Can Renormalization Group effects enhance small neutrino mixing into the observed range?

Amitava Raychaudhuri

University of Calcutta, India

Based on work with Gautam Bhattacharyya and Arunansu Sil

Plan

Neutrino masses and mixings

Effective ν-mass operator in SM $\Rightarrow$ Neutrino masses

RG evolution eqn. for this operator

Mixing angle change under RG evolution for two neutrinos

Two-zero texture three-neutrino mass matrices

Enhancement of mixing in these scenarios

Comment on non-standard models

Conclusions

Motivations

- Experiments indicate that neutrinos oscillate. This means:
  - Neutrinos have mass.
  - Flavour eigenstates are not the mass eigenstates.

  The oscillation probabilities are dependent on $\sin^2 2\theta$ and Δm^2.
- SK Atmospheric ν: $\theta_{23} \sim 45^\circ$ and $\Delta m^2_{23} \sim 2.4 \times 10^{-3}$ eV^2
  K2K in agreement.
- Solar neutrinos: SNO and SK prefer the LMA soln. with $\theta_{12} \sim 32^\circ$ and $\Delta m^2_{12} \sim 7.2 \times 10^{-5}$ eV^2.
  KAMLand confirms.
- CHOOZ reactor expt.: $\theta_{13} < 10^\circ$

ν-mass and the SM

(a) Dirac mass $\Rightarrow$ Add ν_R to the SM

(b) Majorana mass $\Rightarrow$ Lepton no. violation

(b) possible with SM fields through

$$L^{SM} = \frac{\kappa_{ij}}{M_X} \bar{\ell^c}_i \ell_j H H + h.c.$$

$$\ell_i = \begin{bmatrix} \nu_i \\ e_i \end{bmatrix}, \quad H = \begin{bmatrix} H^+ \\ H^0 \end{bmatrix}.$$

i, j generation index, lepton number violated above M_X

ν-mass matrix

$$L^{SM} = \frac{\kappa_{ij}}{M_X} \bar{\ell}_i^c \ell_j H H + h.c.$$

$< H_0 >= \upsilon$ $\Rightarrow$ Neutrino mass matrix

$$m_{ij} \sim \kappa_{ij} \left(\frac{\upsilon^2}{M_X} \right)$$

$\kappa \sim$ 1 and $M_X \sim 10^{15}$ GeV $\Rightarrow$ $m \sim 0.1$ eV

a) M_X close to GUT scale

b) Charged lepton mass basis. Leptonic sector mixing from neutrino mass matrix alone.

ν mixing angle

Two flavour case mixing matrix

$$U = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\tan 2\theta = \frac{2\kappa_{12}}{\kappa_{11} - \kappa_{22}} = \frac{2(\kappa_{12}/\kappa_{22})}{d\kappa}$$

$$d\kappa \equiv \frac{\kappa_{11} - \kappa_{22}}{\kappa_{22}}$$

RG running of ν mixing angle

RG evolution of κ matrix:

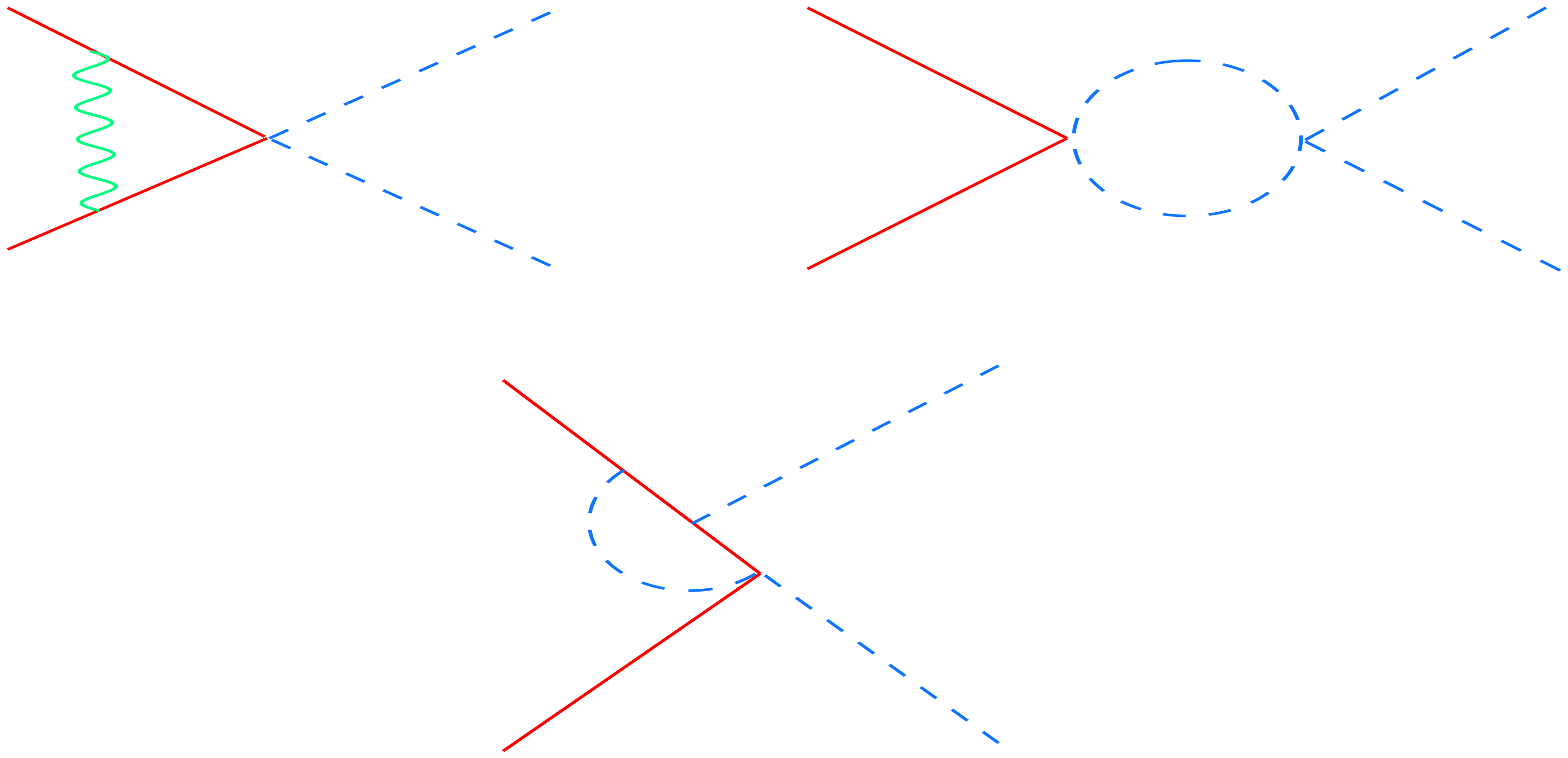
$$16\pi^2 \frac{d\kappa}{d\ln\mu} = \{-3g_2^2 + 2\lambda + 2S\}\kappa - \frac{3}{2}\{\kappa(Y_l{}^\dagger Y_l) + (Y_l{}^\dagger Y_l)^T\kappa\}$$

$g_2 \equiv$ SU(2) coupling, Y_l charged lepton Yukawa coupling matrix (diagonal)

$$S = \mathrm{Tr}\left[3Y_u{}^\dagger Y_u + 3Y_d{}^\dagger Y_d + Y_l{}^\dagger Y_l\right]$$

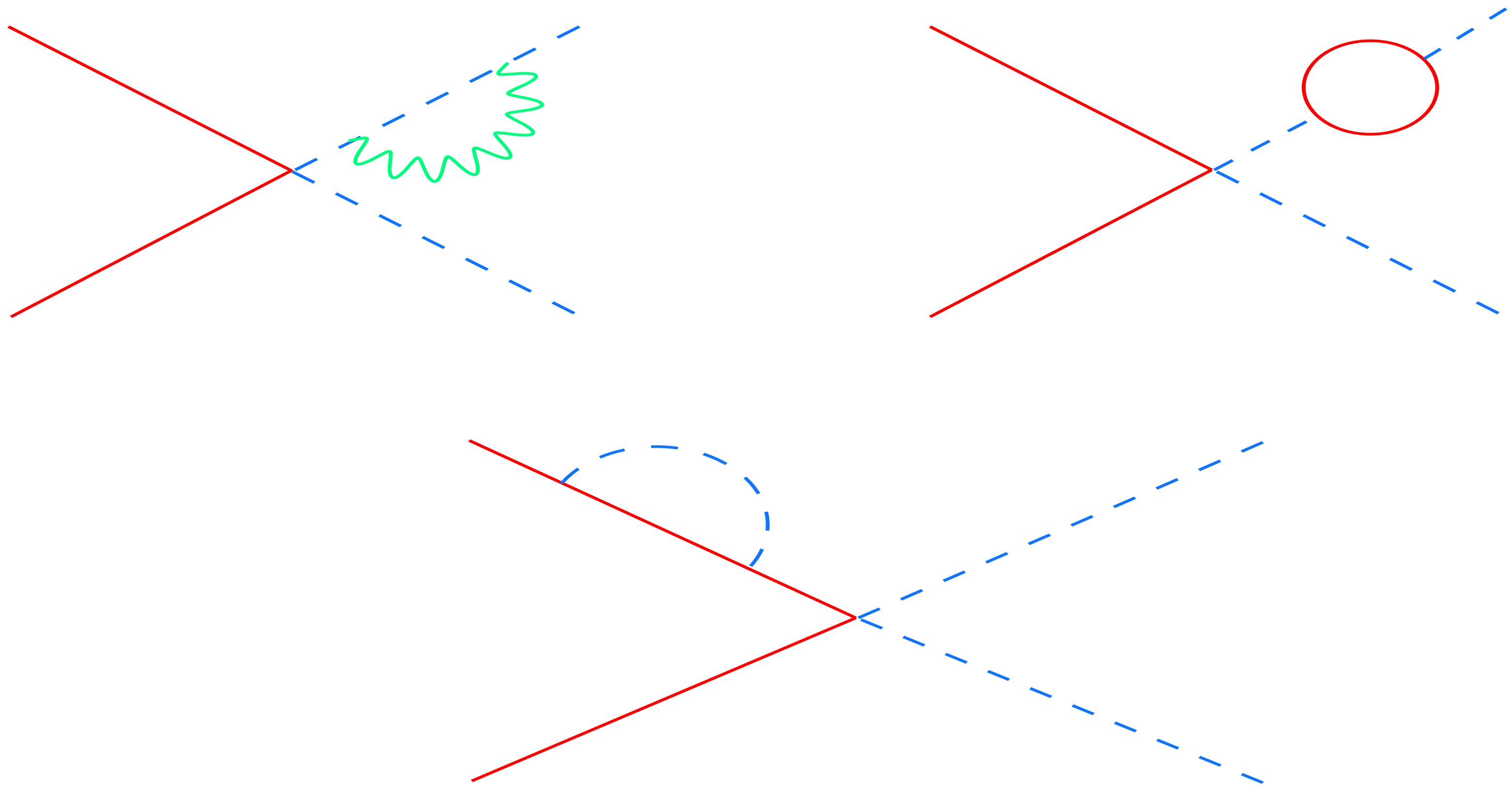
$Y_{u,d}$ u, d sector Yukawa coupling matrices.
Equation linear in κ: net effect of evolution is a multiplicative change.

Vertex correction diagrams



Landau gauge

Self-energy diagrams



Landau gauge

RG running of other couplings

RG equations for the SM couplings are well known

The gauge and Yukawa couplings at the electroweak scale are chosen as input parameters.

Caution required in choosing Higgs mass to fix the quartic scalar coupling λ. Unless $m_H > 150$ GeV, RG evolution drives λ to negative values.

Neutrino mixing angle evolution is quite insensitive to this choice.

RG running of κ

$$16\pi^2 \frac{d\kappa}{d\ln\mu} = \{-3g_2^2 + 2\lambda + 2S\}\kappa - \frac{3}{2}\{\kappa(Y_l^\dagger Y_l) + (Y_l^\dagger Y_l)^T\kappa\}$$

$D \equiv -3g_2^2 + 2\lambda + 2S$, gives universal contribution while Y_l distinguishes between different elements.

$$16\pi^2 \frac{d\kappa_{11}}{d\ln\mu} = D\kappa_{11} - \frac{3}{2}(2Y_1^2)\kappa_{11}$$

$$16\pi^2 \frac{d\kappa_{12}}{d\ln\mu} = D\kappa_{12} - \frac{3}{2}(Y_1^2 + Y_2^2)\kappa_{12}$$

$$16\pi^2 \frac{d\kappa_{22}}{d\ln\mu} = D\kappa_{22} - \frac{3}{2}(2Y_2^2)\kappa_{22}$$

RG running of κ (contd.)

We take $Y_2 = Y_\tau$ and $Y_1 = Y_e$ or Y_μ. To a good approximation:

$$\kappa_{11}(M_X) \rightarrow \kappa_{11}(M_Z) = a\kappa_{11}(M_X)$$

$$\kappa_{12}(M_X) \rightarrow \kappa_{12}(M_Z) = a(1 + r/2)\kappa_{12}(M_X)$$

$$\kappa_{22}(M_X) \rightarrow \kappa_{22}(M_Z) = a(1 + r)\kappa_{22}(M_X),$$

Universal contribution: $a \sim 0.7$ for $M_X = 10^{18} - 10^{19}$ GeV

$$r \simeq -(3Y_\tau^2/16\pi^2)\ln(M_X/M_Z) \sim Y_\tau^2 \sim 10^{-4}$$

r determines the running of the mixing angle

RG running of κ (contd.)

Numerical check: Simultaneous one loop running of all the coupled RG equations indicates that this parametrization works extremely well, to order $r^2 \sim 10^{-8}$.

The size of r is controlled by τ Yukawa coupling, and its order-of-magnitude does not vary appreciably if M_X is altered even by a few orders.

Fine tuning gives large mixing

Recall

$$\tan 2\theta = \frac{2(\kappa_{12}/\kappa_{22})}{d\kappa}, \qquad d\kappa \equiv \frac{\kappa_{11} - \kappa_{22}}{\kappa_{22}}.$$

To a good approximation $d\kappa \tan 2\theta = 2\kappa_{12}/\kappa_{22}$ is scale invariant. Also

$$d\kappa(M_Z) \simeq d\kappa(M_X) - r = d\kappa(M_X)\delta, \qquad \delta \equiv 1 - r/d\kappa(M_X).$$

Thus:

$$\tan 2\theta(M_Z) = \tan 2\theta(M_X)/\delta, \quad \text{where}$$

δ: fine-tuning parameter. Mixing angle enhancement $\sim 1/\delta$.

Numbers: masses and mixings

Assuming $m_1 \sim m_2 >> |m_1 - m_2|$

$$d\kappa(M_Z) = 0.5 \frac{\Delta m^2}{m^2} \cos 2\theta(M_Z)$$

$m = 0.5(m_1 + m_2)$, $\Delta m^2 \equiv m_1^2 - m_2^2$, both at low scale. Also

$$d\kappa(M_Z) = d\kappa(M_X)\delta \simeq r\delta$$

Thus

$$\Delta m^2(\mathrm{eV}^2) = 10^{-3} \left[\frac{\delta}{\cos 2\theta(M_Z)}\right] \left[\frac{m}{2.2 \mathrm{~eV}}\right]^2$$

Numbers: masses and mixings

$$\Delta m^2(\mathrm{eV}^2) = 10^{-3}\left[\frac{\delta}{\cos 2\theta(M_Z)}\right]\left[\frac{m}{2.2\ \mathrm{eV}}\right]^2$$

$$\tan 2\theta(M_Z) = \tan 2\theta(M_X)/\delta \Rightarrow \frac{\delta}{\cos 2\theta(M_Z)} = \frac{\tan 2\theta(M_X)}{\sin 2\theta(M_Z)}$$

$\tan 2\theta(M_X) \sim 0.1 - 0.5 \Rightarrow \theta(M_X) \sim (3^\circ - 13^\circ)$ 'small' initial mixing: $\Delta m^2 \leq 5 \times 10^{-4}\ \mathrm{eV}^2$

LMA admissible. (Tritium beta decay okay. WMAP ν-mass bound: uneasy!! $(\beta\beta)_{0\nu}$?)

Atmospheric neutrinos or $\nu_\mu - \nu_e$ oscillation do not work

3 νs: 2-zero texture mass matrices

We consider two-zero texture neutrino mass matrices defined in terms of six real parameters (No CP-violation)

Three different types are compatible with the present experimental data

$$\mathrm{A-type}: \begin{bmatrix} 0 & 0 & x_1 \\ 0 & x_2 & x_3 \\ x_1 & x_3 & x_4 \end{bmatrix}, \quad \mathrm{B-type}: \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3 \\ 0 & x_3 & x_4 \end{bmatrix}$$

$$\mathrm{C-type}: \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & 0 & x_4 \\ x_3 & x_4 & 0 \end{bmatrix}$$

2-zero texture mass matrices

A-type $\Rightarrow$ hierarchical mases

B-type $\Rightarrow$ quasi-degenerate mases

C-type $\Rightarrow$ inverted-hierarchical mases

Zeros of κ_{ij} are unaffected by RG evolution: texture same at the high and low scales.

Can significant RG evolution of mixing angles take place for these textures?

Quasi-degenerate masses

This is the B-type texture.

At high energy

$$M_\nu^h = \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3 \\ 0 & x_3 & x_4 \end{bmatrix}$$

At the low scale it becomes:

$$M_\nu^l = a \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3(1+\frac{r}{2}) \\ 0 & x_3(1+\frac{r}{2}) & x_4(1+r) \end{bmatrix}$$

Universal scale factor $a \sim \mathcal{O}(1)$ does not affect mixing

Quasi-degenerate masses

$$M_\nu^l = a \begin{bmatrix} x_1 & x_2 & 0 \\ x_2 & 0 & x_3(1+\frac{r}{2}) \\ 0 & x_3(1+\frac{r}{2}) & x_4(1+r) \end{bmatrix}$$

Let

$$V^T M_\nu V = \mathrm{diag}(m_1, m_2, m_3),$$

Maki-Nakagawa-Sakata unitary matrix $V = U_{23}U_{13}U_{12}$, where U_{ij} are the standard rotation matrices.

θ_{23} is given by

$$\tan 2\theta_{23}^l = \frac{2x_3(1+\frac{r}{2})}{x_4(1+r)}.$$

Quasi-degenerate masses

No resonant enhancement of $\tan 2\theta_{23}$ at the low scale possible.

So, keep $\tan 2\theta_{23}$ large for the whole scale of running, i.e.

$$\frac{|x_3|}{|x_4|} \gg 1.$$

After U_{23} rotation,

$$M^l_{\nu 23} = a \begin{bmatrix} x_1 & x_2 c^l_{23} & x_2 s^l_{23} \\ x_2 c^l_{23} & \lambda_1 & 0 \\ x_2 s^l_{23} & 0 & \lambda_2 \end{bmatrix}.$$

Quasi-degenerate masses

$$\lambda_1 \simeq -x_3(1+\frac{r}{2}), \quad \lambda_2 \simeq x_3(1+\frac{r}{2}).$$

For the next rotation U_{13}, we set

$$\tan 2\theta^l_{13} = \frac{2x_2 s^l_{23}}{\lambda_2 - x_1} \simeq \frac{\sqrt{2}x_2}{x_3(1+\frac{r}{2}) - x_1}.$$

Bound on $(V^l)_{e3}$ from CHOOZ $\Rightarrow \theta^l_{13} \to 0$. Significant RG evolution will demand a large $\tan 2\theta^h_{13}$, which is possible if

$$x_3 \simeq x_1$$

Quasi-degenerate masses

This and the smallness of r requires $|x_2|, |x_4| \ll |x_3| \sim |x_1|$.

$$M^l_{\nu 23,13} \simeq a \begin{bmatrix} x_1 & x_2/\sqrt{2} & 0 \\ x_2/\sqrt{2} & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}$$

Next step is to diagonalize the (12)-block:

$$\tan 2\theta^l_{12} = \frac{\sqrt{2} x_2}{\lambda_1 - x_1}$$

From the above $\tan 2\theta^l_{12}$ is small; a direct contradiction of the empirical requirement.

Quasi-degenerate masses

Remaining possibility, $\tan 2\theta_{13}$ *small* and $\tan 2\theta_{23}$ large throughout the energy range. Can there be a prominent RG evolution of θ_{12}? YES!

In this case, $|x_3 - x_1| \gg \sqrt{2}|x_2|$ whence

$$\tan 2\theta_{12}^h \simeq \frac{\sqrt{2}x_2}{-x_3 - x_1}, \quad \tan 2\theta_{12}^l \simeq \frac{\sqrt{2}x_2}{-x_3(1+\frac{r}{2}) - x_1}$$

Resonant enhancement of $\tan 2\theta_{12}$ if $rx_3 \simeq -2(x_3 + x_1)$. Thus,

$$x_3 \sim -x_1$$

and, further, for significant running of θ_{12}, $rx_3 \gg 2\sqrt{2}|x_2|$

Quasi-degenerate masses

Can these relations be satisfied?

For this texture, the mass eigenstates are quasi-degenerate: $m_1 \simeq m_2 \simeq -m_3$ and

$$\Delta m^2_{atm} \sim 2a^2 x_1 x_4, \quad \Delta m^2_{sol} \sim 2\sqrt{2}a^2 x_1 x_2$$

$\Delta m^2_{atm}/\Delta m^2_{sol} \sim 100$ for LMA.

There is no obstruction in achieving the desired ratio by suitably choosing $x_2, x_3,$ and x_4. Since the relationships amongst the x_i are linear, they can all be scaled to achieve the right absolute magnitudes of the mass splittings.

Quasi-degenerate masses

Numerical check:

$x_1 = -1.799196, x_2 = 9.0 \times 10^{-6}, x_3 = 1.8$ and $x_4 = 1.8 \times 10^{-3}$

(all in eV) (Note x_1, x_3 fine tuning!)

Scale	$\tan\theta_{23}$	$\sin 2\theta_{13}$	$\sin 2\theta_{12}$	Δm^2_{atm} (eV^2)	Δm^2_{sol} (eV^2)
High	0.9995	0.0	0.132	6.5$\times 10^{-3}$	3.5$\times 10^{-4}$
Low	0.9995	0.0	0.907	3.2$\times 10^{-3}$	2.5$\times 10^{-5}$

The effective Majorana mass parameter relevant for $(\beta\beta)_{0\nu}$ is predicted in the 1 eV range.

Tritium beta decay bound satisfied.

Hierarchical masses

This is the A-type texture

At high scale:

$$M_\nu^h = \begin{bmatrix} 0 & 0 & x_1 \\ 0 & x_2 & x_3 \\ x_1 & x_3 & x_4 \end{bmatrix},$$

At low scale it becomes:

$$M_\nu^l = a \begin{bmatrix} 0 & 0 & x_1(1+\frac{r}{2}) \\ 0 & x_2 & x_3(1+\frac{r}{2}) \\ x_1(1+\frac{r}{2}) & x_3(1+\frac{r}{2}) & x_4(1+r) \end{bmatrix}.$$

Hierarchical masses

After U_{23} rotation,

$$M^l_{\nu 23} = a \begin{bmatrix} 0 & -x_1 s^l_{23}(1+\frac{r}{2}) & x_1 c^l_{23}(1+\frac{r}{2}) \\ -x_1 s^l_{23}(1+\frac{r}{2}) & \lambda_1 & 0 \\ x_1 c^l_{23}(1+\frac{r}{2}) & 0 & \lambda_2 \end{bmatrix}$$

$$\tan 2\theta^l_{23} = \frac{2x_3(1+\frac{r}{2})}{x_4(1+r) - x_2}.$$

Maximal mixing if: $x_4(1+r) = x_2$

$$\lambda_1 \sim x_2 - x_3(1+\frac{r}{2}), \qquad \lambda_2 \sim x_2 + x_3(1+\frac{r}{2})$$

Hierarchical masses

Next rotate by U_{13}:

$$\tan 2\theta^l_{13} = \frac{2x_1 c^l_{23}(1+\frac{r}{2})}{\lambda_2}.$$

$\tan 2\theta^l_{13} \to 0$ implies:

$$|x_2 + x_3(1+\frac{r}{2})| \gg \sqrt{2}|x_1(1+\frac{r}{2})|.$$

The mass matrix is now:

$$M^l_{\nu 23,13} = a \begin{bmatrix} 0 & -x_1 s^l_{23}(1+\frac{r}{2}) & 0 \\ -x_1 s^l_{23}(1+\frac{r}{2}) & \lambda_1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix}.$$

Hierarchical masses

The remaining (12)-rotation is

$$\tan 2\theta^l_{12} = \frac{-2x_1 s^l_{23}(1+\frac{r}{2})}{x_2 - x_3(1+\frac{r}{2})},$$

large $\tan 2\theta^l_{12}$ is (using $\sin\theta^l_{23} \sim 1/\sqrt{2}$)

$$|x_2 - x_3(1+\frac{r}{2})| \ll \sqrt{2}|x_1(1+\frac{r}{2})|.$$

These are the requirements for compatibility with low energy angles.

Can any of the angles have large RG running?

Hierarchical masses

Several alternatives are possible.

Can θ_{23} have large running? This requires:

$$2|x_3| \ll |x_4 - x_2| = r|x_4|,$$

Recall $x_2 \simeq x_4$: $|x_2| \gg |x_3|$

Hence $|x_2 \pm x_3| \simeq |x_2|$ and one cannot simultaneously satisfy the conditions small θ_{13} and large θ_{23}.

Radiative enhancement of θ_{23} not possible in this texture.

The remaining alternatives examined on a similar case by case basis: Significant running of no angle is allowed.

Inverted hierarchy case

This is the C-type texture. Here, at high scale:

$$M_\nu^h = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_2 & 0 & x_4 \\ x_3 & x_4 & 0 \end{bmatrix}$$

and at low energy:

$$M_\nu^l = a \begin{bmatrix} x_1 & x_2 & x_3(1+\frac{r}{2}) \\ x_2 & 0 & x_4(1+\frac{r}{2}) \\ x_3(1+\frac{r}{2}) & x_4(1+\frac{r}{2}) & 0 \end{bmatrix}$$

Inverted hierarchy case

Maximal mixing in the (23)-sector at all energies.

Here again, a case by case examination shows that running of θ_{13} and/or θ_{23} runs into inconsistency with the solar and atmospheric mass-splittings.

Beyond the SM: SUSY

In MSSM the RG equation for κ is a little different.

Reasons: (a) Two Higgs doublets, (b) The quartic scalar coupling λ is determined by the gauge coupling, and (c) sparticles can be in the internal lines of Feynman diagrams. Upshot:

$$r \simeq -(Y_\tau^2/8\pi^2)\ln(M_X/M_Z)$$

r is of opposite sign from the SM.
Y_τ depends on $\tan\beta$ and can be larger than in SM. $|r|$ can become as large as $\sim 10^{-2}$.

Beyond the SM: Extra Dimensions

Really only one model: fermions live in a brane, bosons in the bulk. (Dienes, Dudas and Ghergetta)

(a) Power law corrections to coupling constants and (b) unification scale is low $\sim \mathcal{O}$(10 TeV).

For δ extra dimensions and compactification radius μ_0^{-1} with power law running above $\mu_0 \sim$ 1 TeV:

$$r \simeq \left(\frac{3Y_\tau^2}{16\pi^2}\right)\frac{X_\delta}{\delta}\left[\left(\frac{M_X}{\mu_0}\right)^\delta - 1\right] \sim 10^{-4} \ \ (\text{for} \ \ \delta = 1)$$

$$X_\delta = \frac{2\pi^{\delta/2}}{\delta.\Gamma(\delta/2)}$$

Conclusions

- Two of the ν mixing angles are large, unlike in the quark sector. Could this be an RG effect?
- 2-flavour: Possible to get LMA *modulo fine tuning*.
- Three classes of 2-zero three ν mass matrices examined.
  - Atmospheric angle cannot be large due to running.
  - Solar LMA can be. But only for one class of texture (quasi-degenerate).
  - Small CHOOZ angle cannot be obtained from a larger one by RG.
- Results similar for MSSM and one extra-dimensional model.