Light-Front Singularities

Some new developments presented

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Outline

Introduction

Light-Front Dynamics (LFD) is ideally suited for a description of relativistic processes because:

(i) A Fock-space expansion of many-particle states is valid owing to the simplicity of the Fock vacuum.

(ii) In LFD one works with physical degrees of freedom only. No negative-energy particles are included and the LF gauge is free of ghosts.

(iii) LFD treats physical systems at the amplitude level: LF wave functions are defined independently of the reference frame. They are boost invariant.

$$\text{Momenta} \quad q^{\pm} = \frac{q^0 \pm q^3}{\sqrt{2}},\ \mathbf{q}_{\perp} = (q^1, q^2)$$

$$\text{Dispersion relation} \quad q^{-} = \frac{m^2 + \mathbf{q}_{\perp}^2}{2q^{+}}, \quad q^{+} \geq 0$$

Components q^{+} and $\mathbf{q}_{\perp}$ are conserved in LFD.

Two approaches to Light-Front Dynamics (LFD):

Kogut and Soper [1]*: project on the light front* $\int dk^-$

Construct the Hamiltonian, see review by Brodsky, Pauli, and Pinsky [2]

There exist many pitfalls, treacherous points, ... in both approaches.

[1] J.B. Kogut and D.E. Soper, Phys. Rev. D **1**, 2901 (1970).

[2] S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, Phys. Rept. **301**, 299 (1998).

Taxonomy of Singularities

Type I singularities

$\int dk^-$ does not converge
(not discussed today)

Type II singularities

covariant amplitude A_{cov} is finite,
LF amplitude A_{LF} diverges

Anomalies

amplitude divergent, after renormalization $A_{\text{cov}} \neq A_{\text{LF}}$

Undefined NEW

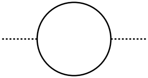
amplitude divergent in some kinematics,
doubt about interpretation

Spin-0 – spin-1/2 Yukawa model

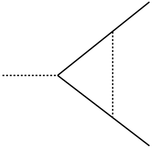
Primitive divergent diagrams

Self energy

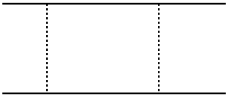
Vacuum polarization



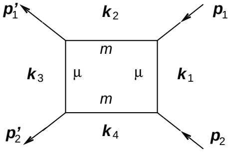
Vertex correction



The box diagram is **finite**



Calculation of the covariant box[a]



$$\mathcal{T} = \bar{u}(p'_1, s'_1)\bar{u}(p'_2, s'_2)\mathcal{M}u(p_1, s_1)u(p_2, s_2).$$

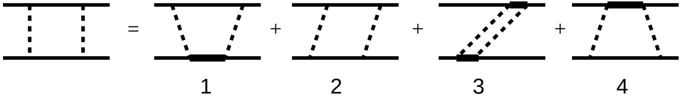
Following the usual procedure–Wick rotation, Feynman parameters, shift–one finds the expression for $\mathcal{M}$

$$\mathcal{M} = N\,6 \int_T d\alpha_1 \ldots d\alpha_4 \int \frac{d^4k}{(2\pi)^4}$$

$$\frac{(\gamma(1)\cdot k_2 + m)\,(\gamma(2)\cdot k_4 + m)}{[\alpha_1(k_1^2-\mu^2)+\alpha_2(k_2^2-m^2)+\alpha_3(k_3^2-\mu^2)+\alpha_4(k_4^2-m^2)]^4}$$

[a] B.L.G. Bakker, J.K. Boomsma, and C.-R. Ji, Phys. Rev. D **75** 065010 (2007)

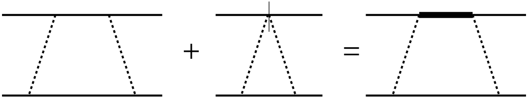
Ligh-front perturbation theory



Perform the usual k^--integration to expand the covariant box in light-front amplitudes

$$\mathcal{T} = \sum_d \mathcal{T}^d, \quad d = 1, \ldots 4.$$

We use the blink construction here to remove the cancelling singularities of the fermion propagators, e.g.



The LF amplitude with blinks is obtained adding the amplitude with an instantaneous fermion propagator to the amplitude with LF fermion propagators only.

Miranda van Iersel, Thesis, (2004)

All LF time ordered boxes are divergent. The divergences are due to the k^- dependence of the fermion propagators.

The sum of **all** divergences

instantaneous parts, stretched box ($d = 3$), ...

vanishes.

Jorn Boomsma (2007)

We need to regularize the LF amplitudes. We used:

DR_2, dimensional regularization in the perpendicular momenta

or

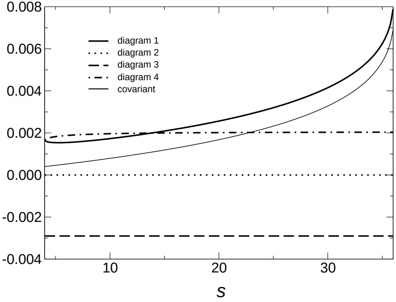
Pauli-Villars for the boson.

Singular parts:

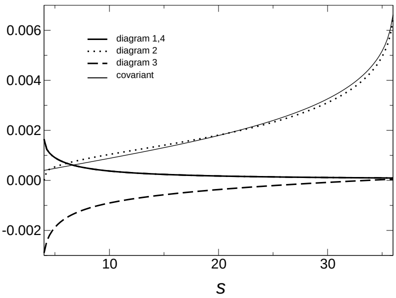
$$C_d \, \frac{1}{\bar{\epsilon}} \, (\mathrm{DR}_2), \; C_d \, \log \Lambda^2, \; (\mathrm{PV}), \; d = 1, \ldots, 4, \quad \sum_d C_d = 0$$

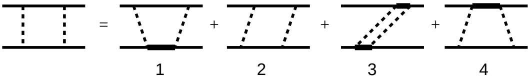
Results for the Yukawa box

Forward matrix element $\mathcal{T}_{11}$



Backward matrix element $\mathcal{T}_{11}$





Light-Front Anomalies[a]

The Yukawa model is not a fundamental theory, so study the renormalization of the CP-even EM weak boson vertex in the Weinberg-Salam sector of the Standard Model.

The Lorentz-covariant and gauge-invariant CP-even electromagnetic $\gamma W^+ W^-$-vertex is defined in the literature.

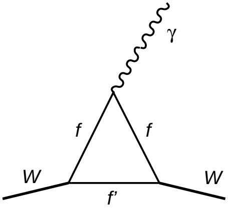
$$\Gamma^\mu_{\alpha\beta} = i\,e\left\{A[(p+p')^\mu g_{\alpha\beta} + 2(g^\mu_\alpha q_\beta - g^\mu_\beta q_\alpha)] + (\Delta\kappa)(g^\mu_\alpha q_\beta - g^\mu_\beta q_\alpha) + \frac{\Delta Q}{2M_W^2}(p+p')^\mu q_\alpha q_\beta\right\},$$

At tree level,

$$A = 1, \quad \Delta\kappa = 0, \quad \Delta Q = 0,$$

for any $Q^2 = -q^2$ because of the point-like nature of $W^\pm$ gauge bosons.

[a] B.L.G. Bakker and C.-R. Ji, Phys. Rev. D **71**, 053005 (2005).



The lowest-order correction beyond beyond tree level is given by the triangle diagram where the particles in the loop are the fermions of the SM. The observables are given by

$$A = F_1(Q^2), \quad -\Delta\kappa = F_2(Q^2) + 2F_1(Q^2), \quad -\Delta Q = F_3(Q^2),$$

F_1, F_2 and F_3 are defined by the relation to the current matrix elements:*i.e.*, $\Gamma^{\mu}_{\alpha\beta} = -i\, eJ^{\mu}_{\alpha\beta}$ and

$$J^{\mu}_{\alpha\beta} = -(p+p')^{\mu} g_{\alpha\beta}\, F_1(Q^2) + (g^{\mu}_{\alpha}\, q_{\beta} - g^{\mu}_{\beta}\, q_{\alpha})\, F_2(Q^2) + \frac{q_{\alpha} q_{\beta}}{2M_W^2}(p+p')^{\mu} F_3(Q^2).$$

One can define helicity matrix elements as follows

$$G^{\mu}_{h'h} = \epsilon^*(p', h')_{\alpha}\, J^{\mu}_{\alpha\beta}\, \epsilon(p, h)_{\beta}.$$

Two regularizations, DR_4, the usual dimensional regularization, and PV_1, Pauli-Villars regularization involving the struck fermion only.
In the manifestly covariant calculation, we first obtain the form factors $F_i (i = 1, 2, 3)$ using dimensional regularization DR_4:

$$
\begin{aligned}
F_1(Q^2) &= \frac{g^2 Q_f}{4\pi^2}\left\{-\frac{2}{3}\left(\frac{1}{\epsilon} - \gamma - \frac{1}{2}\right)\right. \\
&+ \left.\int_0^1 dx \int_0^{1-x} dy \left[-(2-x-y)\ln\frac{4\pi\mu^2}{M_W^2 C_{\rm cov}^2} + \frac{1}{2}\frac{f_1^0}{M_W^2 C_{\rm cov}^2}\right]\right\} \\
F_2(Q^2) &= \frac{g^2 Q_f}{4\pi^2}\left\{\frac{4}{3}\left(\frac{1}{\epsilon} - \gamma - \frac{1}{2}\right)\right. \\
&+ \left.\int_0^1 dx \int_0^{1-x} dy \left[(2+x+y)\ln\frac{4\pi\mu^2}{M_W^2 C_{\rm cov}^2} + \frac{1}{2}\frac{f_2^0}{M_W^2 C_{\rm cov}^2}\right]\right\} \\
F_3(Q^2) &= \frac{g^2 Q_f}{4\pi^2}\int_0^1 dx \int_0^{1-x} dy\, \frac{8xy(x+y-1)}{C_{\rm cov}^2}.
\end{aligned}
$$

x and y are Feynman parameters.

The functions f_1^0 and f_2^0, and f_3^0 are

$$
\begin{aligned}
f_1^0 &= -2[(x+y)(1-x-y)^2 M_W^2 + (2-x-y)xyq^2 + (x+y)m_1^2], \\
f_2^0 &= 2(x+y)[(1-(x+y)^2)M_W^2 + xyq^2 + m_1^2].
\end{aligned}
$$

The parts f_i^0 give finite results.

The red parts came from an integrand containing k'^2, and give divergent results that need regularization.

The physical quantity, $-\Delta\kappa$, corresponds to

$$\begin{aligned} F_2(Q^2) + 2F_1(Q^2) &= \frac{g^2 Q_f}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \\ &\times \left[-(2-3x-3y) \ln \frac{4\pi\mu^2}{M_W^2 C_{\rm cov}^2} + \frac{1}{2} \frac{2f_1^0 + f_2^0}{M_W^2 C_{\rm cov}^2} \right] \\ &= \frac{g^2 Q_f}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \left[(2-3x-3y) \ln C_{\rm cov}^2 \right. \\ &+ \left. \frac{1}{2} \frac{2f_1^0 + f_2^0}{M_W^2 C_{\rm cov}^2} \right]. \end{aligned}$$

The singular part of $2F_1(Q^2) + F_2(Q^2)$ vanishes upon integration over x and y and the dependence on the mass scale μ also vanishes when integrated over x and y.

F_1 and F_2 are given by divergent integrals. After regularization they can be renormalized. Then $F_1(0) = 1$ and $-\Delta\kappa$ becomes a prediction. F_3 is given by a convergent integral.

The results for PV_1 calculated in a manifestly covariant way are

$$F_1(Q^2) = \frac{g^2 Q_f}{4\pi^2}\left\{-\frac{2}{3}\ \ln\frac{\Lambda^2}{M_W^2}+\frac{8}{9}+\int_0^1 dx\int_0^{1-x} dy\left[-(2-x-y)\ \ln\frac{1}{C_{\rm cov}^2}+\frac{1}{2}\frac{f_1^0}{M_W^2 C_{\rm cov}^2}\right]\right\},$$

$$F_2(Q^2) = \frac{g^2 Q_f}{4\pi^2}\left\{\frac{4}{3}\ \ln\frac{\Lambda^2}{M_W^2}-\frac{10}{9}+\int_0^1 dx\int_0^{1-x} dy\left[(2+x+y)\ \ln\frac{1}{C_{\rm cov}^2}+\frac{1}{2}\frac{f_2^0}{M_W^2 C_{\rm cov}^2}\right]\right\},$$

$$2F_1(Q^2)+F_2(Q^2) = \frac{g^2 Q_f}{4\pi^2}\times \left\{\frac{2}{3}+\int_0^1 dx\int_0^{1-x} dy\left[(2-3x-3y)\ \ln C_{\rm cov}^2+\frac{1}{2}\frac{2f_1^0+f_2^0}{M_W^2 C_{\rm cov}^2}\right]\right\}.$$

In LFD the form factors cannot be derived directly from the current tensor $J^{\mu}_{\alpha\beta}$, but must be extracted form the helicity matrix elements $G^{\mu}_{h'h}$.
We have essentially two options for F_2

$$F_2^{+0} = \frac{1}{p^+}\left[-G^+_{++} + \frac{1}{\sqrt{2\eta}}G^+_{+0}\right],$$

$$F_2^{00} = \frac{1}{p^+}\left[(1-2\eta)G^+_{++} + G^+_{+-} - G^+_{00}\right],$$

$$\eta = \frac{-q^2}{4M_W^2}.$$

These options should give the same form factor F_2 if our calculations are correct.

In LFD DR_4 cannot be applied, although PV_1 can. The closest one can come to DR_4 is to use DR_2.

Using the usual techniques to obtain the LF amplitudes we find the observables

$$(F_2^{+0} + 2F_1)^{DR_2} - (F_2 + 2F_1)^{DR_4} = \frac{g^2 Q_f}{4\pi^2} \int_0^1 dx x(1-x),$$

$$(F_2^{00} + 2F_1)^{DR_2} - (F_2 + 2F_1)^{DR_4} = \frac{g^2 Q_f}{4\pi^2} \left(\frac{1}{2\eta}\right) \times$$

$$\left[\int_0^1 dx \int_0^1 dy \left\{1 + 2\eta((3-4y(1-y))x^2 - 2x + 1)\right\} - \frac{2}{3}(1+2\eta)\right]$$
$$= -\frac{g^2 Q_f}{4\pi^2} \left(\frac{1}{2\eta}\right) \left(\frac{1}{3} + \frac{2\eta}{9}\right).$$

Thus, we find that the vector anomaly in LFD breaks the Lorentz symmetry, *i.e.*, $F_2^{+0} \neq F_2^{00}$ if DR_2 is used.

Using PV_1 we find

$$(F_2^{+0} + 2F_1)^{\mathrm{PV}_1} - (F_2 + 2F_1)^{\mathrm{DR}_4} = \frac{2}{3}\,\frac{g^2 Q_f}{4\pi^2}$$
$$(F_2^{00} + 2F_1)^{\mathrm{PV}_1} - (F_2 + 2F_1)^{\mathrm{DR}_4} = \frac{2}{3}\,\frac{g^2 Q_f}{4\pi^2}$$

Also, we note that the PV_1 results in LFD are identical to the PV_1 result from the manifestly covariant calculation because

$$(F_2 + 2F_1)^{\mathrm{PV}_1}_{\mathrm{cov}} - (F_2 + 2F_1)^{\mathrm{DR}_4} = \frac{2}{3}\,\frac{g^2 Q_f}{4\pi^2}$$

so that

$$(F_2^{+0} + 2F_1)^{\mathrm{PV}_1} = (F_2^{00} + 2F_1)^{\mathrm{PV}_1} = (F_2 + 2F_1)^{\mathrm{PV}_1}_{\mathrm{cov}}.$$

Thus, the PV_1 results are absolutely convergent and restore completely the Lorentz symmetry.
However, the fermion-mass-independent difference between the PV_1 results and the manifestly covariant DR_4 result persists.

Conclusions regarding LF Anomalies

(i) In the manifestly covariant calculation the different regularizations give different results for the physical quantity $\Delta\kappa$.

(ii) The difference is proportional to the fermion charge and independent of its mass.

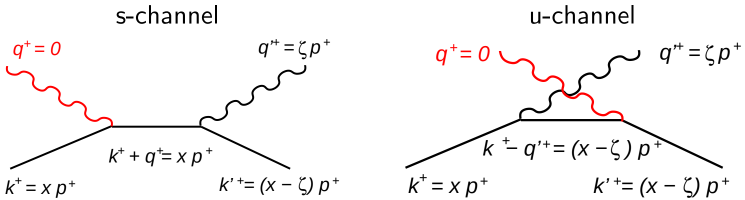
(iii) In LFD the different regularizations give also different results, and these results differ, moreover, from the covariant ones.

(iv) All differences, between LF results for different normalizations, and between LF results and manifestly covariant ones, are independent of the masses of the fermions and proportional to their charges.

(v) The anomaly-free condition $\sum_f Q_f = 0$, that is part of the Standard Model, removes all differences.

Tree-level DVCS

Deeply Virtual Compton Scattering



Shown are the hadronic parts of the DVCS amplitude. The lines are labelled with the plus components of the momenta in terms of a hadronic scale p^+. The complete amplitude is obtained by convoluting with the leptonic amplitude.

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h)\frac{1}{q^2}\mathcal{H}(\{s', s\}\{h', h\})$$

$$\mathcal{H}(\{s', s\}\{h', h\}) = \bar{u}(k'; s')\not{\epsilon}(q'; h')(\mathcal{O}_s + \mathcal{O}_u)\not{\epsilon}(q; h)u(k; s)$$

$$\mathcal{O}_s + \mathcal{O}_u = \frac{\not{k} + \not{q} + m}{(k+q)^2 - m^2} + \frac{\not{k} - \not{q}' + m}{(k-q')^2 - m^2}$$

Kinematics

The kinematics used is taken from S.J. Brodsky, M. Diehl, and D.-S. Hwang, Nucl. Phys. B **596**, 99 (2001).

$$k^\mu = \left(xp^+, 0, 0, \frac{m^2}{2xp^+} \right), \quad q^\mu = \left(0, Q, 0, \frac{Q^2}{2\zeta p^+} + \frac{\zeta m^2}{2(1-\zeta)p^+} \right),$$

$$k'^\mu = \left((x-\zeta)p^+, 0, 0, \frac{m^2}{2(x-\zeta)p^+} \right), \quad q'^\mu = \left(\zeta p^+, Q, 0, \frac{Q^2}{2\zeta p^+} \right),$$

$k^2 = {k'}^2 = m^2$, $q^2 = -Q^2$, and ${q'}^2 = 0$.

Notation: $p^\mu = (p^+, p^1, p^2, p^-)$; reminder: $p^2 = 2p^+p^- - \mathbf{p}_\perp^2$.

Reduction

Factorization is supposed to take place in the limit $Q^2 \to \infty$. Then

$$\mathcal{O}_s|_{\text{Red}} = \lim_{Q\to\infty} \mathcal{O}_s = \frac{\gamma^+}{2p^+}\frac{1}{x-\zeta}, \quad \mathcal{O}_u|_{\text{Red}} = \lim_{Q\to\infty} \mathcal{O}_u = \frac{\gamma^+}{2p^+}\frac{1}{x}.$$

The polarization vectors $\epsilon(q;h)$ are singular if $q^+ = 0$. We write $q^+ = \delta\, p^+$ and expand all matrix elements in powers of δ.

$$\begin{aligned}
\epsilon(q;+1) &= \frac{1}{\sqrt{2}}\left(0,-1,-i,-\frac{q_x + iq_y}{q^+}\right), \\
\epsilon(q;0) &= \frac{1}{\sqrt{q^2}}\left(q^+, q_x, q_y, \frac{\mathbf{q}_\perp^2 - q^2}{2q^+}\right), \\
\epsilon(q;-1) &= \frac{1}{\sqrt{2}}\left(0,1,-i,\frac{q_x - iq_y}{q^+}\right).
\end{aligned}$$

Partial results

Leptonic part $\mathcal{L}$ of the matrix element $\mathcal{M} = \mathcal{L} \otimes \mathcal{H}/q^2$

$\{\lambda', \lambda\}$	h	$\mathcal{L}(\{\lambda', \lambda\}h)$
$\{+1/2, +1/2\}$	$+1$	$-Q\left(1 - \frac{\delta}{2\zeta} + \frac{2\zeta}{\delta}\right)$
$\{+1/2, +1/2\}$	0	$-i2\sqrt{2}Q\,\frac{\zeta}{\delta}$
$\{+1/2, +1/2\}$	-1	$-Q\left(1 - \frac{3\delta}{\zeta} - \frac{2\zeta}{\delta}\right)$

Hadronic part $\mathcal{H}$ of the matrix element $\mathcal{M} = \mathcal{L} \otimes \mathcal{H}/q^2$

$\{h', h\}$ $\{s', s\}$	$\mathcal{H}(\{h', h\}\{s', s\})_{\text{Full}}$	$\mathcal{H}(\{h', h\}\{s', s\})_{\text{Red}}$
$\{1, +1\}$ $\{1/2, 1/2\}$	$2\sqrt{\frac{x}{x-\zeta}}\,\left(1 + \frac{\zeta}{\delta}\right)$	$2\sqrt{\frac{x-\zeta}{x}}$
$\{1, -1\}$ $\{1/2, 1/2\}$	$-2\sqrt{\frac{x-\zeta}{x}}\,\frac{\zeta}{\delta}$	0
$\{1, 0\}$ $\{1/2, 1/2\}$	$i\sqrt{2}\sqrt{\frac{x}{x-\zeta}}\,\left(1 + \frac{2\zeta}{\delta} - \frac{\delta}{4\zeta}\right)$	$i\sqrt{2}\sqrt{\frac{x-\zeta}{x}}\,\left(1 + \frac{\delta}{2\zeta}\right)$

The complete amplitude will be finite if the singular parts, proportional to δ, cancel.

Complete results

Example

$\{\lambda',\lambda\}$	$\{h',h\}$	$\{s',s\}$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\text{Full}}$
$\{1/2,1/2\}$	$\{1,1\}$	$\{1/2,1/2\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(-\frac{4\zeta^2}{\delta^2}-\frac{6\zeta}{\delta}-\frac{3}{2}+\frac{\delta}{4\zeta}\right)$
$\{1/2,1/2\}$	$\{1,0\}$	$\{1/2,1/2\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{8\zeta^2}{\delta^2}+\frac{4\zeta}{\delta}-1+\frac{\delta}{2\zeta}\right)$
$\{1/2,1/2\}$	$\{1,-1\}$	$\{1/2,1/2\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(-\frac{4\zeta^2}{\delta^2}+\frac{2\zeta}{\delta}-\frac{3}{2}+\frac{5\delta}{4\zeta}\right)$
	$\sum_h$		$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(-4+\frac{2\delta}{\zeta}\right)$
$\{\lambda',\lambda\}$	$\{h',h\}$	$\{s',s\}$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\text{Red}}$
$\{1/2,1/2\}$	$\{1,1\}$	$\{1/2,1/2\}$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{2\zeta}{\delta}-1+\frac{\delta}{4\zeta}\right)$
$\{1/2,1/2\}$	$\{1,\ 0\}$	$\{1/2,1/2\}$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(\frac{2\zeta}{\delta}+1-\frac{\delta}{4\zeta}\right)$
$\{1/2,1/2\}$	$\{1,-1\}$	$\{1/2,1/2\}$	0
	$\sum_h$		0

After summing the complete amplitude over the virtual photon polarization, the singular parts cancel, but if the reduced hadronic amplitude is used, the complete amplitude is wrong.

Summary

Summary of the DVCS case

- In the 'convenient kinematics' ($q^+ = 0$) all leptonic matrix elements are singular, namely $\propto 1/q^+$. The singularities are due to the polarization vectors.
- Idem for all full hadronic matrix elements.
- All reduced hadronic matrix elements are finite but wrong.
- The complete matrix elements using the full hadronic part are finite; the ones including the reduced hadronic matrix elements are wrong.

Conclusion

Extracting a GPD from the reduced matrix elements may depend on the kinematics and must be considered with the utmost care.

Summary

LF type II Type II LF singularities cancel once the LF amplitudes corresponding to one and the same covariant diagram are added. If some amplitudes are dropped for 'physical reasons', the singularities persist.

LF Anomalies LF anomalies may be removed by a symmetry. If such a symmetry does not occur, a genuine problem arises. The requirement that a theory must be anomaly free may be applied as a bottom-up test of models.

Summary, continued

Undefined The LF singularity found in tree-level DVCS is not yet fully understood. One may speculate that it is due in the BDH kinematics to the singular nature of the $q^+ = 0$ kinematics. Such a kinematics leads in many LF calculations to drastic simplifications, although they may be fortuitous in those cases where a zero mode is needed to obtain agreement with the covariant amplitude.