Box 9-A

Geometrized Units

It is easy to check that $R_s = \frac{2GM}{c^2}$ has the dimension of a lenght. Indeed the constants G and c, whose values are

$$G = 6.674 \times 10^{-8}\ \frac{\mathrm{cm}^3}{\mathrm{g\,s}^2}, \qquad c = 2.998 \times 10^{10}\ \mathrm{cm/s},$$

have dimensions

$$[G] = \frac{l^3}{m\,t^2}, \qquad [c] = \frac{l}{t};$$

therefore

$$\left[\frac{G}{c^2}\right] = \frac{l}{m}, \quad \text{with} \quad \frac{G}{c^2} = 7.425 \times 10^{-29}\mathrm{cm} \cdot \mathrm{g}^{-1}. \tag{9.32}$$

It is often convenient to use *geometrized units* putting

$$G = c = 1. \tag{9.33}$$

In these units masses, lengths, and time have the same dimensions. To recover a quantity in physical units, it is necessary to multiply it by suitable powers of G and c. For example, the quantities

$$M, \quad \frac{GM}{c^2}, \quad \frac{GM}{c^3}, \tag{9.34}$$

have the same expression in geometrized units, but their physical dimensions are those of a mass, length, and time, respectively. Likewise the quantity

$$m = \frac{GM}{c^2} \tag{9.35}$$

has the dimension of a lenght, and it is usually referred to as the **geometrical mass** or the **gravitational radius**. As an example, the mass of the Sun $M_\odot = 1.989 \cdot 10^{33} g$, in geometrized units is equivalent to

$$m_\odot = \frac{GM_\odot}{c^2} = 7.425 \cdot 10^{-29} \times 1.989 \cdot 10^{33}\ \mathrm{cm} = 1.477 \cdot 10^{5}\ \mathrm{cm} = 1.477\ km. \tag{9.36}$$

Two useful conversion factors are the following:

$$m_\odot \approx 1.477\,\mathrm{km} \approx 4.926 \times 10^{-6}\,\mathrm{s}. \tag{9.37}$$

While the units with $G = c = 1$ are convenient in General Relativity, there exist other possible choices of geometrized units, such as those with $\hbar = c = 1$, used in Quantum Field Theory. We note that in the $G = c = 1$ units the mass has the dimension of a lenght, and Planck's constant has the dimension of a lenght squared; its value is $\hbar = l_P^2$, where $l_P \simeq 1.6 \cdot 10^{-35}$ m is called *Planck lenght*. In the $\hbar = c = 1$ units, instead, the mass has the dimension of an inverse lenght, while Newton's constant has the dimension of a lenght squared (with $G = l_P^2$).