COMPUTING METHODS FOR PHYSICS
18 NOVEMBER 2019

You must submit your exam by **Wednesday Nov 20 at 15:00** following the instruction at http://www.roma1.infn.it/people/rahatlou/cmp/

Complex numbers

Write a class **`Complex`** to implement the algebra of complex numbers. For simplicity, the constructor accepts the real and imaginary part using the cartesian coordinates.

- Provide **`re()`** (real part), **`im()`** (imaginary part), **`r()`** (magnitude), and **`phi()`** (angle in XY plane with respect to x axis) member functions
- Overload the **`*, /, +, -`** operators for operations with both **`Complex`** and **`double`** types (see example below)
- Overload the << operator (see example below) to print the number to screen in the **`(re: xxx, im: yyy)`** format
- Compile and run the following simple program **`app.cc`** to test your class.

```
#include "Complex.h"
#include  <iostream>

int main() {
      Complex z(1.2, -0.6), w, y;
      const Complex i(0,1);

      w = z + 2*i;
      std::cout << w << std::endl;

      Complex y = (w*i)/1.5;
      std::cout << w << std::endl;

      Complex inv = 3.2/w;
      std::cout << inv << std::endl;

      Complex sum = -2.5 + i*2 + Complex(4,-3);
      std::cout << sum << std::endl;

}
```

Provide **`Complex.hh`**, **`Complex.cc`**, and **`app.cc`** for evaluation.

Evaluation will be based on: successful compilation, correct use of C++ syntax, return type and arguments of functions, data members and interface of classes, unnecessary void functions, use of unnecessary C features, and correct mathematical operations.

Electromagnetic Shower in matter

The passage of a high-energy electron or photon in matter produces an electromagnetic shower until all energy of the incident particle is lost through Bremsstrahlung (when energy is above the critical energy E_c in the material) and ionisation for electrons (and positrons) and pair production for photons. Each material is characterised by a radiation length X_0 which causes the energy of the incident particles to decease as

$$E(x) = E_0 e^{-x/X_0}$$

where $\mathbf{E_0}$ is the initial energy of the incident particle and **`E(x)`** is its energy after traversing thickness **x** of material. (See for example William R. Leo, Techniques for Nuclear and Particle Physics Experiments or A. Das and T. Ferbel, Introduction to Nuclear and Particle Physics)

We want to study a simplified electromagnetic shower produced by an electron of 10 GeV hitting a NaI crystal of 5 cm x 5 cm x 50 cm. (NaI properties)

Simulate passage of 10000 electrons through the crystal. For each electron, positrons and photon an interaction can occur after thickness x assume. For simplicity assume steps of x = X_0. For electrons assume a 50% probability for Bremsstrahlung to occur while E > E_c, and for photons assume a 80% probability for pair production until energy is sufficient.

Use python to simulate the shower development for each incident electron and compute

1. maximum number N_{max} of particles produced in each simulation. Make a 1D plot of the distribution of N_{max} for all simulations
2. Depth x_{max} at which the maximum number of particles is in the shower. Make a 1D plot of the distribution of x_{max}
3. Depth x_{tot} where all electrons are below the critical energy E_c and no photon conversion can occur. Make a 1D plot of the distribution of x_{tot}

Save a PDF file for each of the above 3 plots. You need to use comprehensions and dictionaries to implement the simulation and plotting the required plots. Define a function **`Interaction`** (with proper arguments and return values) to simulate the interaction at each step for each particle.

Evaluation will be based on use of python features and data structures, comprehensions (instead of C-style for loops), NumPy objects, labels, units, and clarity of plots.