NON PERTURBATIVE RENORMALIZATION SCHEME IN LIGHT FRONT DYNAMICS

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PLAN

❑ Motivations

❑ General framework : Light-Front dynamics

❑ Practical calculations : Fock space truncation
  ➢ Control of violation of rotational invariance
  ➢ Renormalization scheme / condition
  ➢ Appropriate regularization procedure

❑ First applications in Hadronic Physics
  ➢ Yukawa / QED in the two-body truncation
  ➢ Scalar / Yukawa in the three-body truncation
  ➢ Light Front Chiral Effective field theory

❑ Perspectives
Motivations

Calculation of relativistic bound state systems in Hadronic Physics

(like everybody in this room !!!)

➢ Solution of an eigenvalue equation
➢ Non-perturbative kernel, including self-energy type corrections
➢ Systematic procedure to improve any approximate calculations
General framework: Light Front Dynamics

- **Hamiltonian formulation**
  - In terms of the light-front time
    \[ \tau = t + \frac{z}{c} \]
    \[ \hat{P}^2 \phi(p) = M^2 \phi(p) \]
    with
    \[ \hat{P}_\mu = \hat{P}_\mu^0 + \hat{P}_\mu^{\text{int}} \]
    and
    \[ \langle p' | p \rangle = 2p_0 \delta^3(p' - \vec{p}) \]

- **No vacuum fluctuations**
  - Decomposition of the state vector \(|p\rangle\) in terms of Fock components with exactly \(i\) particles ("particle sector") plus eventually zero modes ("vacuum sector", not considered here)
    \[ |p\rangle = |1\rangle + |2\rangle + \ldots |N\rangle + \ldots \]
    \[ \Gamma_n \propto \langle n | p \rangle \quad \text{vertex function} \]
The boost operator along $z$ is kinematical

- No mixing between the various Fock components by the boost

Treat eventual divergences of amplitudes by appropriate regularization methods

- Very similar to N-body calculations (J. Vary) IF we know how to deal with renormalization

Renormalization conditions

- Impose renormalization condition in order to relate amplitudes on the light front to physical observables in QED (or Yukawa model)

$$\Gamma_2(s = M^2) \equiv g_{phys}$$

How this (very) general framework can work in practice?

- Truncation of the decomposition in Fock components
Fock space truncation

\[ |p\rangle = |1\rangle + |2\rangle + \ldots |N\rangle \]

控制违反旋转不变性的控制

\[ \tau = t + \frac{z}{c} \]

是不旋转不变的

- 或计算在微扰理论中需要恢复旋转不变性的贡献（例如对称或Z贡献）

- 或者识别并用适当的反项消除依赖于光锥位置的伪贡献，这仅可能在非微扰计算中

\[ \tau = t + \frac{z}{c} \rightarrow \tau = \omega \cdot x \hspace{1cm} \text{with arbitrary} \hspace{0.5cm} \omega, \hspace{0.5cm} \omega^2 = 0 \]

- 这是唯一可能的在非微扰计算中的光锥动力学明确守恒形式

Explicitly covariant formulation of light-front dynamics

First application: decomposition of the Fock components in (covariant) spin structures

ex. for the Yukawa model (fermion coupled to scalar boson)

\[ \bar{u}(p)\Gamma_2 u(k_1) = \bar{u}(p) \left[ b_1 + b_2 \frac{m\phi}{\omega.p} \right] u(k_1) \]

\[ \bar{u}(p)\Gamma_3 u(k_1) = \bar{u}(p) \left[ b_1 + b_2 \frac{m\phi}{\omega.p} + C_{ps} \left( b_1 + b_2 \frac{m\phi}{\omega.p} \right) \gamma_5 \right] u(k_1) \]

with \[ C_{ps} = \frac{1}{m^2\omega.p} e^{\mu\nu\rho\gamma} k_{2\mu} k_{3\nu} p_\rho \omega_\gamma \]

Since the Fock components are not observables, they DO depend in general on \( \omega \) (necessary to have the angular condition right)

On the energy shell however, they should be independent on \( \omega \) for an exact calculation

The renormalization condition \( \Gamma_2(s = M^2) \propto g \) implies \( b_2(s = M^2) = 0 \) to be inforced eventually by appropriate \( \omega \) - dependent counterterms
Renormalization scheme

- It should be consistent with the truncation of the Fock space

- Simple example: renormalization of the self-energy

\[ p^2 = m^2 \]

- Couples two different Fock components
  - one should keep track of the physical content of the counterterm as a function of the number of particles it corresponds to: \( \delta m^{(2)} \)
  - and more generally \( \delta m^{(n)} \)

- The same is true for the bare coupling constant \( g_0 \rightarrow g_0^{(n)} \)
In perturbation theory we have also a whole series of bare parameters/counterterms where \( n \) is the order in the perturbative expansion.

Here \( n \) is related to the number of particles “in flight”.

A calculation of order \( N \) involves \( \delta m^{(1)} \ldots \delta m^{(N)} \) and \( g_0^{(1)} \ldots g_0^{(N)} \).

\( \delta m^{(n)} \text{ and } g_0^{(n)} \) are calculated by successive solution of the \( N=1, 2, 3 \ldots N \) systems.

This is a systematic, non-perturbative, procedure which should avoid uncancelled divergences.

but also true if there are no divergences!
Renormalization condition

\[ \Gamma_2(s = M^2) \equiv g \]
if there is no Fock truncation

Otherwise, the dressing of the final fermion is not the same as the dressing of the initial one.

This should be corrected for in the renormalization condition

\[ \Gamma_2(s = M^2) \equiv g \sqrt{1 - I_N} \]

where \( I_N \) is the normalization of the N-th state, with \( I_1 + I_2 + \ldots + I_N = 1 \).

if the Fock expansion converges rapidly,

\[ I_N \approx 0 \]

and one recovers the “exact” renormalization condition.
Adequate regularization procedure

Naive cut-off procedure in x and $k_T$ : violation of rotational invariance

$$\Sigma(p) = A(p^2) + B(p^2) \frac{p}{M} + C(p^2) \frac{M \psi}{\omega \cdot p}$$

and $C(p^2) \neq 0$

Pauli-Villars regularization scheme

- Physical particle $\rightarrow$ Physical particles + Pauli-Villars particles
- Depending on the singularity of the amplitudes, many PV fermions and/or bosons
- Infinite mass limit to perform (numerically)
New regularization scheme: Taylor-Lagrange regularization scheme

H. Epstein - V. Glaser, J.M. Gracla-Bondia, P. Grangé, E. Werner

Field operators are treated as distributions which are defined on specific test functions

\[ T_x \Phi(\rho) = \langle \varphi, \rho \rangle = \int d^D \varphi(y) \rho(x - y) \]

Decomposition in momentum space

\[ \phi(x) = \int \frac{d^{(D-1)}}{(2\pi)^{(D-1)}} \frac{f(\omega_p^2, \vec{p}^2)}{2\omega_p} [a_p^+ e^{ipx} + a_p e^{-ipx}] \]

Adequate choice of test functions

- partition of unity: observables should be independent of the choice of test functions
  \[ f(x) = \sum_{j=0}^{N-1} u(x - j\hbar) \]
- Super regular test functions with all their derivatives equal to zero at the boundaries to treat all types of singularities at once
  \[ f \rightarrow R^k(f) \equiv f \quad \text{Taylor remainder} \]
Scaling properties provided by the boundary condition

In the UV domain

\[ f(X) = 0 \quad \text{at} \quad X = h \]

with \( h(X) = \mu^2 X^\alpha + \alpha - 1 \)

the limit \( f \to 1 \) corresponds to \( \alpha \to 1^- \)

Using the Lagrange formula

\[ f(X) = -\frac{X}{k!} \int_1^\infty \frac{dt}{t} (1 - t)^k \partial_X^{(k+1)} \left[ X^k f(Xt) \right] \]

one can define the extension of any distribution

\[ \langle T, f \rangle \equiv \langle \tilde{T}, f \rangle \to \langle \tilde{T}, 1 \rangle \quad \text{by partial integration,} \quad \tilde{T} \quad \text{is finite} \]

very easy to implement in Light-Front dynamics

\[ \Gamma_n \to \tilde{\Gamma}_n = \Gamma_n f_1 f_2 \ldots f_n \]

Direct relation to BPHZ scheme
First applications in Hadronic Physics

- **Two-body Fock space truncation**
  \[ |p⟩ = |1⟩ + |2⟩ \]
  - Equivalent to (resummed) first order perturbation theory
  - Anomalous magnetic moment of the electron in QED

- **Three-body Fock space truncation**
  \[ |p⟩ = |1⟩ + |2⟩ + |3⟩ \]
  - Scalar model (A. Smirnov)
  - Yukawa model: fermion coupled to two scalar bosons
Eigenvalue equation

\[ \Gamma_1 = \Gamma_2 = \Gamma_3 = g_0 \]

First results

\[ m = 1 \text{ GeV} \]
\[ \mu = 1 \text{ GeV} \]
\[ \alpha = 0.5 \]
Light Front Chiral Effective Field Theory

- A new way to look at chiral perturbation theory (N. Tsirova)

- Based on the Fock expansion of the state vector in terms of the number of pions, with a consistent expansion of the effective Lagrangian

\[ \mathcal{L}_{LFD}^N \equiv \mathcal{L}_{eff}^{p=2(N-1)} \]

- \( N + 2\pi \): \( \Delta \pi \) intermediate state automatic
  \( N - \rho, "\sigma" \) coupling also automatic

- \( NN + 2\pi \): Correlated \( 2\pi \) exchange to the NN potential
  All vertex corrections included
  All self-energy corrections also included consistently
Perspectives

- Light-Front dynamics: a very convenient tool to investigate relativistic bound state systems
  - Non-perturbative
  - Relativistic many-body physics (with 3 dim momenta)
  - Systematic strategy to improve the approximation (Fock expansion)

- Non-trivial - and very interesting! - physics already at the level of one fermion and two bosons
  - Convergence properties in the Yukawa model
  - New physical insight in chiral effective field theory