12.6 HOW DOES A GRAVITATIONAL WAVE AFFECT THE MOTION OF A SINGLE PARTICLE

Consider a particle at rest in flat spacetime before the passage of the wave. We take the x-axis coincident with the direction of propagation of an incoming gravitational wave in the TT gauge. The particle will follow a geodesic of the curved spacetime generated by the wave

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \equiv \frac{du^\alpha}{d\tau} + \Gamma^\alpha{}_{\mu\nu}u^\mu u^\nu = 0\,. \tag{12.78}$$

At $\tau = 0$ the particle is at rest ($u^\alpha = (1, 0, 0, 0)$) and, from the above equation, the initial acceleration produced by the wave is

$$\left(\frac{du^\alpha}{d\tau}\right)_{(\tau=0)} = -\Gamma^\alpha{}_{00} = -\frac{1}{2}\eta^{\alpha\beta}\left[h_{\beta 0,0} + h_{0\beta,0} - h_{00,\beta}\right]\,. \tag{12.79}$$

Since in the TT gauge all time components of $h_{\mu\nu}$ are zero (see Eq. 12.77), Eq. 12.79 gives

$$\left(\frac{du^\alpha}{d\tau}\right)_{(t=0)} = 0. \tag{12.80}$$

Thus, u^α remains constant, which means that, in the TT gauge, the particle is not accelerated by the wave and remains at a *fixed coordinate position*. We conclude that *the motion of a single particle is not affected by the passage of the gravitational wave.*

However the situation changes if we consider the proper distance between two particles A and B, with coordinates x^μ_A, x^μ_B. Let us assume that the particles are initially at rest, and that a plane-fronted gravitational wave, propagating along the x-axis, reaches them at some time $t = 0$. In the TT gauge, the only non-vanishing components of the wave are those on the (y, z)-plane as in Eq. 12.77 and the metric is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h^{TT}_{\mu\nu})dx^\mu dx^\nu\,. \tag{12.81}$$

As shown above, if the two particles are initially at rest, they will remain at the same coordinate position even later, when the wave passes by; therefore their *coordinate* separation $\delta x^\mu = x^\mu_B - x^\mu_A$ will remain constant. However, since the metric changes, the *proper distance between them will change.* For example if the particles are on the y-axis,

$$\Delta l = \int ds = \int_{y_A}^{y_B} |g_{yy}|^{\frac{1}{2}}dy = \int_{y_A}^{y_B} |1 + h^{TT}_{yy}(t - x/c)|^{\frac{1}{2}}dy \neq \text{constant}\,. \tag{12.82}$$

In order to study the relative motion of neighbouring particles induced by a gravitational wave, in the next section we shall solve the equation of geodesic deviation given by Eq. 4.54.

12.7 GEODESIC DEVIATION INDUCED BY A GRAVITATIONAL WAVE

Let us consider two particles A and B initially at rest. To solve the equation of geodesic deviation it is convenient to choose a LIF $\{\xi^\alpha\}$ centered on the geodesic of one of the two particles, say the particle A.

As explained in Box 3-A, in the neighborhood of A the metric differs from Minkowski's metric by terms of order $|\xi|^2$, i.e.

$$ds^2 = \eta_{\alpha\beta}d\xi^\alpha d\xi^\beta + O(|\xi|^2)\,. \tag{12.83}$$

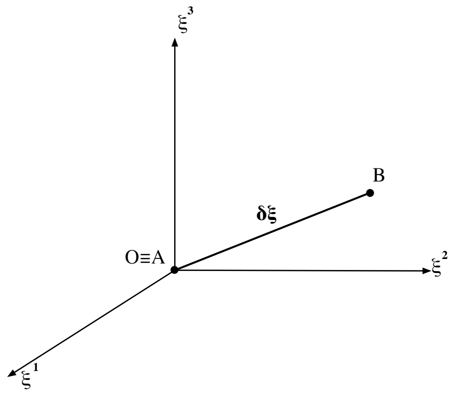

Figure 12.2: Two nearby particles in the LIF $\{\xi^\mu\}$ centered on the particle A.

In this frame the particle A has space coordinates $\xi^i_A = 0$ ($i = 1, 2, 3$), and since it is at rest, its proper time concides with the coordinate time

$$t_A = \tau/c\,, \qquad \frac{d\xi^\mu}{d\tau}_{|A} = (1,0,0,0)\,; \tag{12.84}$$

in addition

$$g_{\mu\nu\,|A} = \eta_{\mu\nu}\,, \qquad g_{\mu\nu,\alpha\,|A} = 0 \quad (\text{i.e.}\, \Gamma^\alpha_{\mu\nu\,|A} = 0)\,, \tag{12.85}$$

where the subscript $_{|A}$ means that the quantities are computed along the geodesic of the particle A. The equation of geodesic deviation is (see Eq. 4.54)

$$\frac{D^2\delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\gamma} t^\alpha t^\beta \delta x^\gamma\,, \tag{12.86}$$

where in the present case $t^\alpha = \frac{d\xi^\mu}{d\tau}_{|A} = (1,0,0,0)$, and $\delta x^\gamma = \delta\xi^\gamma$ is the separation vector between the particles A and B. Thus Eq. 12.86 becomes

$$\frac{D^2\delta\xi^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\gamma}\frac{d\xi^\alpha}{d\tau}_{|A}\frac{d\xi^\beta}{d\tau}_{|A}\delta\xi^\gamma \quad \to \quad \frac{1}{c^2}\frac{d^2\delta\xi^\mu}{dt^2} = R^\mu{}_{00\gamma}\delta\xi^\gamma\,. \tag{12.87}$$

In the LIF attached to the particle A the Riemann tensor takes the simple form (see Eq. 4.24)

$$R_{\alpha\kappa\lambda\mu} = \frac{1}{2}\left(g_{\alpha\mu,\lambda\kappa} + g_{\kappa\lambda,\mu\alpha} - g_{\alpha\lambda,\mu\kappa} - g_{\kappa\mu,\lambda\alpha}\right)\,, \tag{12.88}$$

where

$$g_{\mu\nu} = \eta_{\mu\nu} + h^{LIF}_{\mu\nu}\,. \tag{12.89}$$

However, we know that the metric tensor takes a simple form in the TT gauge $g_{\mu\nu} = \eta_{\mu\nu} + h^{TT}_{\mu\nu}$, where the only non vanishing components are those orthogonal to the direction of propagation of the wave. The Riemann tensor in this frame is

$$R^{TT}_{\alpha\kappa\lambda\mu} = \frac{1}{2}\left(h^{TT}_{\alpha\mu,\lambda\kappa} + h^{TT}_{\kappa\lambda,\mu\alpha} - h^{TT}_{\alpha\lambda,\mu\kappa} - h^{TT}_{\kappa\mu,\lambda\alpha}\right) + O(h^2)\,. \tag{12.90}$$

Thus, it is convenient to find the relation between the Riemann tensor computed in the TT gauge and that computed in the LIF. As explained in Sec. 12.5, in order to choose a TT frame we need to make an infinitesimal coordinate transformation $x^\mu \to x^\mu + \epsilon^\mu$, with the condition $\Box_F \epsilon^\mu = 0$, and impose the transverse-traceless conditions. Under this transformation $h_{\mu\nu}$ transform as $h_{\mu\nu} \to h_{\mu\nu} - \epsilon_{\mu,\nu} - \epsilon_{\nu,\mu}$; replacing this expression in the expression of the Riemann tensor in the LIF given by Eq. 12.88, we find

$$R^{TT}_{\alpha\kappa\lambda\mu} = R_{\alpha\kappa\lambda\mu} - \frac{1}{2}\left(\epsilon_{\alpha,\mu\kappa\lambda} + \epsilon_{\mu,\alpha\kappa\lambda} + \epsilon_{\kappa,\lambda\alpha\mu} + \epsilon_{\lambda,\kappa\alpha\mu}\right.$$
$$\left. - \epsilon_{\alpha,\lambda\kappa\mu} - \epsilon_{\lambda,\alpha\kappa\mu} - \epsilon_{\kappa,\mu\alpha\lambda} - \epsilon_{\mu,\kappa\alpha\lambda}\right) = R_{\alpha\kappa\lambda\mu}\,, \tag{12.91}$$

hence

$$R^{TT}_{\alpha\kappa\lambda\mu} = R_{\alpha\kappa\lambda\mu}\,, \tag{12.92}$$

which shows that the Riemann tensor is *invariant* (and not only covariant) under the infinitesimal coordinate transformation 12.69.

Therefore, we can solve Eq. **??**, which holds in the LIF centered in the particle A, using the Riemann tensor 12.90 computed in the TT gauge. Since in this gauge the only non-vanishing components of $h^{TT}_{\mu\nu}$ for a wave traveling along x^1 are those with $\mu, \nu = 2, 3$, these components only depend on the coordinates x^0, x^1 and - for a progressive wave - $h^{TT}_{\mu\nu,0} = h^{TT}_{\mu\nu,1}$, using Eq. 12.90 it is easy to show that the non-vanishing components of the Riemann tensor are

$$R_{i00m} = R^{TT}_{i00m} = \frac{1}{2} h^{TT}_{im,00}\,, \qquad i, m = 2, 3\,. \tag{12.93}$$

It follows that

$$R^{\mu}{}_{00m} = \eta^{\mu i} R_{i00m} = \frac{1}{2}\,\eta^{\mu i}\,\frac{1}{c^2}\frac{\partial^2 h^{TT}_{im}}{\partial t^2}\,, \qquad i, m = 2, 3\,, \tag{12.94}$$

and the equation of geodesic deviation, Eq. 12.87, becomes

$$\frac{d^2 \delta\xi^\mu}{dt^2} = \frac{1}{2}\,\eta^{\mu i}\,\frac{\partial^2 h^{TT}_{im}}{\partial t^2}\delta\xi^m\,. \tag{12.95}$$

For $t \leq 0$ the two particles are at rest relative to each other, and consequently

$$t \leq 0 \qquad \delta\xi^j = \delta\xi^j_0\,. \tag{12.96}$$

Since $h_{\mu\nu}$ is a small perturbation, when the wave arrives the relative position of the particles will change only by infinitesimal quantities, and therefore we put

$$\delta\xi^j(t) = \delta\xi^j_0 + \delta\xi^j_1(t), \qquad t > 0\,, \tag{12.97}$$

where $\delta\xi^j_1(t)$ has to be considered as a small perturbation with respect to the initial separation $\delta\xi^j_0$. Substituting Eq. 12.97 in Eq. 12.95, remembering that $\delta\xi^j_0$ is constant and retaining only terms of order $O(h)$ Eq. 12.95 becomes

$$\frac{d^2 \delta\xi^j_1}{dt^2} = \frac{1}{2}\,\eta^{ji}\frac{\partial^2 h^{TT}_{ik}}{\partial t^2}\delta\xi^k_0\,. \tag{12.98}$$

This equation can be integrated and the solution is

$$\delta\xi^j = \delta\xi^j_0 + \frac{1}{2}\,\eta^{ji}\,h^{TT}_{ik}\,\delta\xi^k_0\,, \tag{12.99}$$

If the wave propagates along ξ^1 only the components $h_{22} = -h_{33}, h_{23} = h_{32}$ are different from zero, and Eq. 12.99 yields

$$\begin{aligned}
\delta\xi^1 &= \delta\xi_0^1 + \frac{1}{2}\,\eta^{11}h_{1k}^{TT}\ \delta\xi_0^k = \delta\xi_0^1 \\
\delta\xi^2 &= \delta\xi_0^2 + \frac{1}{2}\,\eta^{22}h_{2k}^{TT}\ \delta\xi_0^k = \delta\xi_0^2 + \frac{1}{2}\left(h_{22}^{TT}\ \delta\xi_0^2 + h_{23}^{TT}\ \delta\xi_0^3\right) \\
\delta\xi^3 &= \delta\xi_0^3 + \frac{1}{2}\,\eta^{33}h_{3k}^{TT}\ \delta\xi_0^k = \delta\xi_0^3 + \frac{1}{2}\left(h_{32}^{TT}\ \delta\xi_0^2 + h_{33}^{TT}\ \delta\xi_0^3\right).
\end{aligned} \tag{12.100}$$

Thus, the particles will be accelerated only in the plane orthogonal to the direction of propagation, and this clearly shows the tranverse nature of gravitational waves.

Let us now study the effect of the polarization of the wave, and to simplify the notation let us put $(\xi^1, \xi^2, \xi^3) = (x, y, z)$. Consider a progressive plane wave (see Eq. 12.56). In the TT gauge, its non-vanishing components are (we omit in the following the superscript TT)

$$\begin{aligned}
h_{yy} &= -h_{zz} = 2\Re\left\{A_+ e^{i\omega(t-\frac{x}{c})}\right\}, \\
h_{yz} &= h_{zy} = 2\Re\left\{A_\times e^{i\omega(t-\frac{x}{c})}\right\},
\end{aligned} \tag{12.101}$$

where A_+ and $A_\times$ are constant. Consider two particles, **1** and **2**, initially located at $(0, y_0, 0)$ and $(0, 0, z_0)$, respectively, as indicated in the first plot of the upper panel of Fig. 12.3. The particle A to which the LIF is attached, is sitting at the origin and is not indicated in Fig. 12.3 and in the following figures. Thus the space components of the vectors which separate A from the two particles coincide with the particles coordinates (see Fig. 12.2).

Let us first consider the polarization "+", i.e.

$$A_+ \neq 0 \qquad \text{and} \qquad A_\times = 0\,. \tag{12.102}$$

Assuming the initial phase is zero (i.e. A_+ is real) Eq. 12.101 gives

$$h_{yy} = -h_{zz} = 2A_+ \cos\omega\left(t - \frac{x}{c}\right), \qquad h_{yz} = h_{zy} = 0\,, \tag{12.103}$$

and Eq. 12.100 written for the two particles for $t \geq 0$ gives

$$\begin{aligned}
&\text{particle 1}: \quad z = 0, \quad y = y_0 + \frac{1}{2}h_{yy}\ y_0 = y_0\left[1 + A_+\cos\omega\left(t - \frac{x}{c}\right)\right], \\
&\text{particle 2}: \quad y = 0, \quad z = z_0 + \frac{1}{2}h_{zz}\ z_0 = z_0\left[1 - A_+\cos\omega\left(t - \frac{x}{c}\right)\right].
\end{aligned} \tag{12.104}$$

Since at $t \leq 0$ the particles are at $(y_0, 0)$ and $(0, z_0)$, at $t = 0$ $\omega(t - \frac{x}{c}) = \frac{\pi}{2}$. Therefore, at t=0

$$\begin{aligned}
&\text{particle 1}: \quad z = 0, \qquad y = y_0\,, \\
&\text{particle 2}: \quad y = 0, \qquad z = z_0\,.
\end{aligned} \tag{12.105}$$

After a quarter of a period, i.e. $t = T/4$, $\omega\left(t - \frac{x}{c}\right) = \pi$ and

$$\begin{aligned}
&\text{particle 1}: \quad z = 0, \qquad y = y_0\left(1 - A_+\right), \\
&\text{particle 2}: \quad y = 0, \qquad z = z_0\left(1 + A_+\right).
\end{aligned} \tag{12.106}$$

After half a period, $t = T/2$, $\omega\left(t - \frac{x}{c}\right) = \frac{3}{2}\pi$ and

$$\begin{aligned}
&\text{particle 1}: \quad z = 0, \qquad y = y_0\,, \\
&\text{particle 2}: \quad y = 0, \qquad z = z_0\,.
\end{aligned} \tag{12.107}$$

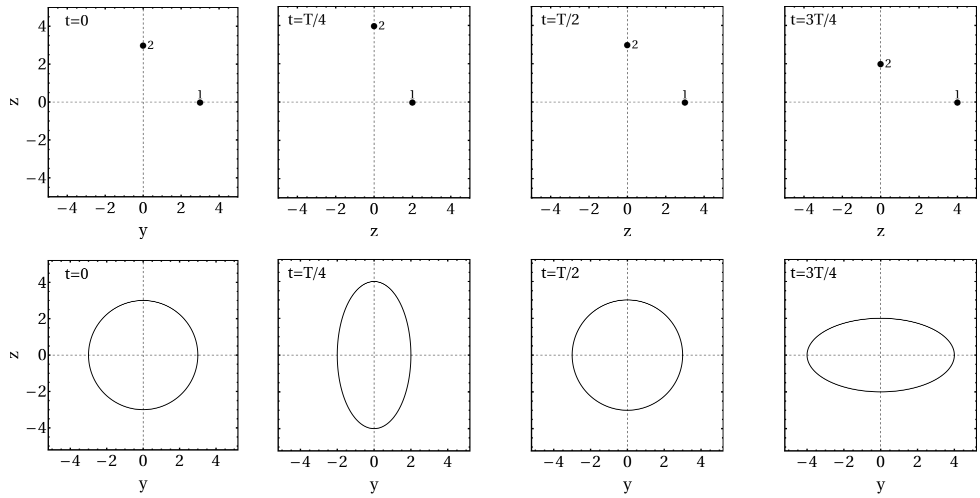

Figure 12.3: In the upper panel we show the displacement of two particles in the (y, z) plane, due to the passage of a gravitational wave propagating along x with "+" polarization. Initially the particles are located at $(3, 0)$ and at $(0, 3)$, respectively, and the gravitational wave amplitude is $A_+ = 1$ (as discussed in the next chapters, this deformation is enormously exaggerated relative to the real effects on a gravitational wave detectable on Earth). The time snapshots correspond to $t = 0$, $T = T/4$, $t = T/2$, $t = 3T/2$ (left upper corner of each plot) where T is the wave period. In the lower panel the deformation of a small ring of particles in the (y, z) plane produced by the same wave is shown for the same values of time.

After three quarters of a period, $t = 3T/4$

$$\begin{aligned} &\text{particle 1}: \quad z = 0, \qquad y = y_0 \left(1 + A_+\right) , \\ &\text{particle 2}: \quad y = 0, \qquad z = z_0 \left(1 - A_+\right) . \end{aligned} \tag{12.108}$$

Similarly, if we consider a small ring of particles centered at the origin, the effect of the wave with polarization '+' is that of deforming the circle in an ellipse prolate (after half a period) and oblate (after three quarters of a period), as shown in the lower panel of Fig. 12.3. Note that in this figure the effect of the gravitational wave was enormously exaggerated: for typical gravitational waves detectable on Earth, a ring of particles with size roughly 1 meter would be deformed by 10^{-21} m only!

Let us now study the effect of the "×" polarization, i.e. $A_\times \neq 0$ and $A_+ = 0$. Consider four particles **1**, **2**, **3** and **4**, initially located at $(0, y_0, z_0)$, $(0, -y_0, z_0)$, $(0, -y_0, -z_0)$, $(0, y_0, -z_0)$, respectively, as indicated in the first plot of the upper panel of Fig. 12.4.

Assuming the initial phase is zero (i.e. $A_\times$ is real) Eq. 12.101 gives

$$h_{yy} = h_{zz} = 0, \qquad h_{yz} = h_{zy} = 2A_\times \cos\omega\left(t - \frac{x}{c}\right) \tag{12.109}$$

and Eq. 12.100 written for each of the four particles for $t \geq 0$ gives

$$y = y_0 + \frac{1}{2}h_{yz}\ z_0 = y_0 + A_\times \cos\omega\left(t - \frac{x}{c}\right) z_0 \tag{12.110}$$

$$z = z_0 + \frac{1}{2}h_{zy}\ y_0 = z_0 + A_\times \cos\omega\left(t - \frac{x}{c}\right) y_0 \,. \tag{12.111}$$

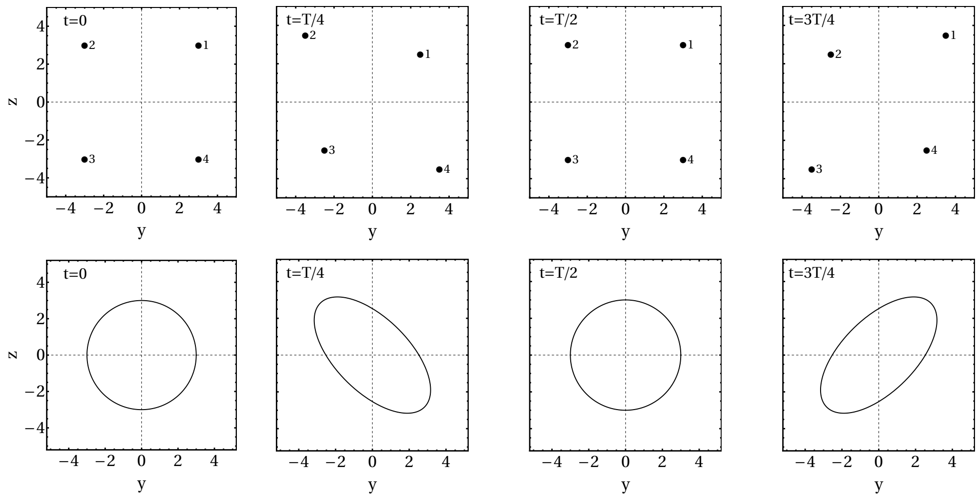

Figure 12.4: Upper panel: displacement produced by a wave with the "×"polarization, propagating along x on four particles in the (y, z) plane. Initially the particles are located at $(y_0, z_0) = (3, 3)$, $(y_0, z_0) = (-3, 3)$, $(y_0, z_0) = (-3, 3)$, and $(y_0, z_0) = (3, -3)$ respectively, and the gravitational wave amplitude is $A_\times = 1$ (again, this deformation is enormously exaggerated relative to the real effects on a gravitational wave detectable on Earth!). As in Fig. 12.3 the time snapshots correspond to $t = 0$, $T = T/4$, $t = T/2$, $t = 3T/2$. Lower panel: deformation produced by the same wave on a small ring of particles in the (y, z), for the same values of time.

If, for simplicity, we assume that $y_0 = z_0 = r$, we find that at $t = 0$ the positions of the four particles are:

$$\begin{aligned} &\text{particle 1}: \quad y = r, \qquad z = r\,, \\ &\text{particle 2}: \quad y = -r\,, \qquad z = r\,, \\ &\text{particle 3}: \quad y = -r, \qquad z = -r\,, \\ &\text{particle 4}: \quad y = r, \qquad z = -r\,. \end{aligned} \tag{12.112}$$

At $t = T/4$ $\cos\omega(t - \frac{x}{c}) = -1$, and

$$\begin{aligned} &\text{particle 1}: \quad y = r\,(1 - A_\times)\,, \qquad z = r\,(1 - A_\times)\ , \\ &\text{particle 2}: \quad y = -r\,(1 + A_\times) \qquad z = r\,(1 + A_\times)\ , \\ &\text{particle 3}: \quad y = -r\,(1 - A_\times) \qquad z = -r\,(1 - A_\times)\ , \\ &\text{particle 4}: \quad y = r\,(1 + A_\times) \qquad z = -r\,(1 + A_\times)\ . \end{aligned} \tag{12.113}$$

At $t = T/2$, $\cos\omega(t - \frac{x}{c}) = 0$ and the particles go back to the initial positions. At $t = 3T/4$, $\cos\omega(t - \frac{x}{c}) = 1$ and

$$\begin{aligned} &\text{particle 1}: \quad y = r\,(1 + A_\times)\,, \qquad z = r\,(1 + A_\times)\ , \\ &\text{particle 2}: \quad y = -r\,(1 - A_\times) \qquad z = r\,(1 - A_\times)\ , \\ &\text{particle 3}: \quad y = -r\,(1 + A_\times) \qquad z = -r\,(1 + A_\times)\ , \\ &\text{particle 4}: \quad y = r\,(1 - A_\times) \qquad z = -r\,(1 - A_\times)\ . \end{aligned} \tag{12.114}$$

The snapshots of the particles position at these times is shown in the upper panel of Fig. 12.4.

It follows that a small ring of particles centered at the origin, will again become an ellipse after a quarter of a period, but rotated at 45° respect to the case previously analysed (see the lower panel of Fig. 12.3). In conclusion, we can define A_+ and $A_\times$ as the **polarization amplitudes** of the wave. The wave will be linearly polarized when only one of the two polarization amplitudes is different from zero. The effect produced by a general wave containing both polarizations will be a superposition of the effects shown in Fig. 12.3 and Fig. 12.4.

12.8 GRAVITATIONAL WAVES AND MICHELSON INTERFEROMETERS

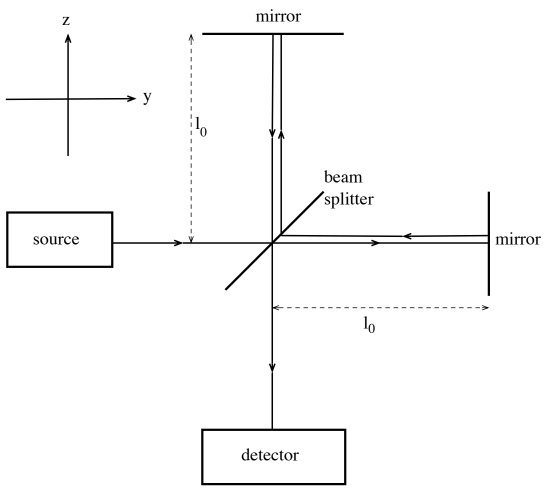

Figure 12.5: Schematic structure of a Michelson interferometer.

The Michelson inteferometer is a device consisting of two tubes (“arms”) orthogonal to each other. A source of light sends a light beam to a beam splitter (e.g. a half-silvered mirror), and the two parts of the beam are reflected by mirrors put at the end of the arms (see Fig. 12.5). These beams go back and forth along the arms, and when they reach the screen (the detector) they produce the interference pattern. During the XIX century, this instrument played a fundamental role in the crisis of Classical Physics, since it was used to prove that the speed of light is a universal constant, eventually leading to the formulation of Special Relativity. After one century and a half, in 2015, a similar device has been used in the LIGO experiment to detect - for the first time - the gravitational waves emitted by an astrophysical source [1]: the coalescence of a binary system composed by two black holes. In the following years, LIGO (and a similar interferometric detector, Virgo) detected several gravitational wave signals emitted by compact sources like neutron stars and black holes.

The LIGO and Virgo interferometers are of course much more sophisticated instruments than that used by Michelson in the XIX century: for instance the light beams are laser beams, they cross the arms back and forth tens of times before reaching the detector, where a photodetector replaces the screen; moreover in order to detect the incredibly small variation of the interference pattern induced by a gravitational wave, the interferometers must be accurately isolated from any source of noise. However, they work on the same basic principles of the Michelson instrument.

Let us assume, for instance, that the arms of the inteferometer lie in the y and z directions, and that a gravitational wave propagates in the x direction, with polarization '+' in the plane yz (see Eq. 12.103). When the wave crosses the interferometer, the proper lenghts of the two arms change, and the paths of the light rays change as well. The difference of the paths determines a shift in the interference pattern on the detector.

This description may appear too simplistic. One could remark, for instance, that the number of light wavelenghts contained in an arm does not change when the gravitational wave passes through the interferometer, because the arm and the wavelength are stretched by the same amount. Does the gravitational wave affect at all the interference pattern?

The answer to this question is "yes!", because the interference pattern is affected by the *time delay* in the light propagation, produced by the gravitational wave. In order to estimate this delay, we describe the interferometer and the gravitational wave (with $'+'$ polarization) in the TT gauge (see Eq. 12.81):

$$ds^2 = (\eta_{\mu\nu} + h^{TT}_{\mu\nu})dx^\mu dx^\nu = -c^2dt^2 + dx^2 + (1 + h_+)dy^2 + (1 - h_+)dz^2 \,. \qquad (12.115)$$

Let l_0 be the proper length of the two arms (between the beam splitter and the mirrors), measured in the frame 12.115 before the arrival of the wave, and let ω be the frequency of the gravitational wave. We assume, for simplicity, that the wavelength of the gravitational wave is much larger than the arm length l_0,[2] i.e. $2\pi c/\omega \gg l_0$. Thus, the gravitational perturbation h_+ can be considered constant as the light ray crosses the arm.

A light ray moving in the y direction follows a null geodesic with $c^2dt^2 = (1 + h_+)dy^2$, thus $dt = c^{-1}\,(1 + h_+/2)\,dy + O(h^2)$ and the time to cross back and forth the y-arm is

$$t_{(y)} = \left(1 + \frac{h_+}{2}\right)\frac{2l_0}{c} \,. \qquad (12.116)$$

A light ray moving in the z direction, instead, follows a null geodesic with $c^2dt^2 = (1 - h_+)dz^2$, therefore it crosses back and forth the z-arm in the time

$$t_{(z)} = \left(1 - \frac{h_+}{2}\right)\frac{2l_0}{c} \,. \qquad (12.117)$$

Therefore, although - as discussed in section 12.6 - the coordinate positions of the arm points in the TT gauge are not affected by the gravitational wave, the time needed to cross the arms *is* affected by the wave. When the rays join in the detector, there is a time delay

$$\Delta t = t_{(y)} - t_{(z)} = \frac{2l_0}{c}h_+ \qquad (12.118)$$

between them, which produces a shift $\sim c\Delta t = 2l_0h_+$ in the interference fringes (this shift was measured on a screen in the original Michelson-Morley experiment, while in modern gravitational-wave interferometers it is measured with a photodetector). If the amplitude of the wave is large enough, as we shall discuss in the the next chapters, this shift can be directly measured.

[2]For the existing interferometers this assumption is only marginally satisfied, i.e. $2\pi c/\omega \gtrsim l_0$.