Three-nucleon Force: A Comparative Study

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Motivation

- Realistic NN potentials describe 2N data with $\chi^2 \approx 1$
- Realistic NN potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 3N data with $\chi^2 >> 1$
- Realistic NN+3N potentials describe 4N data with $\chi^2 >> 1$

Potential	Method	^{3}H[MeV]	^{4}He[MeV]	$^2a_{nd}$[fm]
AV18	HH	7.624	24.22	1.258
	FE/FY Bochum	7.621	24.23	1.248
	FE/FY Lisbon	7.621	24.24	
CDBonn	HH	7.998	26.13	
	FE/FY Bochum	8.005	26.16	0.925
	FE/FY Lisbon	7.998	26.11	
	NCSM	7.99(1)		
N3LO-Idaho	HH	7.854	25.38	1.100
	FE/FY Bochum	7.854	25.37	
	FE/FY Lisbon	7.854	25.38	
	NCSM	7.852(5)	25.39(1)	
AV18/UIX	HH	8.479	28.47	0.590
	FE/FY Bochum	8.476	28.53	0.578
CDBonn/TM	HH	8.474	29.00	
	FE/FY Bochum	8.482	29.09	0.570
N3LO-Idaho/N2LO	HH	8.474	28.37	0.675
	NCSM	8.473(5)	28.34(2)	
Exp.		8.48	28.30	0.645±0.010

It is possible to describe simultaneously the ^{3}H and ^{4}He binding energies and the $n-d$ scattering lenth using the available Three-Nucleon Force models?

The 3N Potential

Urbana, TM, N2LO

$$W_{3N} = \sum_{i,j,k} W(i,j,k)$$

$$\begin{aligned}
W(1,2,3) = {} & C_1 (\tau_1 \cdot \tau_2)(\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{23}) y(r_{31}) y(r_{23}) \\
& + C_3 \{X_{23}, X_{31}\}\{\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\} \\
& + C_4 \left[X_{23}, X_{31}\right] \left[\tau_2 \cdot \tau_3, \tau_3 \cdot \tau_1\right] \\
& + C_E \, (\tau_1 \cdot \tau_2) Z_0(r_{23}) Z_0(r_{31}) \\
& + C_D \, (\tau_1 \cdot \tau_2)\{(\sigma_1 \cdot \sigma_2)[y(r_{31}) Z_0(r_{23}) + y(r_{23}) Z_0(r_{31})] \\
& \qquad + (\sigma_1 \cdot r_{31})(\sigma_2 \cdot r_{31}) t(r_{31}) Z_0(r_{23}) \\
& \qquad + (\sigma_1 \cdot r_{23})(\sigma_2 \cdot r_{23}) t(r_{23}) Z_0(r_{31})\}
\end{aligned}$$

$$X_{ij} = t(r_{ij})(\sigma_i \cdot r_{ij})(\sigma_j \cdot r_{ij}) + y(r_{ij})(\sigma_i \cdot \sigma_j)$$

$$Z_0(r) = \frac{12\pi^2}{M_\pi^3}\frac{1}{2\pi^2}\int dq q^2 j_0(qr) f(q,\Lambda)$$

$$f_0(r) = \frac{12\pi^2}{M_\pi^3}\frac{1}{2\pi^2}\int dq q^2 j_0(qr)\frac{1}{q^2+M_\pi^2} f(q,\Lambda)$$

$$y(r) = \frac{1}{r} f_0'(r) \qquad Y(r) = T(r) - y(r)$$

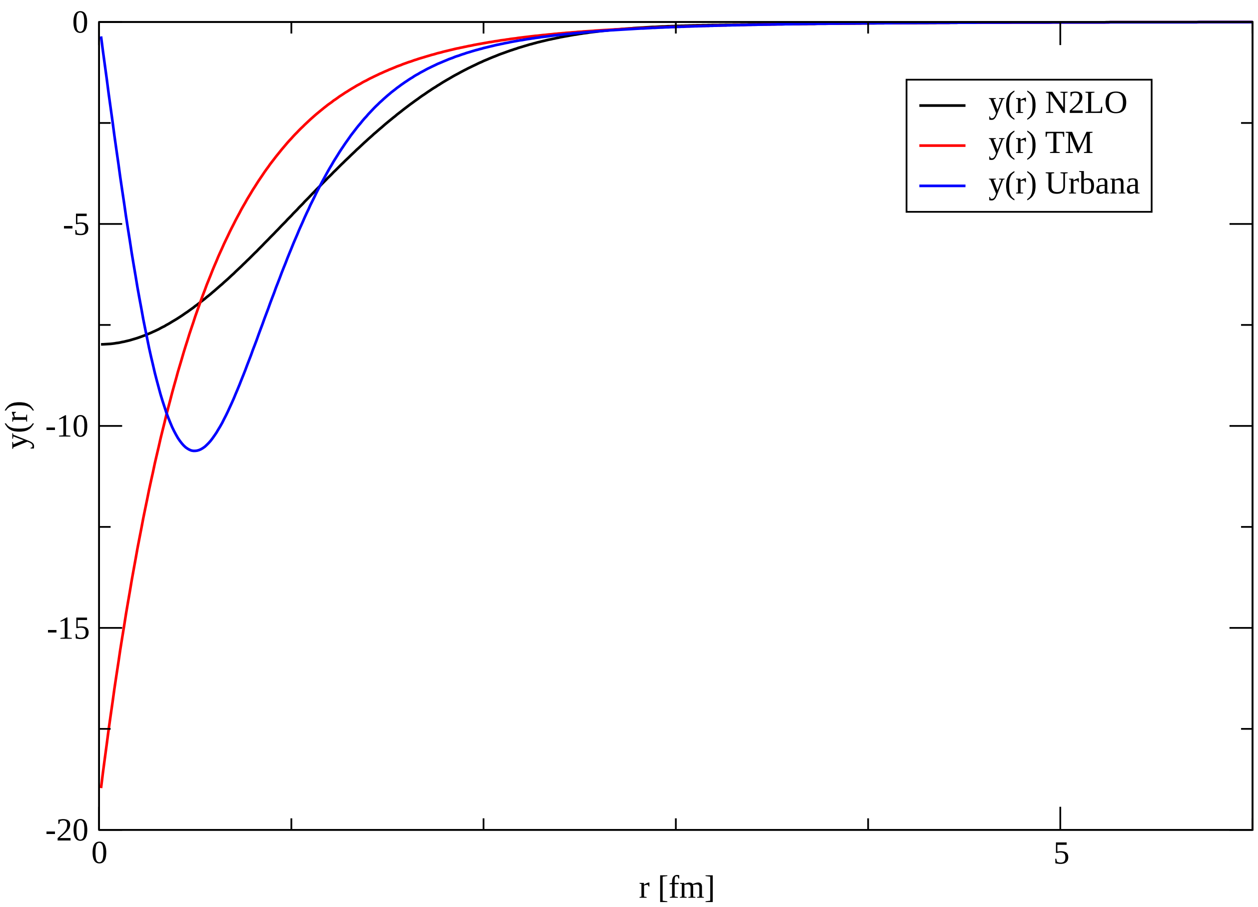
$$t(r) = \frac{1}{r} y'(r) \qquad T(r) = \frac{r^2}{3} t(r)$$

TM : $f(q,\Lambda) = \left(\frac{\Lambda^2 - M_\pi^2}{\Lambda^2 + q^2}\right)^2$ N2LO : $f(q,\Lambda) = \mathrm{e}^{-q^4/\Lambda^4}$

Urbana : $Y(r) = \frac{\mathrm{e}^{-x}}{x}\zeta(r)$; $T(r) = (1 + \frac{3}{x} + \frac{3}{x^2})Y(x)\zeta(r)$

$Z_0(r) = T^2(r)$; $\zeta(r) = (1 - \mathrm{e}^{-cr^2})$

$(x = M_\pi r;\ c = 2.1\mathrm{fm}^{-2})$



y(r) N2LO
y(r) TM
y(r) Urbana
y(r)
0
-5
-10
-15
-20
0
5
r [fm]

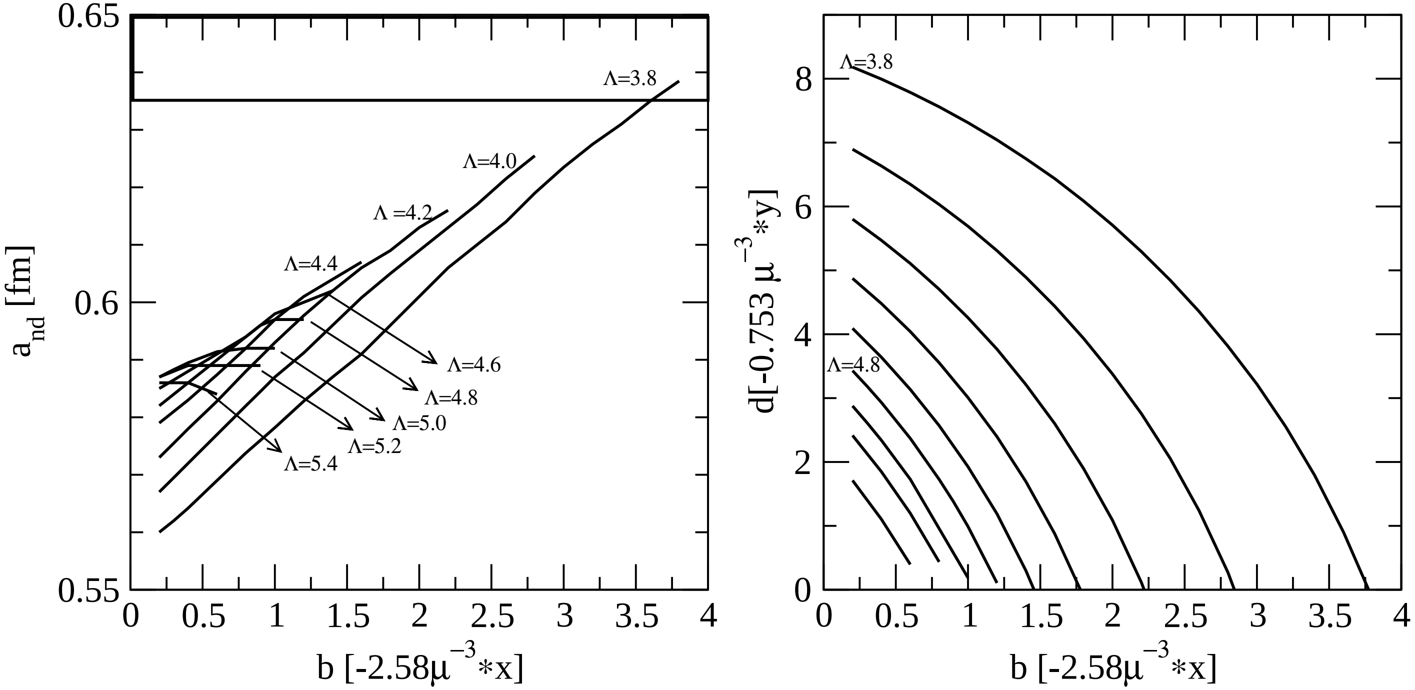
Fixing the 3N potential

	C_1 [MeV]	C_3 [MeV]	C_4 [MeV]	C_E [MeV]	C_D [MeV]	Λ [MeV]
Urbana	0	-0.029	$\frac{1}{4}C_3$	0.0048	0	–
TM'	-0.76	-0.063	-0.018	0	0	$4.8M_\pi$
N2LO	-0.67	-0.043	-0.037	-0.0028	0.015	500

	^{3}H [MeV]	a_{nd} [fm]	^{4}He [MeV]
AV18+Urbana	-8.479	0.590	28.47
AV18+TM'	-8.478	0.595	28.52
AV18+(1.4)*N2LO	-8.478	0.654	28.55
N3LO+N2LO	-8.474	0.675	28.37

$$TM : C_1 = V_0[a' M_\pi]; C_3 = V_0[bM_\pi^3]; C_4 = V_0[dM_\pi^3]$$

$$a' = -0.87 M_\pi^{-1}$$

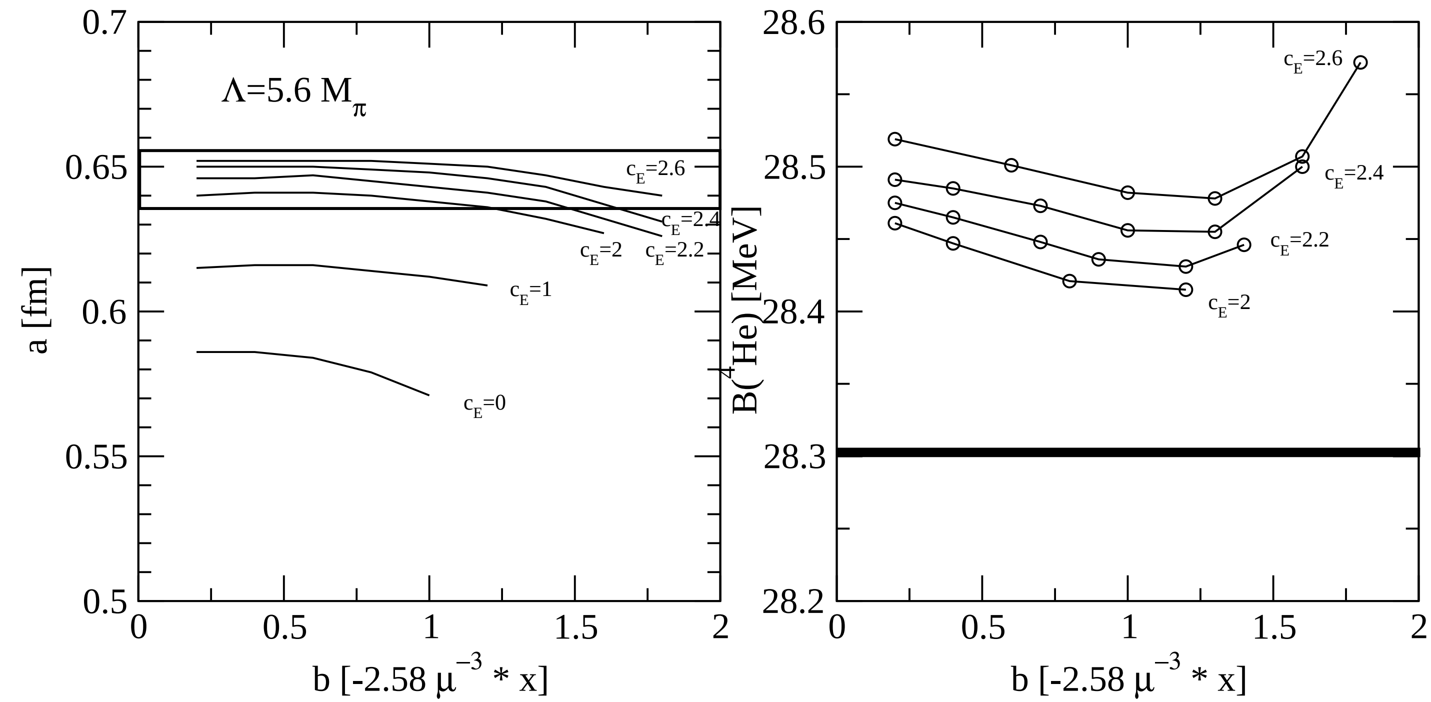


With the TM potential it is no possible to describe the ^{3}H and $n-d$ scattering lenth with reasonable values of the parameters

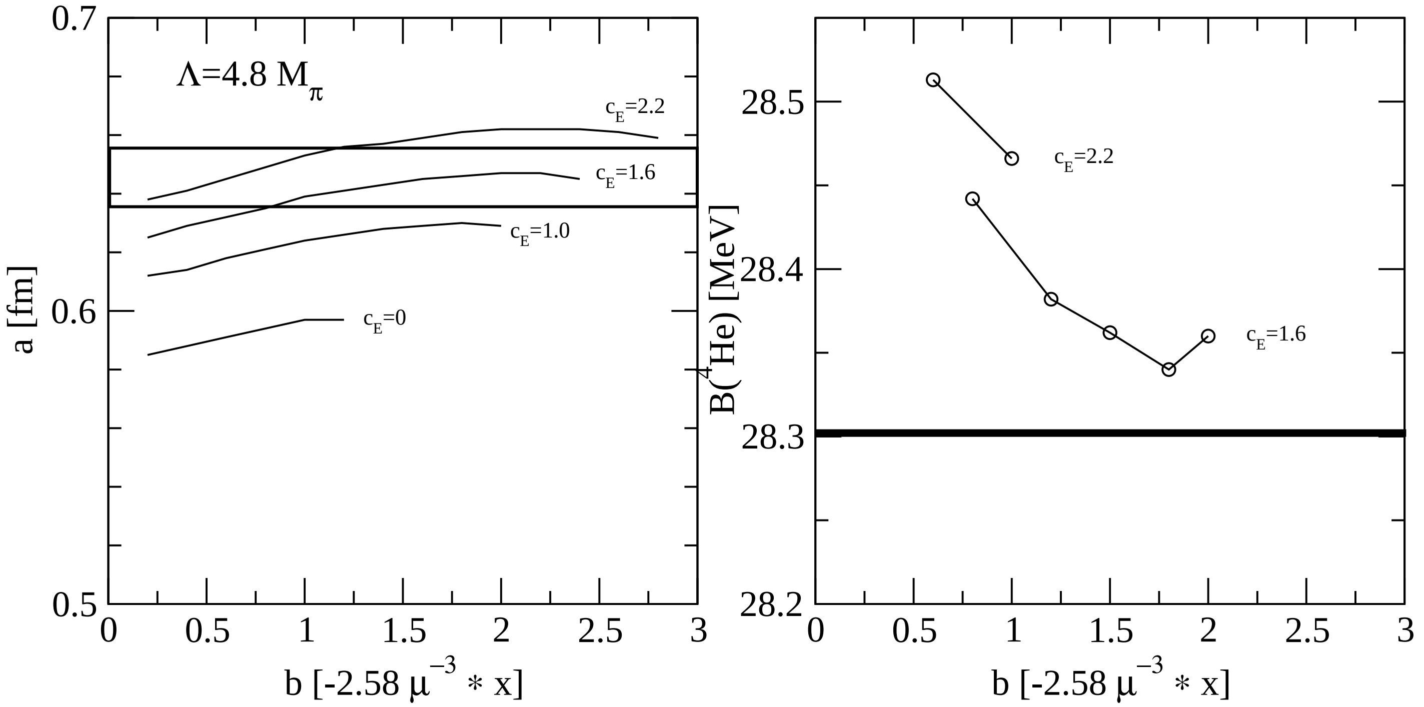
Now we consider the term: $V_0 C_E Z_0(r_{23}) Z_0(r_{31})$ with

$$Z_0(r) = \frac{12\pi^2}{M_\pi^3}\frac{1}{2\pi^2}\int dq q^2 j_0(qr)\left(\frac{\Lambda^2 - M_\pi^2}{\Lambda^2 + q^2}\right)^2$$

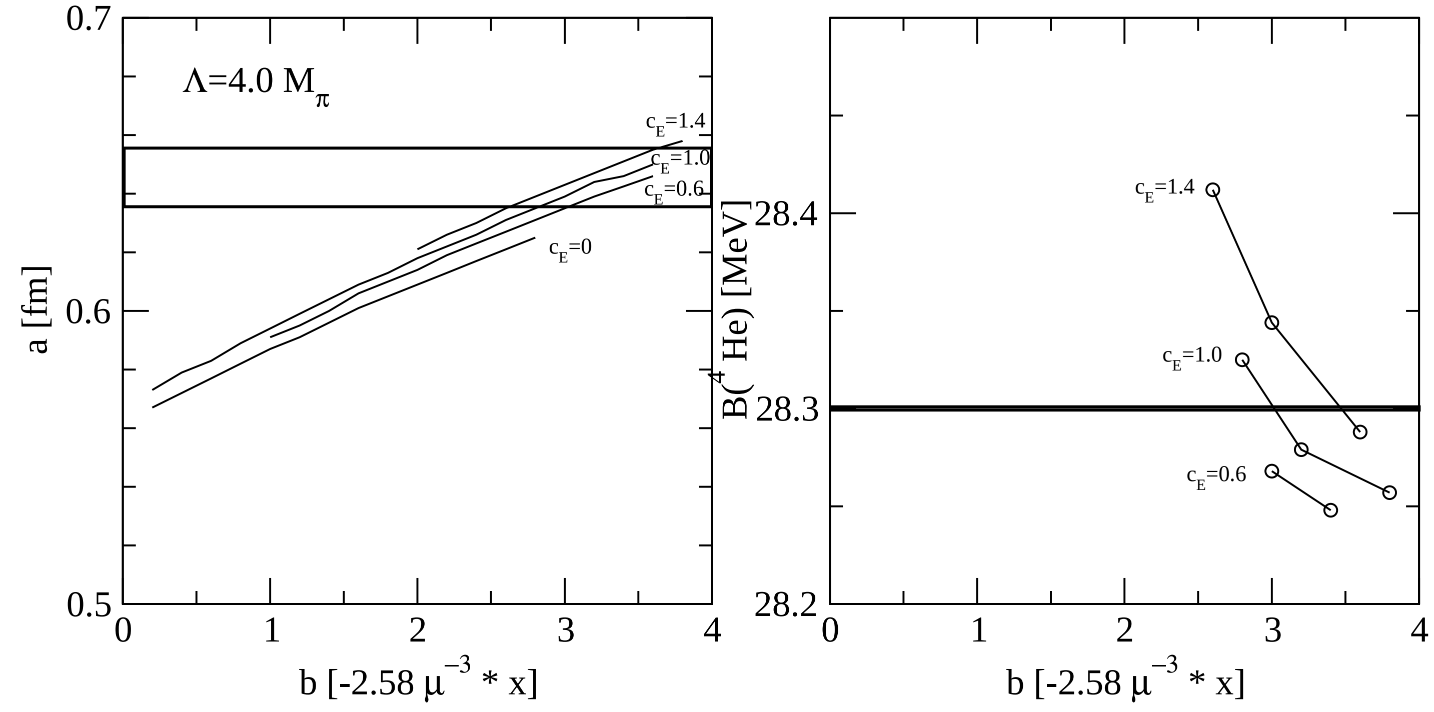
$$Z_0(r) = \frac{3}{2}\pi\frac{M_\pi}{\Lambda}\left(\frac{\Lambda^2}{M_\pi^2} - 1\right)^2 \mathrm{e}^{-\Lambda r}$$



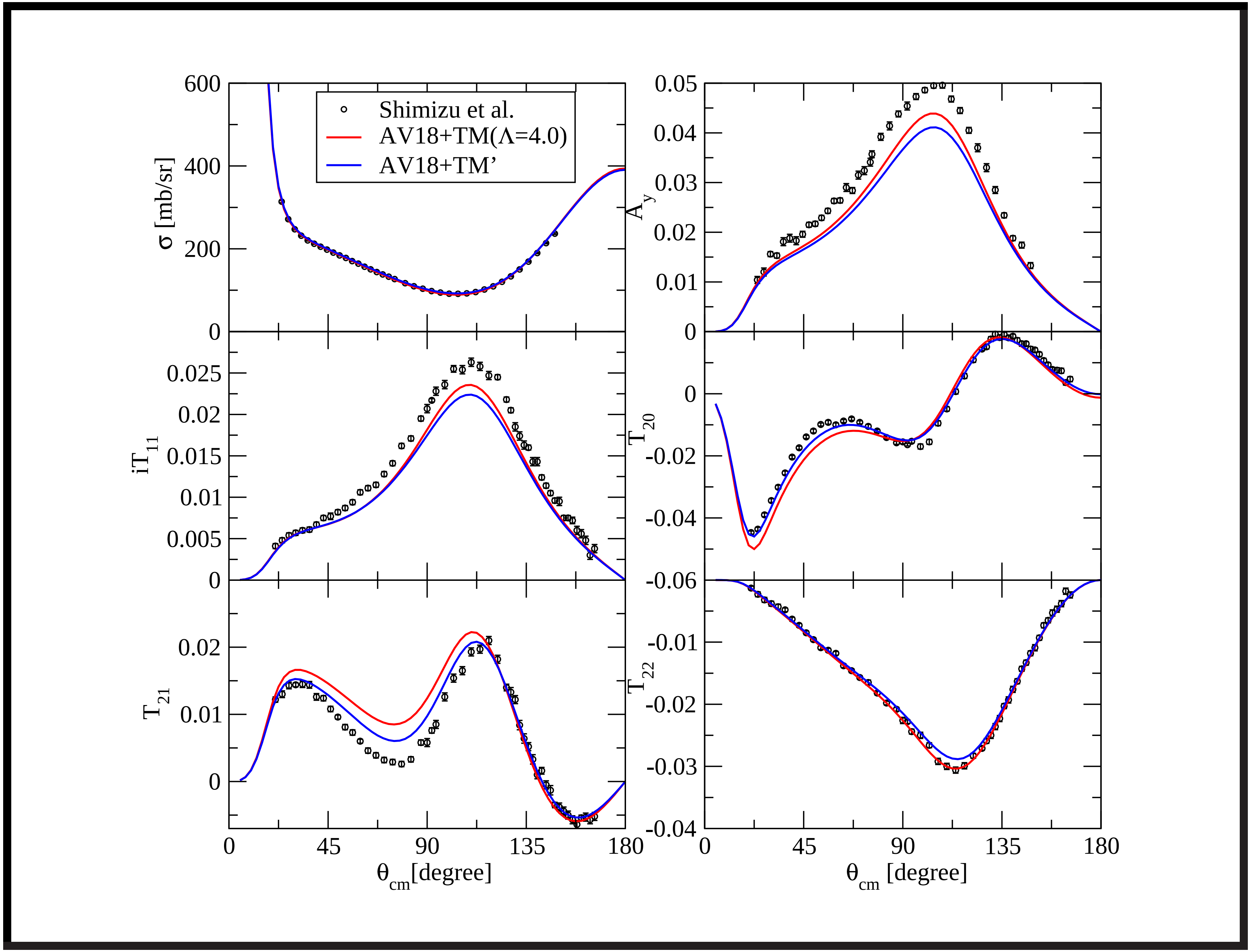
Λ=5.6 Mπ
a [fm]
0.7
0.65
0.6
0.55
0.5
0
0.5
1
1.5
2
b [-2.58 μ^−3 * x]
cE=2.6
cE=2.4
cE=2
cE=2.2
cE=1
cE=0
B(4He) [MeV]
28.6
28.5
28.4
28.3
28.2



Λ=4.8 Mπ
a [fm]
b [-2.58 μ⁻³ * x]
cE=2.2
cE=1.6
cE=1.0
cE=0
B(4He) [MeV]
0.7
0.6
0.5
28.5
28.4
28.3
28.2
0
0.5
1
1.5
2
2.5
3



Λ=4.0 M_π
c_E=1.4
c_E=1.0
c_E=0.6
c_E=0
a [fm]
0.7
0.6
0.5
0
1
2
3
4
b [-2.58 μ^{-3} * x]
c_E=1.4
c_E=1.0
c_E=0.6
B(^{4}He) [MeV]
28.4
28.3
28.2
0
1
2
3
4
b [-2.58 μ^{-3} * x]



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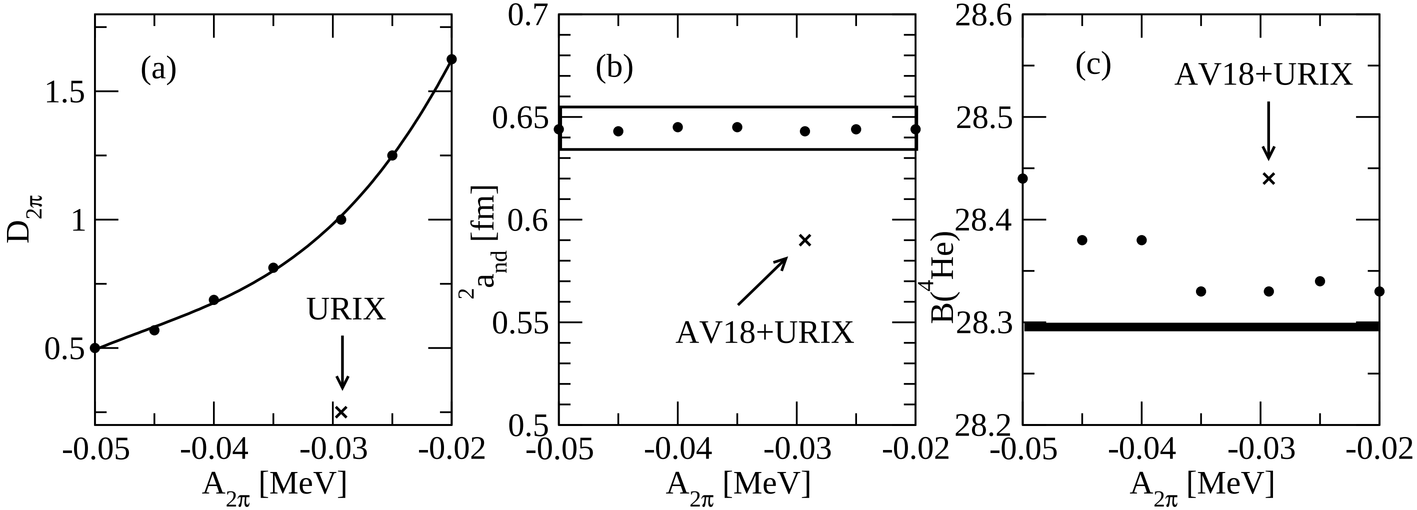
AV18+TM(Λ=4.0)
AV18+TM'
σ [mb/sr]
600
400
200
0
A_y
0.05
0.04
0.03
0.02
0.01
0
iT_{11}
0.025
0.02
0.015
0.01
0.005
0
T_{20}
0
-0.02
-0.04
-0.06
T_{21}
0.02
0.01
0
T_{22}
-0.01
-0.02
-0.03
-0.04
0
45
90
135
180
θ_{cm}[degree]
θ_{cm} [degree]

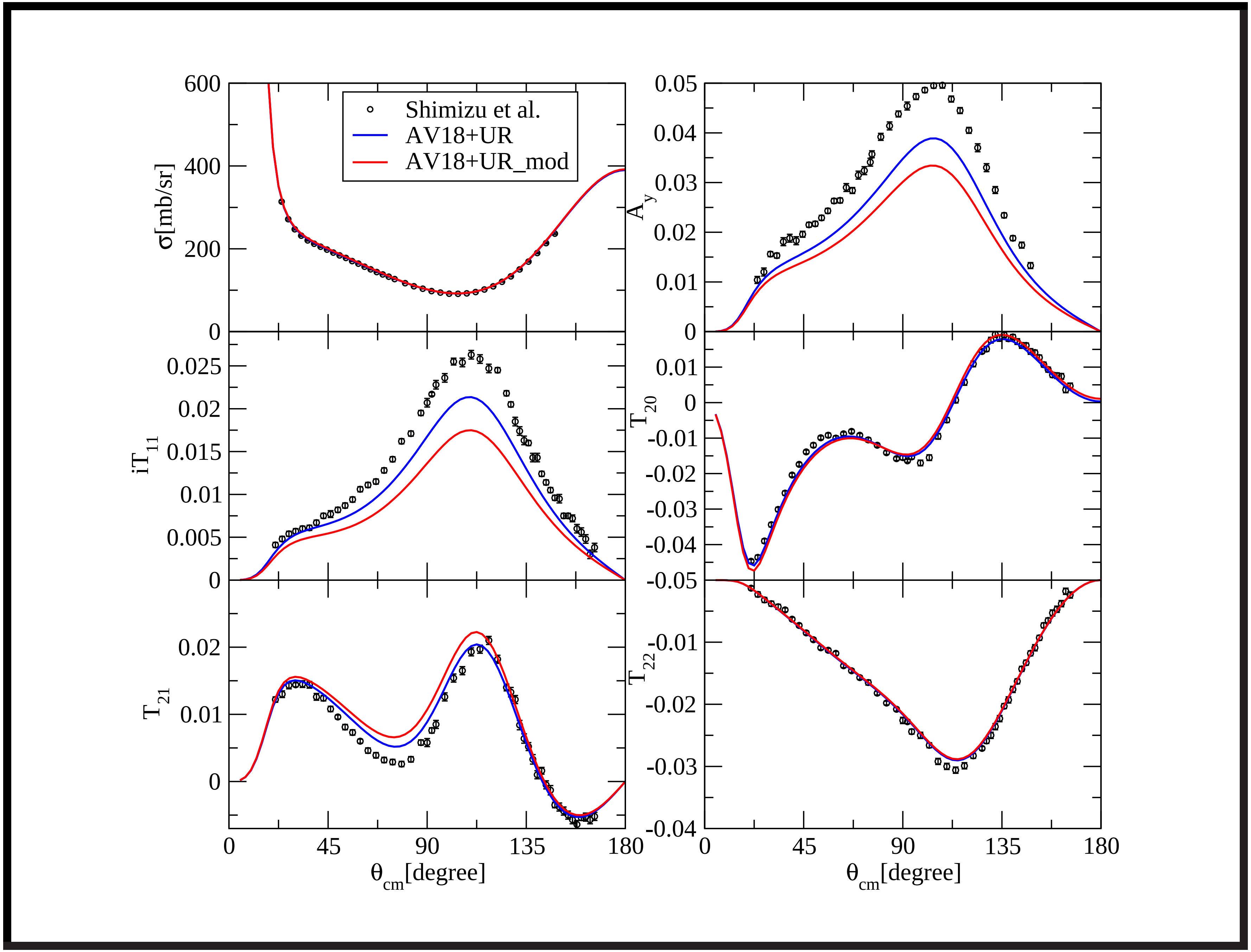
$$\text{URIX}: C_3 \rightarrow A_{2\pi} = -0.0293\text{MeV}; \qquad C_1 = C_D = 0$$

$$C_4 \rightarrow D_{2\pi} = \frac{1}{4}A_{2\pi}; \qquad C_E \rightarrow U_0 = 0.0048\text{MeV}$$



AV18+ Urbana

C_3 [MeV]	C_4	U_0 [MeV]	^{3}H [MeV]	a_{nd} [fm]	^{4}He [MeV]
-0.0293	$\frac{1}{4}C_3$	0.0048	-8.475	0.590	-28.47
-0.020	$\frac{6.5}{4}C_3$	0.018	-8.475	0.644	-28.33
-0.025	$\frac{5}{4}C_3$	0.018	-8.475	0.644	-28.34
-0.029	$\frac{4}{4}C_3$	0.018	-8.475	0.643	-28.33
-0.035	$\frac{3.25}{4}C_3$	0.019	-8.475	0.645	-28.33
-0.040	$\frac{2.5}{4}C_3$	0.018	-8.475	0.643	-28.38
-0.045	$\frac{2.25}{4}C_3$	0.020	-8.475	0.643	-28.38
-0.050	$\frac{2}{4}C_3$	0.021	-8.475	0.645	-28.44



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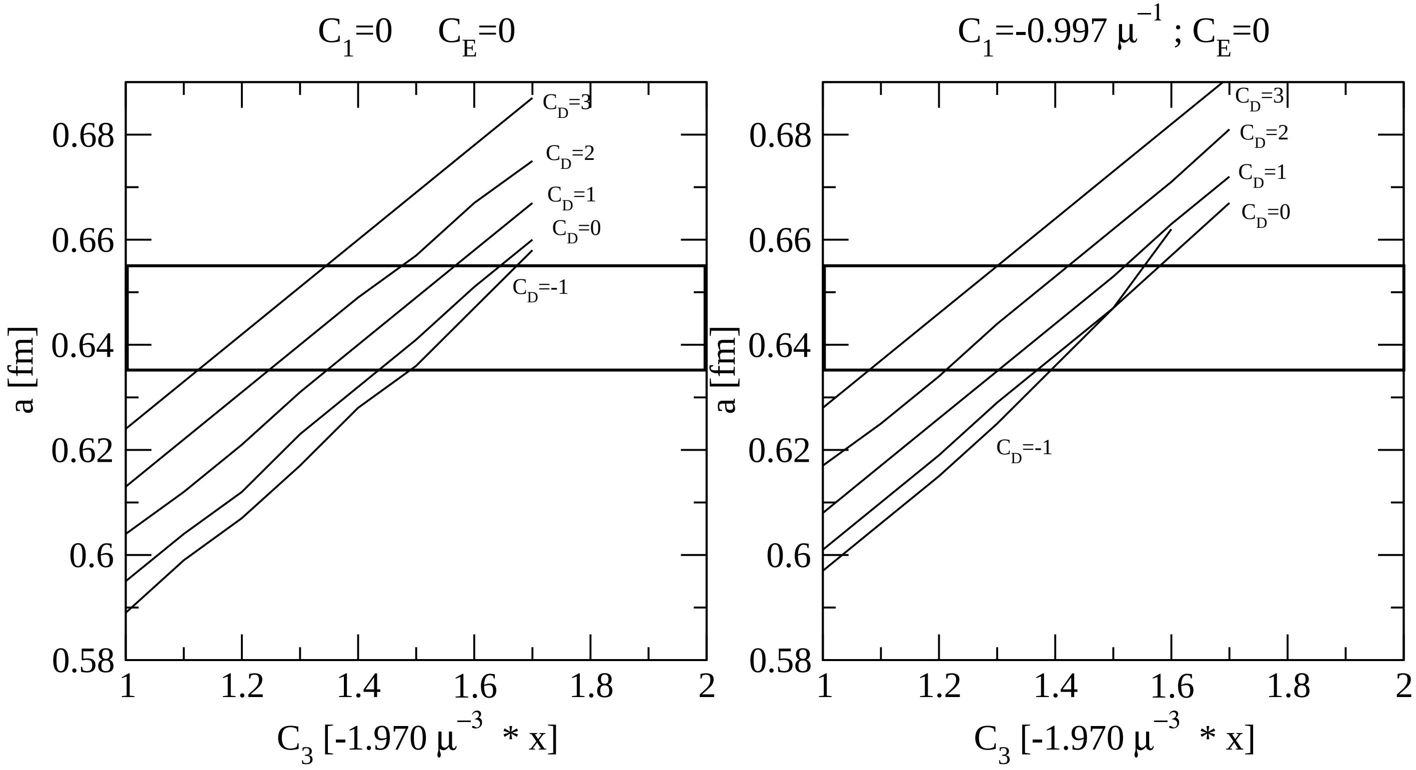
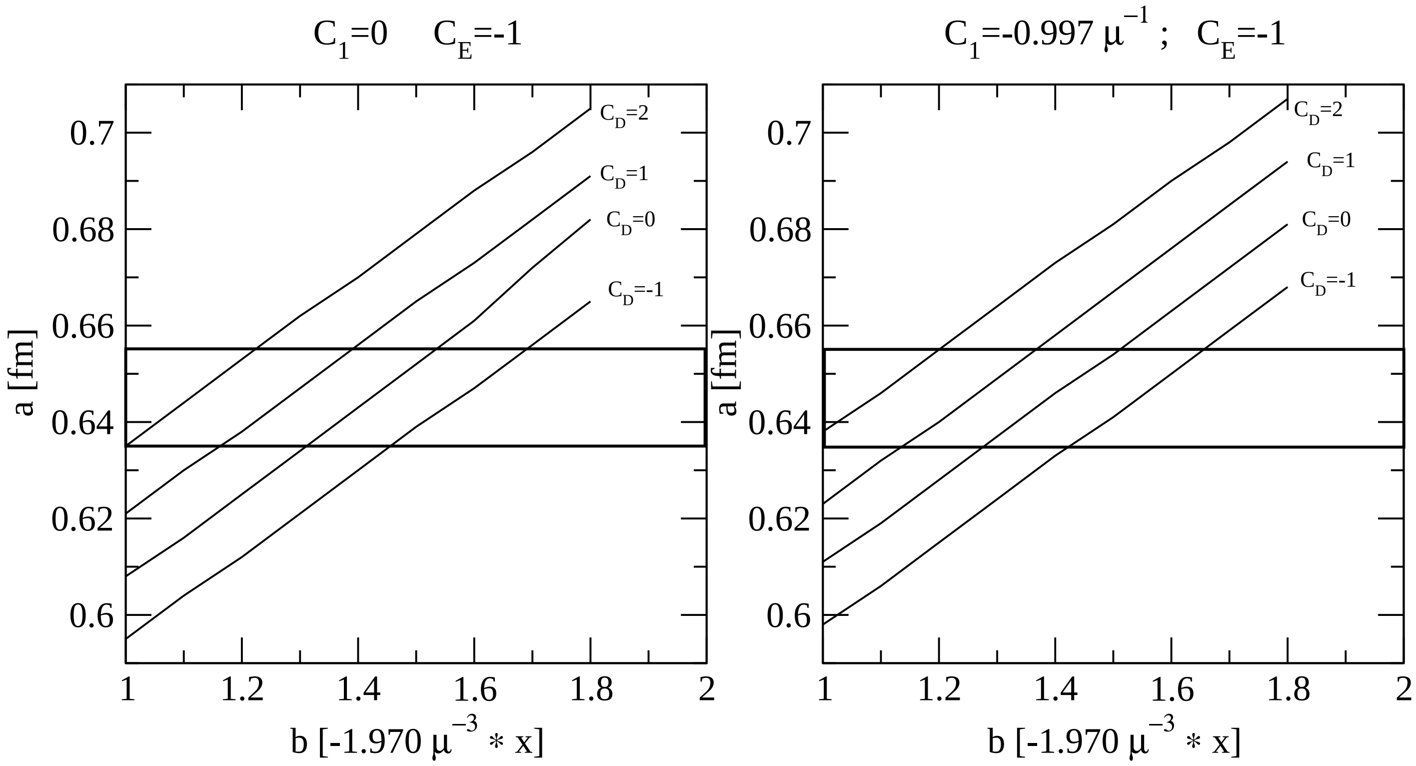
AV18+UR
AV18+UR_mod
σ[mb/sr]
600
400
200
0
A_y
0.05
0.04
0.03
0.02
0.01
0
iT_{11}
0.025
0.02
0.015
0.01
0.005
0
T_{20}
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
T_{21}
0.02
0.01
0
T_{22}
-0.01
-0.02
-0.03
-0.04
0
45
90
135
180
θ_{cm}[degree]
0
45
90
135
180
θ_{cm}[degree]

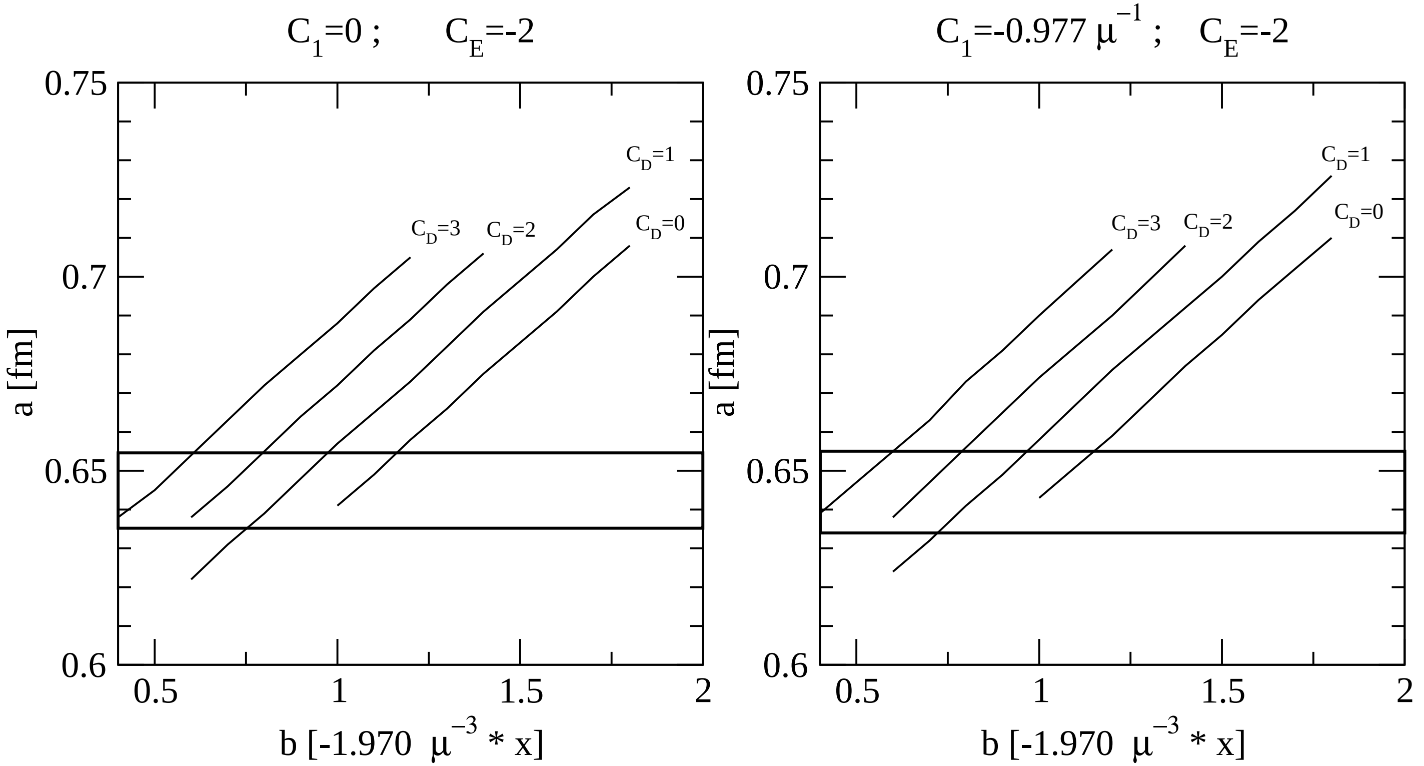
N2LO : $C_1 = -0.997 M_\pi^{-1}$; $C_3 = -1.97 M_\pi^{-3}$; $C_4 = -1.66 M_\pi^{-3}$

$C_D = 1 * 0.095\text{MeV}$; $C_E = -0.029 * 0.015\text{MeV}$





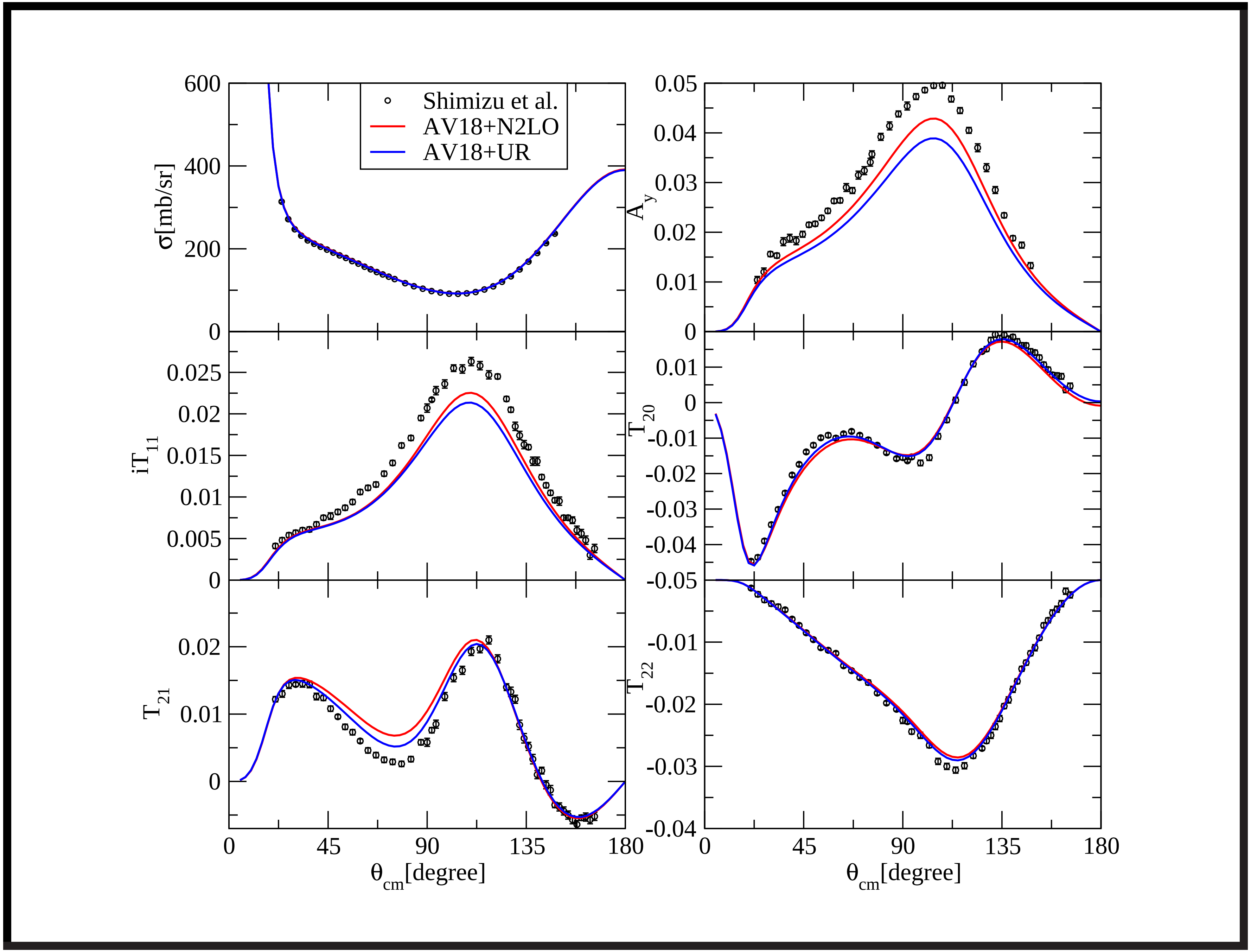
C1=0 CE=-1
C1=-0.997 μ−1 ; CE=-1
CD=2
CD=1
CD=0
CD=-1
0.7
0.68
0.66
0.64
0.62
0.6
a [fm]
1
1.2
1.4
1.6
1.8
2
b [-1.970 μ−3 ∗ x]



C1=0 ; CE=-2
C1=-0.977 μ−1 ; CE=-2
a [fm]
b [-1.970 μ−3 * x]
CD=3
CD=2
CD=1
CD=0
0.75
0.7
0.65
0.6
0.5
1
1.5
2

AV18+ N2LO

C_1	C_3	C_4	C_D	C_E	^{4}He [MeV]
$-0.997 M_\pi^{-1}$	$-1.97 M_\pi^{-3}$	$-1.66 M_\pi^{-3}$	1	-0.029	
1.4	1.4	2.10	0	-0.837	28.99
1.4	1.4	1.0	0.6	0.046	28.42
1.4	1.4	0.84	0.4	0.076	28.37
1.4	1.4	0.70	0.2	0.091	28.33



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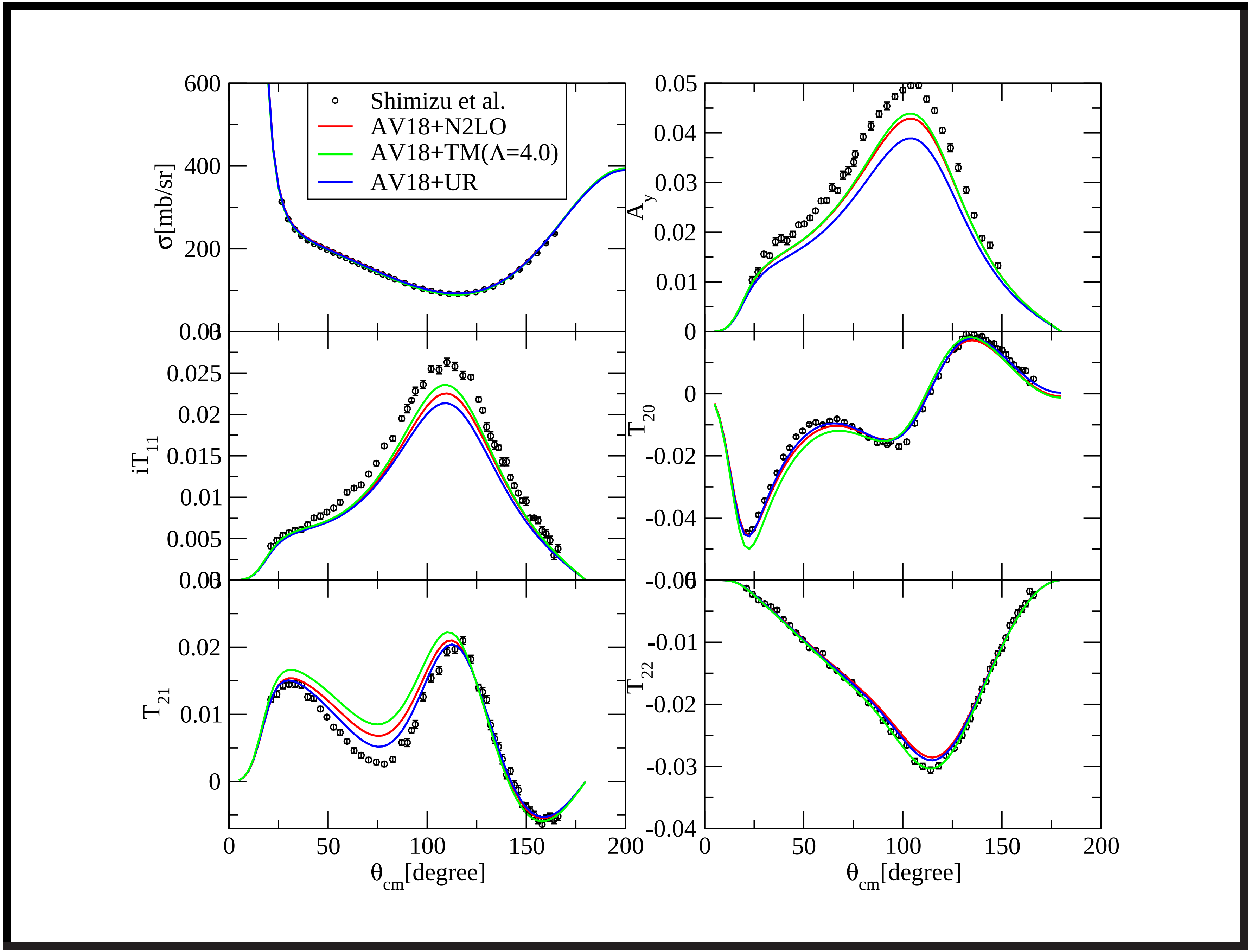
AV18+N2LO
AV18+UR
σ[mb/sr]
A_y
iT_{11}
T_{20}
T_{21}
T_{22}
θ_{cm}[degree]
θ_{cm}[degree]



Shimizu et al.
AV18+N2LO
AV18+TM(Λ=4.0)
AV18+UR
σ[mb/sr]
iT11
T21
Ay
T20
T22
θcm[degree]
θcm[degree]
0
50
100
150
200
600
400
200
0.03
0.025
0.02
0.015
0.01
0.005
0
0.05
0.04
0.03
0.02
0.01
-0.02
-0.04
-0.06
-0.01
-0.03
-0.04

Conclusions

- The actual NN+3N potential models do not fit simultaneously $B(^3\mathrm{H})$, $B(^4\mathrm{He})$, and $^{(2)}a_{nd}$
- 3N potentials are not "phase equivalent"
- The parameters in the 3N potentials can be varied to fit those quantities
- To perform the fit for the TM potential, a repulsive term has been included
- For the Urbana potential the fit worsened some polarization observables due to the large repulsion introduced
- For the N2LO (local) potential the fit was possible without varying too much the original parameters.
- Work is still in progress