A relativistic coupled-channels approach to the electromagnetic hadron structure

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Overview

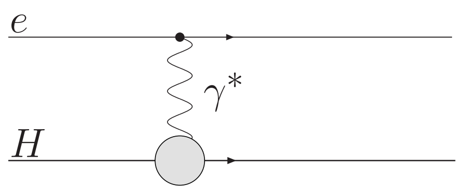
- Motivation, remarks on construction of currents within relativistic QM
- Relativistic coupled-channels formalism for π form factor
- Results and generalizations of the coupled-channel formalism
- Hadron dressing within a simple constituent-quark model
  $\longrightarrow$ decaying resonances

Motivation

Elastic electron-hadron scattering $\Longrightarrow$ information on internal hadron structure

1-γ-exchange approximation

$$\mathcal{M}_{1\gamma} \propto j^{\nu} \frac{g_{\nu\mu}}{Q^2} J^{\mu}$$



with hadron current

$$J^{\mu} = \sum_i F_i(Q^2)\, L_i^{\mu}(p_H, p'_H)$$

Form factors $F_i(Q^2)$ are observables $\Longrightarrow$ encode the e.m. structure of H

Fundamental question for microscopic description of FFs:

How is the e.m. current of H related to the e.m. currents of the constituents?

Try to give an answer within the framework of relativistic QM

- J^μ cannot be a simple sum of constituent currents!

  (A.J.F. Siegert, Phys. Rev. 52 (1937) 787)

  J^μ has to be interaction dependent if some of PC generators are

  $\Longrightarrow$ binding forces must show up in J^μ

Constraints on J^μ:

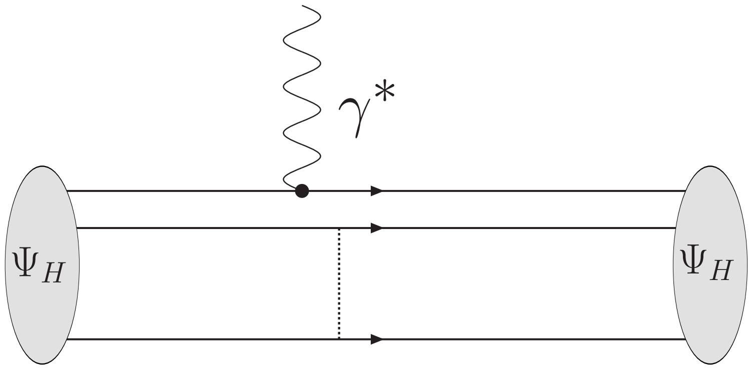
- Poincaré covariance $\rightarrow$ not very restrictive

  (cf. W.N. Polyzou and W.H. Klink, Ann. Phys. 185 (1988) 369)

- current conservation, i.e. $q_\mu J^\mu = 0$

- $Q_H = \sum_i Q_i$ (no renormalization of hadron charge by interaction)

Simplest ansatz for $J^\mu \longrightarrow$ spectator current (impulse approximation)



only 1 constituent active
couples to γ^* like a free particle

- can only be formulated consistently in point form of relativistic QM ($\Longrightarrow$ spectator character survives Lorentz transformations);
  but not the full photon virtuality Q^2 is transferred to the constituent
  (cf. R.F. Wagenbrunn et al., Phys. Lett. B 511 (2001) 33)
- in instant and front form a spectator current depends nontrivially on the choice of frame in which it is formulated!
  (it will not remain a spectator current in a boosted frame)

“Exact construction of e.m. current operators in relativistic QM”
by *F.Lev, Ann Phys. 237 (1995) 355*

$\Longrightarrow$ whole class of current operators for 2- and 3-particle systems
(Poincaré covariant, current conserved, right cluster properties)

- spectator current (for independent components) belongs to this class
- construction carried out in point form ($\rightarrow$ cluster properties)
- results transferred to instant and front form via unitary transformations
- construction repeated in front form and applied to π and deuteron FF
  (cf. F.M. Lev, E. Pace, G. Salmé, Nucl. Phys. A 641 (1998) 229

Ingredients: appropriate ansatz for J^μ + bound-state wave function

Our strategy: derive an electromagnetic current which is consistent with the binding forces

- work within a Poincaré invariant framework in which covariance properties are obvious (i.e. within point-form QM)
- treat electron scattering off a bound few-body system as multichannel problem in which the photon dynamics is explicitly taken into account
- analyze the one-photon exchange optical potential to single out the e.m. current J^μ of the bound system
- check whether J^μ has all the required properties
- extract electromagnetic form factors

E.m. pion form factor from a CQM

(E.P. Biernat et al., Phys. Rev. C 79 (2009) 055203)

Framework:

Point-form QM $\longrightarrow$ P^μ interaction dependent, $M^{\mu\nu}$ interaction free

Bakamjian-Thomas construction $\longrightarrow$ guarantees Poincaré invariance

$$P^\mu = P^\mu_{\text{free}} + P^\mu_{\text{int}} = (M_{\text{free}} + M_{\text{int}})\, V^\mu_{\text{free}} = M\, V^\mu_{\text{free}} \quad \text{with} \quad [M_{\text{int}}, V^\mu_{\text{free}}] = 0$$

Coupled-channels formulation of e-π scattering:

$$\begin{pmatrix} M_{eq\bar{q}} + V_{\text{conf}} & K^\dagger \\ K & M_{eq\bar{q}\gamma} + V_{\text{conf}} \end{pmatrix} \begin{pmatrix} |\Psi_{e\pi}\rangle \\ |\Psi_{e\pi\gamma}\rangle \end{pmatrix} = m \begin{pmatrix} |\Psi_{e\pi}\rangle \\ |\Psi_{e\pi\gamma}\rangle \end{pmatrix}$$

V_{conf} ... instantaneous confinement potential

Optical potential:

$$\Big(\overbrace{M_{eq\bar{q}} + V_{\rm conf}}^{M_{eC}} + \underbrace{K^\dagger \big(m - \big(\overbrace{M_{eq\bar{q}\gamma} + V_{\rm conf}}^{M_{eC\gamma}}\big)\big)^{-1} K}_{=V_{\rm opt}^{\rm mic}(m)}\Big)|\Psi_{e\pi}\rangle = m|\Psi_{e\pi}\rangle$$

$\mathcal{M}_{1\gamma} \longrightarrow$ (on-shell) matrix elements of $V_{\rm opt}^{\rm mic}(m = \sqrt{s})$ between e-π states

▶ Use a velocity-state representation:

$$|v; \vec{k}_1, \mu_1; \ldots; \vec{k}_n, \mu_n\rangle = \hat{U}_{B_c(v)}\, |\vec{k}_1, \mu_1; \ldots; \vec{k}_n, \mu_n\rangle \quad \text{with} \quad \textstyle\sum_{i=1}^n \vec{k}_i = 0\,.$$

▶ Insert eigenstates $M_{eq\bar{q}(\gamma)}$ and $M_{eC\gamma}$ at appropriate places

$$\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_\pi |\, 1'_{eq\bar{q}}\, \hat{K}^\dagger_{q\gamma} 1'''_{eq\bar{q}\gamma} \left(\hat{M}_{eC\gamma} - m\right)^{-1} 1''_{eC\gamma}\, 1''_{eq\bar{q}\gamma}\, \hat{K}_{q\gamma} 1_{eq\bar{q}}\,|\, v; \vec{k}_e, \mu_e; \vec{k}_\pi\rangle$$

$\Longrightarrow$ need to know matrix elements

$$\langle v; \vec{k}_e, \mu_e; \vec{k}_q, \mu_q; \vec{k}_{\bar{q}}, \mu_{\bar{q}} | \underline{v}; \underline{\vec{k}}_e, \underline{\mu}_e; \underline{\vec{k}}_C, n, j, \tilde{m}_j, [\tilde{l}, \tilde{s}] \rangle \ \ldots \ \text{bound-state wave fcts.}$$

$$\left(\propto \delta^3(\vec{v} - \underline{\vec{v}})\, \delta^3(\vec{k}_e - \underline{\vec{k}}_e)\, \delta_{\mu_e \underline{\mu}_e}\, \psi_{n,j,\tilde{m}_j,[\tilde{l},\tilde{s}]}(\vec{\tilde{k}}_q) \right)$$

$$\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_q, \mu'_q; \vec{k}'_{\bar{q}}, \mu'_{\bar{q}}; \vec{k}'_\gamma, \mu'_\gamma | \hat{K} | v; \vec{k}_e, \mu_e; \vec{k}_q, \mu_q; \vec{k}_{\bar{q}}, \mu_{\bar{q}} \rangle \ \ldots \ \text{vertex interact.}$$

$$\left(\propto \delta^3(\vec{v}' - \vec{v})\, \langle \vec{k}'_e, \mu'_e; \vec{k}'_q, \ldots \vec{k}'_\gamma, \mu'_\gamma | \left(\hat{\mathcal{L}}^{e\gamma}_{\text{int}}(0) + \hat{\mathcal{L}}^{q\gamma}_{\text{int}}(0) \right) | \vec{k}_e, \mu_e; \vec{k}_q, \mu_q; \vec{k}_{\bar{q}}, \mu_{\bar{q}} \rangle \right)$$

(cf. W.H. Klink, Nucl. Phys. A 716 (2003) 123)

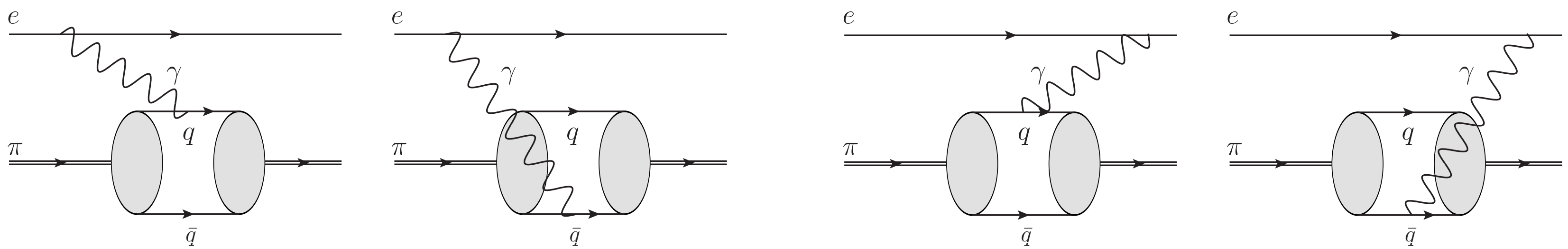
$\Longrightarrow$ E.m. vertex has been approximated to fit into the BT framework!

Assumption: total 4-velocity of the $eq\bar{q}$ system is conserved at e.m. vertices

(in general not true in interacting PFQFTs)

$$\langle v'; \vec{k}'_e, \mu'_e; \vec{k}'_\pi \,|\, V^{\mathrm{mic}}_{\mathrm{opt}}(\sqrt{s}) \,|\, v; \vec{k}_e, \mu_e; \vec{k}_\pi \rangle_{\mathrm{on-shell}}$$

$$\propto \delta^3(\vec{v}\,' - \vec{v}\,)\, j^\nu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e)\, \frac{g_{\nu\mu}}{Q^2}\, J^\mu(\vec{k}'_\pi; \vec{k}_\pi) \quad \text{for } |\vec{k}_e| = |\vec{k}'_e|,\ |\vec{k}_\pi| = |\vec{k}'_\pi|$$



$J^\mu(\vec{k}'_\pi; \vec{k}_\pi)$ ... integral over bound-state wfs. and Wigner rotation factors;
does not transform like a 4-vector under Lorentz trfs.!

BUT

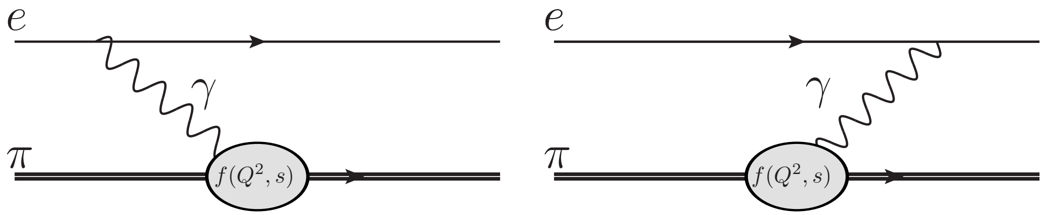
$J^\mu(\vec{p}\,'_\pi; \vec{p}_\pi) := (B_c(v))^\mu{}_\rho\, J^\rho(\vec{k}_\pi{}'; \vec{k}_\pi)$ .... transforms like a 4-vector
and it is a conserved current!

Renormalization of the charge by the binding force?

$\longrightarrow$ determine the e.m. π form factor (at $Q^2 = 0$)

Calculate the one-photon exchange optical potential on the hadronic level along the same lines as on the constituent level with a form factor $f(Q^2, s)$ at the π-γ vertex:

$V_{\rm opt}^{\rm had}$:



Form factor f can, in principle, depend on Mandelstam s and $t = -Q^2$ without spoiling Poincaré invariance ($\rightarrow$ but wrong cluster properties!)

$\Longrightarrow$ $f(Q^2, s)$ can be uniquely identified via $V_{\rm opt}^{\rm had} = V_{\rm opt}^{\rm mic}$:

$$f(Q^2, s) = \frac{j_\mu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e) J^\mu(\vec{k}'_M; \vec{k}_M)}{j_\mu(\vec{k}'_e, \mu'_e; \vec{k}_e, \mu_e)(\vec{k}'_\pi + \vec{k}_\pi)^\mu}$$

Model calculation for harmonic confinement

$\Longrightarrow$ Gaussian w.f. with 2 parameters $m_q = 0.21$ GeV, $a = 0.35$ GeV^2

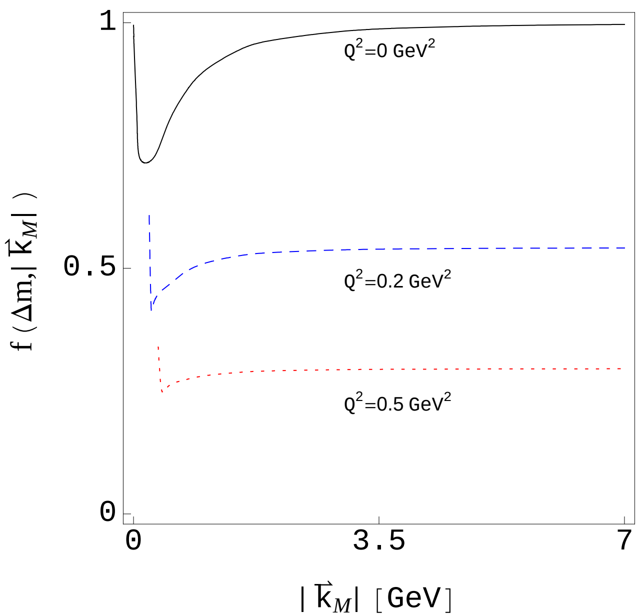
(taken from: P.L. Chung, F. Coester, and W.N. Polyzou, Phys. Lett. B 205 (1988) 545)

Dependence on $|\vec{k}_\pi|$:

$$\Delta m = Q = \sqrt{-t}$$

$$s = \sqrt{m_e^2 + \vec{k}_\pi^2} + \sqrt{m_\pi^2 + \vec{k}_\pi^2}$$

$$2|\vec{k}_\pi| > Q$$



s-dependence of $f \Longrightarrow$ wrong cluster properties!

BUT: s-dependence vanishes rather fast!

$\Longrightarrow$ take s large enough for a sensible extraction of the e.m. form factor!

Simple analytical expressions for current and form factor for $s \to \infty$

Final result:

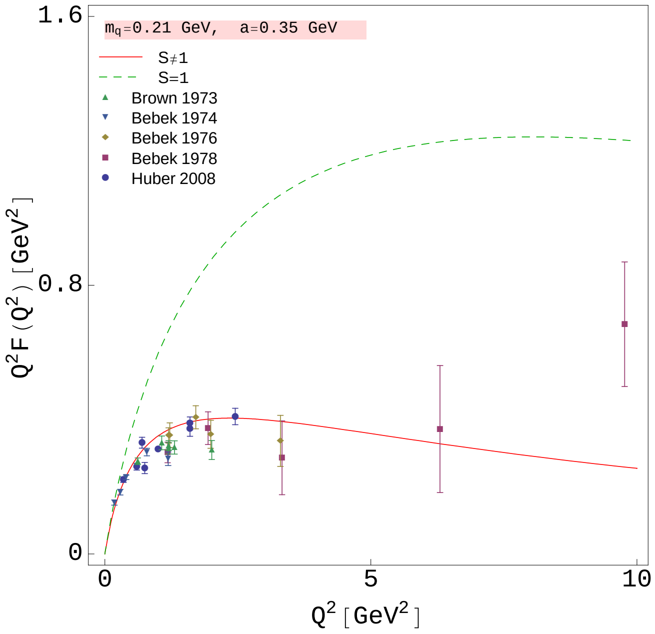
$$J^\mu(\vec{k}'_M; \vec{k}_M) \stackrel{s\to\infty}{\longrightarrow} (Q_q + Q_{\bar{q}})\,(k'_M + k_M)^\mu\, F_\pi(Q^2)\,.$$

with

$$F_\pi(Q^2) = \lim_{s\to\infty} f(Q^2, s) = \frac{1}{4\pi}\int \mathrm{d}^3\tilde{k}'_q \sqrt{\frac{m_{q\bar{q}}}{m'_{q\bar{q}}}}\,\mathcal{S}\, u^*\,(|\vec{\tilde{k}}'_q|)\, u\,(|\vec{\tilde{k}}_q|)$$

$\mathcal{S}$ ... spin-rotation factor coming from Wigner D functions

Numerical results:



Numerical results agree with front-form calculation of Chung, Coester and Polyzou

Spin rotation important!

Analytical comparison with front form:

Change of the integration variables

$$(\tilde{k}_q^{1\prime}, \tilde{k}_q^{2\prime}, \tilde{k}_q^{3\prime}) \to (\xi, \kappa_\perp^1, \kappa_\perp^2)$$

gives front-form expression for the spectator approximation in $q^+ = 0$ frame

$$F(Q^2) = \frac{1}{4\pi} \int_0^1 \mathrm{d}\xi \int_{\mathbb{R}^2} \mathrm{d}^2\kappa_\perp \frac{\sqrt{m_{q\bar{q}}\, m'_{q\bar{q}}}}{4\xi(1-\xi)} \times \mathcal{M}\, u^* (|\vec{\tilde{k}}'_q|)\, u\, (|\vec{\tilde{k}}_q|)$$

$\mathcal{M}$ ... Melosh rotation factor ($\mathcal{S}$ in front-form variables)

Summary

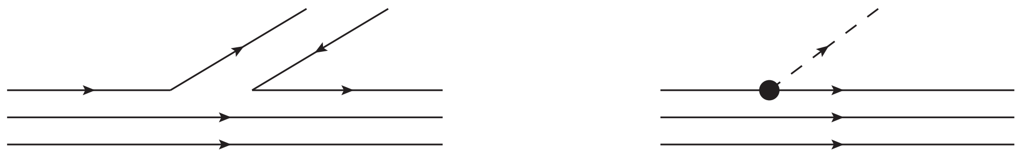
- We have considered electron scattering off a confined quark-antiquark system (with π quantum numbers) within a Poincaré-invariant multichannel framework.
- A conserved 4-vector current has been uniquely determined by comparing the 1γ-exchange optical potentials on the hadronic and the constituent levels
- Use of Bakamjian-Thomas framework $\Longrightarrow$ approximation to e.m. vertex and violation of cluster separability $\Longrightarrow$ e.m. current cannot be expressed only in terms p_M^μ, $p_M^{\mu\prime}$ and Q^2, but we need s-dependence of form factor
- s dependence gives us control on the unwanted features of our approach $\Longrightarrow$ can be overcome if s is taken large enough
- limit $s \to \infty$ provides a simple analytical expression for the π form factor which is equivalent to the front-form result obtained from a spectator current in a $q^+ = 0$ frame

Generalizations and open questions

- generalization to other few-body systems bound by instantaneous interaction is quite obvious
  - mesons with $S \neq 0$, other multiquark systems, few-nucleon systems, ...
  - equivalence with front-form $\Longrightarrow$ solution of angular-condition problem for deuteron form factors
- exchange currents can be treated on the same footing by including additional channels for the exchanged particles
- time-like form factors?
  have to study $e^+e^- \to \gamma^* \to H\bar{H}$, but unwanted features cannot be easily removed by taking $s \to \infty$ (s is argument of f.f.)
- weak and strong form factors

Decaying resonances within CQMs

- Within constituent quark models resonances usually come out as stable (∞ long lived) particles
- Decays are then described in leading-order perturbation theory by assuming a particular form of the decay vertex



- Decay widths notoriously too small!

Attempt to overcome these problems:

A. Krassnigg, W.H. Klink, and W.S., Phys. Rev. C 67 (2003) 064003

Vector-meson spectrum and (non-perturbative) widths calculated within the chiral constituent-quark model emplyoing the kind of coupled-channels approach just presented

χCQM: scalar confinement potential + hyperfine interaction mediated by pseudoscalar meson exchange ($\rightarrow$ couple directly to quarks)

- Good results for baryon spectra (with inst. approx. for meson exchange)

L.Y. Glozman et al., Phys. Rev. D 58 (1998) 094030

- Perturbative decay widths too small (like in any other CQMs)

B. Sengl et al., Phys. Rev. D 76 (2007) 025204

Way out(?): take dynamics of p.s. meson exchange fully into account within a coupled-channels framework and solve the eigenvalue equation for the mass operator

$$\Big(\overbrace{M_{q\bar{q}} + V_{\rm conf}}^{M_C} + \underbrace{K^\dagger \big(m - \big(\overbrace{M_{q\bar{q}\pi} + V_{\rm conf}}^{M_{C\pi}}\big)\big)^{-1} K}_{=V_{\rm opt}(m)}\Big)|\Psi_H\rangle = m|\Psi_H\rangle$$

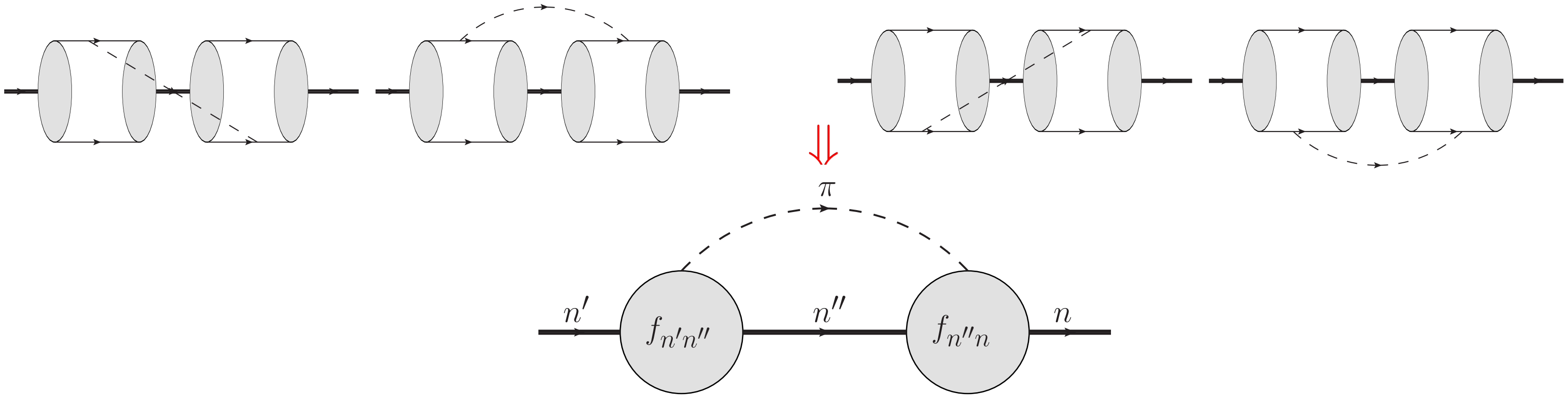
Expand $|\Psi_H\rangle$ in terms of eigenstates of M_C (bare hadrons): $|\Psi_H\rangle \sum_n A_n|n\rangle$

$|\Psi_H\rangle$ ... physical hadrons, $|n\rangle$ ... bare hadrons

$\Longrightarrow$ Nonlinear algebraic equation:

$$(m - m_n)A_n = \sum_{n'} \langle n|V_{\text{opt}}(m)|n'\rangle \, A_{n'}$$

with $\langle n|V_{\text{opt}}(m)|n'\rangle$:

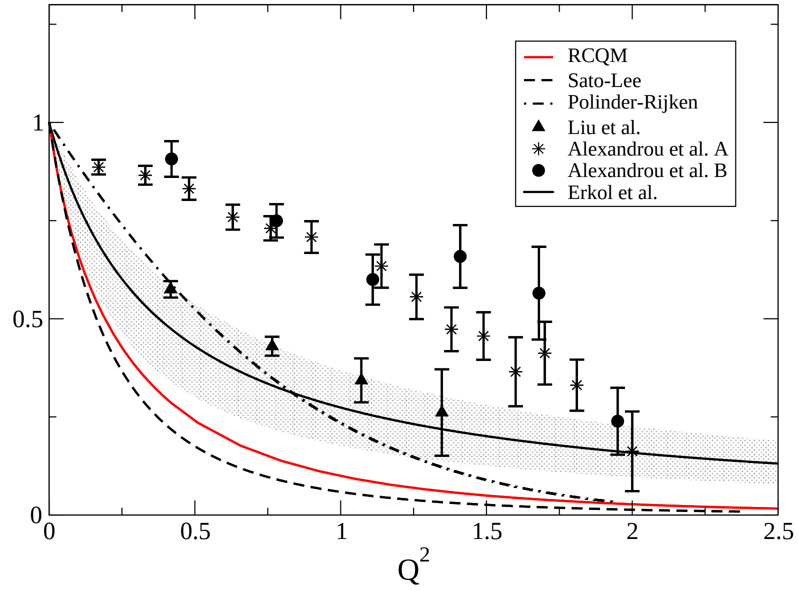


$\Longrightarrow$ eigenvalue problem for χCQM with instantaneous confinement and dynamical meson exchange between quarks can be reformulated as a purely hadronic eigenvalue problem for bare hadrons coupled via meson loops by introducing appropriate strong vertex form factors!

⟹ Hyperfine splitting + decays can be calculated (non-perturbatively) on the pure hadronic level

⟹ only strong vertex form factors have to be calculated on the quark level

→ first attempts to calculate strong vertex form factors within χCQM by *T. Melde, L. Canton, and W. Plessas, Phys. Rev. Lett 102 (2009) 132002*



Numerical studies of the full mass eigenvalue problem along these lines under way