weakness" of the coupling constant but because of the high value of the W boson mass.

- Since g ≈ e it is not necessary to introduce a new charge to understand the weak interactions.
- We have a new mass scale: the Fermi scale, equal to the W boson mass ≈ 100 GeV
- Something similar happens in the electromagnetism: $F = e\vec{E} + e_m \vec{v} x \vec{B}$ $(e = e_m \Rightarrow \text{unification})$
- The magnetic effects become relevant when v is big and they become comparable with the electric ones.
- Whenever there is a unification of two distinct phenomena usually it appears a new scale; in the electromagnetism it is the speed of light.

It is the scale that determines the relative strength of two forces.

Charged pion decay



$$\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$$

- The amplitude has the form: $\mathcal{M} = \frac{G}{\sqrt{2}}(\cdots)^\mu \left(\bar{u}(p)\gamma_\mu(1-\gamma^5)v(k)\right)$; $(\cdots)^\mu$: quark weak current
- If the quarks were free particle, we would have: $(\cdots)^\mu = \left(\bar{u}_d\gamma^\mu(1-\gamma^5)v_{\bar{u}}\right)$

  this is not correct because the quarks ū and d are not free but they are bound within the π^- meson.
- However:
  1. $\mathcal{M}$ is a Lorentz invariant, therefore $(\cdots)^\mu$ must be a vector or an axial vector;
  2. the π^- has spin zero, therefore the quadrimomentum q^μ is the only quadrimomentum available to build $(\cdots)^\mu$

$$\Rightarrow \quad (\cdots)^\mu = q^\mu \cdot f(q^2) = q^\mu \cdot f_\pi \quad ; \quad f_\pi \text{ is a constant}$$

[f is function only of q^2 because there is no other scalar that can be constructed $\Rightarrow$ $q^2 = m_\pi^2$ $\Rightarrow$ $f(m_\pi^2) \equiv f_\pi$]

$$\Rightarrow \mathcal{M} = \frac{G}{\sqrt{2}}\left(p^\mu + k^\mu\right) f_\pi \left(\bar{u}(p)\gamma_\mu(1-\gamma^5)v(k)\right) \qquad \left(q^\mu = p^\mu + k^\mu\right)$$

- memento: eq. di Dirac $\left(\gamma^\mu p_\mu - m\right)u = 0$; $\left(\gamma^\mu p_\mu + m\right)v = 0$ $\Rightarrow$ $\bar{u}(p)\gamma^\mu p_\mu = m_\mu \bar{u}(p)$; $\gamma^\mu k_\mu v(k) = 0$

$$\boxed{\Rightarrow \mathcal{M} = \frac{G}{\sqrt{2}} f_\pi \cdot m_\mu \cdot \left(\bar{u}(p)(1-\gamma^5)v(k)\right)}$$

Charged pion decay

$$\boxed{\Rightarrow \mathcal{M} = \frac{G}{\sqrt{2}} f_\pi \cdot m_\mu \cdot \left(\bar{u}(p)(1-\gamma^5)v(k)\right)}$$

- In the pion center of mass reference frame, the transition probability per unit of time is equal to:

$$d\Gamma = \frac{1}{2m_\pi}\overline{\mathcal{M}^2}\frac{d^3p}{(2\pi)^3 2E}\frac{d^3k}{(2\pi)^3 2\omega}(2\pi)^4\delta(q-p-k)$$

Quadrimomentum conservation

Sum on the final spin and average on the initial spin

Muon phase space

Neutrino phase space

- The pion has spin zero, therefore no average on the initial spin; the sum on the muon and neutrino spin is done with the "traceology" mechanism of the γ matrix.

$$\overline{\mathcal{M}^2} = \frac{G^2}{2} f_\pi^2 m_\mu^2 \cdot Tr\left[(\not{p}+m_\mu)(1-\gamma^5)\not{k}(1+\gamma^5)\right] = 4G^2 f_\pi^2 m_\mu^2 (p\cdot k)$$

- In the pion center of mass we have $\vec{k} = -\vec{p}$, therefore: $p\cdot k = E\omega - \vec{k}\cdot\vec{p} = E\omega + k^2 = \omega(E+\omega)$

- putting all together we have: $\Gamma = \frac{G^2 f_\pi^2 m_\mu^2}{(2\pi)^2 2m_\pi}\int \frac{d^3p\,d^3k}{E\omega}\omega(E+\omega)\delta(m_\pi - E - \omega)\delta^{(3)}(\vec{k}+\vec{p})$

- The integration in d^3p is taken into account by the $\delta^{(3)}$, and since there is no angular dependency, we are only left with the integration in dω:

- The final result is: $\boxed{\Gamma = \frac{1}{\tau} = \frac{G^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2}$

N.B. actually we have computed the partial width of the pion decay in the muon-neutrino channel, but since this is the dominant channel, it is almost equal to the total width, then to the inverse of the lifetime.

$$\boxed{\Gamma_{tot} = \sum \Gamma_{parz.} \Rightarrow \tau = \frac{1}{\Gamma_{tot}}}$$

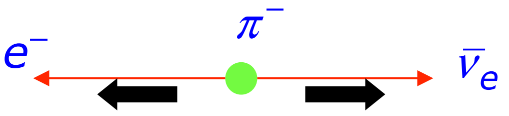
Charged pion decay

$$\Gamma = \frac{1}{\tau} = \frac{G^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- If we take the value of G measured in the β decay or in the muon decay and we assume that $f_\pi = m_\pi$ (at least for dimensional reasons) we find the pion lifetime: $2.6 \cdot 10^{-8}$ s
- This is not a real test of the theory since the assumption $f_\pi = m_\pi$ is not justified. However we can do a quantitative test by comparing the B.R. of the decay in the muon channel with the one in electron: : $p^- \rightarrow e^- \nu_e$. The computation is identical, except that we have to replace the muon mass with the electron mass:

$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.2x10^{-4}$$

- The numerical value obtained inserting the masses in the formula coincides with the one obtained with the measured B.R.
- The pion prefers to decay into muons rather than in electrons. This is not what one would expect based on phase space considerations, since the one of the electron channel is much bigger that the one of the muon. On the other hand the coupling constant is the same for electron and muon (universality of weak interactions). The explanation depends on the helicity of the two particles.
- The pion has spin zero; in the decay we must conserv the total angular momentum, therefore:

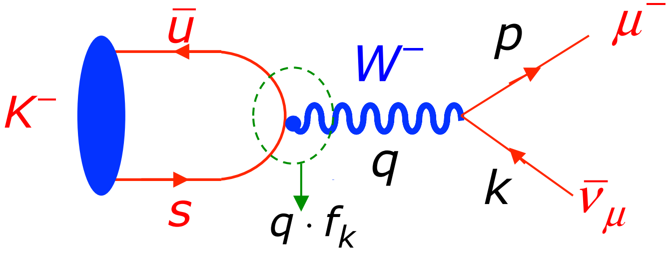


The electron is forced to be righthanded →

← The antineutrino must be righthanded

- This is the state of "wrong" helicity of the electron, because in the limit of zero mass it would have been lefthanded (and the decay would not be possible).
- Since the muon has a mass bigger than the electron one, it is easier that it goes in the state of "wrong" helicity.

Decay of the charged K±



$K^- \to \mu^- + \bar{\nu}_\mu$

The B.R. is 64%

It is similar to the pion decay, with a quark s replacing a quark d

$$\mathcal{M} = \frac{G}{\sqrt{2}}\left(p^\mu + k^\mu\right) f_k \left(\bar{u}(p)\gamma_\mu(1-\gamma^5)v(k)\right) = \frac{G}{\sqrt{2}} f_k m_\mu \bar{u}(p)(1-\gamma^5)v(k)$$

$$\boxed{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu) = \frac{G^2}{8\pi} f_k^2 m_k m_\mu^2 \left(1 - \frac{m_\mu^2}{m_k^2}\right)^2}$$

We replace the pion mass with the K mass, and the constant f_π with f_k

- Since π and K belong to the same SU(3) octet, if this would be an exact simmetry → $f_\pi = f_k$
- Since the simmetry is broken (but not too much), they are diffent, but not so different: f_π=130 MeV, f_k=160 MeV.
- If we do the ratio of the transition probability K/π , assuming $f_\pi = f_k$, we have:

$$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = \frac{m_k}{m_\pi} \left(\frac{1-\left(\frac{m_\mu}{m_k}\right)^2}{1-\left(\frac{m_\mu}{m_\pi}\right)^2} \right)^2 = 17.67$$

- while the experimental value is: $\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = 1.336 \pm 0.004$

- The discrepancy can not be explained by the difference between f_π e f_k due to the broken SU(3) simmetry.

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Decay of the charged K±

- The discrepancy can be explained by a different coupling constant for the hadron current that changes strangeness.

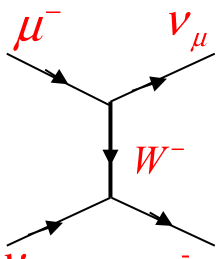
- Let’s call G_s the Fermi coupling constant with the quark s and G_d the Fermi coupling with the quark d.

$$1.336 \pm 0.004 = \frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} = (17.67) \cdot \frac{G_s^2}{G_d^2}\left(\frac{160}{130}\right)^2 \leftarrow \frac{f_K}{f_\pi} \Rightarrow \boxed{\frac{G_s}{G_d} = 0.223}$$

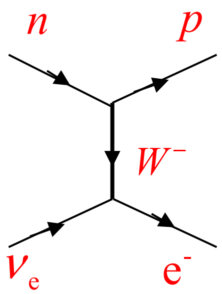
- **This implies a breaking of the weak interactions universality.**

- An explanation of this phenomenon that preserves the weak interactions universality was given by Cabibbo in 1963.

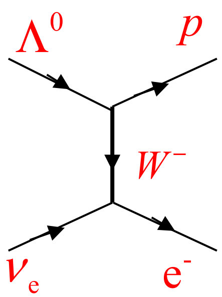
- Besides the chargd K decays, we can also take into account other weak decays:



muon decay



neutron decay



$$\begin{pmatrix} \Delta S = 1 \\ \Delta I = \frac{1}{2} \end{pmatrix}$$

Λ^0 decay

- In the three cases we are dealing with weak charged currents. In the first two cases does not change the strangeness (ΔS=0) while in the third case there is a change in the strangeness (ΔS=1).

- To be noticed that the hadronic current with ΔS=0 is slightly “smaller” than the leptonic current (C_V=0.98) while it is about 5 times higher of the hadronic current with a strangeness change.

Parenthesis: doublet structure of the w.i.

- In 1962 Schwartz, Lederman e Steimberger found that in the interaction of a neutrino beam, obtained from the pion and K decays, the result was:

$$\nu_\mu + N \to \mu^- + N \quad \text{but never} \quad \nu_\mu + N \to e^- + N$$

- This experiment proved the existence of a second type of nutrino, associated with the muon decay, different from the one present in the β decay
- We introduce the conservation of the lepton number separetely for each lepton. In this way we explain the absence of the muon decay in electron plus photon.
- Leptons are organized in a doublet structure:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$$

+ the antiparticles

To be fair, we should put in the doublets only the lefthanded state of the fermions.

- The charged weak interactions (that is with a W exchange) make the transition between the two components of a doublet but never from a doublet to the other one (lepton number conservation).

  N.B. since the neutrinos have a mass, this is no longer true (flavour oscillations) but the probability is so small that as a matter of fact it never happens.

- **As far as the quarks are concerned, the picture was less clear, because we did observe transitions of the quark d toward the quark u, as well as transition of the quark s toward the quark u. Therefore it was not evident what is the doublet involved in the transition.**

To be noticed that we do not observe transitions between the quark d and the quark s (flavour changing neutral current – FCNC) . This was explained by Glashow-Iliopoulos e Maiani (GIM) nel 1970.

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The Cabibbo angle

- **The solution to recover the weak interaction universality was found by Cabibbo in 1963. He proposed that the mass eigenstates, that are also eigenstates of the strong interactions, are NOT eigenstates of the weak interactions.**

- **The original publication of Cabibbo was based on the current-current interaction model of the weak interactions, but in what follows we present a "modern" version of the theory based on quarks, that it is easier to understand (in 1963 the quarks were not yet "invented").**

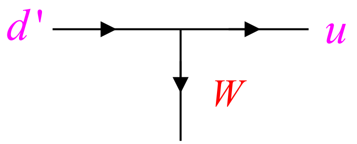
- We recall that experimentally we observe particles with a definite mass and lifetime, in other words we observe only the mass eigenstates.

- To go from the mass eingenstate base to the weak interaction base we need a unitary transformation that preserves the vector normalization. In a two dimensional space it is sufficient a "rotation" matrix characterized only by one parameter, that is one angle: **the Cabibbo angle θ_C**

- The eigenstate of the weak interactions is a linear combination of the mass eigenstates:

$$\boxed{d' = d\cos\theta_c + s\sin\theta_c}$$

- we can then construct a weak isospin doublet (that has the same algebra of the strong isospin but it is a completely different concept):



$$\boxed{\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}}$$

The W couples the state d' with the quark u

Cabibbo hadronic current

- The structure of the Cabibbo hadronic current that “raises the charge” (it makes the transition from the lower component of the doublet to the top one) is of this type (we do not write the factor ½(1-γ^5)):

$$\boxed{J^+_\mu(quarks) \approx g\left(\bar u, \bar d\cos\theta_c + \bar s\sin\theta_c\right)\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}u\\ d\cos\theta_c + s\sin\theta_c\end{pmatrix} = g(\bar u d\cos\theta_c + \bar u s\sin\theta_c)}$$

- The matrix $\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix} = \frac{1}{2}(\tau_1 + i\tau_2) = \frac{1}{2}\tau_+$ is the charge raising operator for a weak isospin doublet

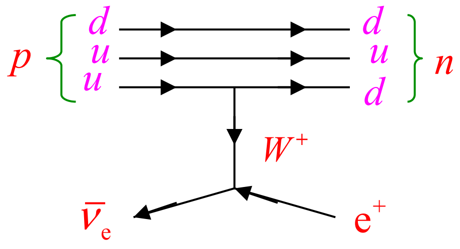
- In a compact way we can write the raising current as follows:

$$J^+_\mu(q) \approx g\bar q_L \tau_+ q_L \quad \text{, where } q_L = \begin{pmatrix}u\\ d'\end{pmatrix}$$

We consider only the quark lefthanded components

- In a similar way we can write the “lowering” current J^- (with a W^+ exchange) :

$$\boxed{J^-_\mu(quarks) \approx g\left(\bar u, \bar d\cos\theta_c + \bar s\sin\theta_c\right)\begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\begin{pmatrix}u\\ d\cos\theta_c + s\sin\theta_c\end{pmatrix} = g(\bar d u\cos\theta_c + \bar s u\sin\theta_c)}$$



- in a compact way: $J^-_\mu(q) \approx g\bar q_L \tau\_q_L$ $\left(\frac{1}{2}\tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\right)$

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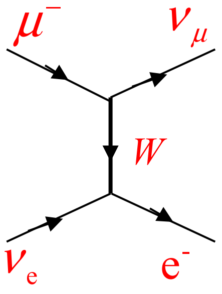
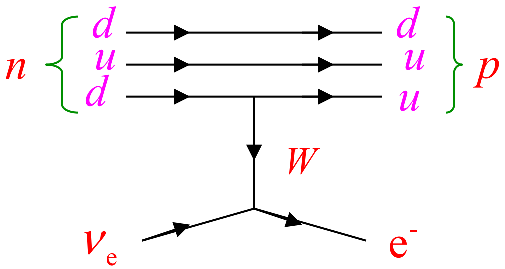
Quarks β decays

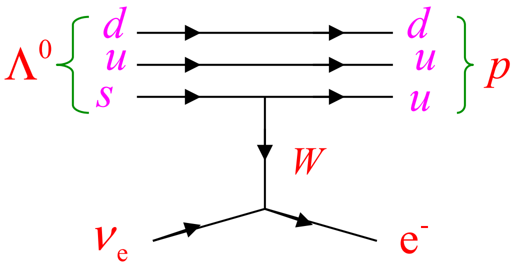
- Let's examine the hadronic β decays, taking into account that hadrons are not elementary particles; at a more fundamental level the β decay involves the quarks that constitute the hadrons:



muon decay



neutron decay



Λ⁰ decay

- Let's write the lepton current and the hadron current, then we compute the matrix element and the decay rate for the three processes:

$$J_{lepton} = \frac{g}{\sqrt{2}} \bar{u}_{\nu_e} \gamma_\mu \frac{1-\gamma^5}{2} u_e$$

$$J_{hadron} = \left[\frac{g}{\sqrt{2}} \bar{u}_u \gamma^\mu \frac{1-\gamma^5}{2} u_d \right] \cos\theta_C + \left[\frac{g}{\sqrt{2}} \bar{u}_u \gamma^\mu \frac{1-\gamma^5}{2} u_s \right] \sin\theta_C$$

$$\mathcal{M}_{\mu \to e \bar{\nu}_e \nu_\mu} = J_{lepton} \frac{1}{M_W^2 - q^2} J_{lepton}$$

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \propto g^4$$

$$\mathcal{M}_{n \to p e \bar{\nu}_e} = J_{d \to u} \frac{1}{M_W^2 - q^2} J_{lepton}$$

$$\Gamma(n \to p e^- \bar{\nu}_e) \propto g^4 \cos^2\theta_c$$

$$\mathcal{M}_{\Lambda^0 \to p e \bar{\nu}_e} = J_{s \to u} \frac{1}{M_W^2 - q^2} J_{lepton}$$

$$\Gamma(\Lambda^0 \to p e^- \bar{\nu}_e) \propto g^4 \sin^2 \theta_c$$

In every vertex the W must conserve the electric charge.
The W only couples to lefthanded fermions and to righthanded antifermions.

weak interaction universality: g is the same coupling constant everywhere.

The Cabibbo angle: the value

- In the Cabibbo's theory all particles (quarks and leptons) carry a weak charge g, but the quarks are mixed:

$J^+_\mu(q) \propto g\cos\theta_c$ for the current where $\Delta S=0$; $J^+_\mu(q) \propto g\sin\theta_c$ for the current where $\Delta S=1$

- therefore:

$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) \propto g^4$ pure lepton current

$\Gamma(n \to pe^-\bar{\nu}_e) \propto g^4\cos^2\theta_c$ $\Delta S=0$ semi-leptonic

$\Gamma(\Lambda^0 \to pe^-\bar{\nu}_e) \propto g^4\sin^2\theta_c$ $\Delta S=1$ semi-leptonic

⇒ $$\frac{\Gamma(\Lambda^0 \to pe^-\bar{\nu}_e)}{\Gamma(n \to pe^-\bar{\nu}_e)} = \tan^2\theta_c$$

Data are consistent with a Cabibbo angle of **$\theta_c \approx 13°$**

$\cos^2\theta_c = 0.949$; $\sin^2\theta_c = 0.050$; $\text{tag}^2\theta_c = 0.053$

- Processes proportional to $\cos^2\theta_c$ are called "Cabibbo favored" while the ones proportional to $\sin^2\theta_c$ are called "Cabibbo suppressed"
- To be noticed that cos 13°=0.974 and indeed experimentally it has been found $C_V \approx 0.98$.
- If we go back to the comparison between the K decay and the pion decay, we have:

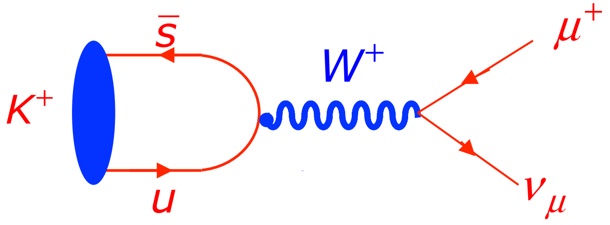
$$\frac{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} \Rightarrow \frac{G_s}{G_d} = 0.223 = \text{tag}\,\theta_c \Rightarrow \theta_c = 12.57°$$

that it is consistent with the value obtained from the comparison between the neutron and Λ^0 decays.
Only one Cabibbo angle is "sufficient" for all weak processes.

Absence of FCNC

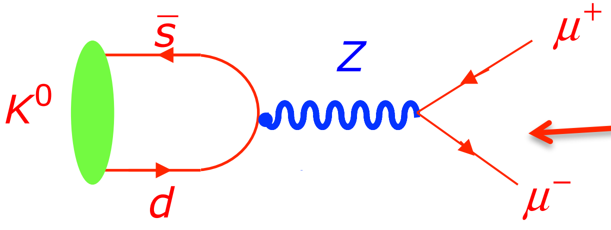
- Experimentally we observe that do not exist neutral weak currents that change the flavour of the quarks.
  This statement can be explained, for instance, by looking at two K decays, one about the charged K and the other one about the neutral K.

$$K^+ \to \mu^+ + \nu_\mu$$



$$\text{B.R.} = (63.51 \pm 0.19)\%$$

$$K^0 \to \mu^+ + \mu^-$$



$$\text{B.R.} = (7.27 \pm 0.14) \cdot 10^{-9}$$

N.B. actually this graph does not exist. We have an empirical rule that says that, at the first order, **ΔS=ΔQ**.
In this case we have ΔS=-1; ΔQ=0

- The W$^\pm$ is a charged boson (negative and positive) that acts as a mediator in the weak interactions due to charged currents.
- For other reasons due to unitarity violation of the weak interactions, in this case in the process of W pair production ($u\bar{u} \to W^+W^-$), we need to introduce another intermediate vector boson, neutral, called Z, that is the mediator of weak interactions with neutral currents.

(The Z boson will be a by product of the unification of weak interactions and electromagnetic interactions, as we will see later when we will study the Standard Model. In any case let's state since now that the Z coupling is different from the W coupling).

- **Based on what we said we should expect that the second process should exist with a rate comparable with the first one, but as a matter of fact it is not so. It is highly suppressed: why?**

Neutral weak currents

- Let's recall the raising and lowering charged currents (we omit the factors $\gamma^\mu(1\text{-}\gamma^5)$ and the coupling constant):

$$\boxed{J^+_\mu(q) \approx g\left(\bar{u}, \bar{d}\cos\theta_c + \bar{s}\sin\theta_c\right)\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} = g(\bar{u}d\cos\theta_c + \bar{u}s\sin\theta_c)}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \frac{1}{2}(\tau_1 + i\tau_2) = \frac{1}{2}\tau_+$$

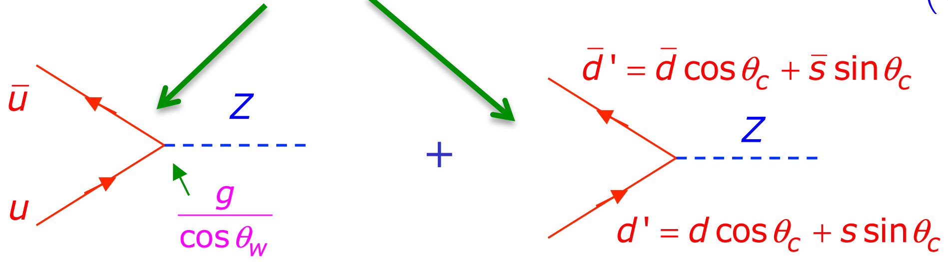
$$\boxed{J^-_\mu(q) \approx g\left(\bar{u}, \bar{d}\cos\theta_c + \bar{s}\sin\theta_c\right)\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} = g(\bar{d}u\cos\theta_c + \bar{s}u\sin\theta_c)}$$

$$\left(\frac{1}{2}\tau_- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right)$$

- Formally it should exist also the third component of the currents, due to a Z exchange:

$$J^0_\mu(q) \approx g\bar{q}\tau_3 q \approx \bar{u}u - \bar{d}'d' \quad \text{where} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad q = \begin{pmatrix} u \\ d' \end{pmatrix}$$

$$\boxed{d' = d\cos\theta_c + s\sin\theta_c}$$



(N.B. the vertex is more complicated than the W one; θ_W = Weinberg angle)

- If we write the current in terms of mass eigenstates we get:

$$\Rightarrow J^0_\mu(q) \approx \underbrace{\bar{u}u - \bar{d}d\cos^2\theta_c - \bar{s}s\sin^2\theta_c}_{\Delta S\, =\, 0} - \underbrace{(s\bar{d} + \bar{s}d)\sin\theta_c\cos\theta_c}_{\Delta S\, =\, 1}$$

The last term is responsible of the flavour changing neutral current processes, with amplitude proportional to $\sin\theta_c\cos\theta_c$. **This term is highly suppressed in Nature.**

3/5

The GIM effect (the quark charm)

- The explanation of the suppression of the flavour changing neutral current was proposed by Glashow, Iliopoulos and Maiani (GIM) in 1970.
- They introduced a new quark, the charm, that has the same charge of the quark u. Then they proposed a second doublet of quark weak isospin:

$$\boxed{\begin{pmatrix} c \\ s' \end{pmatrix}}$$

The W connects s' with c

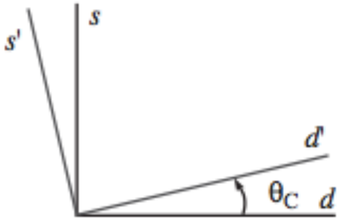
- It was fully restored the symmetry with the lepton doublets known in 1970:

$$\boxed{\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}}$$

The charge difference between the upper and the lower components is +1.

- The weak eigenstate s' can be found using the Cabibbo theory that connects the mass eigenstates to the weak eigenstates by a unitary matrix

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

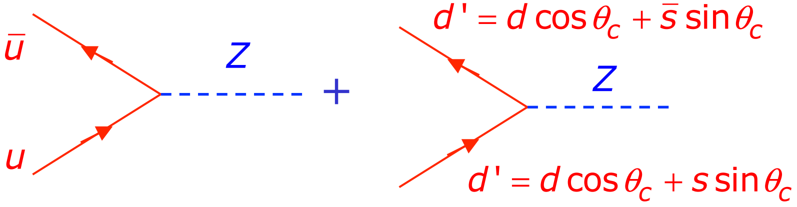


$$\boxed{s' = s\cos\theta_c - d\sin\theta_c}$$

By convention we rotate the down-type quarks and we leave unchanged the up-type quarks. It would be absolutey equivalent the other choice, namely to rotate the up-type quarks.

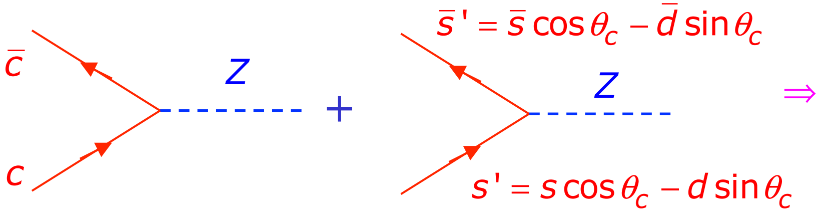
Z couplings

- Let's see how the introduction of c and s' will solve our problems. We saw the coupling of the Z with the u and d':



$$J^0_\mu(q) \approx \underbrace{\bar{u}u - \bar{d}d\cos^2\theta_c - \bar{s}s\sin^2\theta_c}_{\Delta S = 0} - \underbrace{(s\bar{d}+\bar{s}d)\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

- Now we need similar graphs for the coupling of the Z with the quarks c and s':



$$\Rightarrow J^0_\mu(q) \approx \underbrace{\bar{c}c - \bar{d}d\sin^2\theta_c - \bar{s}s\cos^2\theta_c}_{\Delta S = 0} + \underbrace{(s\bar{d}+\bar{s}d)\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

- If we sum up the amplitudes of the four graphs we get:

$$J^0_\mu(q) \approx \bar{u}u - \bar{d}'d' + \bar{c}c - \bar{s}'s' = \underbrace{\bar{u}u + \bar{c}c - (\bar{d}d + \bar{s}s)\cos^2\theta_c - (\bar{d}d + \bar{s}s)\sin^2\theta_c}_{\Delta S = 0} + \underbrace{(s\bar{d}+\bar{s}d-s\bar{d}-\bar{s}d)\sin\theta_c\cos\theta_c}_{\Delta S = 1}$$

$$\boxed{J^0_\mu(q) \approx \bar{u}u - \bar{d}d - \bar{s}s + \bar{c}c}$$

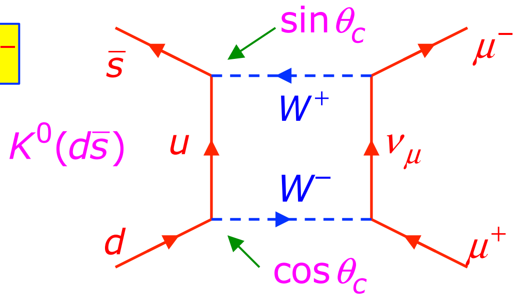
The Z couples to the mass eigenstates.
No need of the Cabibbo angle.

With the introduction of the quark c are disappeared the neutral current with a flavour changing. As we saw the Z couples only to quark-antiquark pairs of the same flavour.

FCNC: the true graphs

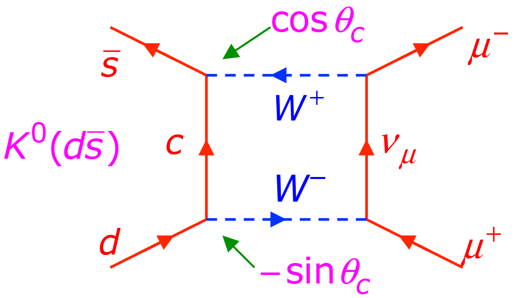
- Experimentally we have seen that the FCNC are highly suppressed but nevertheless they do exist. They can not be due to a Z exchange but they are due to a second order W exchanges (box diagrams):

$K^0 \to \mu^+\mu^-$



- The amplitude is proportional to $\sin\theta_C\cos\theta_C$
- The quark u intervenes as a virtual particle in the box

- With the “invention” of the quark charm we have another graph:



- The amplitude is proportional to $-\sin\theta_C\cos\theta_C$
- If the mass of the quark c was equal to the one of quark up, there would be an exact cancellation between the two graphs and the B.R. of this decay would have been zero.

From the measured B.R. of this decay, G.I.and M. predicted than the mass of the charm should lay in the range 1−3 GeV. **The hunt for the quark charm was officially launched.**

The search for highly suppressed FCNC decays, like for instance $B_s^0 \to \mu^+\mu^-$ is still a powerful tool in the search for new physics, because we could have new particles in the loop that will enhance the Standard Model Branching Ratio.

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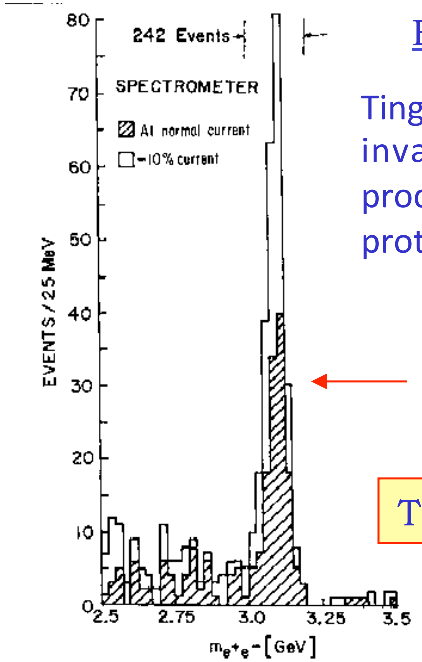
J/Ψ discovery

- In November 1974 there was the "November revolution" with the discovery of a resonance very peculiar, because its lifetime was about three order of magnitude bigger than what one would expect.
- The discovery was done in an independent manner by the Ting's group at Brookhaven and by the Richter's one at Slac, and just afterward also at the Adone e^+e^- collider in the Frascati INFN Laboratory.



Brookhaven

Ting was looking for a peak in the invariant mass of the e^+e^- pairs produced in the collision of 28 GeV protons on a berillium target.

$$p + Be \rightarrow J + anything$$

He found a very narrow peak at ≈3.1 GeV

Ting called this resonance J

Everybody else call it J/Ψ

PDG 2016

$$m_{J/\Psi} = 3096.900 \pm 0.006 \text{ MeV}$$

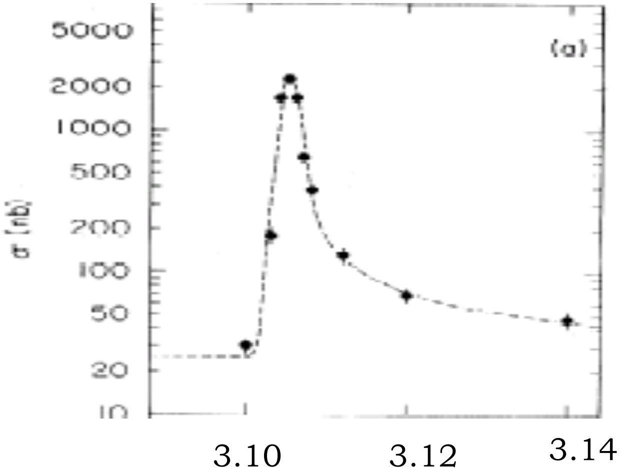
Richter → Ting
e+e- → J/Ψ → hadrons
Richter ← Ting

SLAC

Richter et al. used the e+e-collider Spear at SLAC. They measured the cross-section of the process e+e-→hadrons as a function of the center of mass energy.

They also found a peak at≈3.1 GeV

Richter called this resonance Ψ



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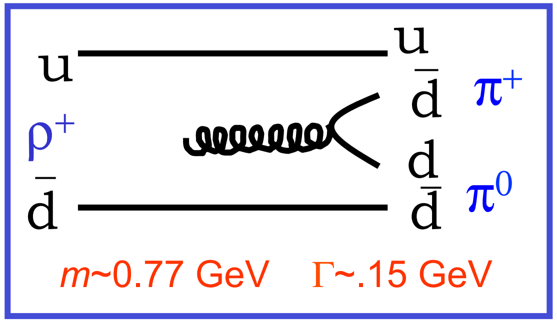
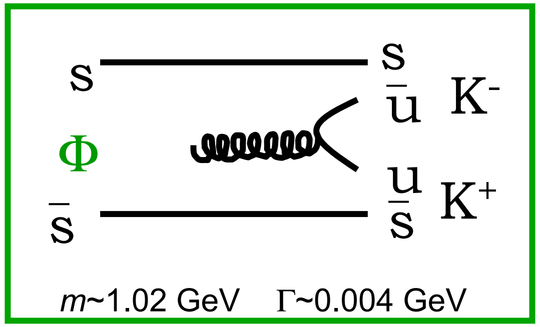
What was “wrong” with the J/Ψ?

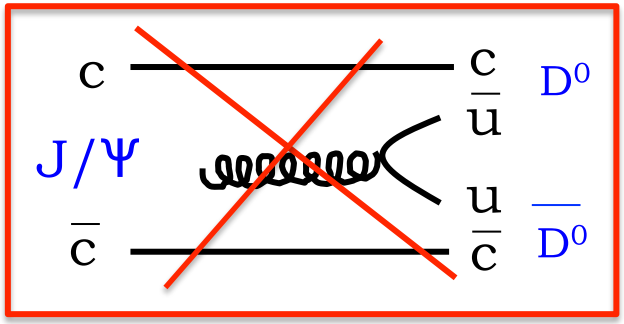
$\Gamma_{tot}^{J/\Psi} = 91.0 \pm 3.2 \text{ keV} \quad \Rightarrow \quad \tau = \frac{h}{\Gamma} \approx 7 \cdot 10^{-21} s$

$m_{J/\Psi} = 3096.900 \pm 0.006 \text{ MeV}$

- One would expect a lifetime typical of strong interactions (10^{-23}), similar to other strong decays, like:



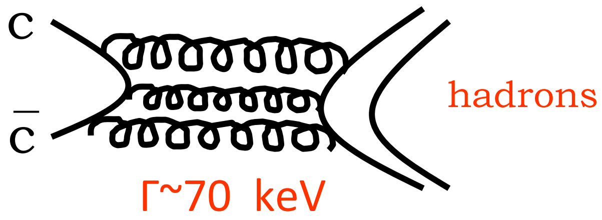




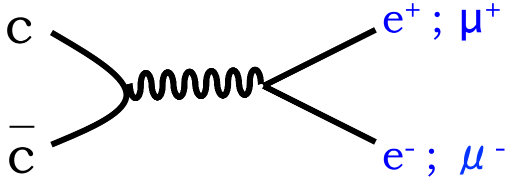
- **The J/Ψ can not do this decay because the lightest charmed meson D⁰ has:** $m_{D^0} = 1864.6 \pm 0.5 \text{ MeV}$

- Then the strong decay must go through the three gluons channel (OZI rule) and becomes of the same order of the e.m. decay channels:

$\Rightarrow \quad m_{J/\psi} < 2m \cdot D_0$



Γ~70 keV



Γ(ee)~5 KeV; Γ(μμ)~5 KeV

- Of course what we said it is only true if the quarks of this new resonance carry a new quantum number that can not be violated by the strong interactions (otherwise there were a lot of mesons lighter than the J/Ψ)
- Since the decay was not due to weak interactions (the lifetime would have been much longer) was an indication that the resonance itself was not carrying this new quantum number, hence the hypothes that the J/Ψ was a meson composed by a charm-anticharm pair.

Cabibbo-Kobayashi-Maskawa (CKM) matrix

- In 1973 Kobayashi and Maskawa wanted to introduce the CP violation in the Standard Model. For this it was necessary to introduce a complex number in the Hamiltonian; let's recall that if $H^* \neq H$ then the Time Reversal T is violated while CPT is always true.
- The simplest way was to introduce a phase in the quark mixing matrix.
- A NxN unitary matrix has:

$\frac{1}{2}N(N-1)$ real parameters (Euler angles)

$\frac{1}{2}(N-1)(N-2)$ non trivial phases (you can't get rid of by changing the phase of the quarks)

- With N=2 one can not introduce any phases, therefore K. and M. proposed in 1973 that should exist a third family of quarks, because with N=3 we have three angles and one phase:

$$\Rightarrow \begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

+ their antiparticles

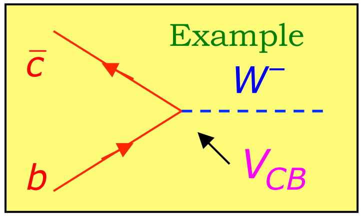
where:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Eigenstates of the weak interactions

CMK matrix of the quark mixing

Mass eigenstates



$$s' = V_{cd} \cdot d + V_{cs} \cdot s + V_{cb} \cdot b$$

The CKM matrix is unitary and it can be parametrized in several ways. Its parameter must be determined experimetally. There is no theory (yet) that can foresee the CKM values.

CKM matrix

- As we said the CKM matrix can be written in several ways, for instance:

1. In terms of 3 angles and 1 phase:

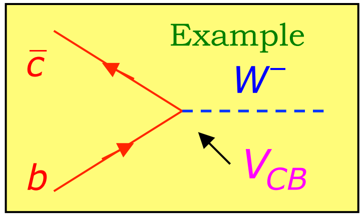
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

From PDG

The four real parameters are: δ_{13}, θ_{12}, θ_{23}, and θ_{13}. The abbreviation is: s=sin, c=cos and the numbers refer to the quark generation. For instance $s_{12}=\sin\theta_{12}$.

2. In terms of quark couplings (this is the best one to understand the “physics”)

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$



3. In terms of a series expansion of the Cabibblo angle θ_{12} (exploting the fact that $s_{12}>>s_{23}>>s_{13}$)

“Wolfenstein” representation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$\lambda=\sin\theta_{12}$, while A, ρ and η are real and close to 1. This parameterization is suitable to relate the CP violation to some specific processes and their decay rate.

CKM matrix

- Let’s see the value of matrix elements taken from the PDG2016:

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$

- Looking at the matrix numerical values we can point out a few things:
  1. The CKM matrix is almost diagonal (the off diagonal elements are small)
  2. The more off we are from a family, the smaller the matrix elements is (for instance $V_{ub} << V_{ud}$)
  3. By making use of points 1. and 2. we deduce that some decays are preferred with respect to some other ones, for instance:

     $c \to s$ over $c \to d$ $D^0 \to K^-\pi^+$ over $D^0 \to \pi^-\pi^+$ (exp. find 3.8% vs 0.15%)

     $b \to c$ over $b \to u$ $B^0 \to D^-\pi^+$ over $B^0 \to \pi^-\pi^+$ (exp. find 3×10^{-3} vs 1×10^{-5})

  4. Since the matrix is supposed to be a unitary matrix, we have several constraints among the elements, for instance:

$$V_{ud}^* V_{ud} + V_{cd}^* V_{cd} + V_{td}^* V_{td} = 1$$

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

so far the experimental results are consistent with a unitary CKM; however we continue to look for deviation from unitary as an indication for new physics signal with respect to what is foreseen by the Standard Model.

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Measurement of the CKM matrix elements

- At the moment there is no theory that is able to predict the values of the CKM matrix elements, therefore they must be determined experimentally.
- The “cleanest” way to do it is to use decays or processes where are present leptons, so that the CKM matrix intervenes only at one vertex. For instance:

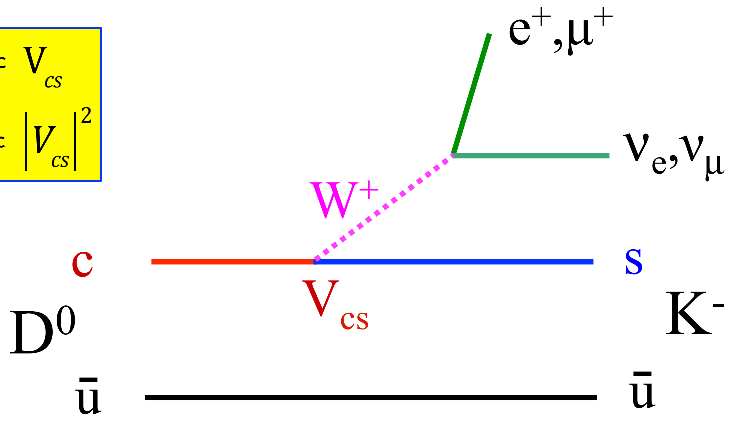
V_{ud}: neutron decay:	$n \rightarrow pev$	$d \rightarrow uev$
V_{us}: kaon decay:	$K^0 \rightarrow \pi^+ e^- v_e$	$s \rightarrow uev$
V_{bu}: B-meson decay:	$B^- \rightarrow (\rho^0 \text{ or } \pi^0) e^- v_e$	$b \rightarrow uev$
V_{bc}: B-meson decay:	$B^- \rightarrow D^0 e^- v_e$	$b \rightarrow cev$
V_{cs}: charm decay:	$D^0 \rightarrow K^- e^+ v_e$	$c \rightarrow sev$
V_{cd}: neutrino interactions:	$v_\mu d \rightarrow \mu^- c$	$d \rightarrow c$

$D^0 = c\bar{u}$; $B^- = \bar{u}b$

“Spectator” model of the $D^0 \rightarrow K^- e^+ v_e$ decay

Amplitude $\propto V_{cs}$

Decay rate $\propto |V_{cs}|^2$



It is called “spectator” model because only one quark participates to the decay, while the others just “go round, look and do nothing”.

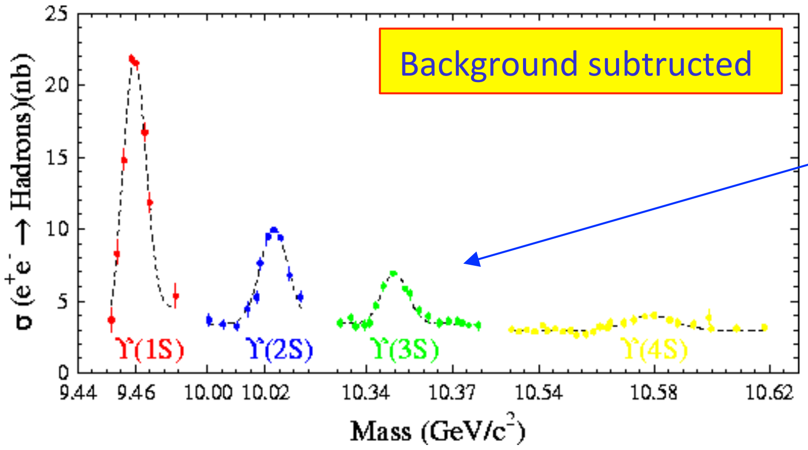
N.B. For the massive neutrinos exist a matrix similar to the CKM one called PMNS matrix. If the neutrinos were massless it would have been diagonal.

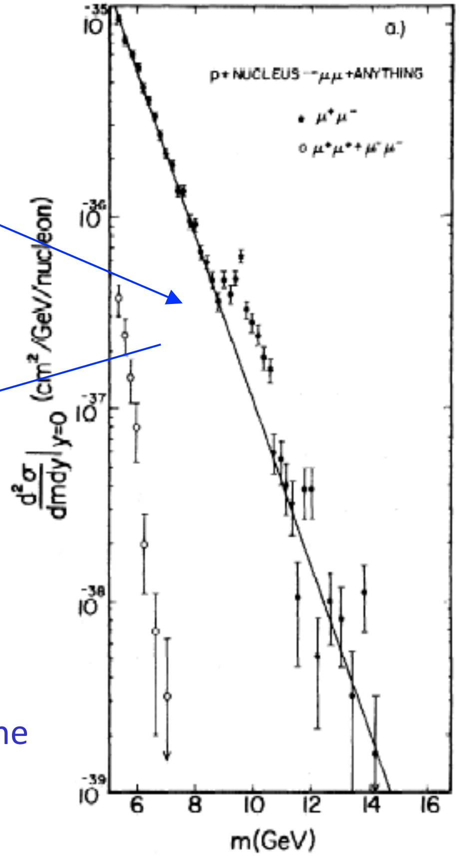
Discovery of the Yupsilon (Y)

- In 1977 at Fermilab (FNAL) in Chicago were discovered other narrow resonances with masses between 9 e 10.5 GeV that showed the same peculiarities of the J/Ψ, that is a lifetime too long with respect to the one expected. This was the hint of a new quantum number: the beauty.

Lederman, in a similar manner to the Ting's experiment, was looking for resonances in the invariant mass of the muon pairs produced in the reaction:

$$p(400\ \text{GeV}) + \text{nucleus} \to \mu^+\mu^- + anything$$





There are several peaks. The Y(1S), Y(2S) and Y(3S) are below the threshold to decay in pairs of meson with open beauty.

- In 1975 at Slac was discovered the third lepton, the tau: $m_\tau = 1776.99 \pm 0.39$ MeV
- In 1994 a FNAL was discovered the quark top: $m_t = 174 \pm 5$ GeV
- And finally, again FNAL, in 2000 was detected in a direct way the neutrino tau.

The three fermion families are complete !

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End of chapter 7