Nuclear Electromagnetic Currents in χEFT with Applications

- Conventional approach: a review
- Nuclear χEFT approach
- Currents up to one loop (or N^{3}LO)
- EM observables at N^{3}LO in A=2–4 systems
- Summary and Outlook

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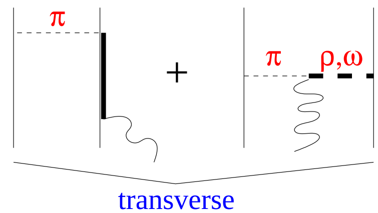
References: Pastore *et al.*, PRC**78**, 064002 (2008); PRC**80**, 034004 (2009)

Conventional Approach: EM Currents

Marcucci *et al.*, PRC**72**, 014001 (2005)

$$\mathbf{j} = \mathbf{j}^{(1)}$$

$$+\ \mathbf{j}^{(2)}(\mathrm{v}) +$$



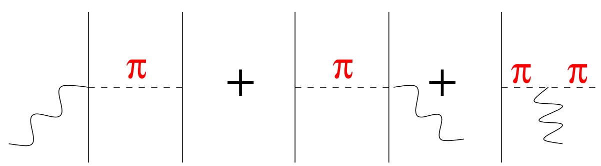
$$+\ \mathbf{j}^{(3)}(\mathrm{V}^{2\pi})$$

- Static part v_0 of v from π-like (PS) and ρ-like (V) exchanges
- Currents from corresponding PS and V exchanges, for example

$$\begin{aligned}\mathbf{j}_{ij}(v_0; PS) &= \mathrm{i}\left(\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j\right)_z \Big[v_{PS}(k_j)\boldsymbol{\sigma}_i \left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right) \\ &+ \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} v_{PS}(k_i)\left(\boldsymbol{\sigma}_i \cdot \mathbf{k}_i\right)\left(\boldsymbol{\sigma}_j \cdot \mathbf{k}_j\right)\Big] + i \rightleftharpoons j\end{aligned}$$

with $v_{PS}(k) = v^{\sigma\tau}(k) - 2\, v^{t\tau}(k)$ projected out from v_0 components

$$\mathbf{j}^{(2)}(\mathrm{v}) \xrightarrow[\text{long range}]{}$$



- Currents from v_p via minimal substitution in i) explicit and ii) implicit p-dependence, the latter from

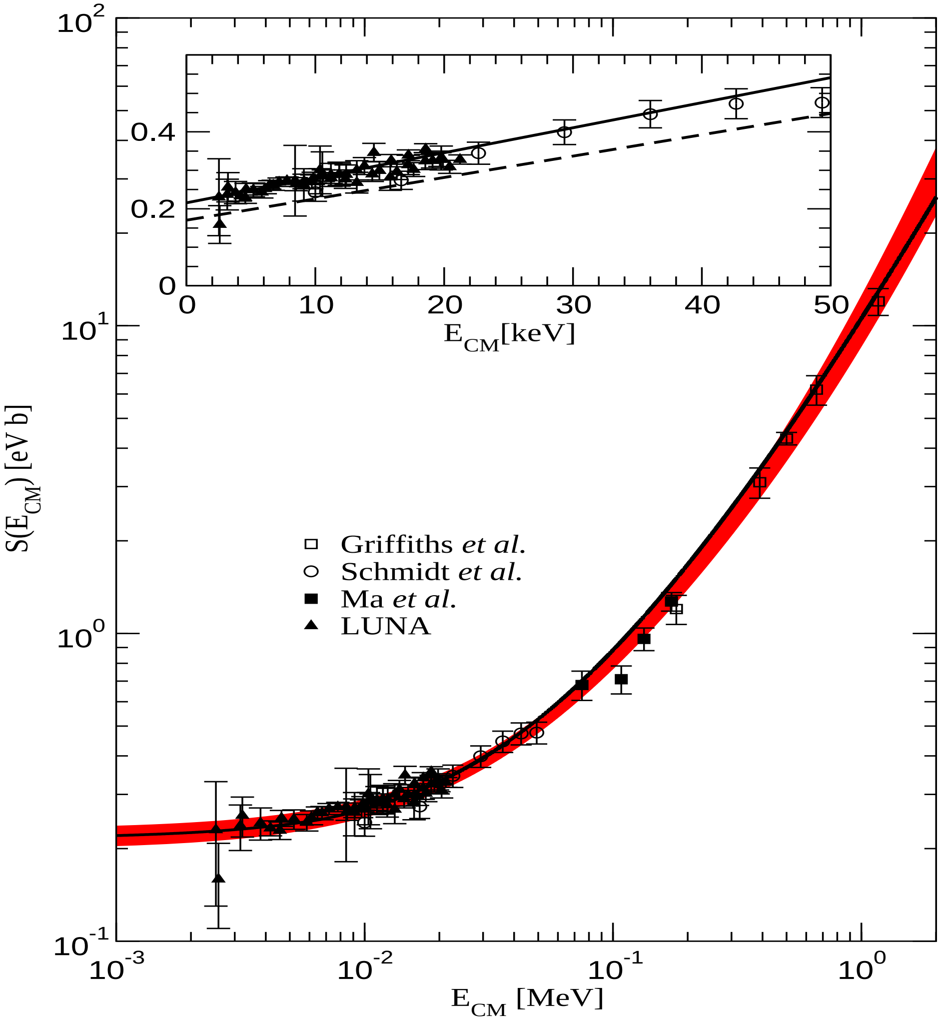
$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 + (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, \mathrm{e}^{\mathrm{i}(\mathbf{r}_{ji}\cdot\mathbf{p}_i + \mathbf{r}_{ij}\cdot\mathbf{p}_j)}$$

- Currents are conserved, contain no free parameters, and are consistent with short-range behavior of v and $V^{2\pi}$, but are not unique

Variety of EM observables in A=2–7 nuclei well reproduced, including μ's and $M1$ widths, elastic and inelastic f.f.'s, inclusive response functions, ...

but ^{2}H$(n,\gamma)^3$H and ^{3}He$(n,\gamma)^4$He cross-sections too large by $\approx 10\%$ and $\approx 60\%$, isoscalar μ's are a few % off (10% in A=7 nuclei), ...

$^{2}\mathrm{H}(p,\gamma)^{3}\mathrm{He}$ capture at low energies



Nuclear χEFT Approach

Weinberg, PLB**251**, 288 (1990); NPB**363**, 3 (1991); PLB**295**, 114 (1992)

- χEFT exploits the χ-symmetry exhibited by QCD to restrict the form of π interactions with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum Q, and $\mathcal{L}_{\rm eff}$ can be systematically expanded in powers of Q/Λ_χ ($\Lambda_\chi \simeq 1$ GeV)

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

- χEFT allows for a perturbative treatment in terms of a Q–as opposed to a coupling constant–expansion
- The unknown coefficients in this expansion–the LEC's–are fixed by comparison with experimental data
- Nuclear χEFT provides a practical calculational scheme, susceptible (in principle) of systematic improvement

Previous Work

Since Weinberg's papers (1990–92), nuclear χEFT has developed into an intense field of research. A *very* incomplete list:

- NN potentials:
  - van Kolck *et al.* (1994–96)
  - Kaiser, Weise *et al.* (1997–98)
  - Epelbaum, Glöckle, Meissner (1998–2005)
  - Entem and Machleidt (2002–03)
- Currents and nuclear electroweak properties:
  - Rho, Park *et al.* (1996–2009), hybrid studies in A=2–4
  - Epelbaum, Meissner *et al.* (2001, 2009)
  - Phillips (2003), deuteron static properties and f.f.'s

Lots of work in pionless EFT too ...

Preliminaries

- Degrees of freedom: pions (π) and nucleons (N)
- Time-ordered perturbation theory (TOPT):

$$
\begin{aligned}
-\frac{\hat{\mathbf{e}}_{\mathbf{q}\lambda}}{\sqrt{2\,\omega_q}}\cdot\mathbf{j} &= \langle N'N' \mid T \mid NN;\gamma\rangle \\
&= \langle N'N' \mid H_1 \sum_{n=1}^{\infty}\left(\frac{1}{E_i - H_0 + i\,\eta}H_1\right)^{n-1} \mid NN;\gamma\rangle
\end{aligned}
$$

- H_0 = free π and N Hamiltonians; H_1 = interacting π, N, and γ Hamiltonians implied by $\mathcal{L}_{\rm eff}$
- In general, a term with M H_1's leads to $M!$ time-ordered diagrams
- Irreducible and recoil-corrected reducible contributions retained in T expansion

Power Counting

- In the chiral expansion the transition amplitude is expressed as

  $$T = T^{LO} + T^{NLO} + T^{N^2LO} + \ldots\,, \text{ and } T^{N^nLO} \sim (Q/\Lambda_\chi)^n\, T^{LO}$$

  and power counting allows one to arrange contributions to T in powers of Q

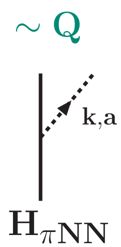
- A contribution with N interaction vertices and L loops scales as

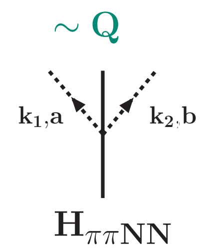
  $$\underbrace{e\left(\prod_{i=1}^{N} Q^{\alpha_i - \beta_i/2}\right)}_{H_1 \text{ scaling}} \times \underbrace{Q^{-(N-1)}}_{\text{denominators}} \times \underbrace{Q^{3L}}_{\text{loop integrations}}$$

  α_i = number of derivatives (momenta) and β_i = number of π's at each vertex

- This power counting also follows from considering Feynman diagrams, where loop integrations are in four dimensions

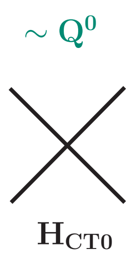
Strong Interaction Vertices up to Q^2

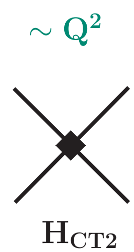




$$H_{\pi NN} = -i \frac{g_A}{F_\pi} \frac{\boldsymbol{\sigma} \cdot \mathbf{k}}{\sqrt{2\,\omega_k}} \tau_a \qquad H_{\pi\pi NN} = -\frac{i}{F_\pi^2} \frac{\omega_{k_1} - \omega_{k_2}}{\sqrt{4\,\omega_{k_1}\omega_{k_2}}} \epsilon_{abc}\,\tau_c$$

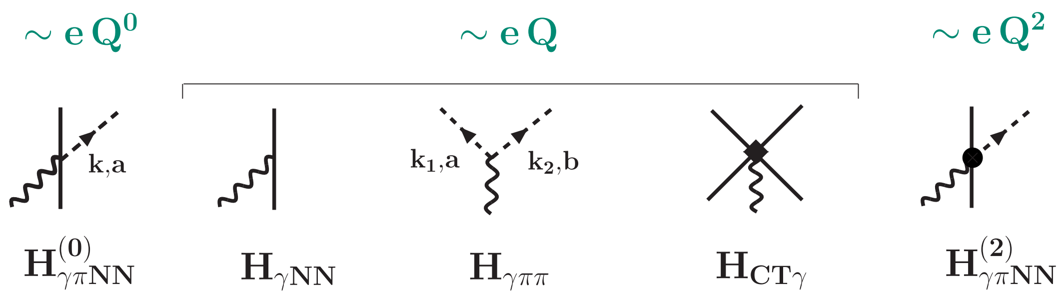
- $g_A = 1.29$ (via GT-relation) and $F_\pi = 184.8$ MeV





- $H_{\rm CT0}$: $4N$ contact terms, 2 LEC's
- $H_{\rm CT2}$: $4N$ contact terms with two gradients, 12 LEC's

Electromagnetic Interaction Vertices up to Q^2



- $H^{(0)}_{\gamma\pi NN}$, $H_{\gamma NN}$, and $H_{\gamma\pi\pi}$ known: depend on g_A, F_π, and proton and neutron μ's ($\mu_p = 2.793\,\mu_N$ and $\mu_n = -1.913\,\mu_N$)

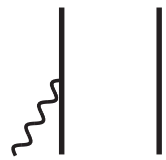
- $H_{\mathrm{CT}\gamma}$: terms from minimal substitution in $H_{\mathrm{CT}2}$ known, but 2 additional LEC's enter due non-minimal couplings

- $H^{(2)}_{\gamma\pi NN}$ from $\mathcal{L}_{\gamma\pi N}$ of Fettes *et al.* (1998): depends on 3 LEC's, two multiplying isovector structures ($\sim \gamma N\Delta$-excitation current) and one isoscalar structure ($\sim \gamma\rho\pi$ transition current)

Two-Body Currents up to N^{2}LO

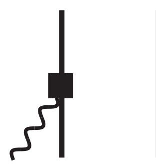
- Up to N^{2}LO

LO : $\mathbf{eQ^{-2}}$



NLO : $\mathbf{eQ^{-1}}$

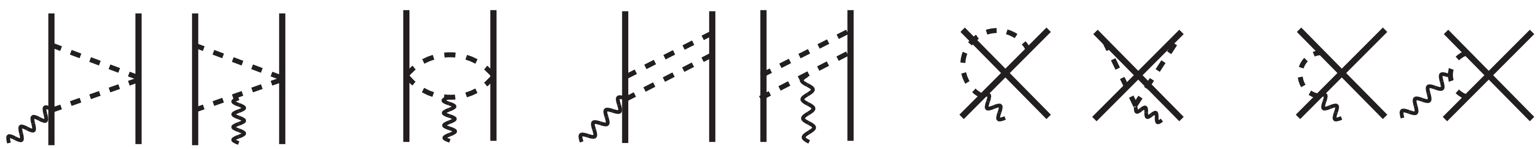
N^{2}LO : $\mathbf{eQ^{0}}$



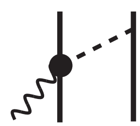
- One-loop corrections to one-body current absorbed into μ_N and $\langle r_N^2 \rangle$

Two-Body Currents at N^{3}LO

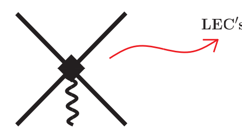
- One-loop corrections:



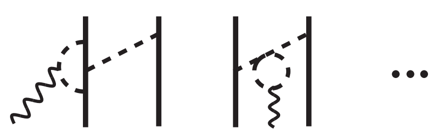
- Tree-level current with one $e\,Q^2$ vertex (3 LEC's):



- Currents from contact interactions (12 LEC's from minimal and 2 LEC's from non-minimal couplings):



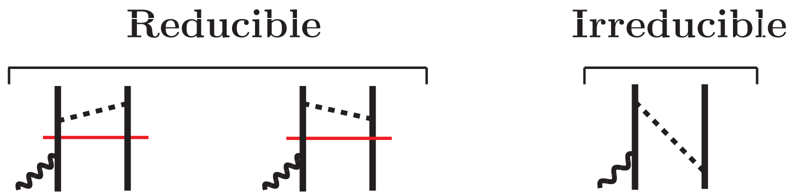
- One-loop renormalization of tree-level currents:



Technical Issues I: Recoil Corrections at N^{2}LO

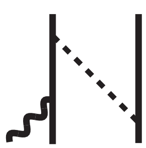
- N^{2}LO reducible and irreducible contributions in TOPT

$\mathbf{j}^{\mathrm{N^2LO}} =$



- Recoil corrections to the reducible contributions obtained by expanding in powers of $(E_i - E_I)/\omega_\pi$ the energy denominators

$$= v^\pi \left(1 + \frac{E_i - E_I}{2\,\omega_\pi}\right) \frac{1}{E_i - E_I}\, \mathbf{j}^{\mathrm{LO}}$$

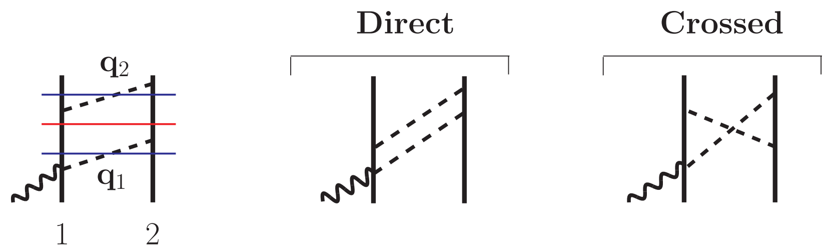


$$= -\frac{v^\pi}{2\,\omega_\pi}\, \mathbf{j}^{\mathrm{LO}}$$

- Recoil corrections to reducible diagrams cancel irreducible contribution

Technical Issues II: Recoil Corrections at N^3LO

$$\mathbf{j}^{\mathrm{N^3LO}} =$$



- Reducible contributions

$$\mathbf{j}_{\mathrm{red}} = \int v^{\pi}(\mathbf{q}_2)\,\frac{1}{E_i - E_I}\,\mathbf{j}^{\mathrm{NLO}}(\mathbf{q}_1)$$
$$-2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Irreducible contributions

$$\mathbf{j}_{\mathrm{irr}} = 2\int \frac{\omega_1+\omega_2}{\omega_1\,\omega_2}\,V_{\pi NN}(2,\mathbf{q}_2)\,V_{\pi NN}(2,\mathbf{q}_1)\,V_{\pi NN}(1,\mathbf{q}_2)\,V_{\gamma\pi NN}(1,\mathbf{q}_1)$$
$$+\; 2\int \frac{\omega_1^2+\omega_2^2+\omega_1\,\omega_2}{\omega_1\,\omega_2(\omega_1+\omega_2)}\,[V_{\pi NN}(2,\mathbf{q}_1), V_{\pi NN}(2,\mathbf{q}_2)]_-\;V_{\pi NN}(1,\mathbf{q}_2)V_{\gamma\pi NN}(1,\mathbf{q}_1)$$

- Partial cancellations between recoil corrections to reducible diagrams and irreducible contributions

Comparing to Park *et al.* (1996) and Kölling *et al.* (2009)

Expressions for pion-loop corrections in agreement with those of Bonn group (derived via the unitary transformation method)

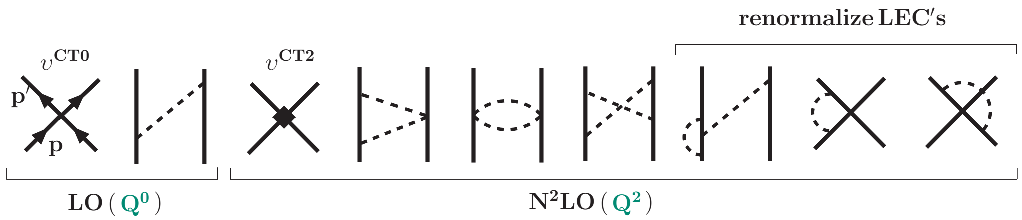
Differences relative to the expressions derived by Park *et al.*:

- Treatment of box diagrams (only irreducible diagrams retained in Park *et al.*) leads to different isospin structure for $\boldsymbol{\mu}$
- Short-range terms proportional to LEC's, for example loop corrections to contact C_S and C_T

$$\boldsymbol{\mu}^{\mathrm{N^3LO}}_{\mathrm{CT,loop}} = \frac{e\, g_A^2}{2\,\pi^2 F_\pi^2}\, \tau_{2,z}\, \left(C_S\, \boldsymbol{\sigma}_2 - C_T\, \boldsymbol{\sigma}_1\right) + 1 \rightleftharpoons 2$$

are ignored in Park *et al.*

Determining LEC's: NN Potential at $\mathrm{N^2LO}$

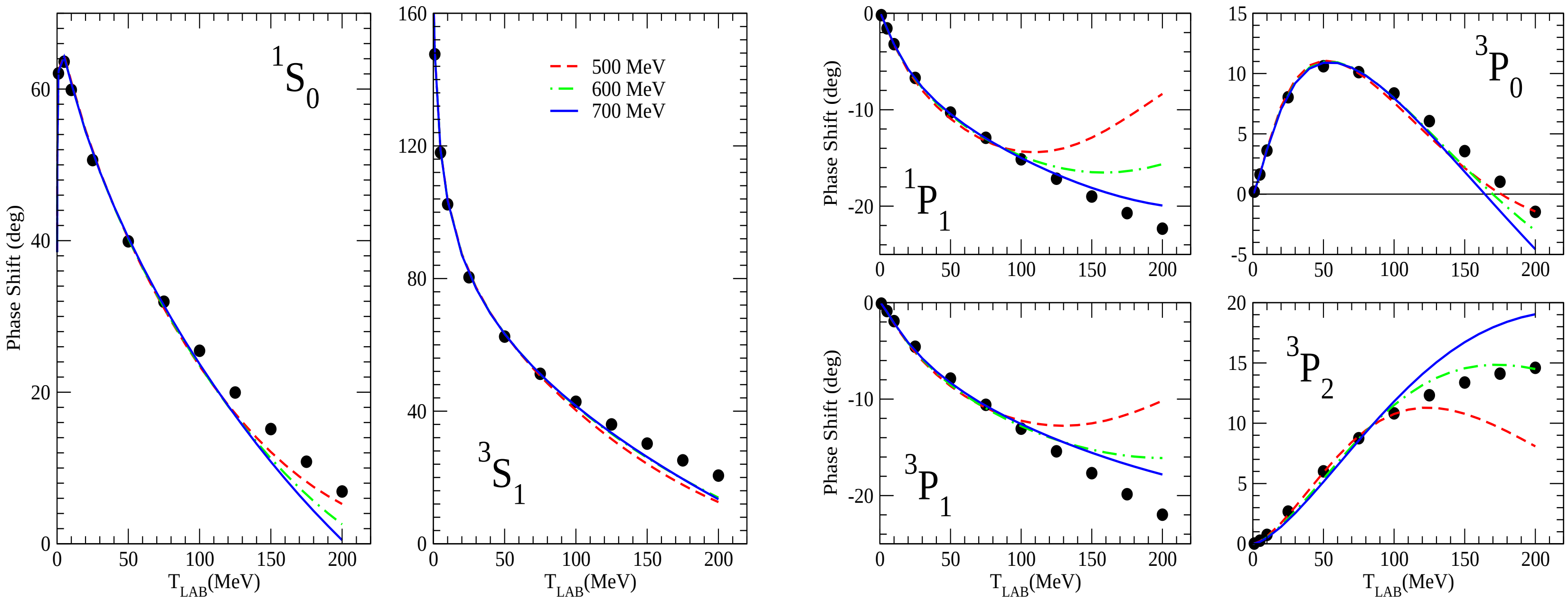


- Contact potential at $\mathrm{N^2LO}$: $v^{\mathrm{CT2}}(\mathbf{k},\mathbf{K}) + v^{\mathrm{CT2}}_{\mathbf{P}}(\mathbf{k},\mathbf{K})$
  - Galilean-invariant term v^{CT2} depends on 7 LEC's
  - Pair-momentum dependent term $v^{\mathrm{CT2}}_{\mathbf{P}}$ depends on 5 LEC's:

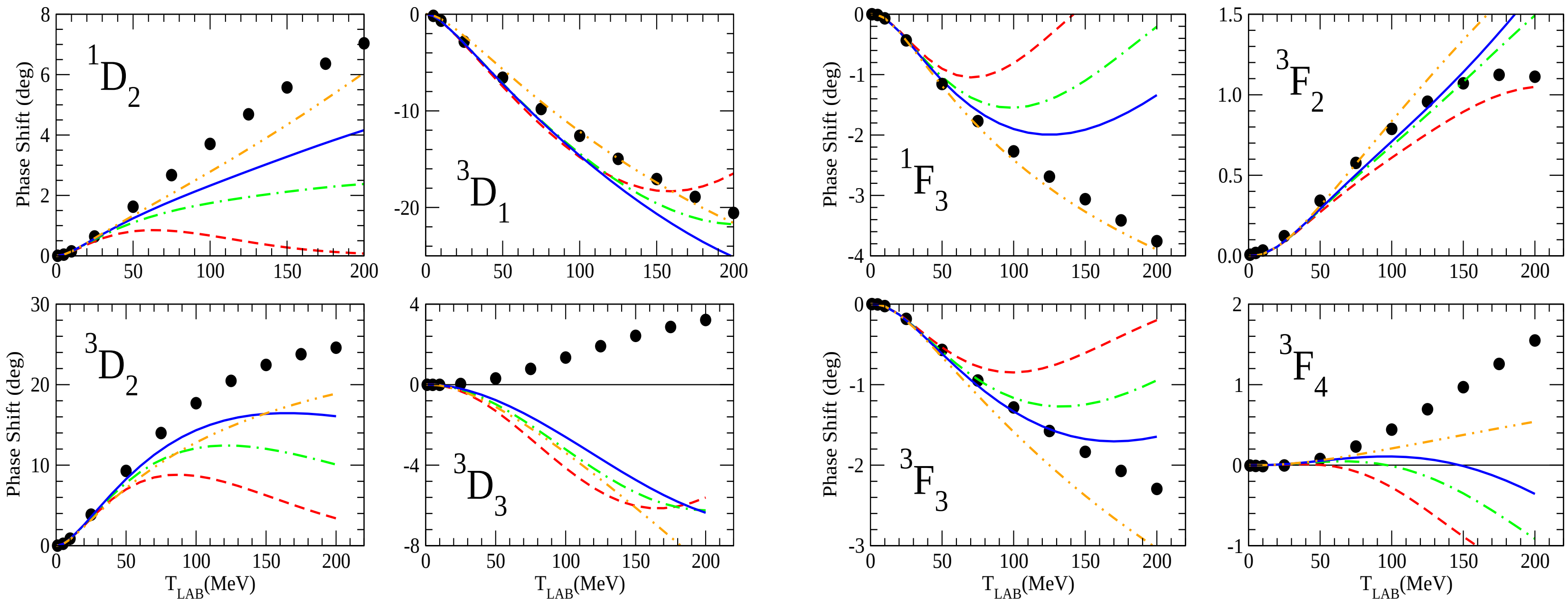
$$
\begin{aligned}
v^{\mathrm{CT2}}_{\mathbf{P}} &= i\, C_1^* \,\frac{\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2}{2} \cdot \mathbf{P} \times \mathbf{k} + C_2^* \,(\boldsymbol{\sigma}_1 \cdot \mathbf{P}\ \boldsymbol{\sigma}_2 \cdot \mathbf{K} - \boldsymbol{\sigma}_1 \cdot \mathbf{K}\ \boldsymbol{\sigma}_2 \cdot \mathbf{P}) \\
&+ \quad (C_3^* + C_4^*\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\, P^2 + C_5^*\, \boldsymbol{\sigma}_1 \cdot \mathbf{P}\ \boldsymbol{\sigma}_2 \cdot \mathbf{P}
\end{aligned}
$$

- Interpretation of $v^{\mathrm{CT2}}_{\mathbf{P}}$: boost correction to LO (rest-frame) v^{CT0}, then $C_1^* = (C_S - C_T)/(4m_N^2)$, $C_2^* = C_T/(2m_N^2)$, ...
- Retaining recoil corrections in both v and $\mathbf{j}$ ensures current conservation up to $\mathrm{N^3LO}$

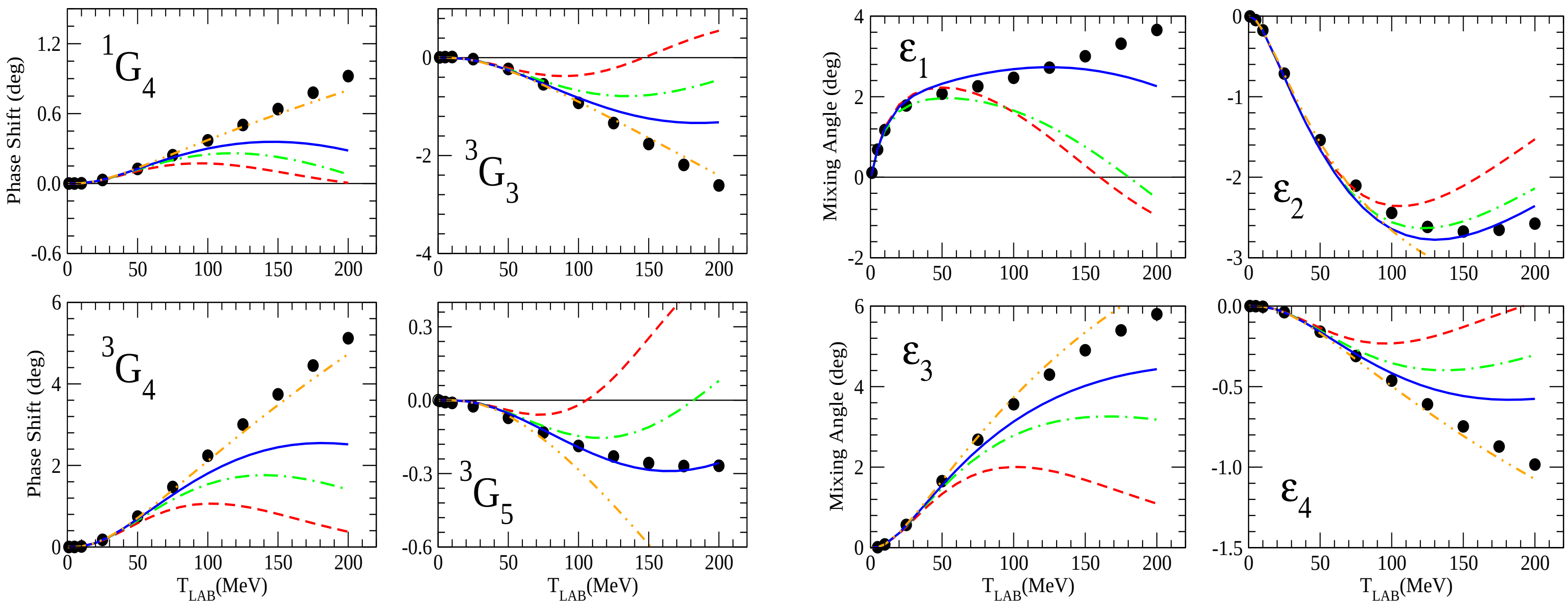
Fits to np Phases up to $T_{\rm LAB} = 100$ MeV



LS-equation regulator $\sim \exp(-2\,Q^4/\Lambda^4)$ with Λ=500, 600, and 700 MeV (cutting off momenta $Q \gtrsim 3$–$4\ m_\pi$)

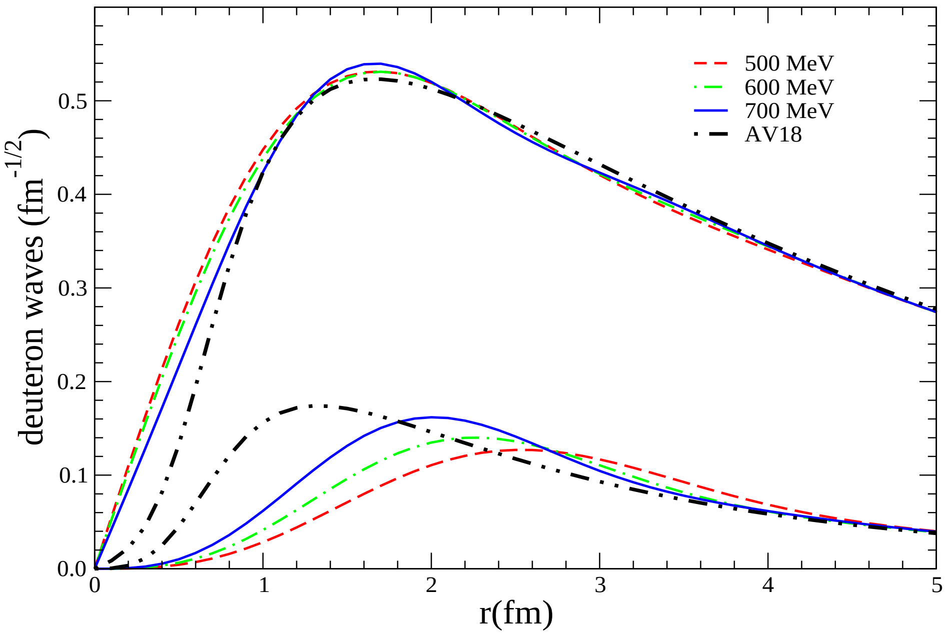


OPE+TPE chiral potential in first order PT, after Kaiser *et al.* (1997): orange dash-double-dot line



$^1G_4$
$^3G_3$
ε_1
ε_2
$^3G_4$
$^3G_5$
ε_3
ε_4
Phase Shift (deg)
Mixing Angle (deg)
T_{LAB}(MeV)

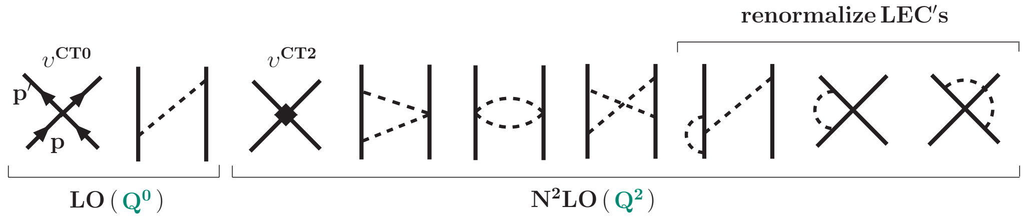
Deuteron Properties



	Λ (MeV)			
	500	600	700	Expt
B_d (MeV)	2.2244	2.2246	2.2245	2.224575(9)
η_d	0.0267	0.0260	0.0264	0.0256(4)
r_d (fm)	1.943	1.947	1.951	1.9734(44)
μ_d (μ_N)	0.860	0.858	0.853	0.8574382329(92)
Q_d (fm^2)	0.275	0.272	0.279	0.2859(3)
P_D (%)	3.44	3.87	4.77	

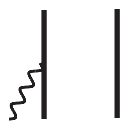
Nuclear χEFT

NN potential:

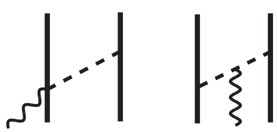


and consistent EM currents:

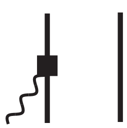
LO : eQ^{-2}



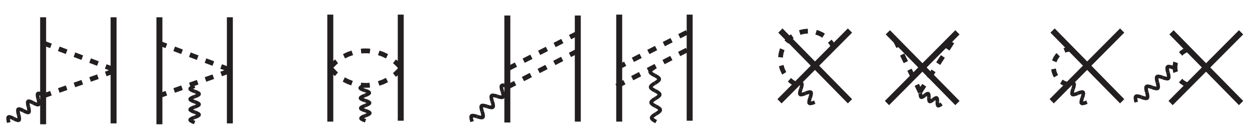
NLO : eQ^{-1}



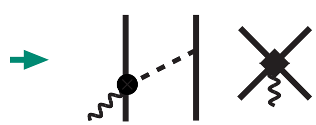
N^{2}LO : eQ^0



N^{3}LO : eQ

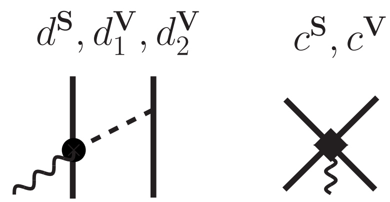


unknown LEC′s →

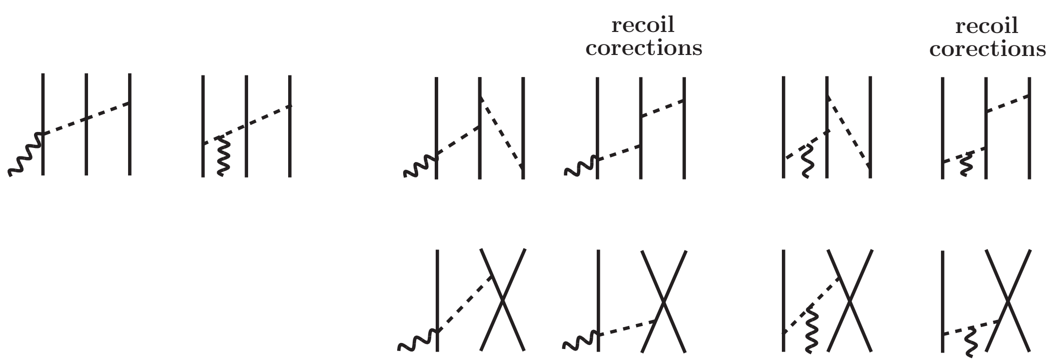


EM Observables at N^3LO

- Pion loop corrections known (g_A and F_π)
- Five LEC's: d^S, d_1^V, and d_2^V could be determined by pion photo-production data on the nucleon

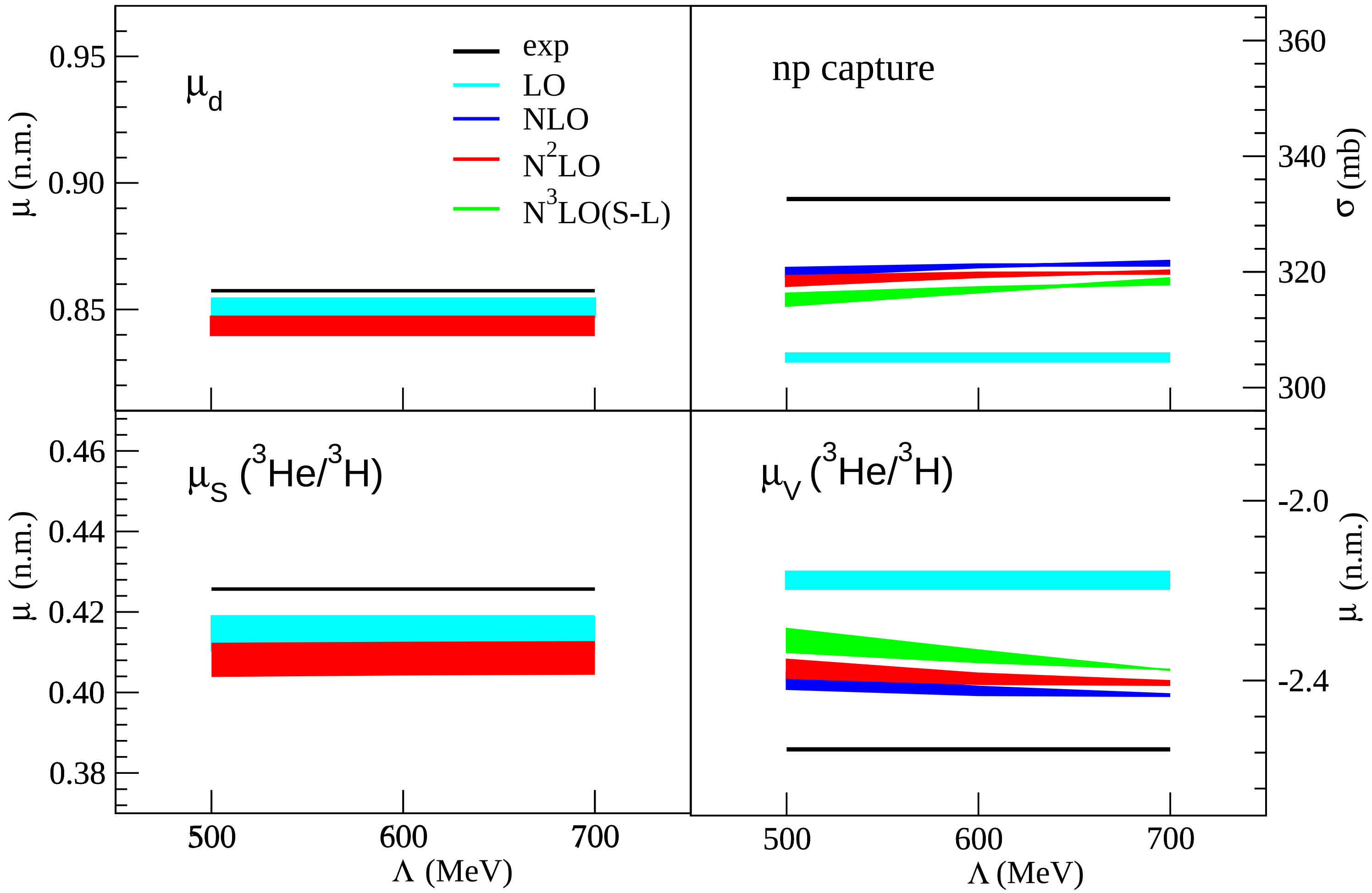


- $d_2^V/d_1^V = 1/4$ assuming Δ-resonance saturation
- Three-body currents at N^3LO vanish:

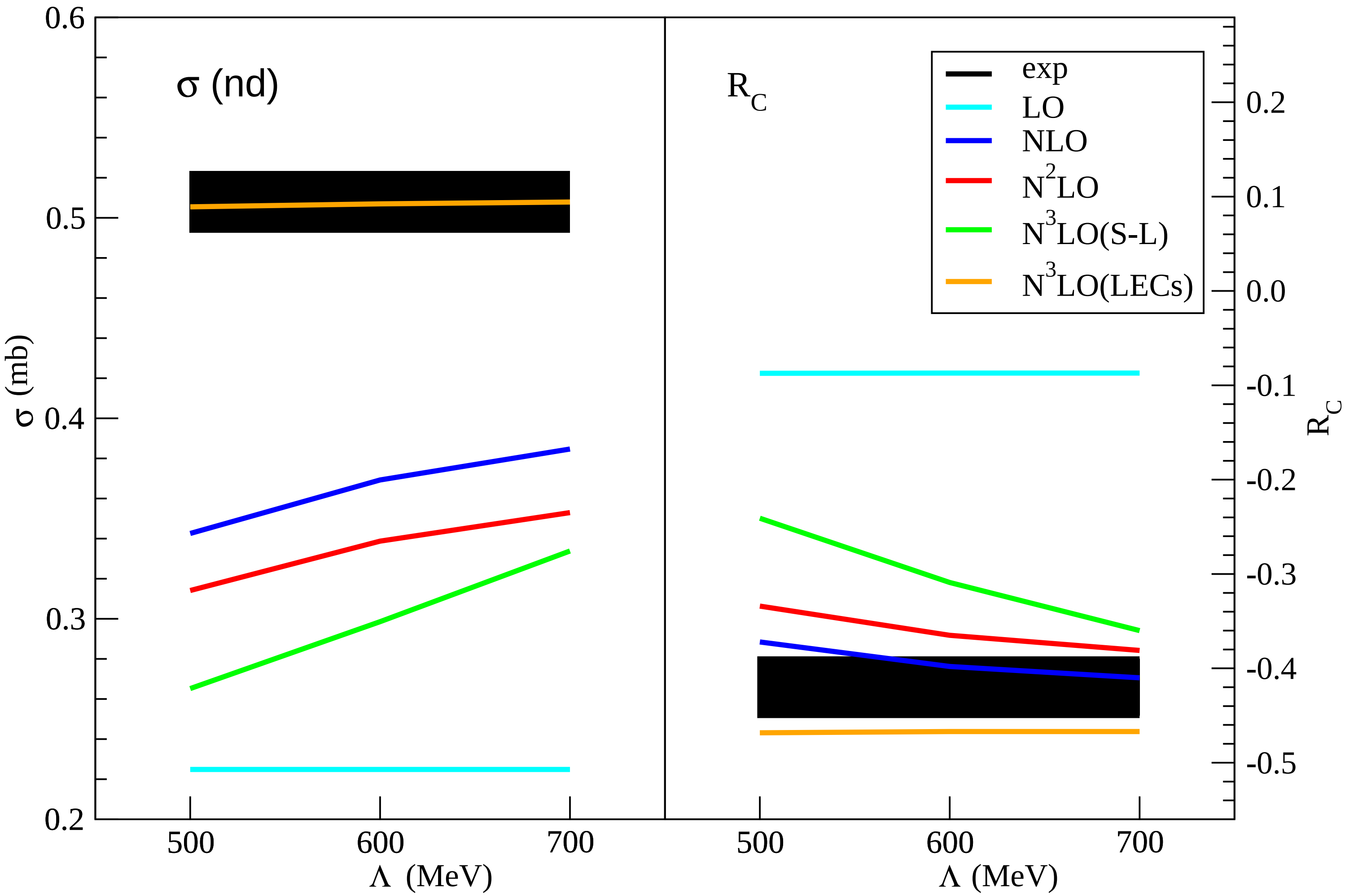


Fixing LEC's in EM Properties of A=2 and A=3 Nuclei

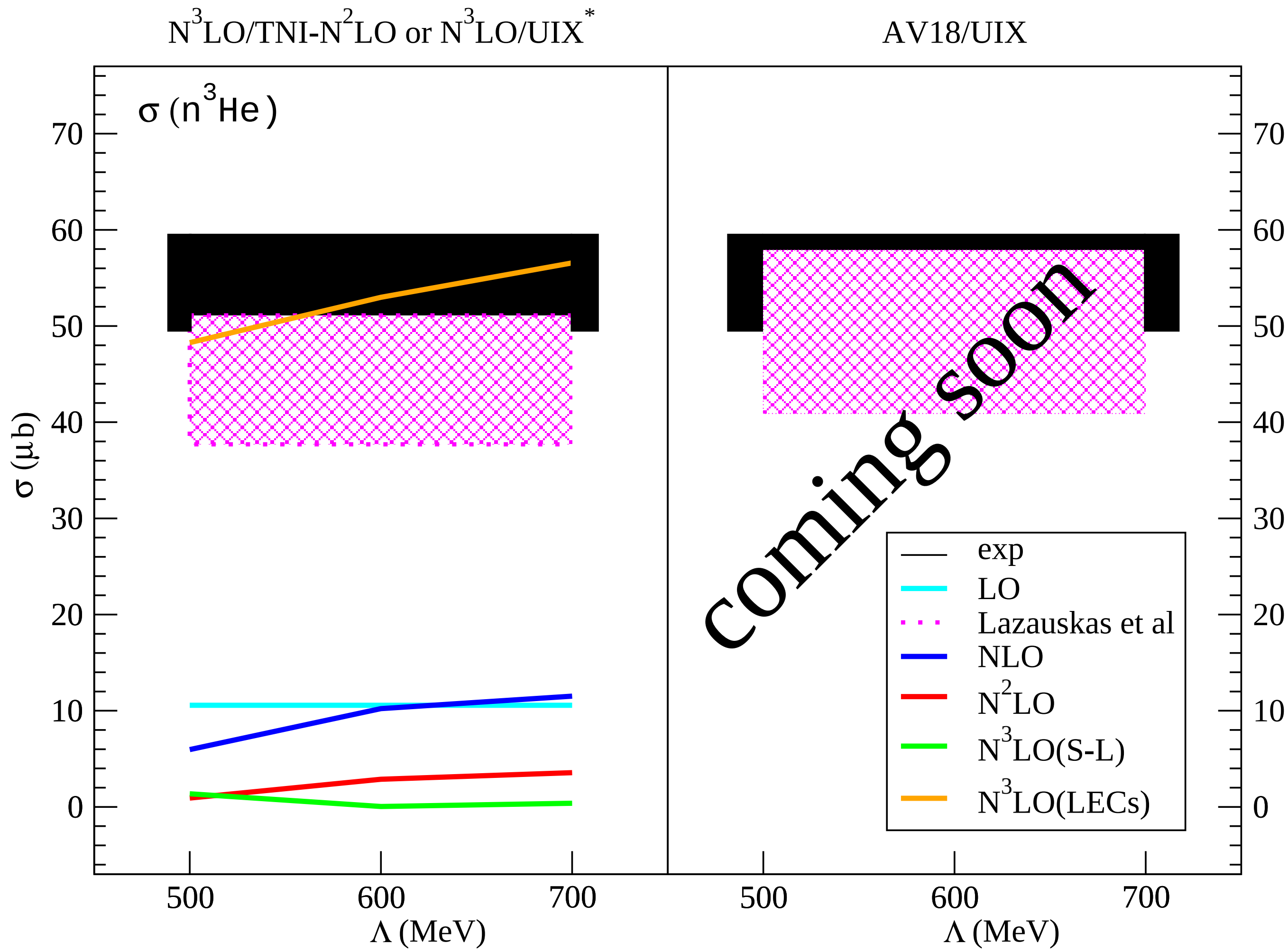
AV18/UIX or N^{3}LO/TNI-N^{2}LO (band)



Predictions (AV18/UIX) for nd Capture



Predictions for n ^{3}He Capture



Summary and Outlook

- Currents up to N^3LO derived in χEFT: in agreement with Kölling *et al.* (2009), but differences with Park *et al.* (1996)
- Hybrid predictions for nd ($n\,^3$He) capture in (reasonable) agreement with exp, and exhibit weak ($\simeq 10\%$) Λ-dependence
- Future work:
  1. Extend hybrid studies to different combinations of 2N and 3N potentials and up to $A = 7$ systems (in progress)
  2. Carry out consistent calculation—based on N^2LO potential—of A=2–4 observables (in progress)
  3. Include Δ-isobars in theory (should improve fits to phase shifts and reduce cutoff dependence)