Collider Physics - Chapter 3

LEP — e^+e^- physics

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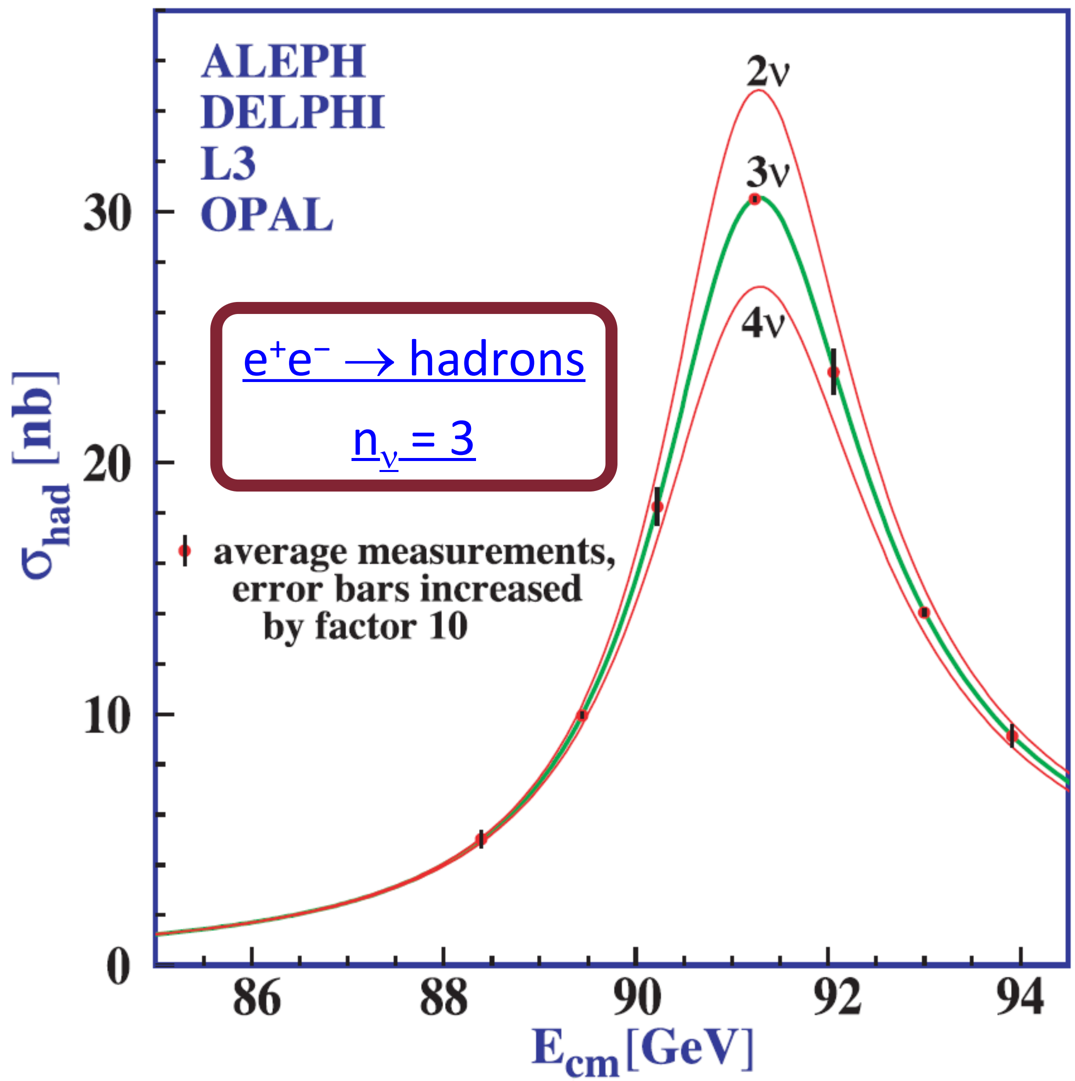
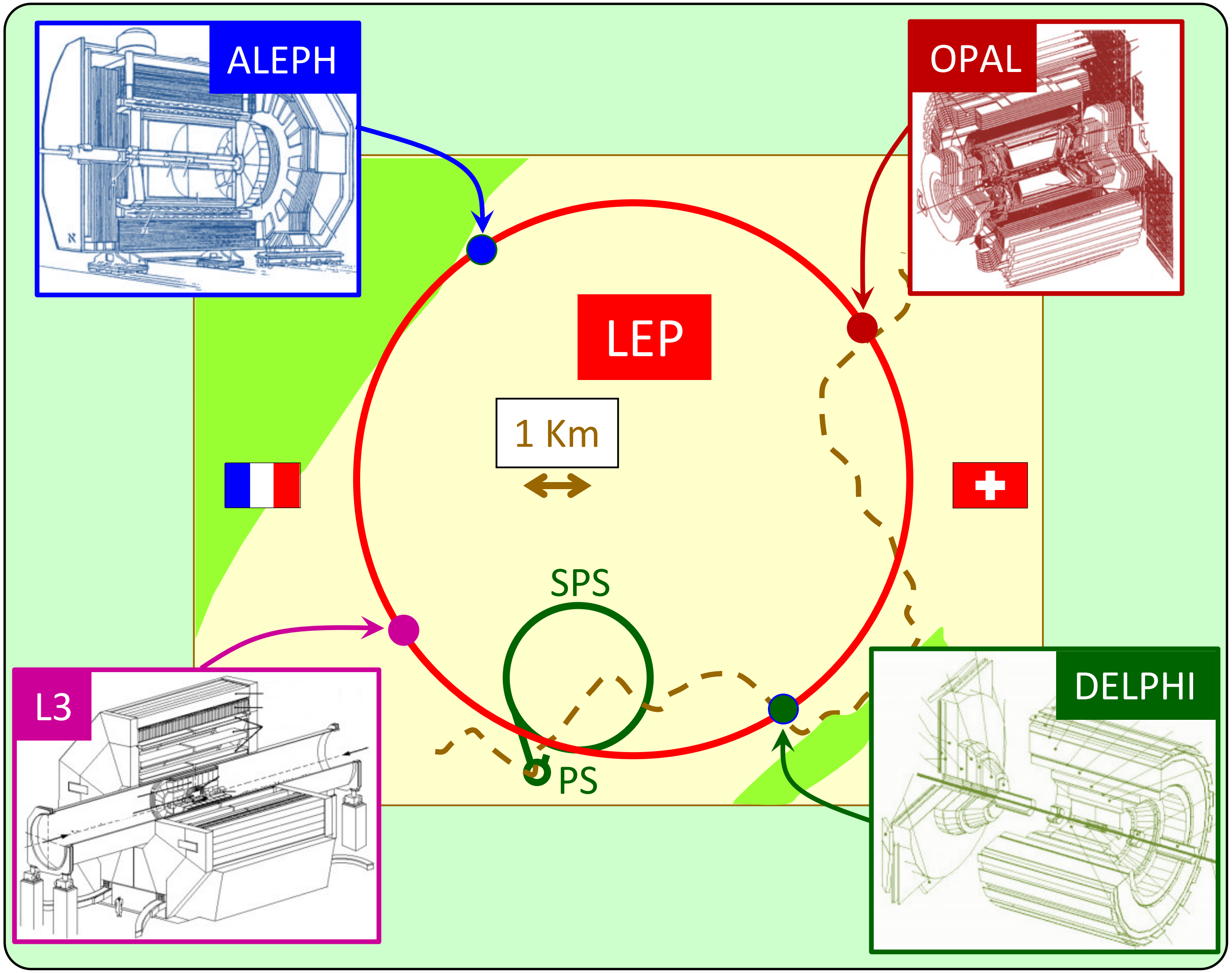
3 – LEP – e^+e^- physics

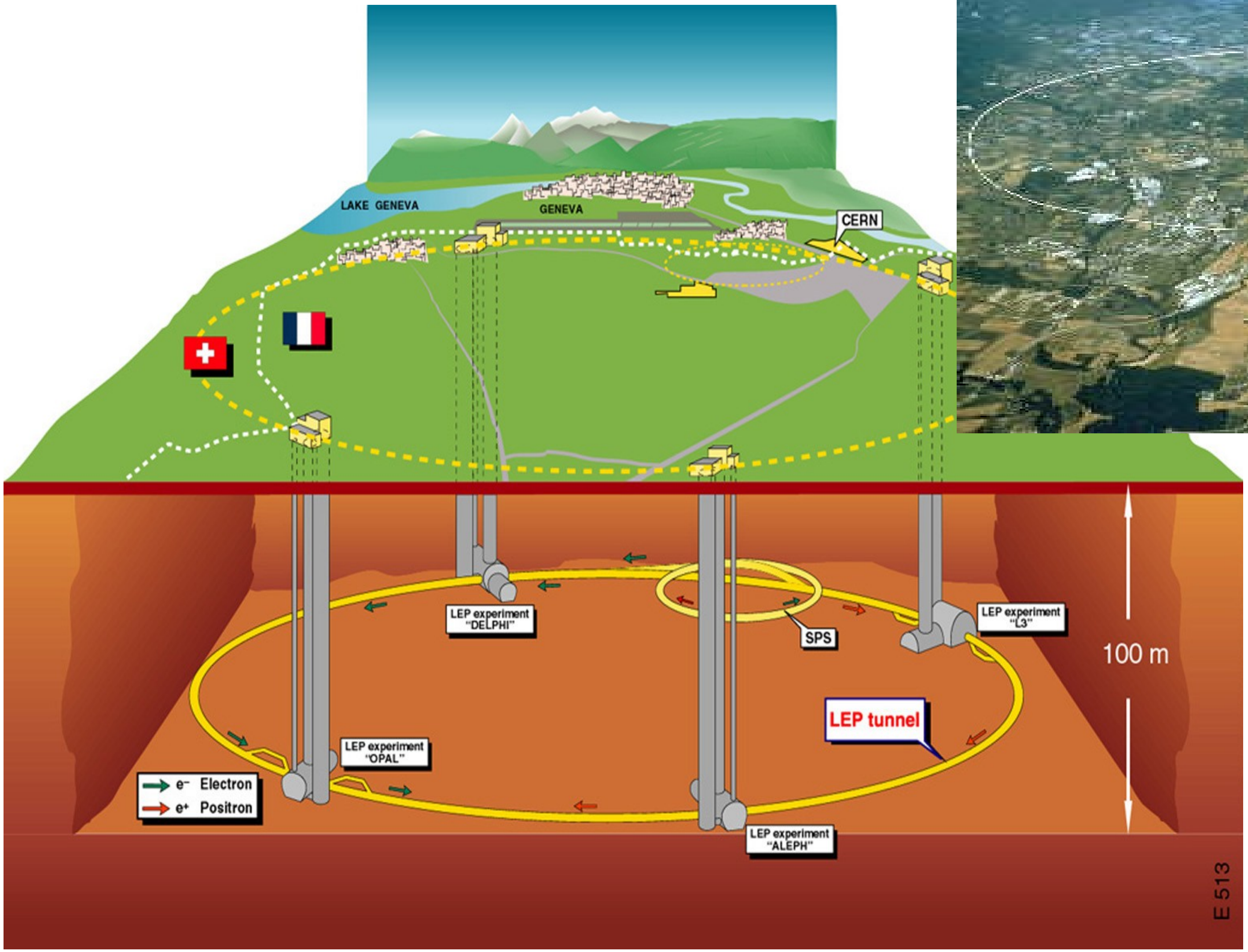


i. Machine and detectors

1. The LEP Collider
2. Detectors
3. The L3 detector
4. LEP events
5. – 16. […]



The LEP collider



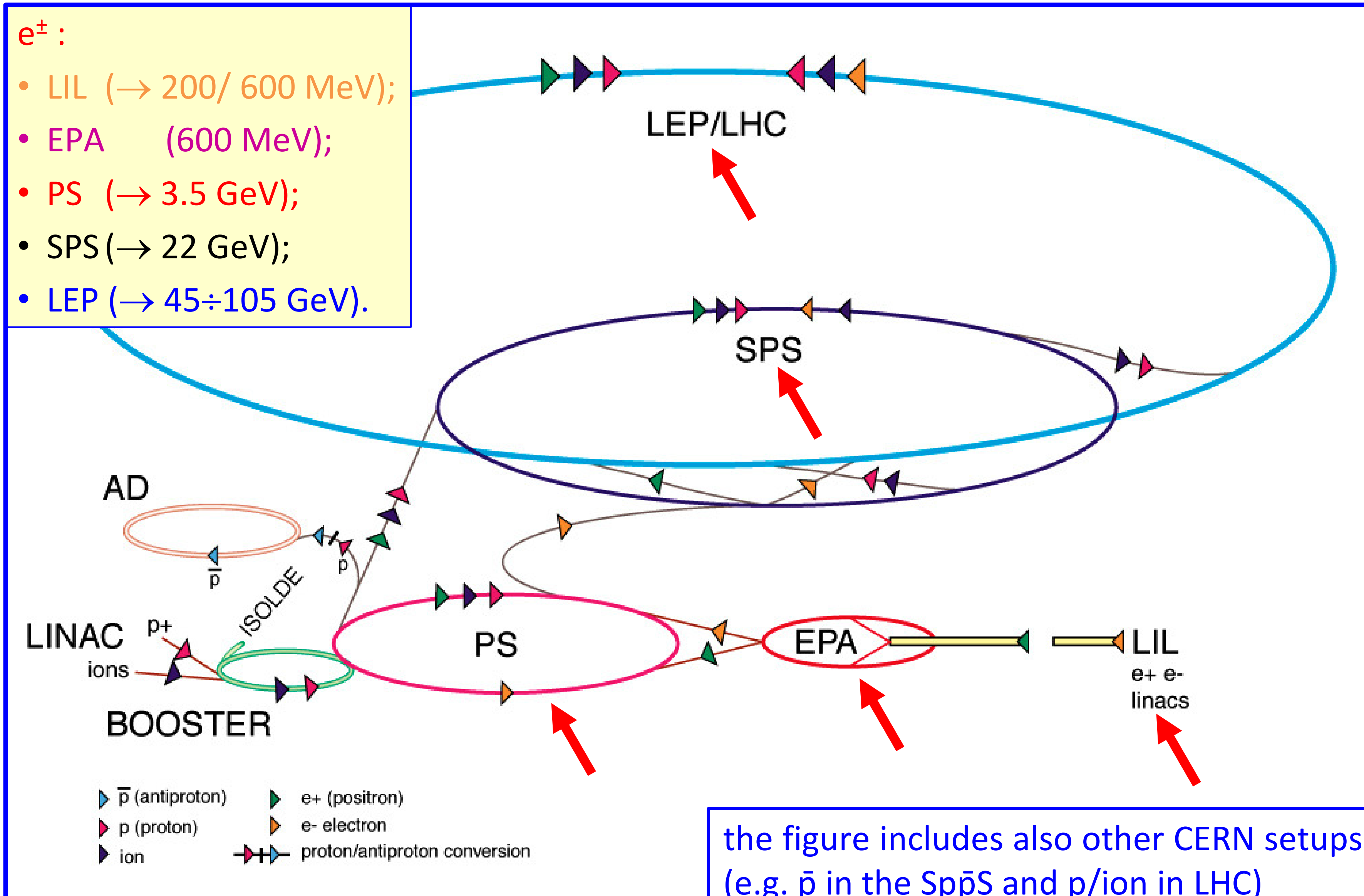
$2\pi R \approx 27$ km

$\sim$100 m underground

planar, slightly tilted wrt surface, because of geology.

The LEP collider : e$^\pm$ acceleration

$e^\pm$:
- LIL ($\to$ 200/ 600 MeV);
- EPA (600 MeV);
- PS ($\to$ 3.5 GeV);
- SPS ($\to$ 22 GeV);
- LEP ($\to$ 45÷105 GeV).

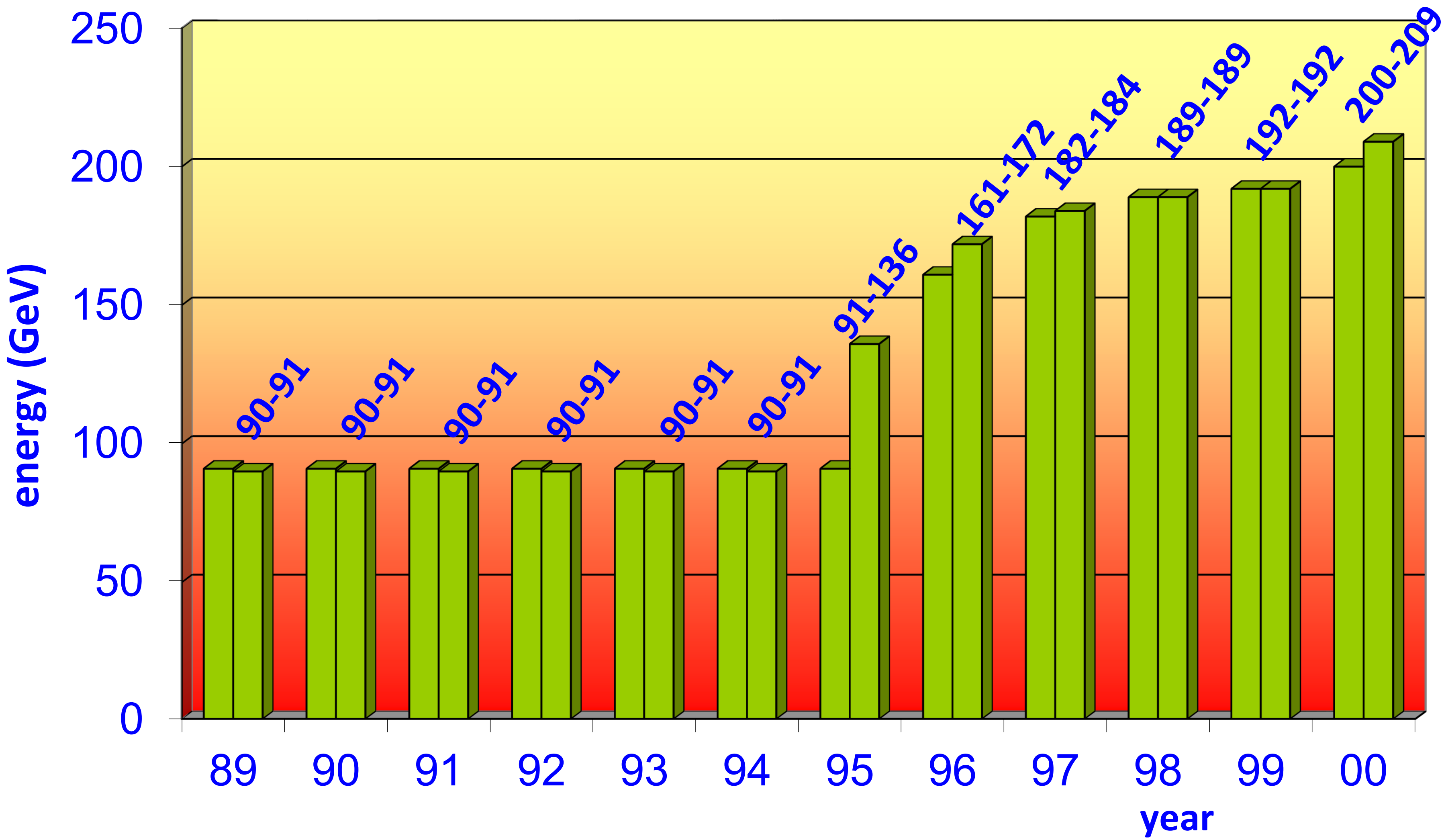


the figure includes also other CERN setups (e.g. p̄ in the Spp̄S and p/ion in LHC)

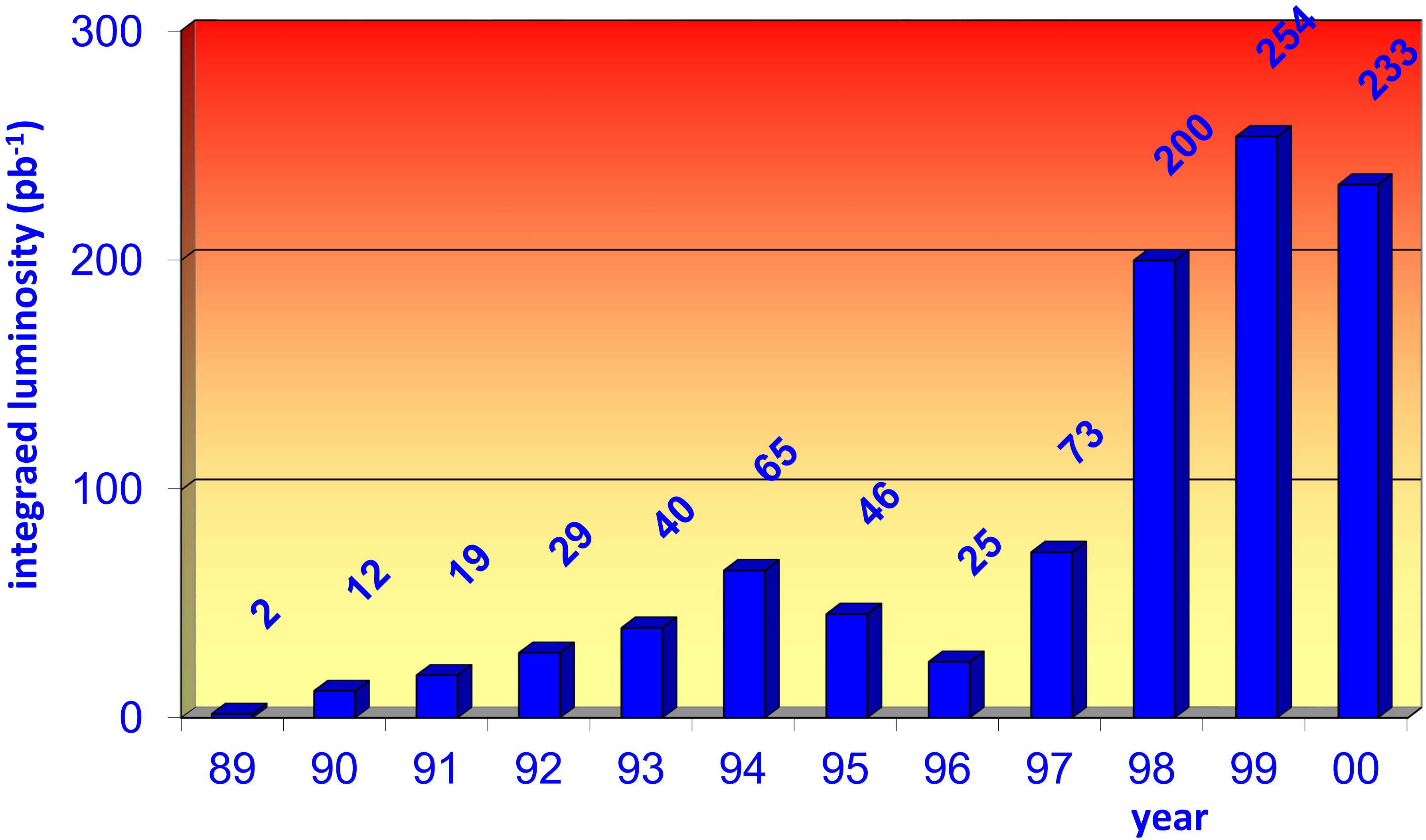
The LEP collider : parameters

	LEP 1	LEP 2
Circumference (Km)	26.66	
E_{max} / beam (GeV)	50	105
max lumi $\mathscr{L}$ (10^{30} cm^{-2} s^{-1})	~25	~100
time between collisions (μs)	22 (11)	22
bunch length (cm)	1.0	
bunch radius (hori.) (μm)	200÷300	
bunch radius (vert.) (μm)	2.5÷8	
injection energy (GeV)	22	
particles/packet (10^{11})	4.5	
packet number	4+4 (8+8)	4+4
years	1989-1995	1996-2000

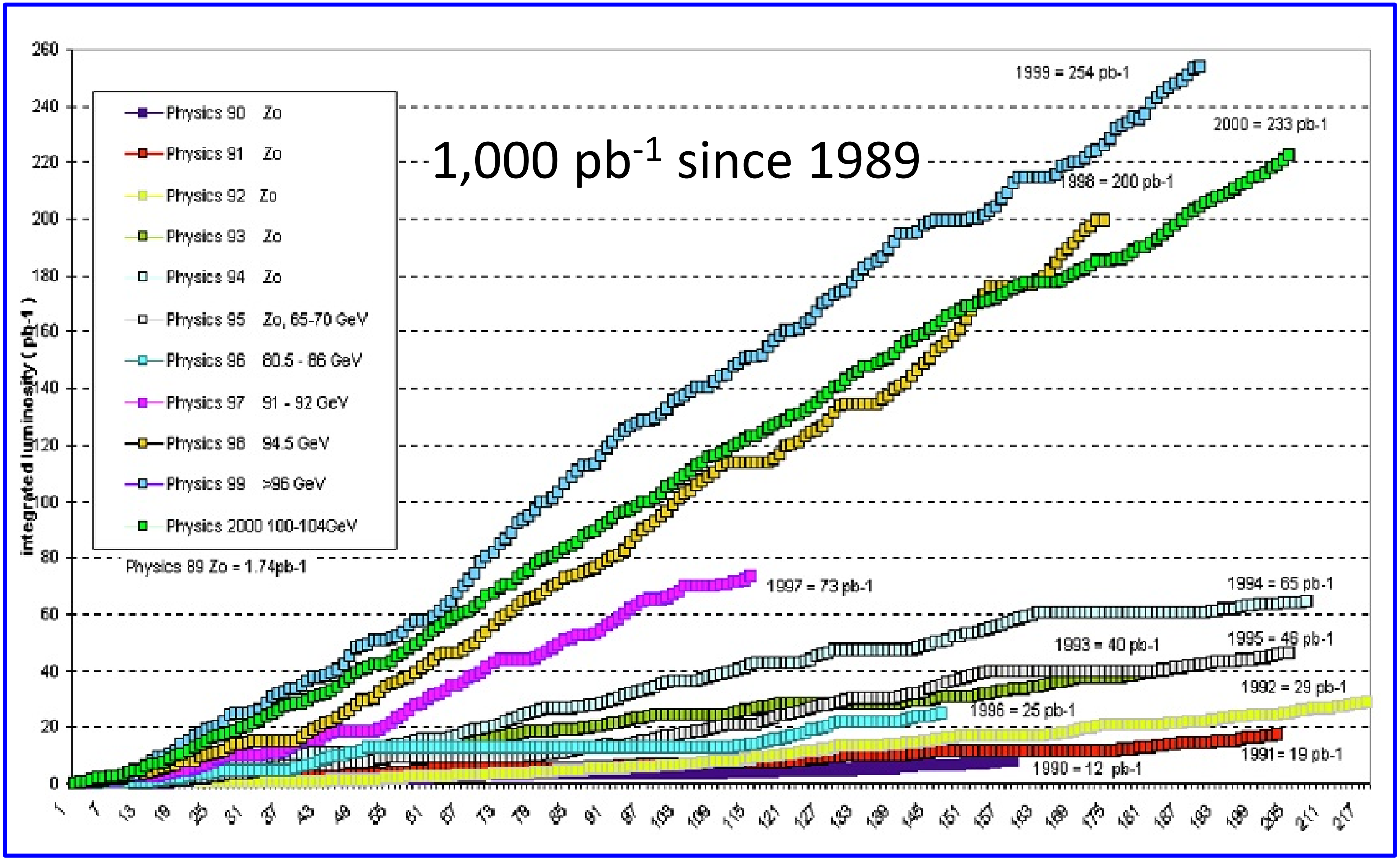
The LEP collider : √s vs year



The LEP collider : $\mathcal{L}_{\text{integrated}}$



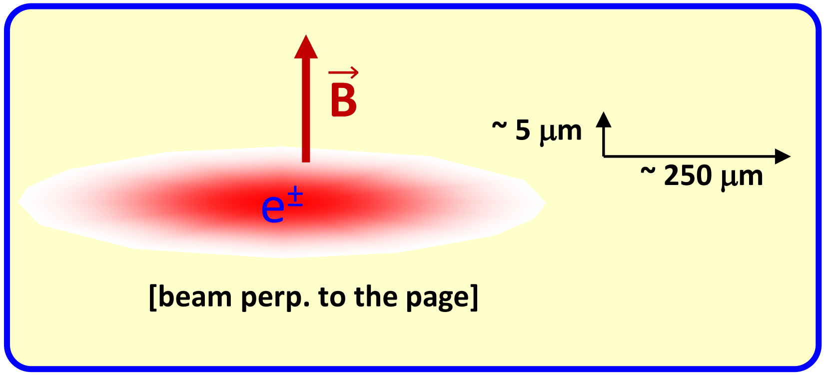
The LEP collider: $\mathcal{L}_{int}$ vs day



The LEP collider: $e^{\pm}$ brem

- $\Delta E_{orbit} \propto e^2 E^4 / (M^4 R)$;   [§ 1]
- ➢ $\Delta E^{e\pm}_{orbit}$(MeV) = 8.85×10^{-5} E^4 (GeV) / R (Km);
- $\langle R_{LEP} \rangle = 4.25 \times 10^3$ m ($\rightarrow$ see table);
- in QED, the bremsstrahlung is not deterministic; the formula gives the average; a further (annoying) effect is the increase of emittance, i.e. the increase of the packets both in space and momentum; this effect is greater in the horizontal plane, as an effect of the magnetic bending:
  - ➢ σ_{hori} = $200 \div 300$ μm;
  - ➢ σ_{vert} = $2.5 \div 8$ μm.

E_{beam} (GeV)	$\sqrt{s}$ (GeV)	ΔE_{orbit} (GeV)
45	90	~0.1
90	180	~1.4
100	200	~2.1



The LEP collider: $\mathscr{L}$ effective

Assume $\mathscr{L}_{max} = 2\times10^{31}cm^{-2}s^{-1}$:

- $\sigma_{tot}(e^+e^- \to Z, \sqrt{s}=m_Z) \approx 40$ nb :
  - $R_{max}(e^+e^- \to Z, \sqrt{s}=m_Z) = \mathscr{L}\,\sigma_{tot} = 0.8$ Hz;
  - ~~6×10^4 events / day $\to$ 10^7 events/~~ year;
  - [??? no !!!];

... because ...

- the luminosity normally quoted corresponds to the "peak lumi.", i.e. the first minutes after acceleration and squeezing;

  $\mathscr{L}(t) = \mathscr{L}_{max}\exp(-t/\tau)$ (stochastic effects + optics corrections)

  $\to \langle\mathscr{L}\rangle \approx ½\,\mathscr{L}_{max}$

  \+ techn. stops, maintenance, mistakes, ...

- global efficiency ~ ¼

- also data @ $\sqrt{s} \neq m_Z$ (e.g. to measure the lineshape), where σ much smaller.

$\Rightarrow$ @ LEP 1 (many years) :

$\boxed{4 \times 10^6 \text{ hadronic events} \times 4 \text{ exp} = 15.5 \times 10^6 \text{ hadronic events}}$

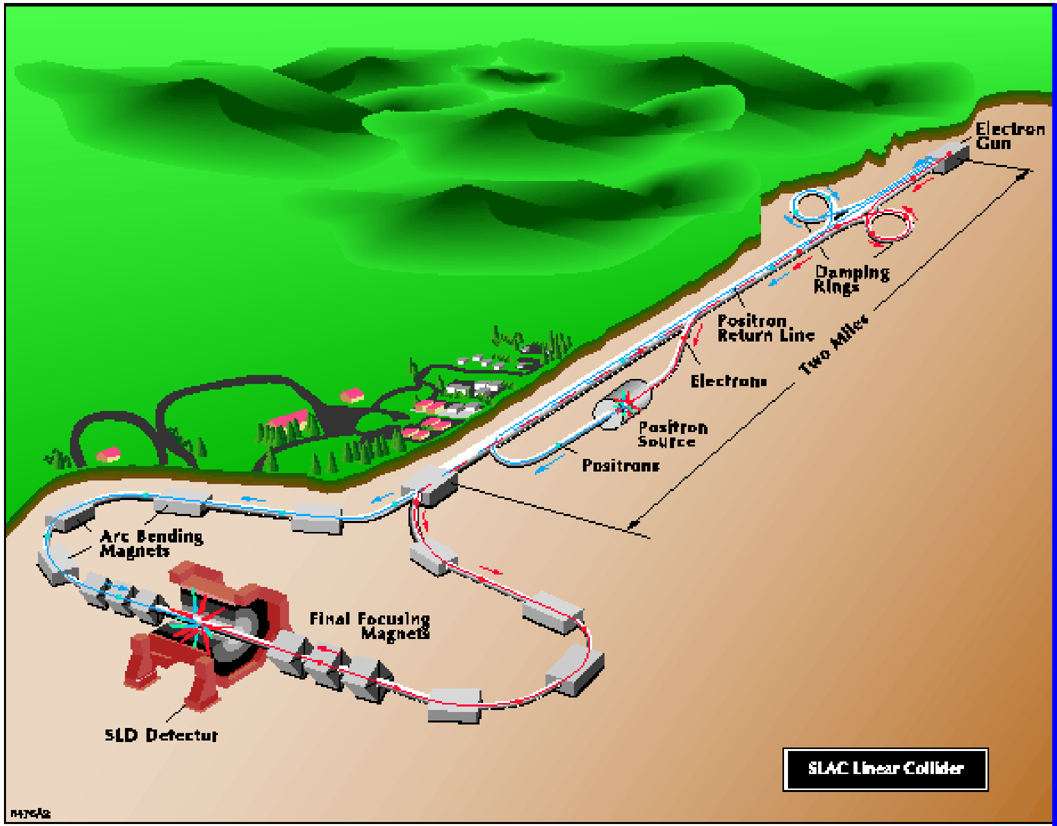
\+ the corresponding leptons.

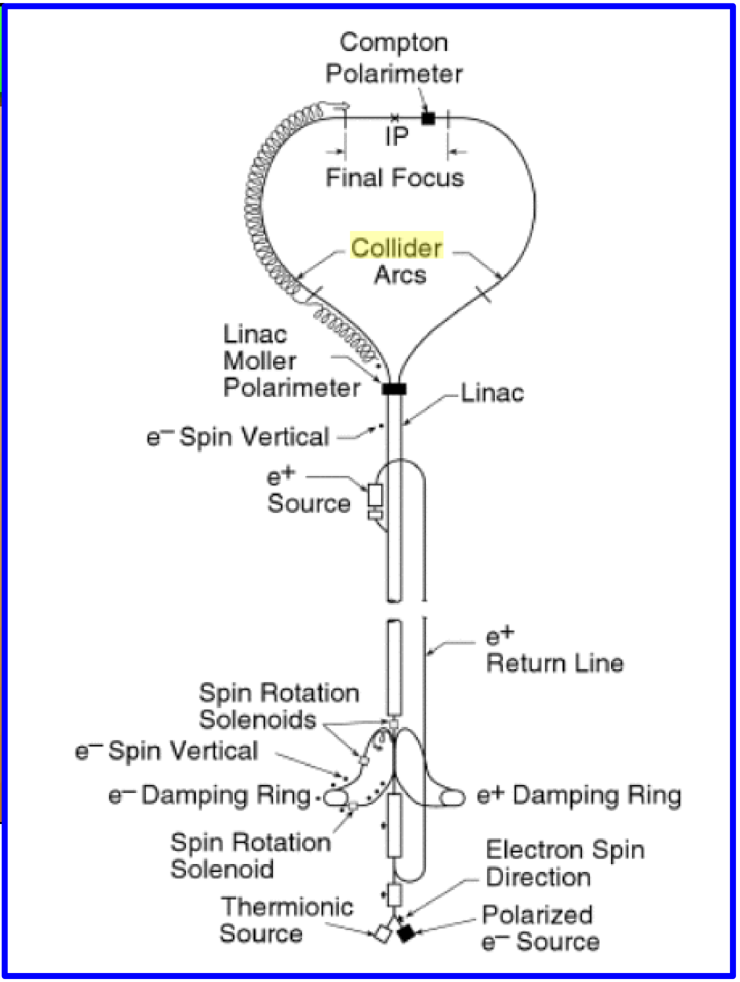
Problem: use the formulæ of § 1 and the LEP parameters to compute $\mathscr{L}_{bc}$ and μ $(=\mathscr{P}_{int})$.

Comment on TDAQ requirements. Is LEP trigger/DAQ "easy" or "difficult" ?

[*please think before answering*]

The LEP collider : the competition - SLC

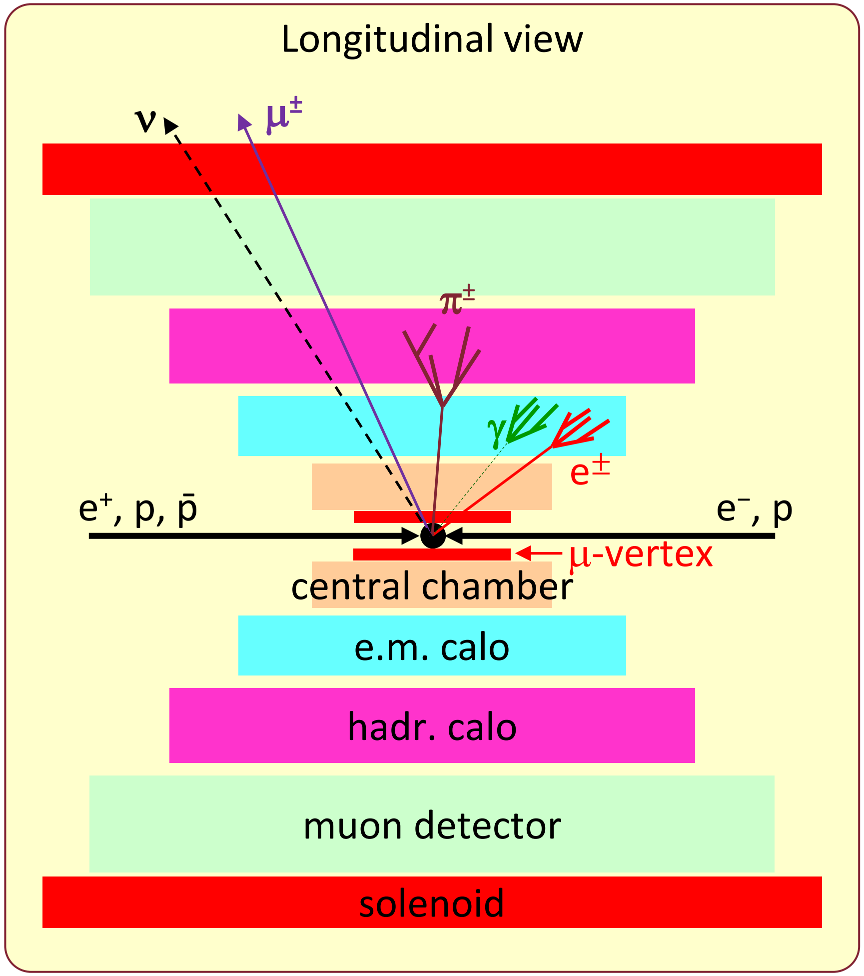


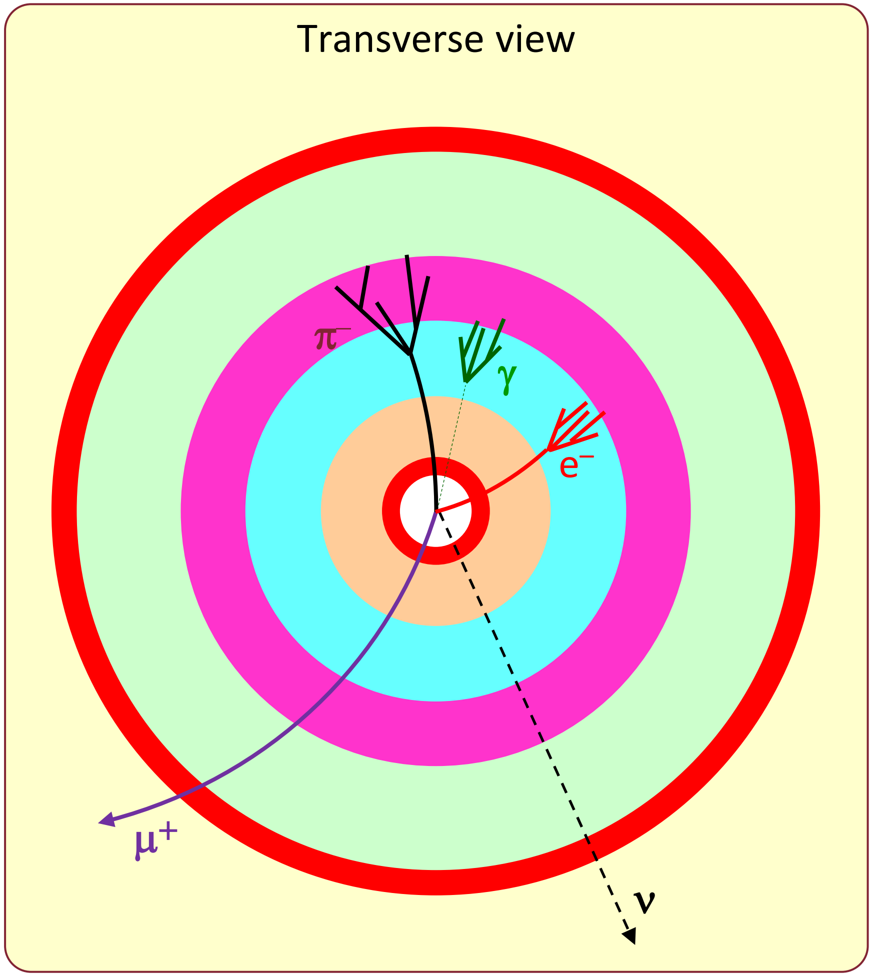


SLC : Stanford Linear Collider (1989-98):

- the first example of linear e^+e^- collider;
- lower energy (only Z pole) and less intense;
- polarized beams;
- promising new technique ($\sqrt{s}$ > 500 GeV $\rightarrow$ a circular e^+e^- requires a huge ring).

Detectors

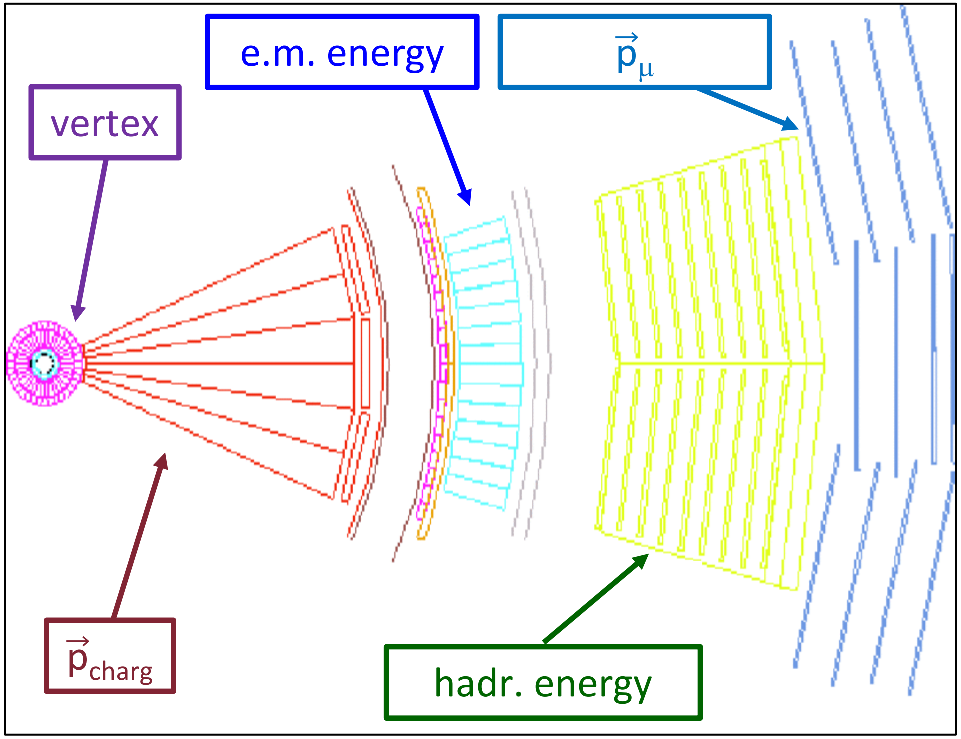




A typical detector of LEP / TeVatron / LHC (ATLAS is the only remarkable exception).

Please, figure out how exp.'s measure E, $\vec{p}$ and identify all particles.

Detectors: principles



	$\vec{p}_{charg}$	E_{em}	E_h	$\vec{p}_\mu$	**sec. vtx. ?**
$e^\pm$	yes	yes	~no	no	yes
γ	no	yes	~no	no	no
$\pi^\pm$, $K^\pm$	yes	← yes →		no	yes
n, K^0	no	← yes →		no	no
$\mu^\pm$	yes	mip	mip	yes	yes
ν	no (but *hermeticity*)				

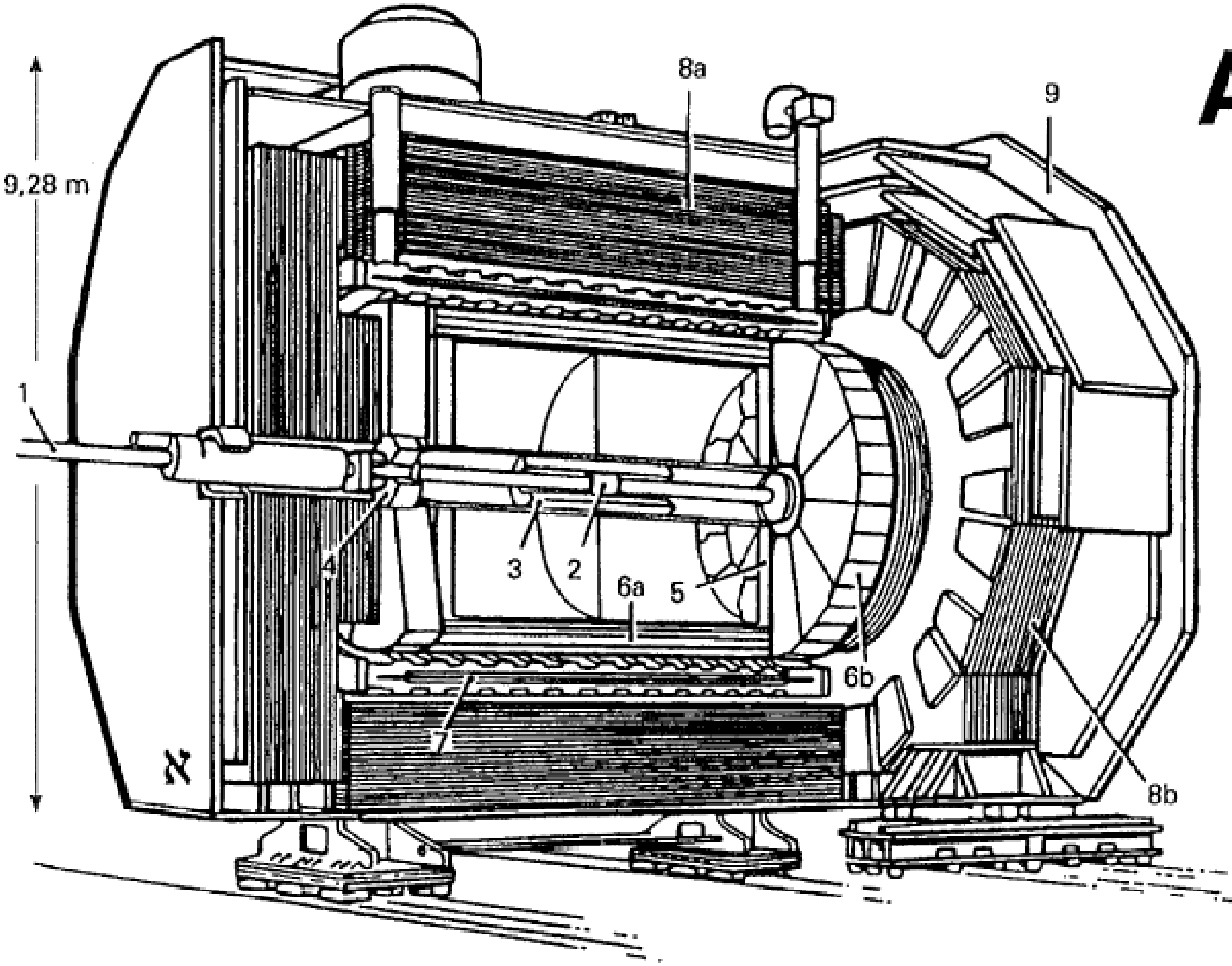
The ν's are "detectable" from the conservation of the 4-momentum, i.e. :

$$\begin{cases} \vec{p}_\nu = -\sum_{all} \vec{p}_j; \\ E_\nu = \sqrt{s} - \sum_{all} E_j; \end{cases} \quad \left[\oplus\ m_\nu^2 = E_\nu^2 - |\vec{p}\,|_\nu^2 = 0 \right].$$

A detector fully operational allows for both the measurement of the 4-momenta of all the particles and their identification ("*part.id*"). The charge is measured by the sign of the bending.

Problem : what happens if there are two ν's in the final state ?
An interesting question … and not uncommon $[Z\to\tau\bar{\tau},\ ZH\to\nu\bar{\nu}b\bar{b}]$.

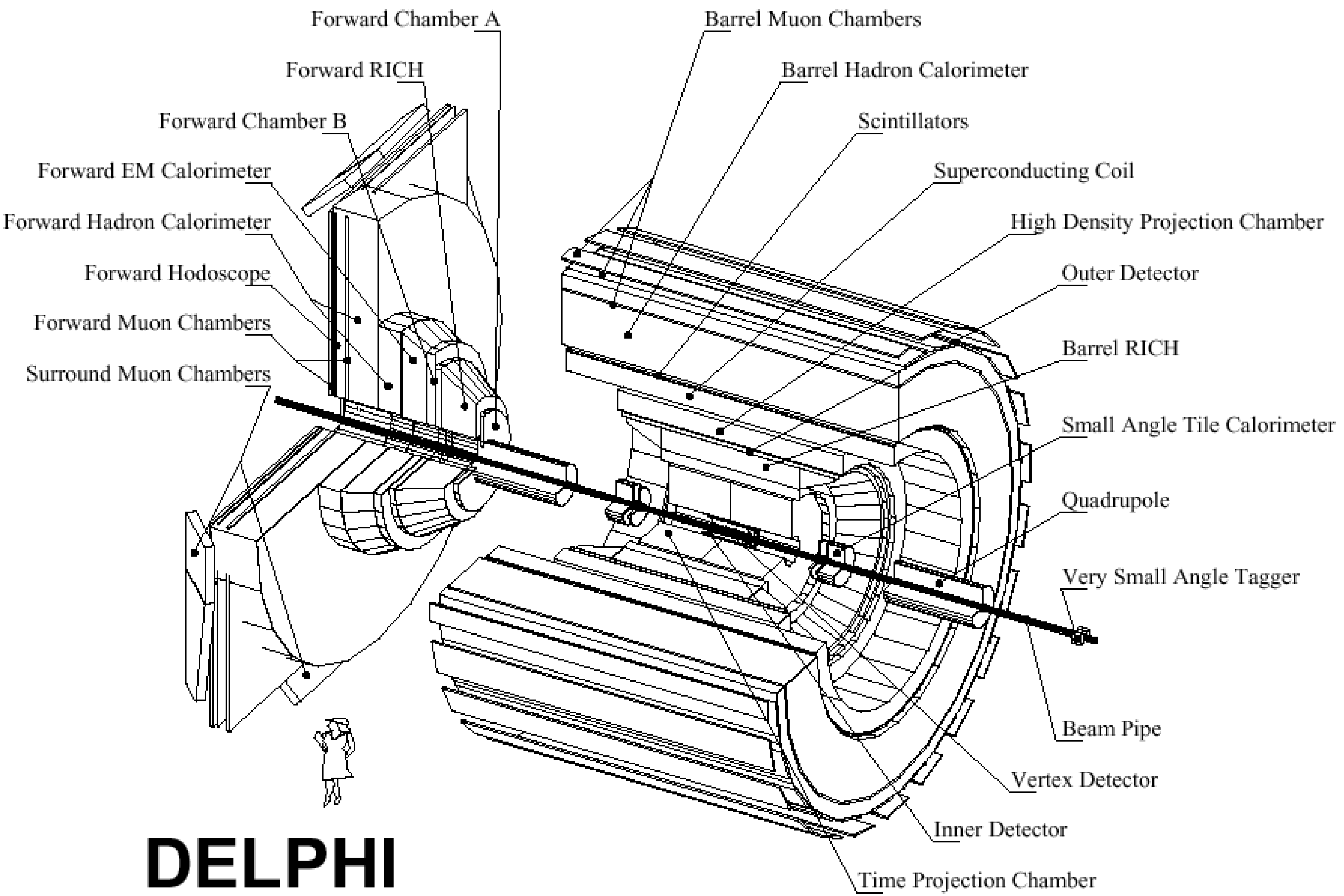
Detectors : א



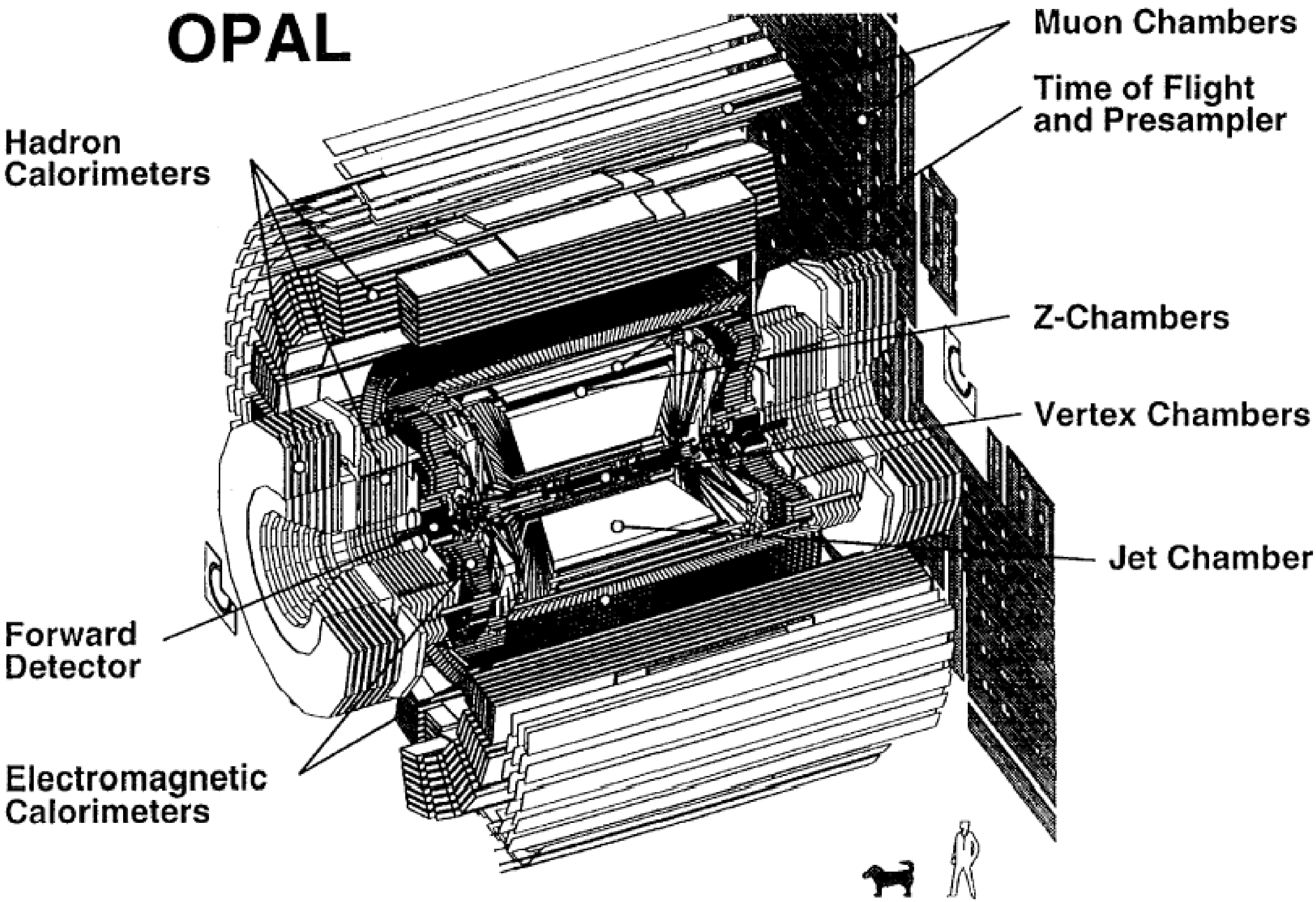
ALEPH

1 Beam Pipe
2 Silicon Vertex Detector
3 Inner Tracking Chamber
4 Luminosity Monitor
5 TPC Endplate
6 Electromagnetic Calorimeter
  6a Barrel
  6b Endcap
7 Superconducting Coil
8 Hadron Calorimeter
  8a Barrel
  8b Endcap
9 Muon Chambers

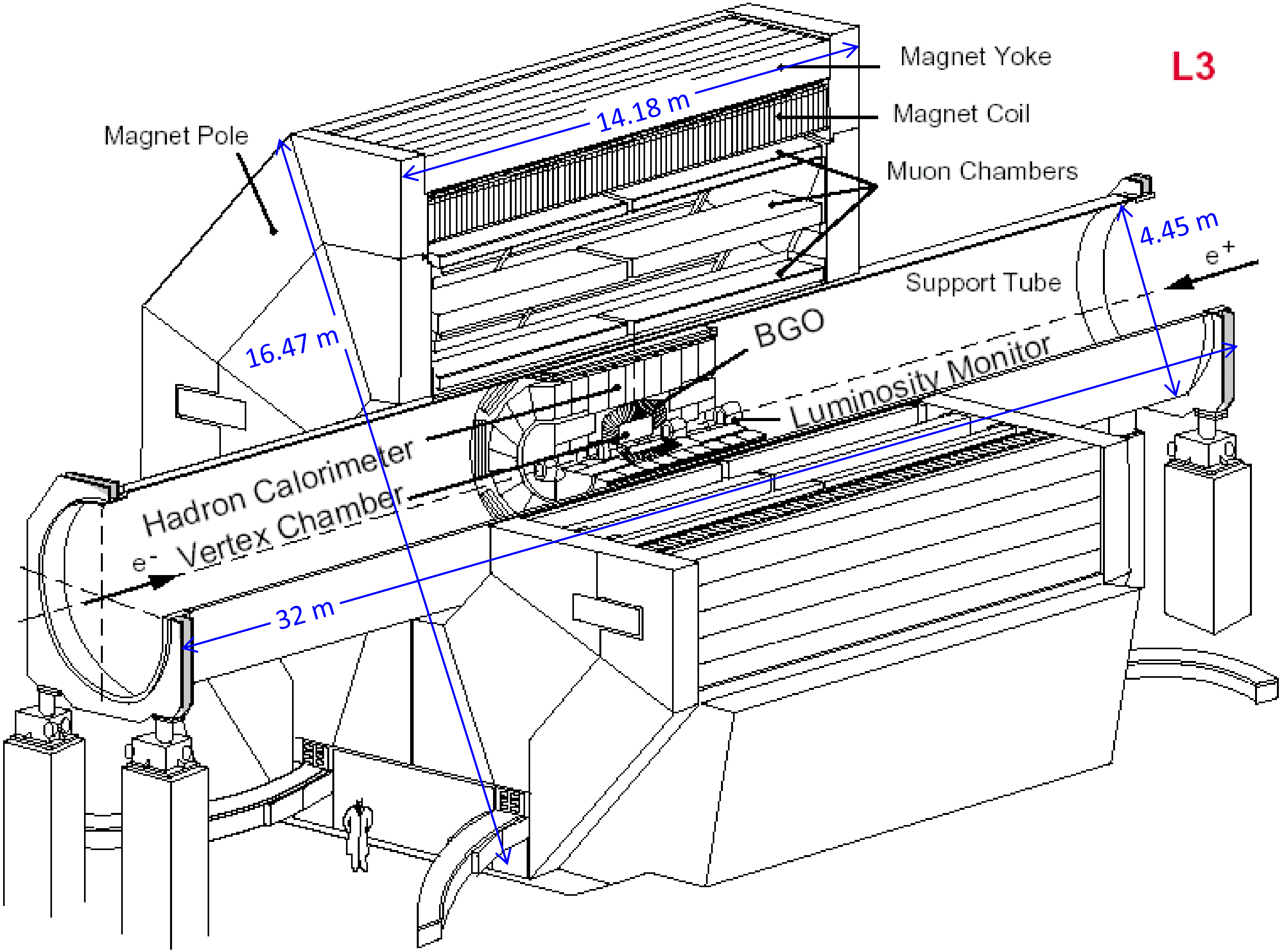
Detectors : DELPHI



Detectors : OPAL

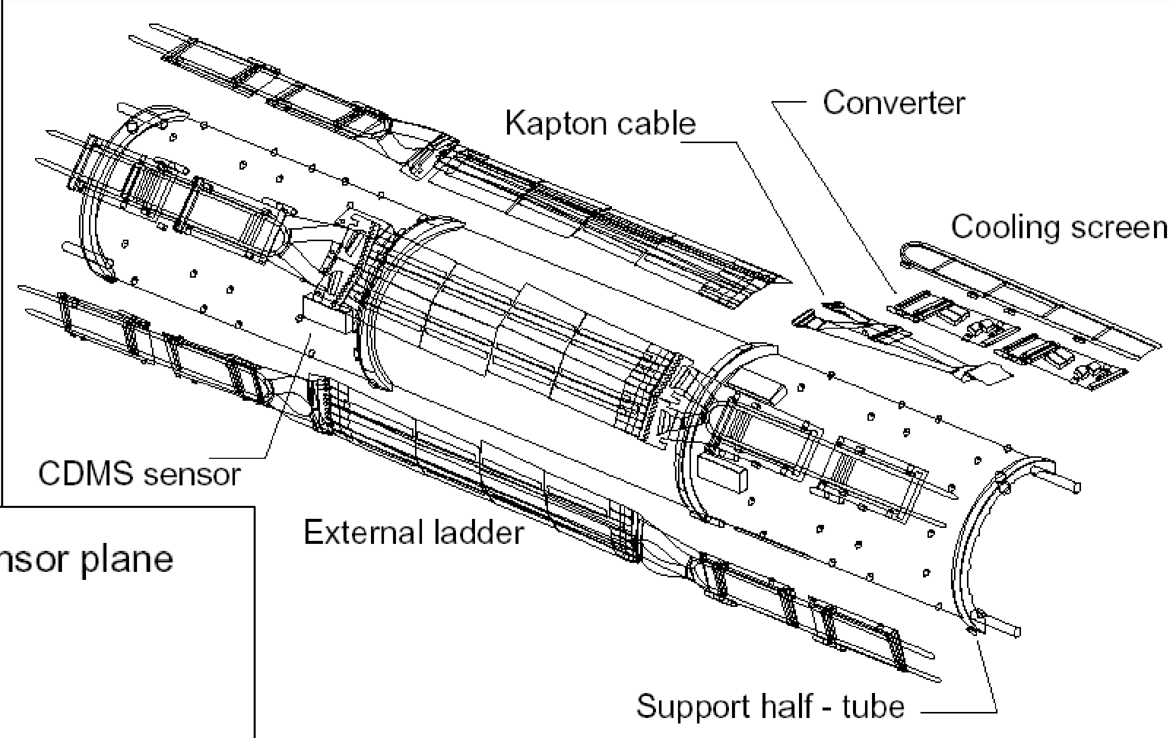


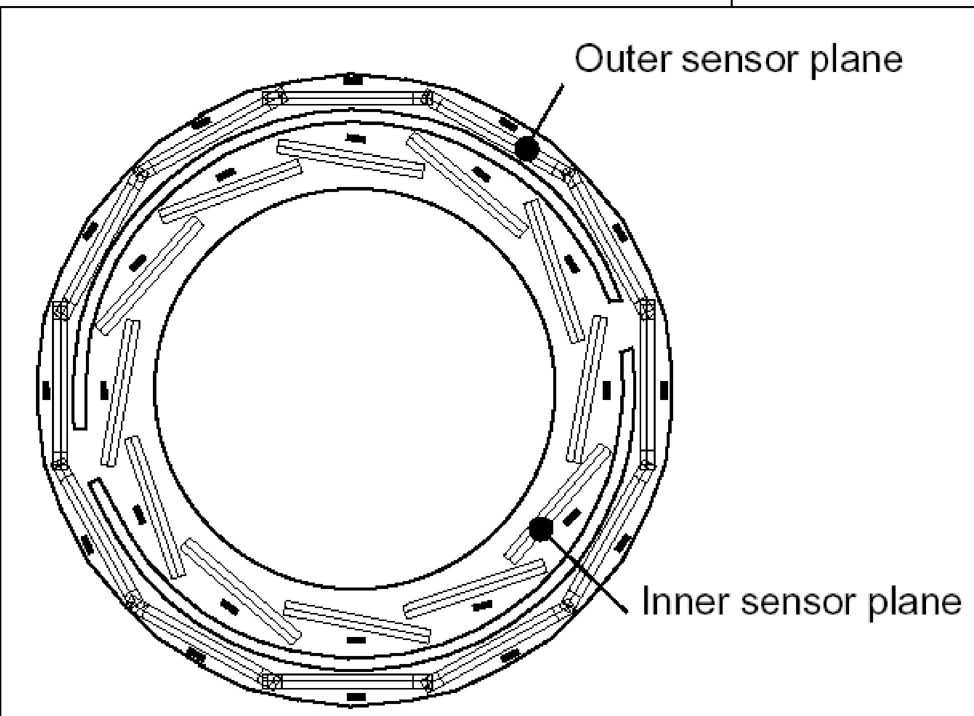
Detectors : L3



The L3 detector: SMD

- 96 silicon wafers
- 70 mm × 40 mm × 300 μm
- two layers:
  - $\varnothing$ inner layer : 120 mm
  - $\varnothing$ outer layer : 150 mm
  - zenith coverage : $|\cos\theta| < 0.93$.

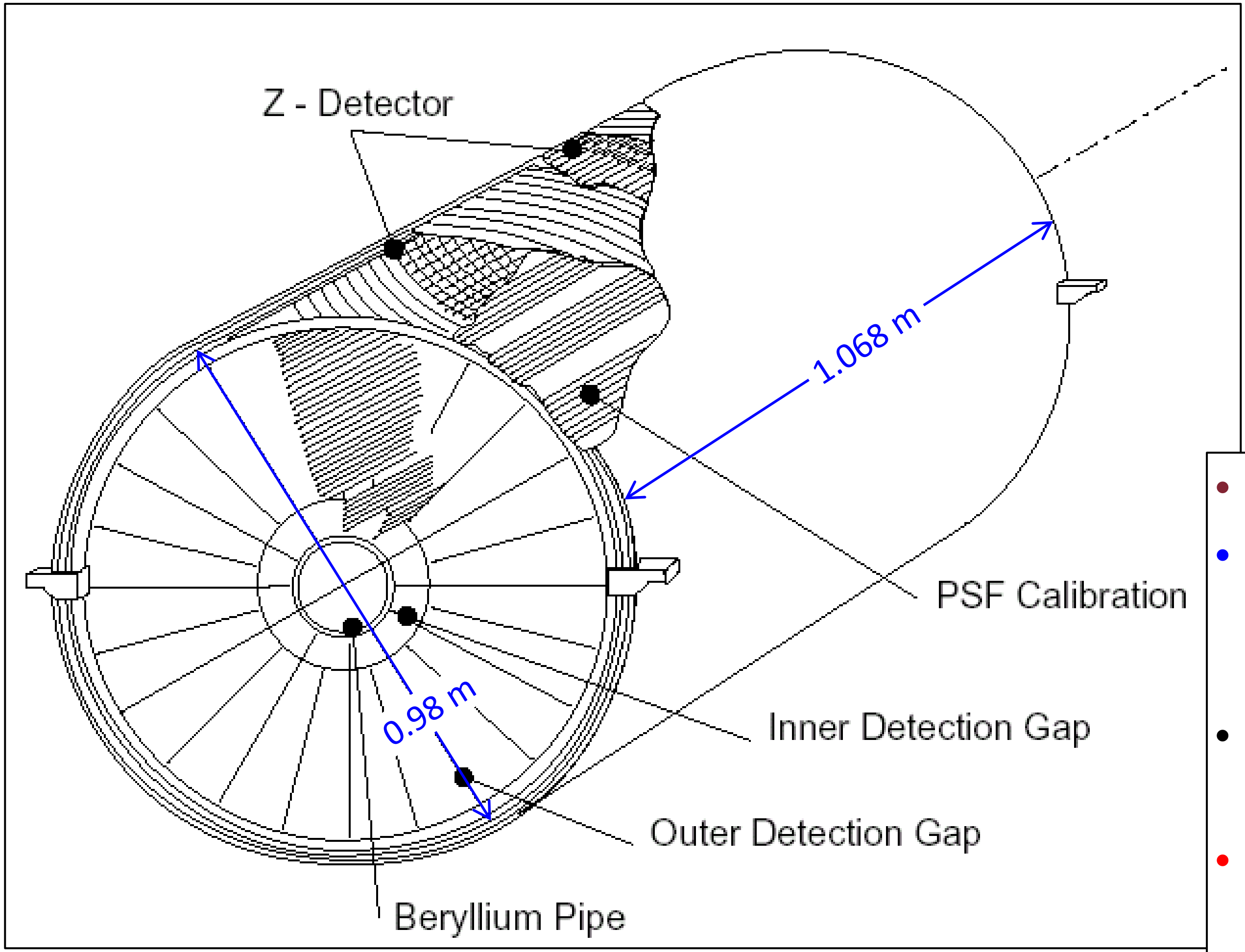




2 read outs :

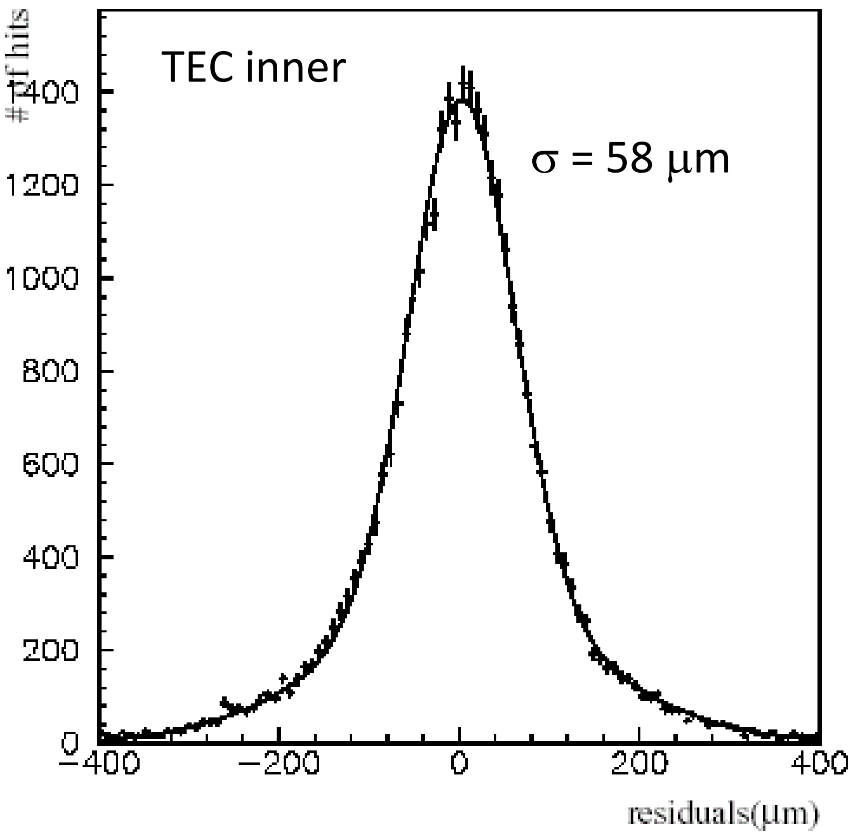
- 50 μm in rϕ;
- 150÷200 μm in z

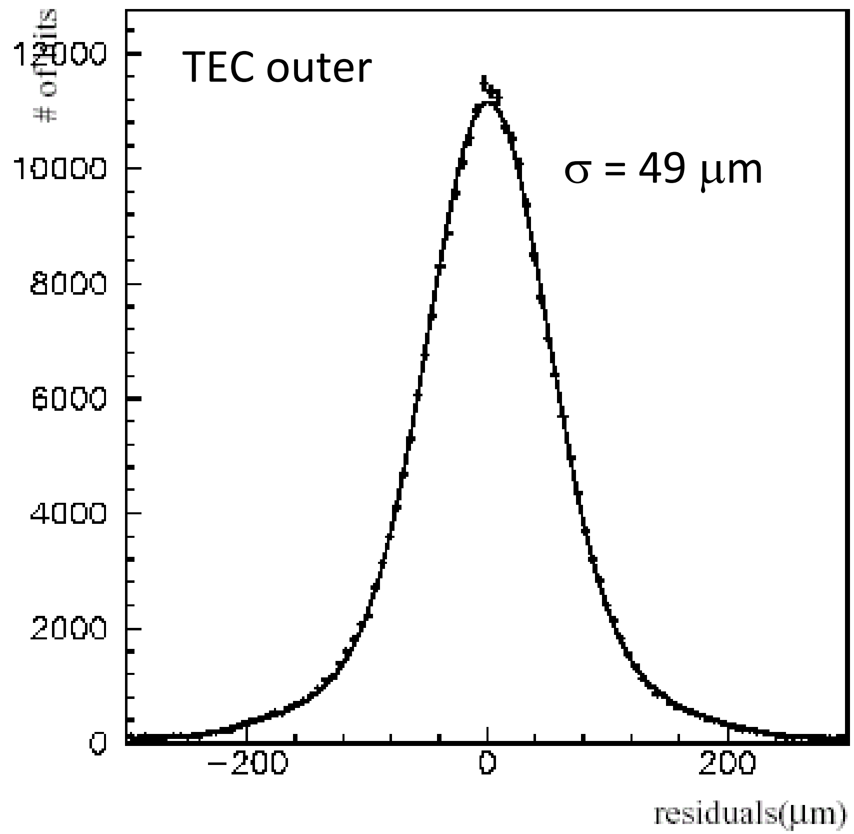
The L3 detector: TEC



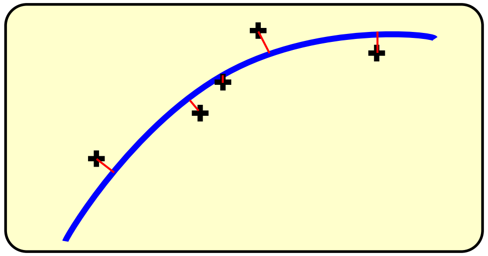
- ext. – int. radius = 317 mm;
- two separate concentrical regions : inner 8 wires + outer 54 wires;
- 80% CO_2, 20% iC_4H_{10}, 1.2 bar (abs);
- v_{drift} = 6μm / ns ("TEC" = Time *Expansion* Chamber);
- $\alpha_{Lorentz}$ = 2.3°;
- z-detector (σ= 320μm).

The L3 detector: TEC results





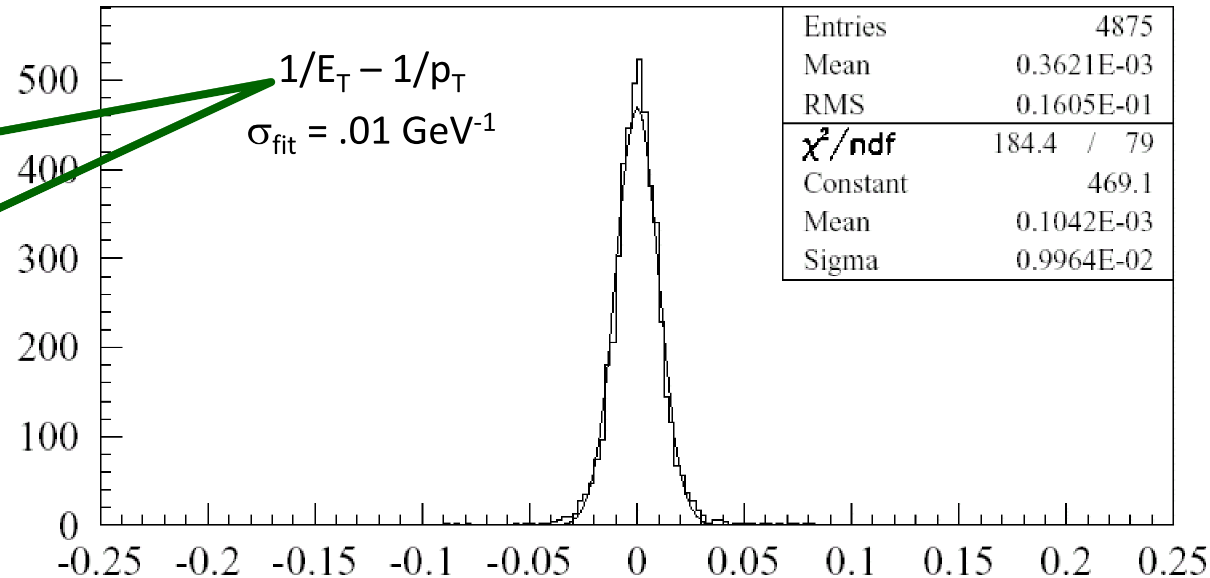
The ***residuals*** are the distances (with sign) between the measurements and the fitted trajectory. Assuming "many" measurements with the same resolution, their distribution is expected to be gaussian with mean=0 and RMS=resolution.

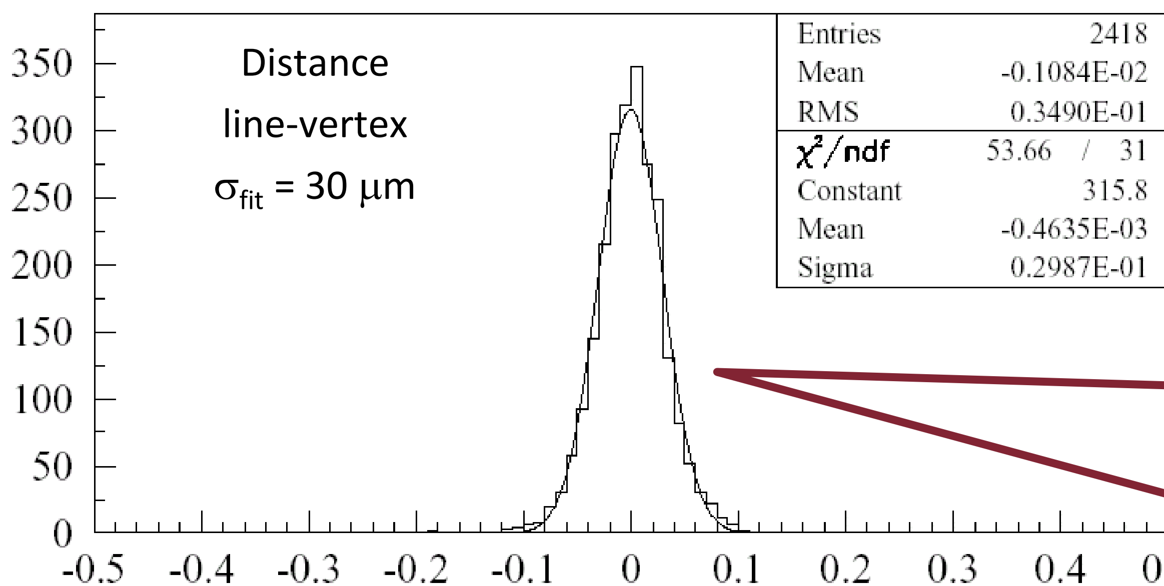


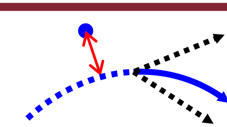
The L3 detector: SMD + TEC

Why plot (1/E – 1/p), instead of (E–p) ?

Answer in few slides, but you should be able to understand yourself.

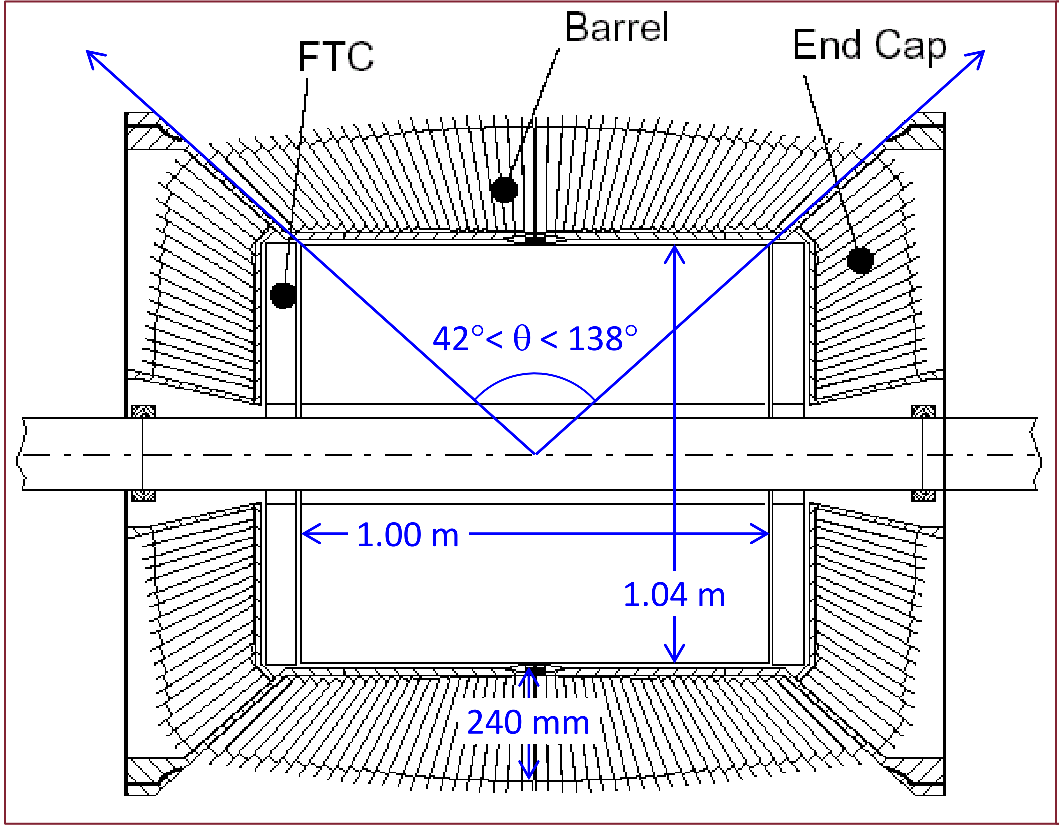


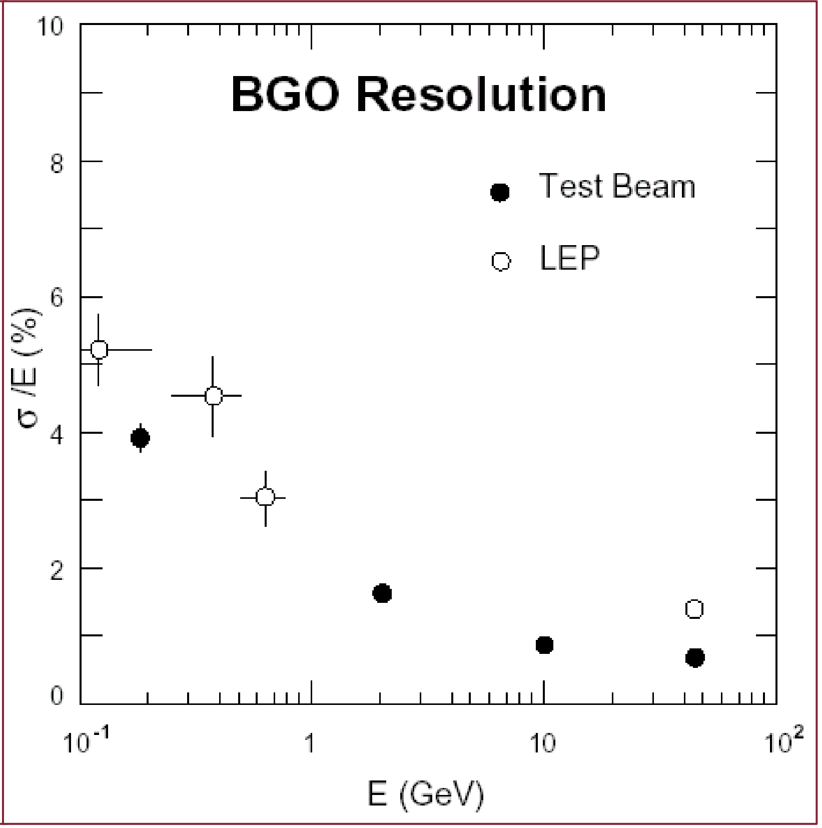




Tracks, which miss the interaction point, are a signal of secondary verteces (τ's, heavy flavors…) → the resolution on the "impact parameter" is important.

The L3 detector: BGO

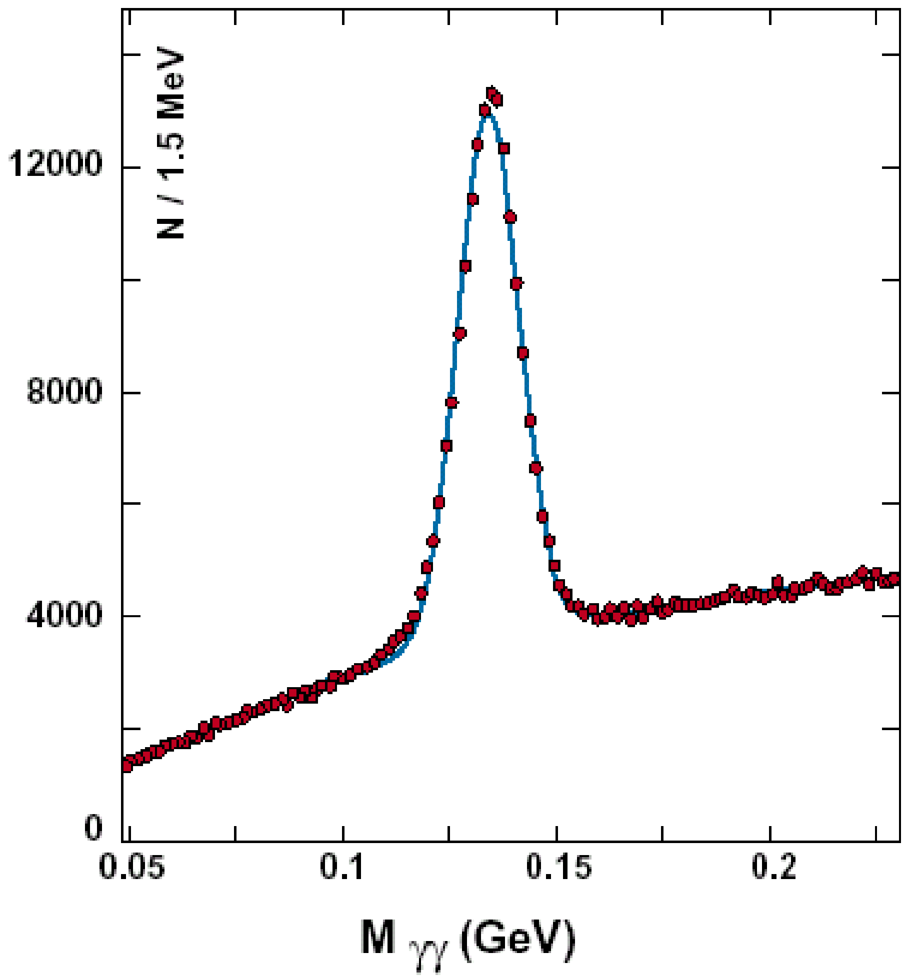




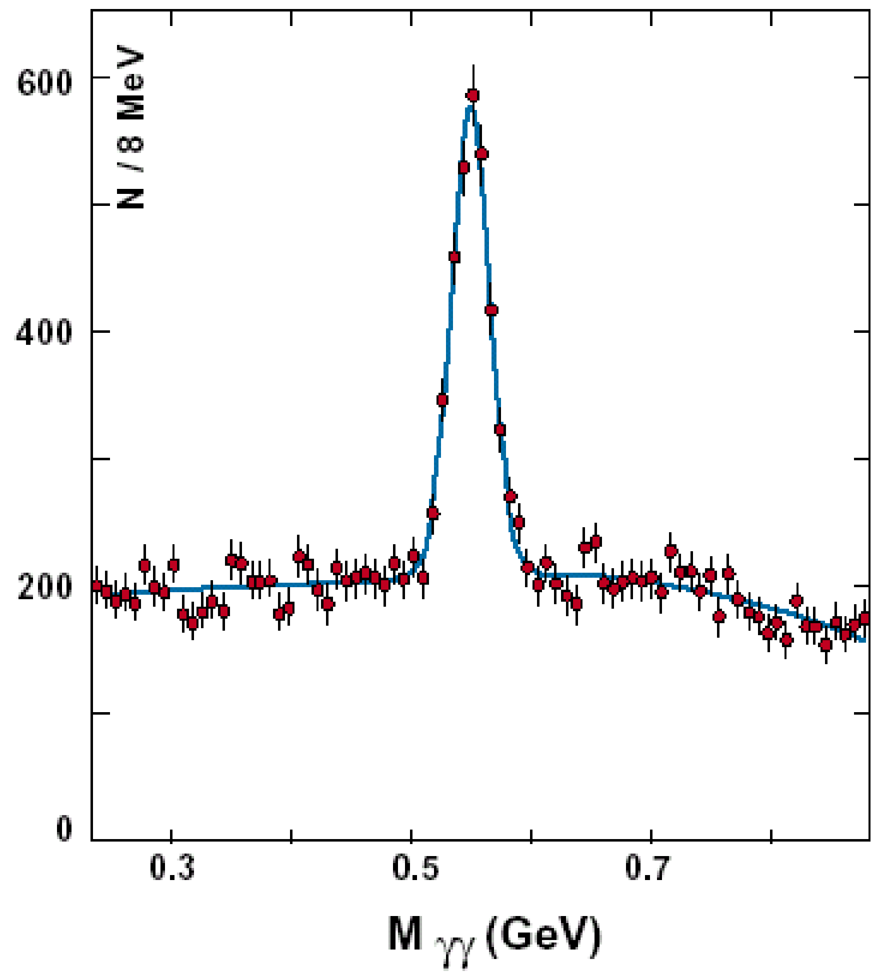
- 11,000 BGO (Bismuth germanium oxide $Bi_4 Ge_3 O_{12}$) scintillating crystals;
- pyramids 20×20 → 30×30 mm², length 240 mm;
- X_0 = 11.3 mm → 21 X_0.

The L3 detector: BGO results

π°, σ = 7 MeV



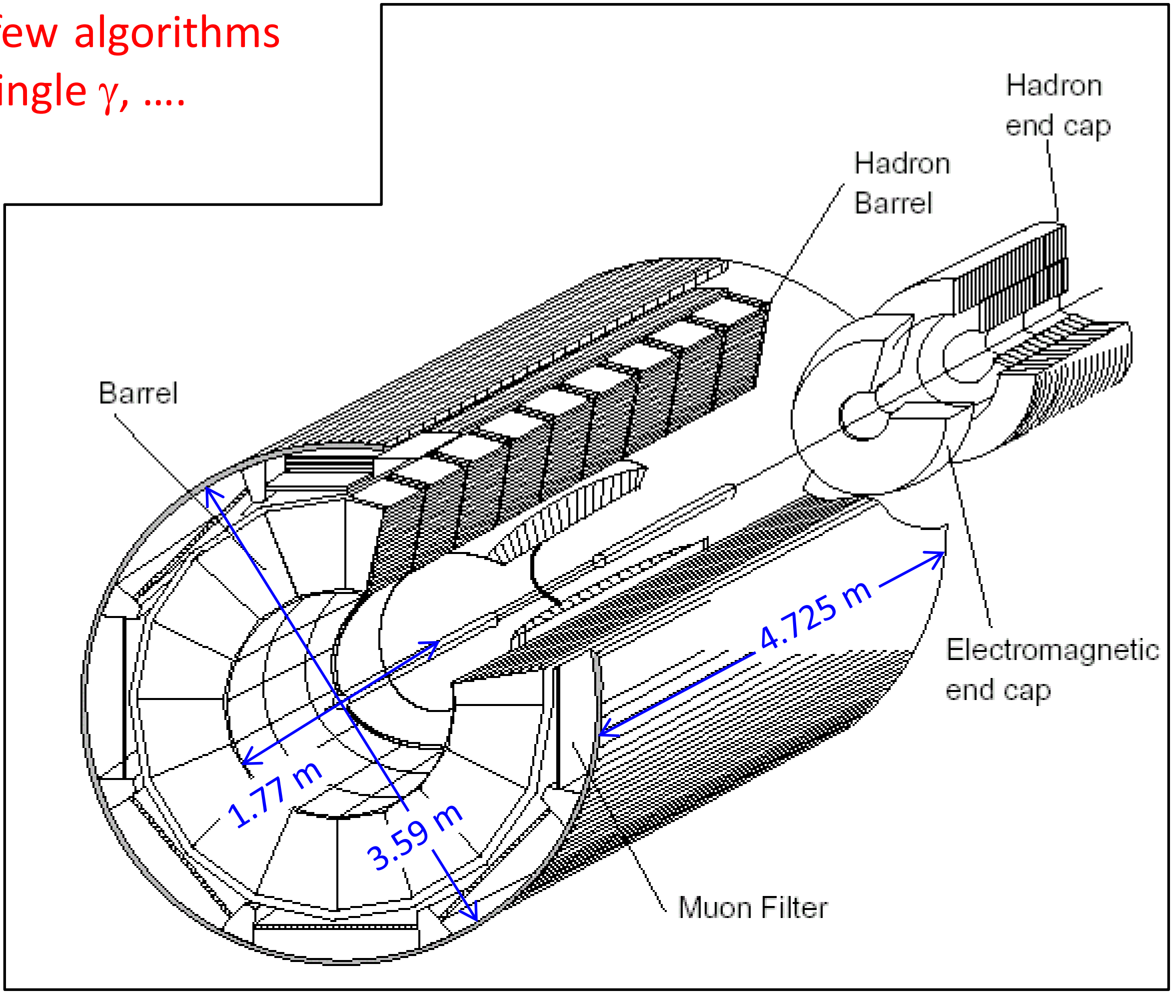
η , σ = 16 MeV



the mass resolution for particles decaying into γ's is the traditional figure of merit of the e.m. calo (also for $H \to \gamma\gamma$ at LHC !!!).

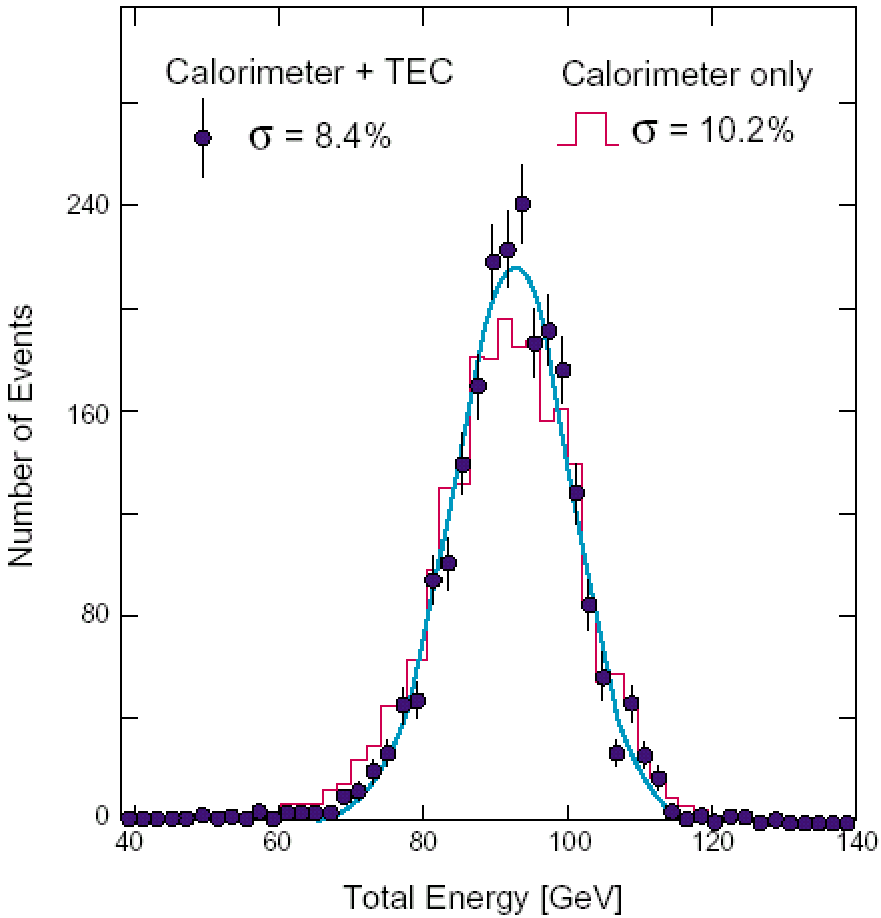
The L3 detector: HadCal

- plates of depleted U (U_{238}) + proportional wire chambers (370,000 wires);
- brass μ-filter (65%Cu, 35% Zn) + prop. tubes;
- BGO + hadcal in calo trigger (few algorithms in .OR., e.g. E_{tot}, E_{tot}^{BGO}, cluster, single γ, ….



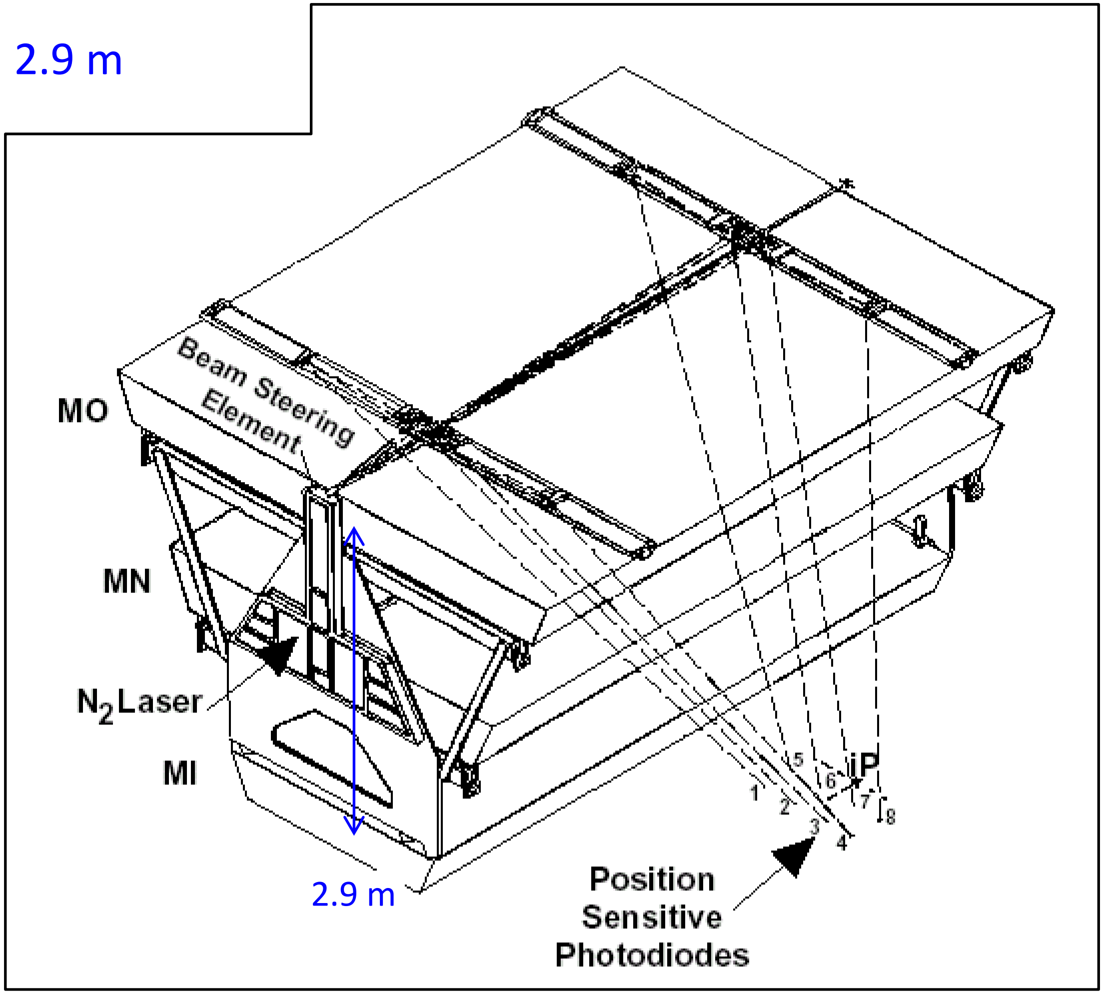
The L3 detector: HadCal results

- $Z \to q\bar{q}$ at $\sqrt{s} = m_Z$;
- E_{tot} is known and used to calibrate the detector;
- $E_{vis} / \sqrt{s} = \sum_i E_i / \sqrt{s}$ in two cases :
  - calo e.m. + had;
  - calo e.m. + had + TEC (no double-counting);
  - resolution = 10.2% with calos only;
  - resolution = 8.4% , when TEC is also used (avoiding double counting).

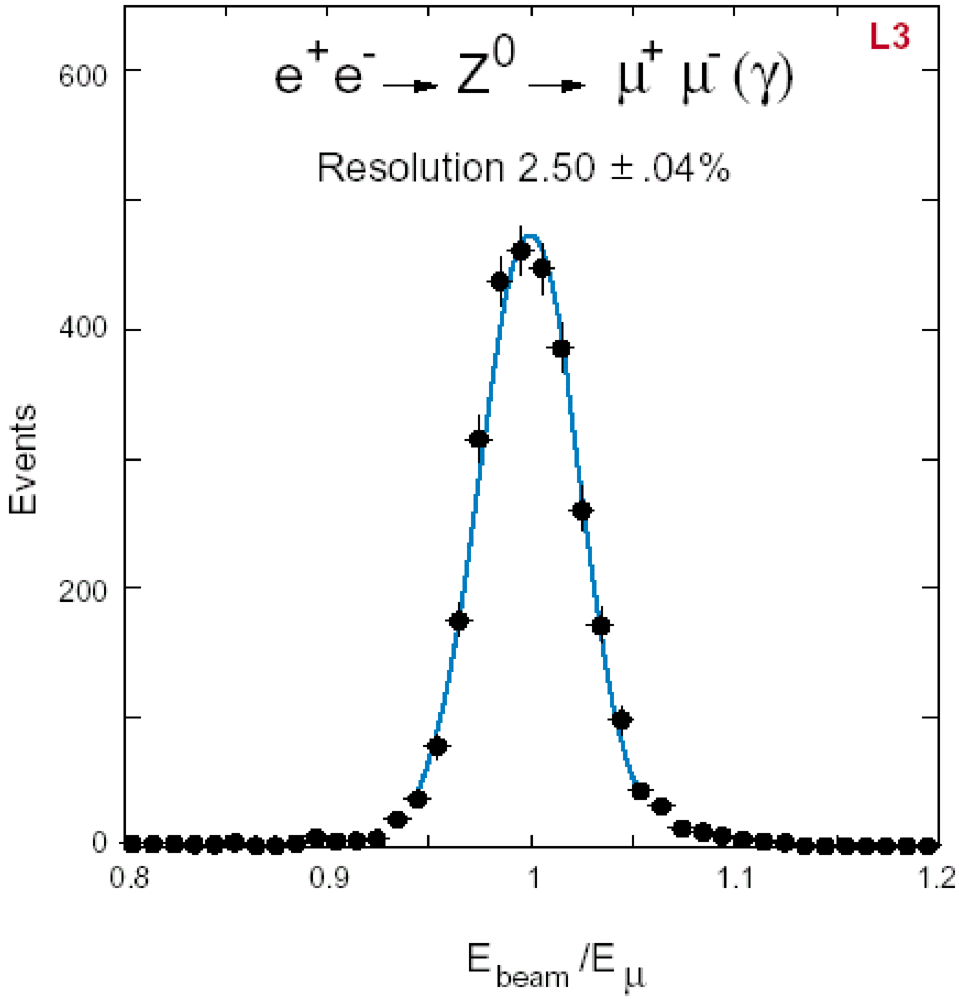


The L3 detector: μ chambers

- octants, each with three chamber types : MO + MN + MI (16 + 24 + 16 wires);
- effective length of measurement: 2.9 m
- mechanical accuracy: ~10μm;
- alignment with optical sensors.



The L3 detector: μ chambers results



Why plot E_{beam} / $E_{measured}$? [i.e. $\sqrt{s}/(2E_\mu)$]

- the sagitta ($\propto$ 1/p) is the measured parameter;
- therefore 1/p ($\approx 1/E_\mu$) expected gaussian, while p is asymmetric in the tails;
- E_{beam} / E_μ = $\sqrt{s}$ / (2 p_μ);
- $\sigma(m_Z)/m_Z = \sigma\,[E_{beam} / E_\mu]\,/\sqrt{2}$ [show it !!!]

For Z events, error from the machine, i.e. $\sigma(m_Z) = \sigma(\sqrt{s})$ = few MeV.

This method is used to check $\vec{p}_\mu$, which is used in other channels (e.g. Higgs search).

And why (1/E – 1/p), or (1/E_T – 1/p_T) ?

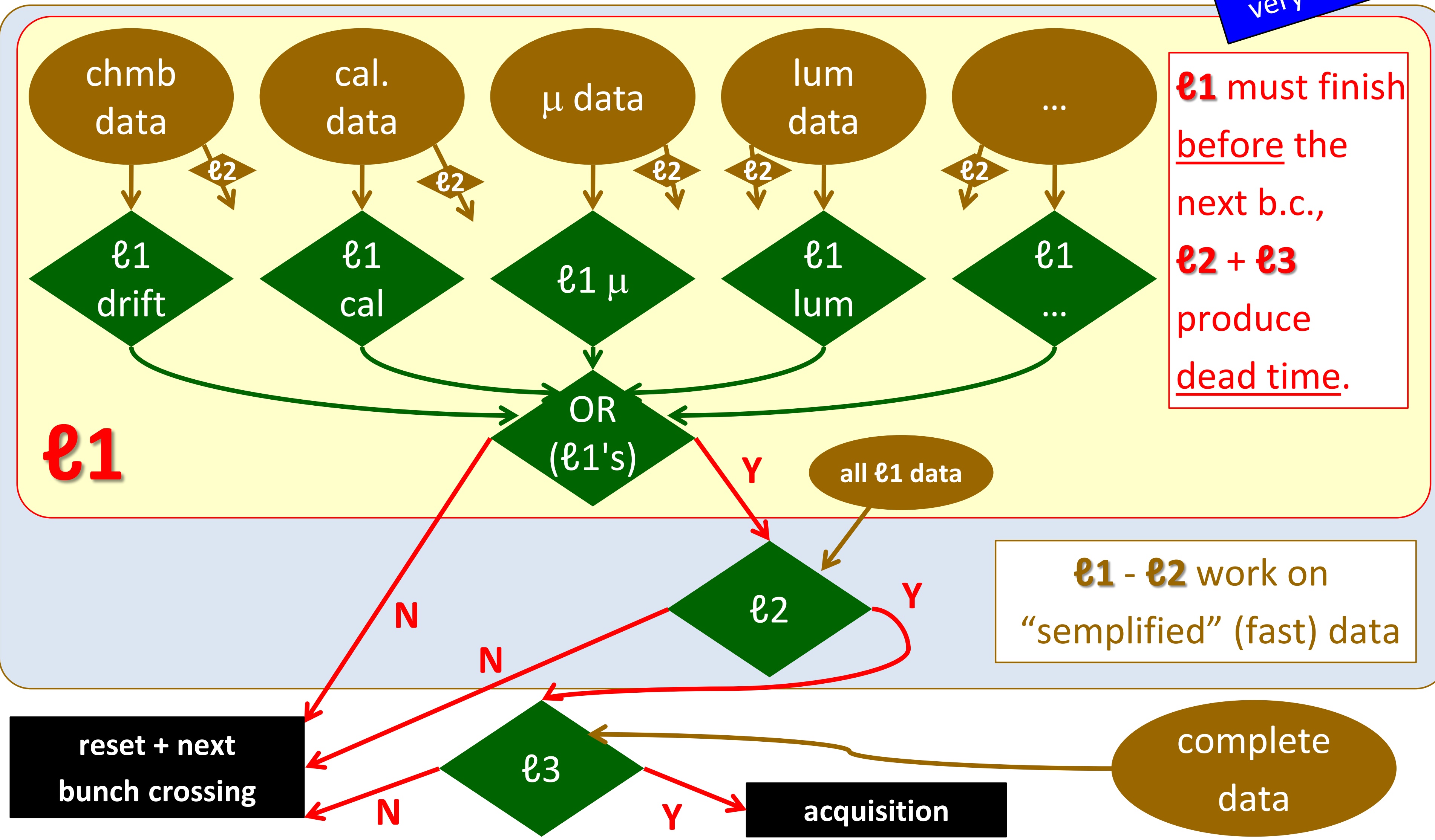
Similar, but more elaborated.

E (and E_T) comes from a calorimeter, so it is ~gaussian, while p (and p_T) comes from a spectrometer, so 1/p is ~gaussian.

Plot (E – p) if $\sigma(E) >> \sigma(p)$, but (1/E – 1/p) if $\sigma(p) >> \sigma(E)$.

The L3 detector: trigger / DAQ

other exp.'s very similar



ℓ1 must finish <u>before</u> the next b.c., **ℓ2** + **ℓ3** produce <u>dead time</u>.

ℓ1 - **ℓ2** work on "semplified" (fast) data

The L3 detector: trigger requirements

- crossing @ 44/88 KHz $\leftrightarrow$ physics $\leq$ 1 Hz, i.e. "μ" $\approx 10^{-4} \div 10^{-5}$;
- event trigger (no selection on process type, unlike LHC);
- 3 levels of trigger;
- 1st level: simplified readout (e.g. faster ADC less precise), logical OR among:
  - TEC (e.g. 2 opposite tracks);
  - μ (at least one candidate);
  - …
  - energy (see next slides);
- 2nd level: same data as 1st lvl, but combine different detectors (e.g. a track + corresponding calo deposit);
- 3rd level: final data.

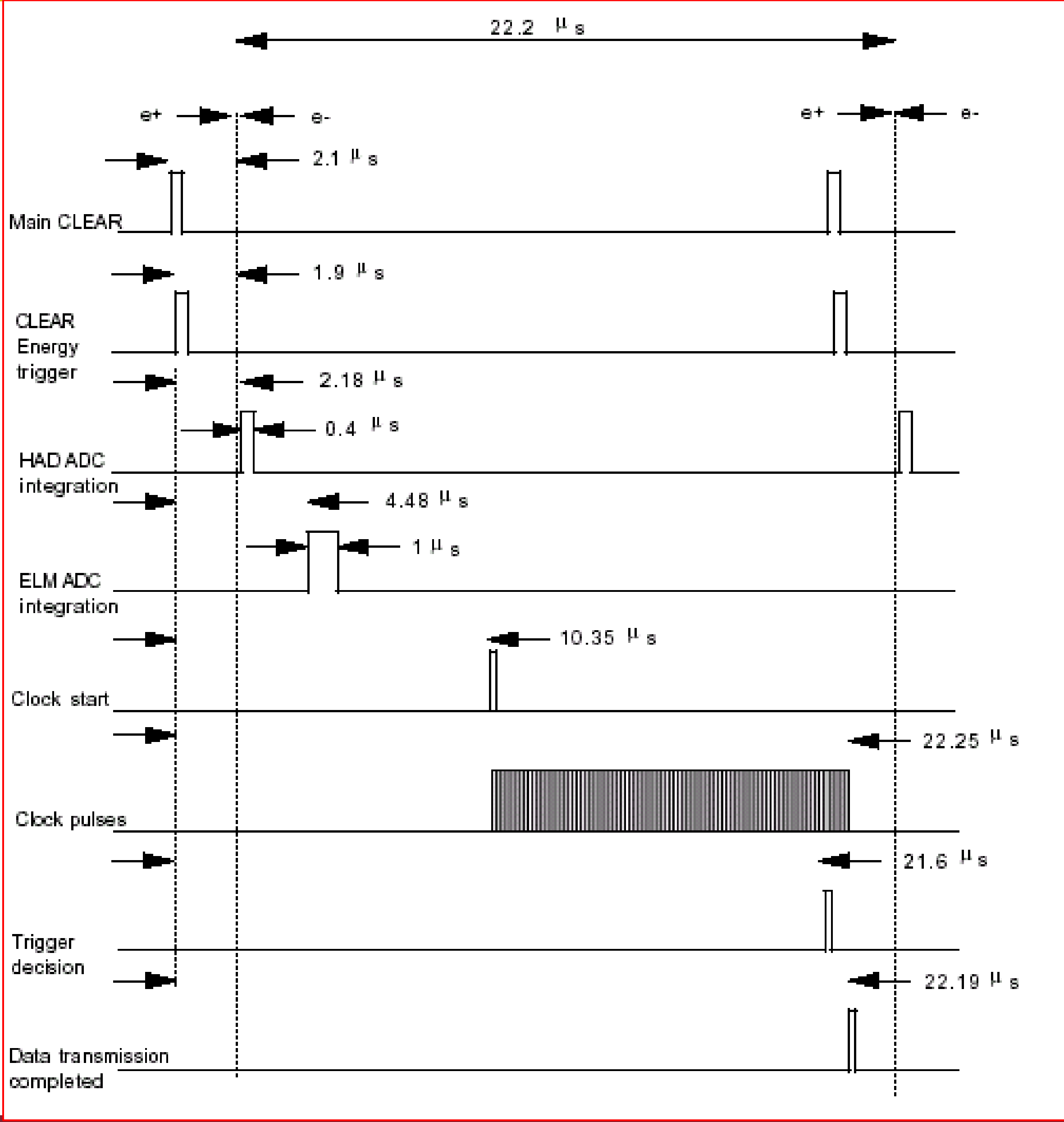
- fake triggers sources (~10÷20 Hz at 1st level) :
  - electronic noise;
  - beam halo + "beam-gas" interactions , brem photons, …;
  - cosmics, …;
- 1st level is cabled + home-made processors [*home : THIS building*];
- 2nd level: (quasi-)commercial processor;
- 3rd level: standard computer (vax-station at the time, today would use pc server + LINUX).

$\rightarrow$ inefficiency $\leq 10^{-3}$ for $Z \rightarrow e^+e^-, \mu^+\mu^-$, hadrons;

$\rightarrow$ dead time $\approx$ 5%.

The L3 detector: energy trigger

- Roma : 1989-2000;
- CAMAC(*) processor, built by "Sezione INFN" (this building, ground floor);
- fast digitization of calo signals;
- decision algorithm based on a digital programmable processor, realized with logic and arithmetic units;
- ~200 CAMAC modules;
- decision in ~22 μs →



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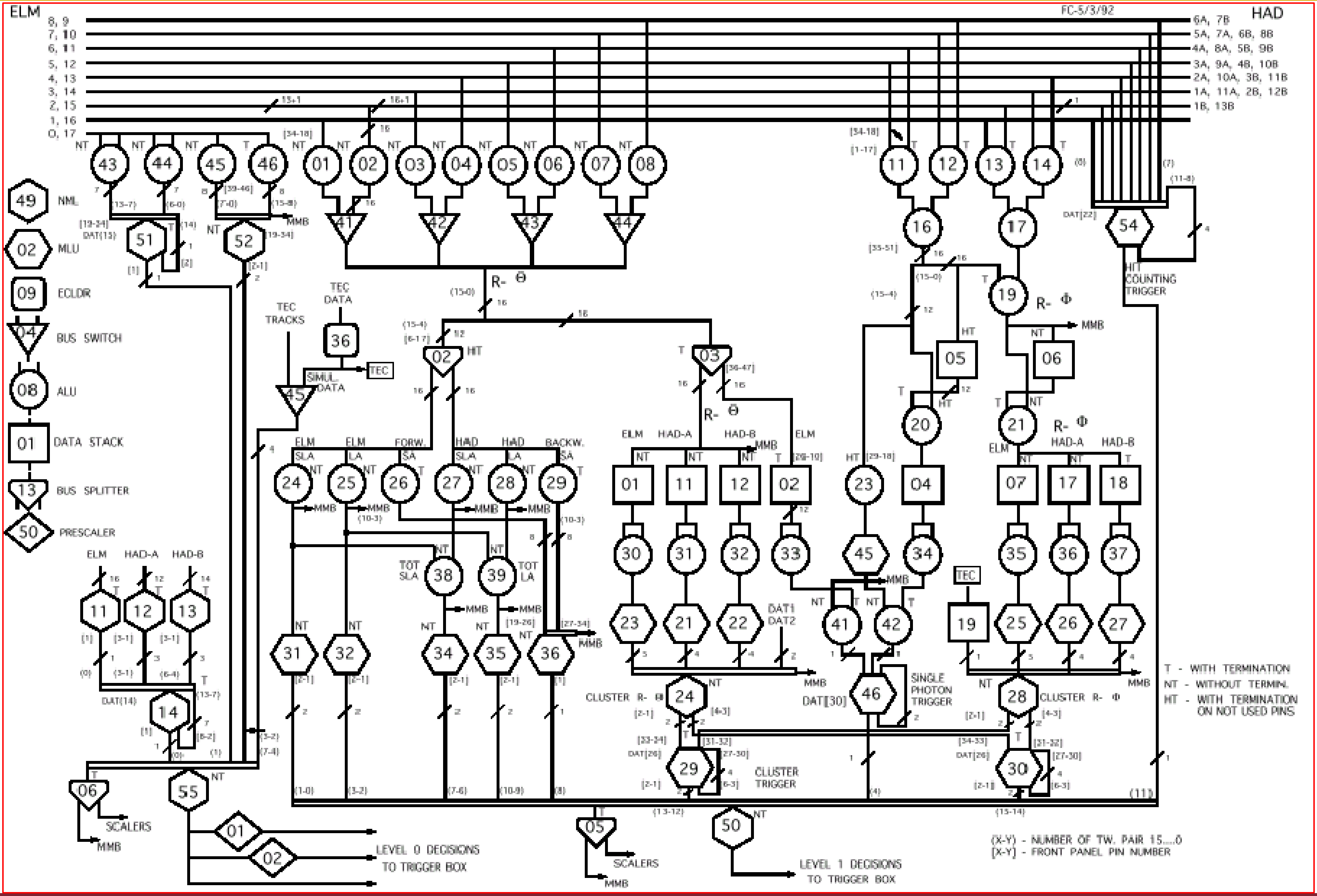
(*) CAMAC was an electronic standard, widely used in the '70s – '90s, now almost completely replaced by VME and other systems.

The L3 detector: energy trigger scheme



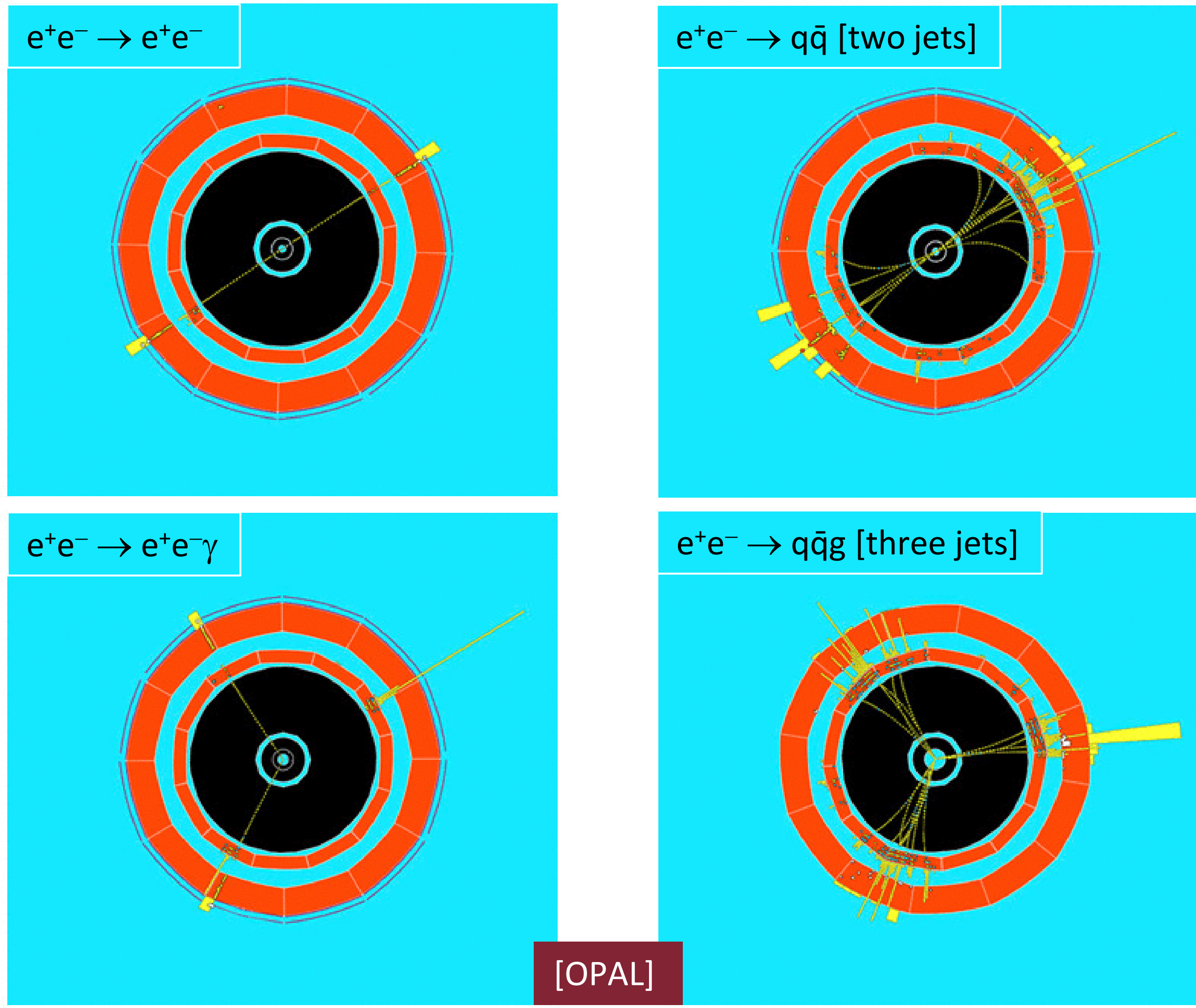
LEP events

The e^+e^- initial state produces very clean events (parton system = CM system = laboratory, no spectators).

In these four LEP events the beams are perpendicular to the page.

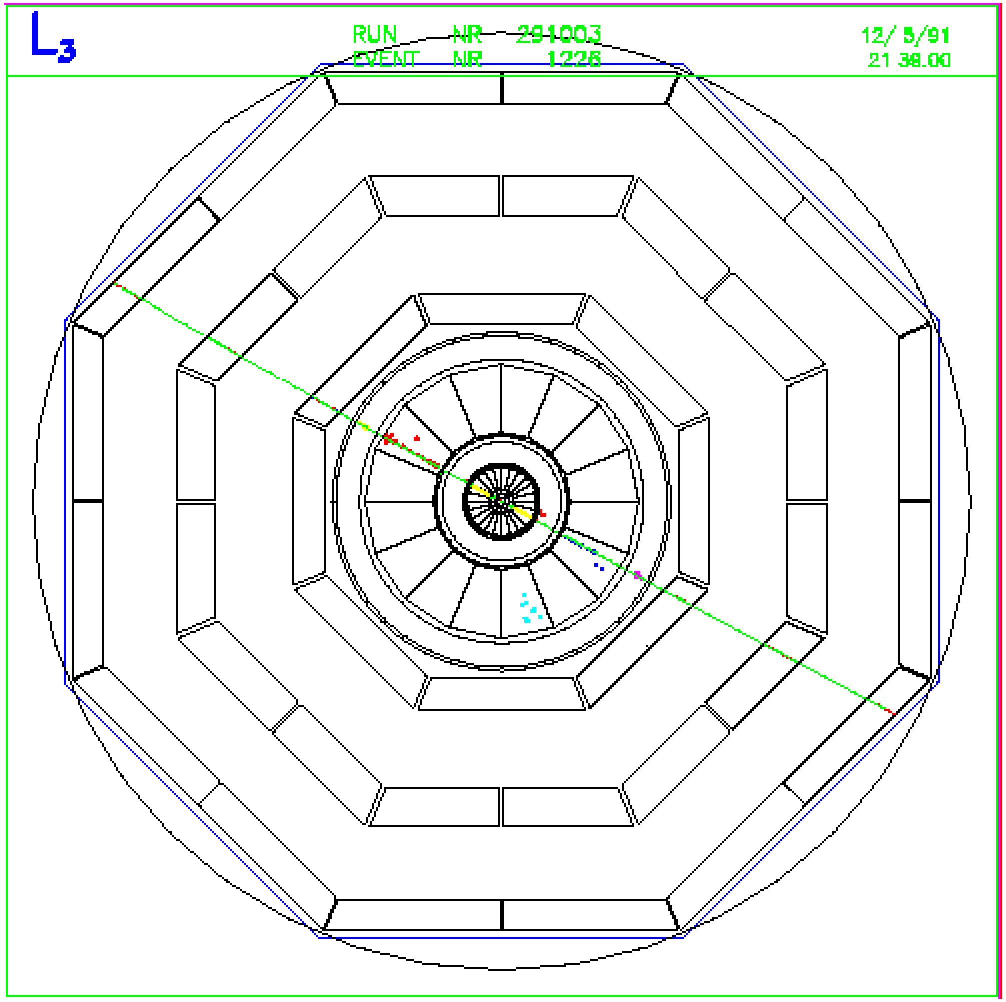
The recognition of the events is really simple, also for non-experts.

Great machines for high precision physics …



LEP events: $\mu^+\mu^-$

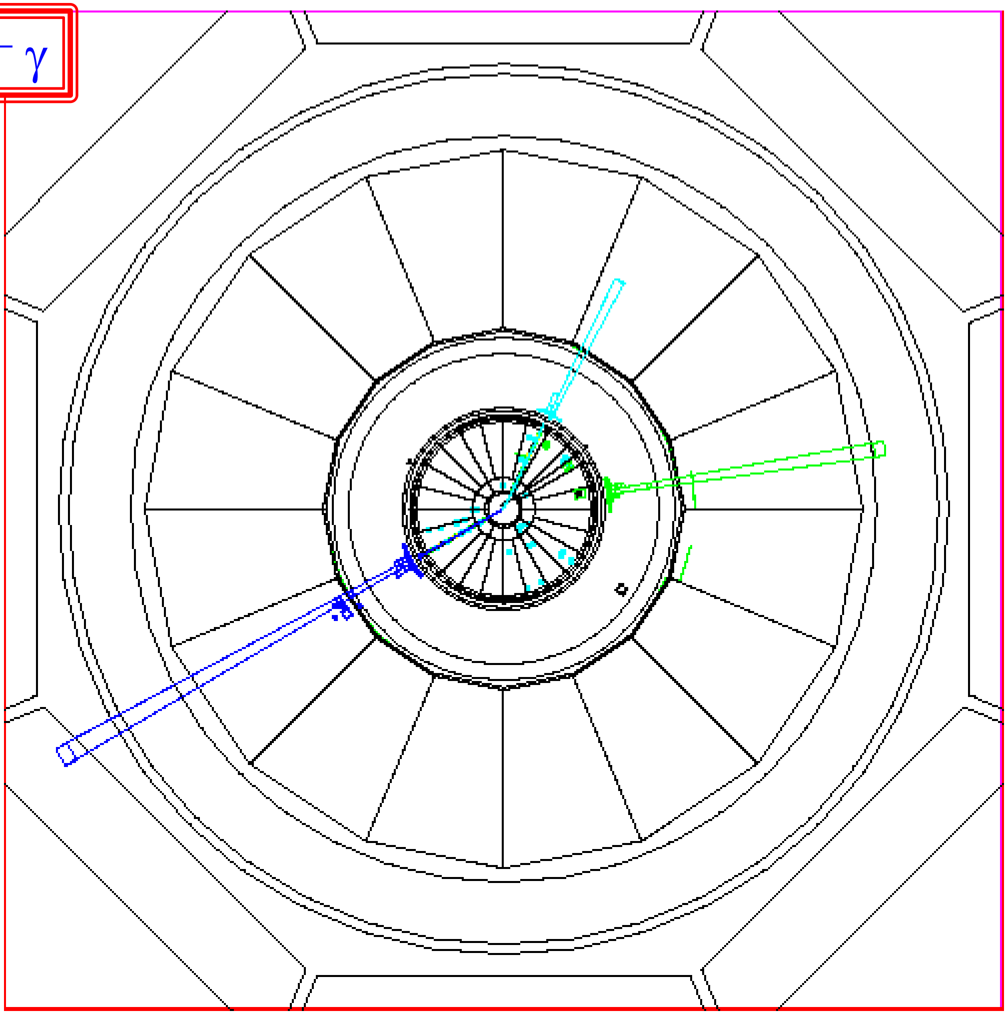
$e^+ e^- \to \mu^+ \mu^-$



+ signals in SMD
+ track in TEC ($\to$ momentum and charge)
+ mip in calos
+ signals in μ chambers ($\to$ momentum and charge)
= identified and measured $\mu^\pm$.

LEP events : $e^+e^-\gamma$

$e^+ e^- \to e^+ e^- \gamma$



+ signals in SMD
+ track in TEC ($\to$ momentum and charge)
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in μ chambers

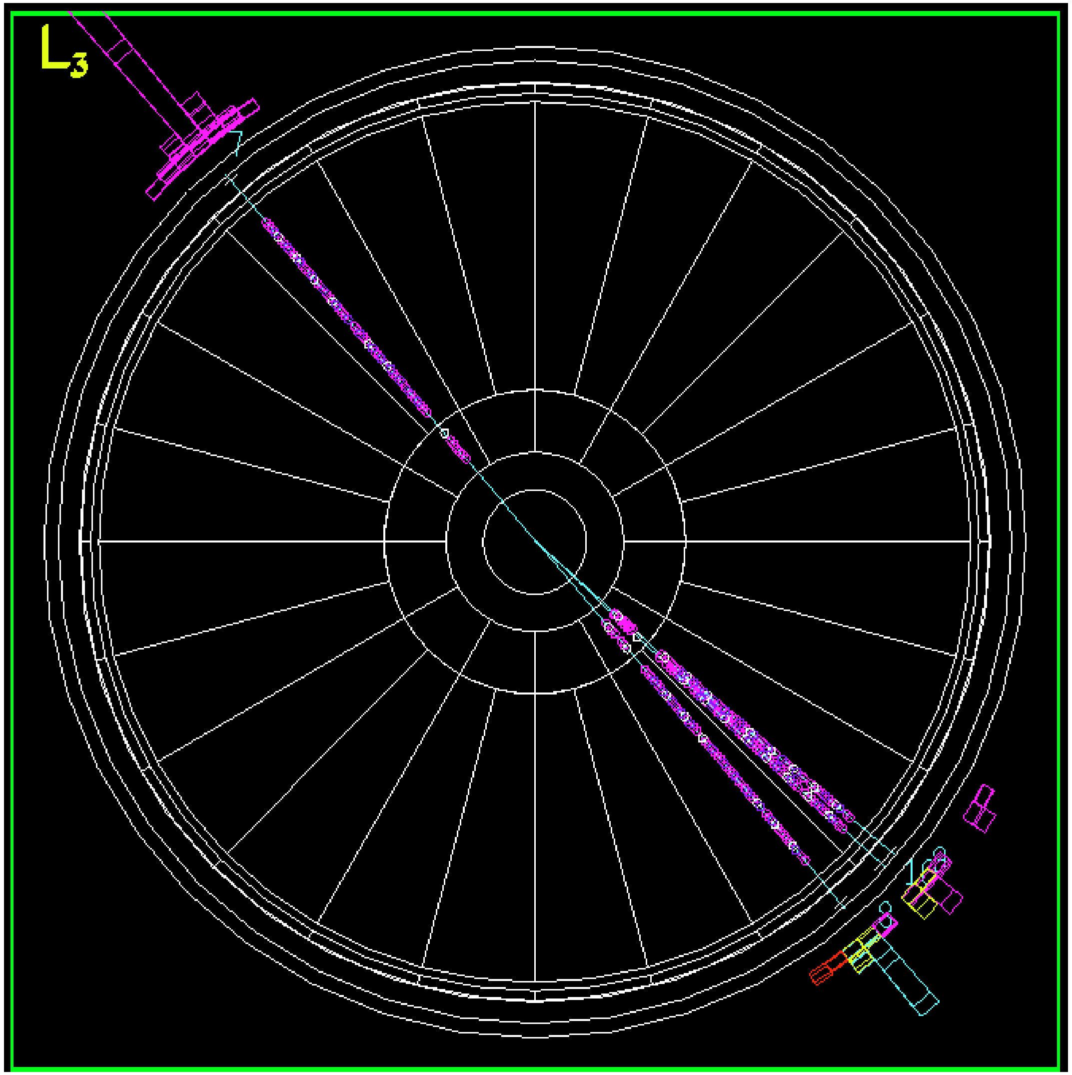
= identified and measured $e^{\pm}$.

+ no signal in SMD
+ no signal in TEC
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in μ chambers

= identified and measured γ.

LEP events : $\tau^+\tau^-$

$e^+ e^- \rightarrow \tau^+ \tau^-$



$\tau^\pm$ id. does depend on decay:

- 1/3/5 had tracks;
- [or identified single $\ell^\pm$;]

+ $\not{E}$ (i.e. a ν_τ /$\bar{\nu}_\tau$)

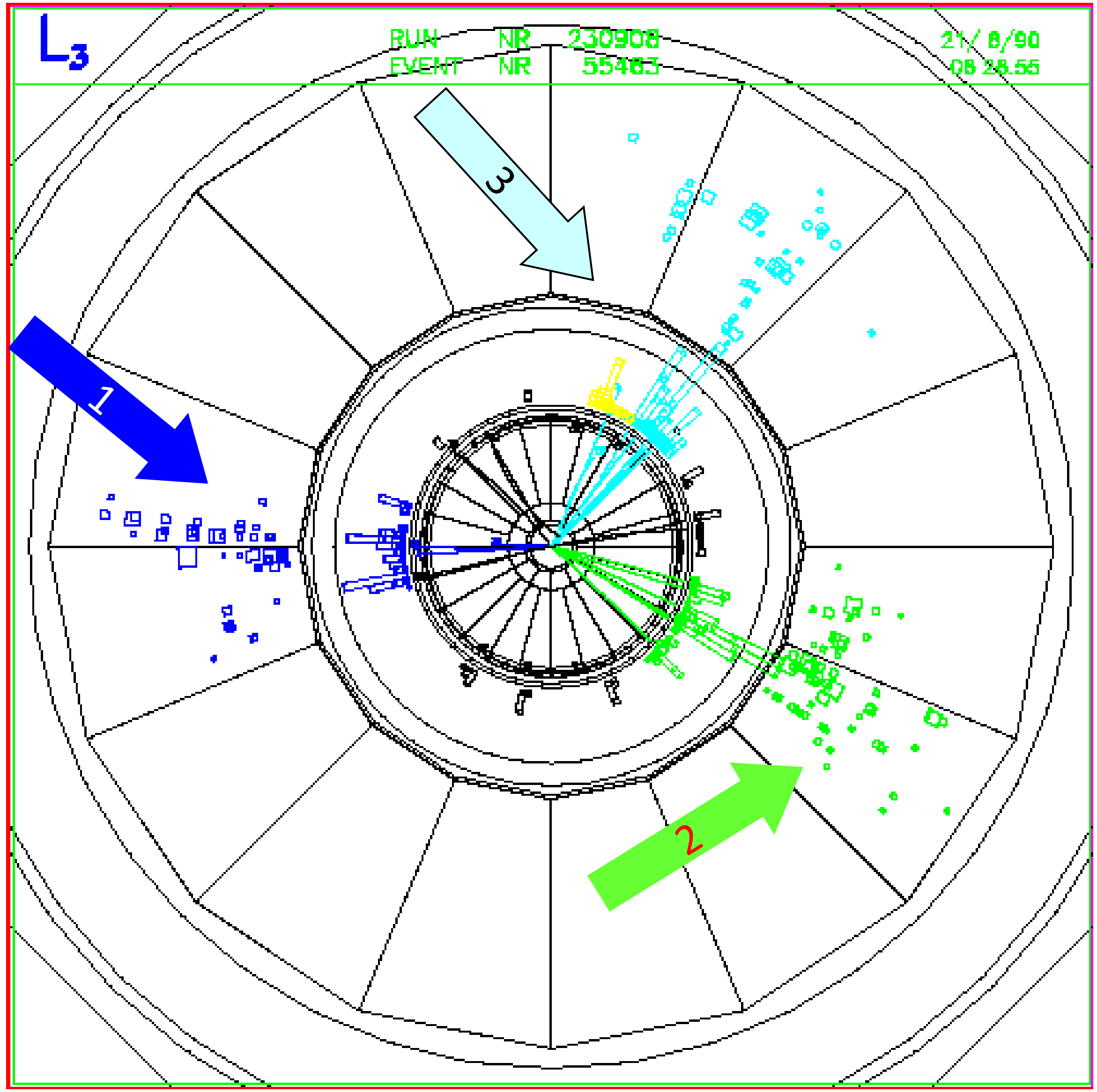
(the evidence comes from the combination of the two decays in the opposite emispheres).

LEP events : 3 jets

$e^+ e^- \to q\, \bar{q}\, g$

a (anti-)quark or a gluon gives a hadronic jet:

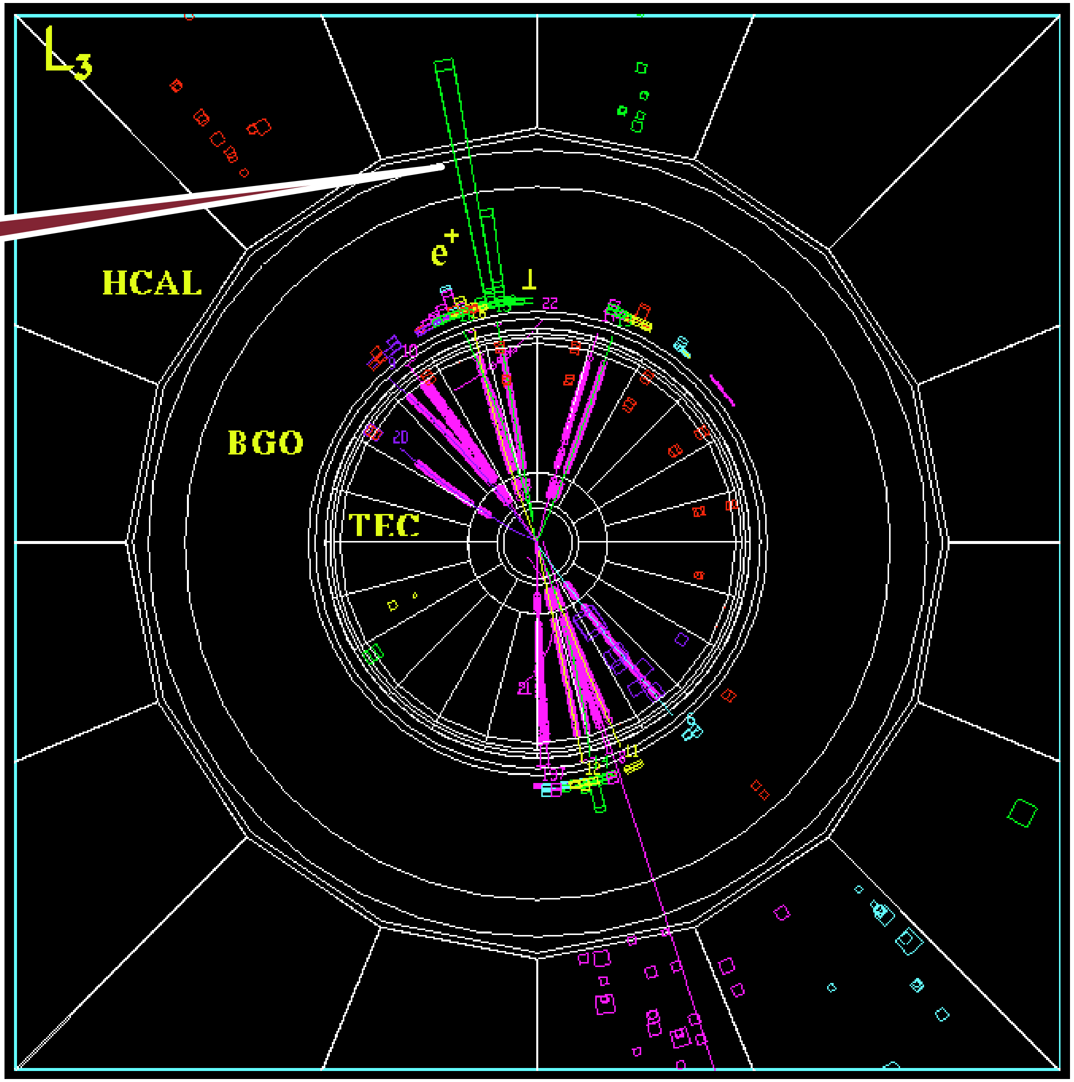
+ many collimated tracks
+ large splashes in e.m. and had calos
+ (possibly) low momentum associated $e^\pm/\mu^\pm$



LEP events : $b\bar{b}$, $b \to e^-$

$e^+ e^- \to b\,\bar{b}$

identified e^+



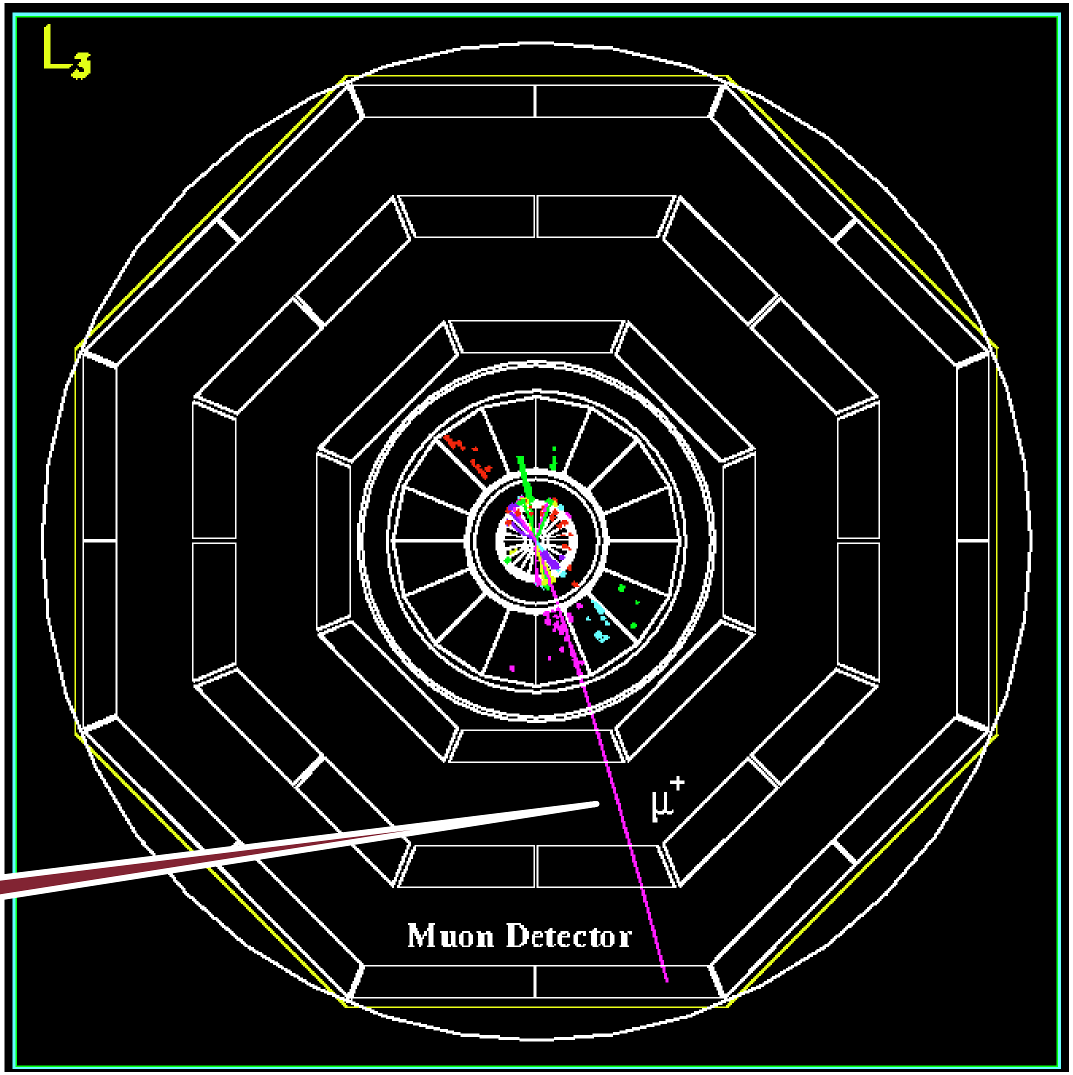
a heavy flavor quark is a quark (i.e. a jet) with:

+ displaced secondary verteces (SMD)
+ high momentum leptons from quark semileptonic decays

[not all h.f. have one or both characteristics → h.f. id. efficiency not complete (see next)]

LEP events : $b\bar{b}$, $\bar{b} \to \mu^+$

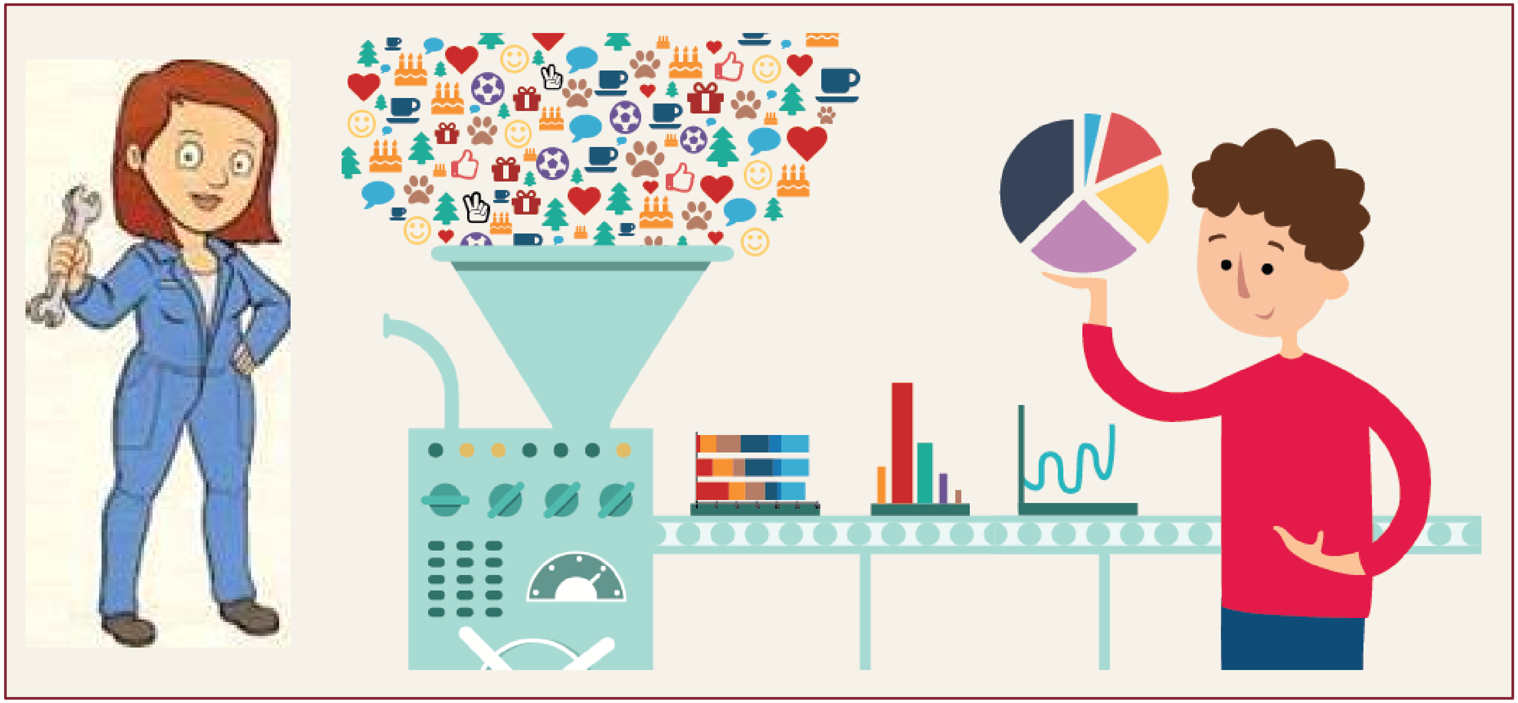
$e^+ e^- \to b\,\bar{b}$



identified μ^+

ii. Exp. methods

1. – 4. […]
5. Measure the luminosity
6. Secondary verteces
7. Efficiency and purity
8. Data analysis
9. – 16. […]



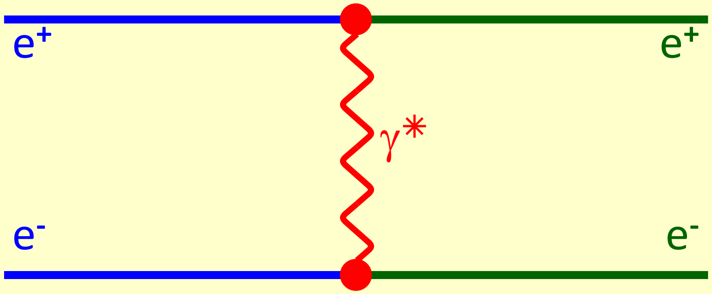
measure the luminosity

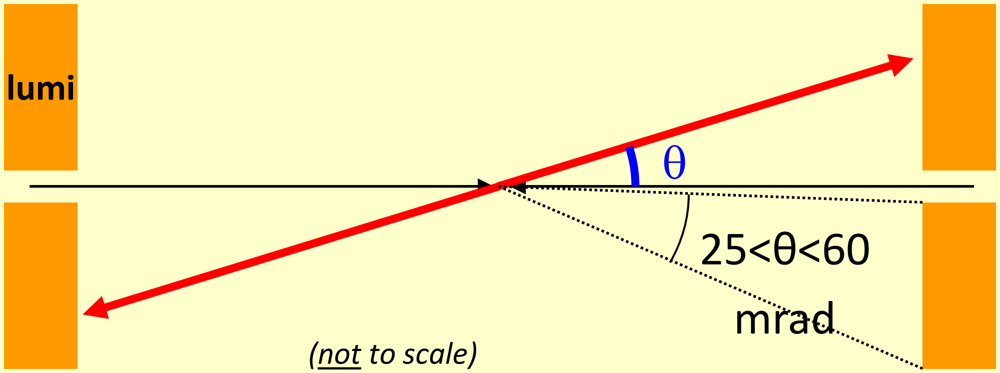
[in a few slides:
LEP measures $\mathscr{L}_{int}$ from a process (...):
$\mathscr{L}_{int} = N_{lumi} / (\varepsilon_{lumi}\, \sigma_{lumi} + \varepsilon_{b\text{-}lumi}\, \sigma_{b\text{-}lumi})$ *]*

- the chosen "lumi" process is $e^+e^- \to e^+e^-$ (Bhabha scattering) at small θ;
- we **assume** that, when $\theta \to 0°$, the Bhabha scattering is dominated by the γ^* exchange in the t-channel, while both (a) the γ^*/Z exchange in the s-channel; (b) the $Z^{(*)}$ exchange in the t-channel are negligible;
- therefore, the LEP experiments have e.m. calorimeters at small θ, to both identify and measure $e^\pm$ ("luminometers", ring-shaped ♦);
- it is essential that the "ring" reaches very small θ, to minimize □ stat ($d\sigma_{Rutherford} / d\cos\theta \propto \theta^{-4}$);
- their position and efficiency must be known (= measured) very reliably, in order to minimize systematics;
- typically at LEP, $25 \le \theta_{lumi} \le 60$ mrad :

$$\sigma_{lumi}(\theta \to 0) \approx \frac{16\pi\alpha_{em}^2}{s}\left(1/\theta_{min}^2 - 1/\theta_{max}^2\right);$$

$$\Delta\mathscr{L}/\mathscr{L} \approx \Delta\sigma_{lumi}/\sigma_{lumi} \approx 2\Delta\theta_{min}/\theta_{min}.$$



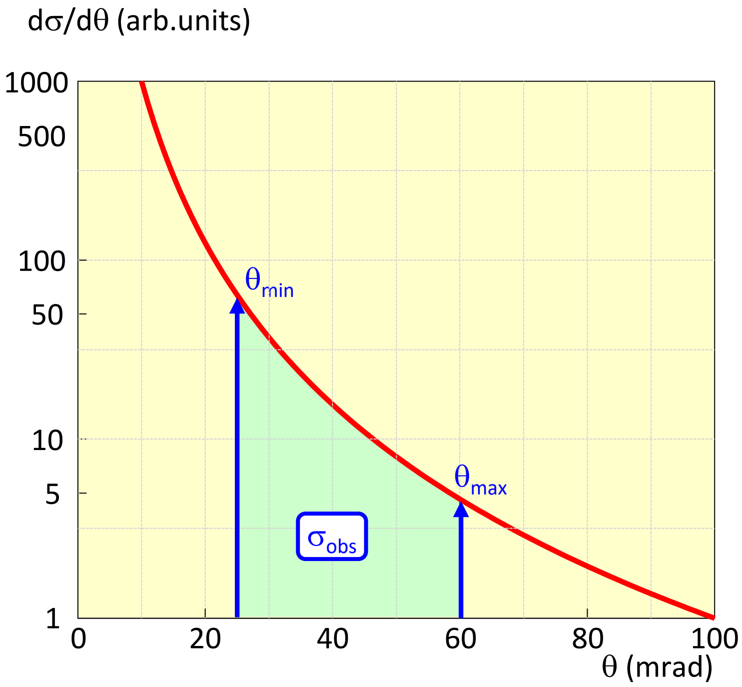


(not to scale)

measure the luminosity: computation

An exercise for dummies:

[notice: e.m. only, small θ only]



$$\frac{d\sigma_{Bhabha}\left(e^+e^- \to e^+e^-\right)}{d\cos\theta} = \frac{2\pi\alpha^2}{s}\left(\frac{3+\cos^2\theta}{1-\cos^2\theta}\right)^2;$$

only 1^{st} order in $\theta \to \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2;$

$\cos^2\theta \approx 1-\theta^2; \qquad \left|\frac{d\cos\theta}{d\theta}\right| \approx \theta;$

$\sigma_{Bhabha}\left(e^+e^- \to e^+e^-; \text{ small } \theta\right) \equiv \text{"}\sigma\text{"};$

$$\frac{d\sigma}{d\theta} \approx \frac{d\sigma}{d\cos\theta}\times\theta \approx \frac{2\pi\alpha^2\theta}{s}\left(\frac{3+\left[1-\theta^2\right]}{1-\left[1-\theta^2\right]}\right)^2 =$$

$$= \frac{2\pi\alpha^2\theta}{s}\left(\frac{4-\theta^2}{\theta^2}\right)^2 \approx \frac{2\pi\alpha^2\theta}{s}\left(\frac{16}{\theta^4}\right) = \frac{32\pi\alpha^2}{s\theta^3}$$

[← see plot].

$$\boxed{\sigma_{observed} = \int_{\theta_{min}}^{\theta_{max}} \frac{32\pi\alpha^2}{s\theta^3}d\theta = \frac{16\pi\alpha^2}{s}\left(\frac{1}{\theta^2_{min}} - \frac{1}{\theta^2_{max}}\right).}$$

measure the luminosity: results

- at the end of LEP, using sophisticated silicon calos, the final results on luminosity was :

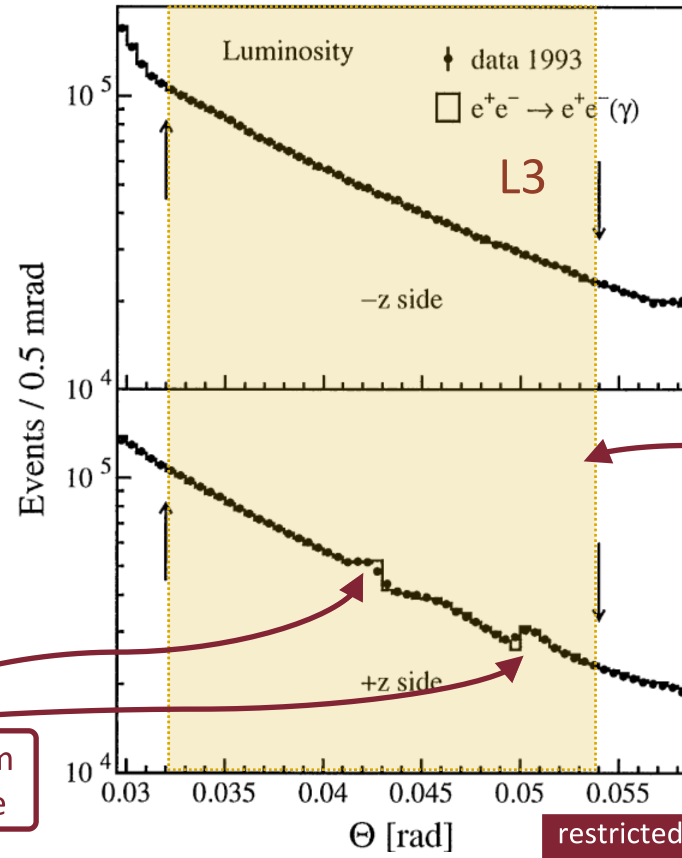
  $\Delta\mathscr{L}_{int}/\mathscr{L}_{int}$ = [see box] (statistical);

  $\oplus$ [0.03 ÷ 0.1 %] (syst. exp : $\Delta\theta$, alignment, ...);

  $\oplus$ [0.11 %] (theory, higher orders like $e^+e^- \to e^+e^-\gamma_{unseen}$);

- some of the LEP measurements, as <u>number of ν's</u>, <u>asymmetries</u>, do NOT depend on $\Delta\mathscr{L}_{int}$: because can be expressed as ratios "σ_1/σ_2 [$=N_1/N_2$]";

- [the luminosity data are an important fraction of all LEP1 data].



fake Bhaba's from beam pipe shape

restricted fiducial region in θ

An estimate of the importance of the statistical error comes from the comparison :

- $\sigma(e^+e^- \to$ hadrons, $\sqrt{s} = m_Z) \approx 30$ nb, the 2nd largest cross-section among all LEP1 processes;
- $\sigma(e^+e^- \to e^+e^-$, $25 \le \theta \le 60$ mrad$) \approx 100$ nb.

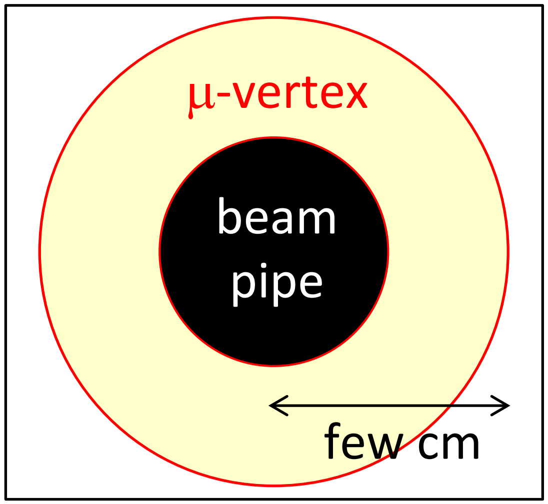
Therefore the statistical error on the luminosity is negligible, but for the <u>hadronic cross section</u> at $\sqrt{s} = m_Z$, where it is $\sim\sqrt{3/10}$ of the statistical error on the hadron data [but for this process the stat. error is irrelevant wrt systematics].

secondary verteces

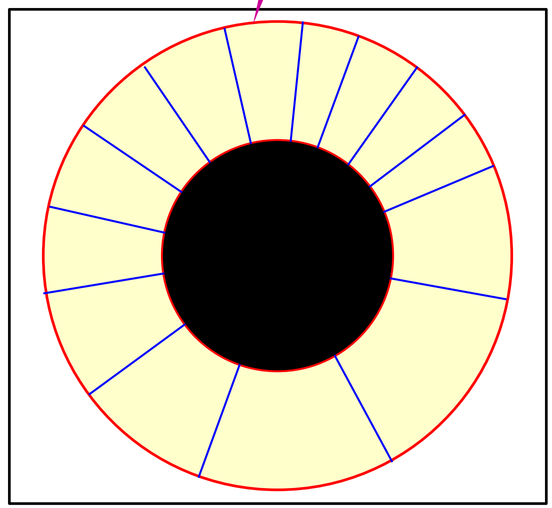
how to detect and identify c / b / τ's with a

do you see the difference ?

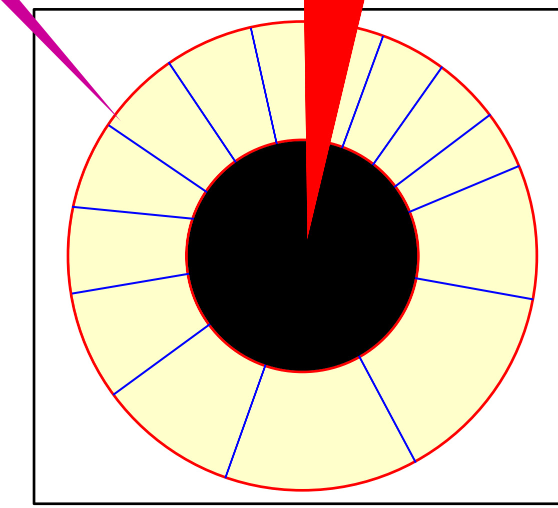
heavy quark (e.g. b) decay



the detector

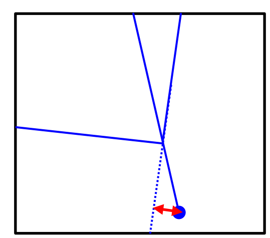


typical event: case 1



typical event: case 2

it needs a great accuracy in the "impact parameter" measurement.



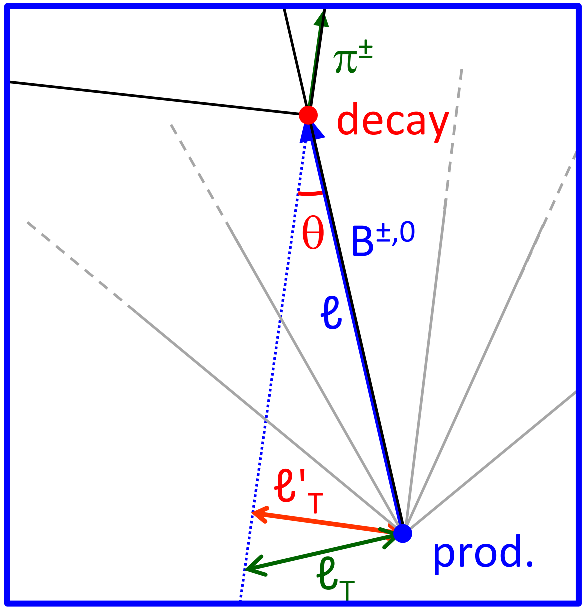
secondary verteces: kinematics

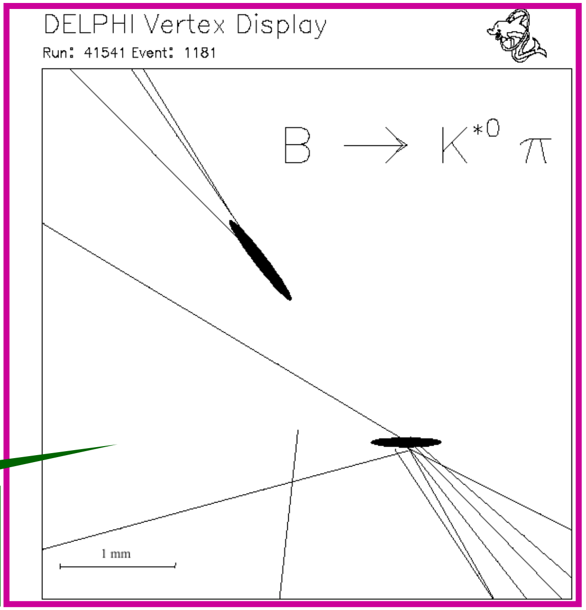
Analysis method (B meson as an example, similar for other b-particles, c-mesons/baryons, $\tau^\pm$] :

- [B conservation $\to$ 2 B in the event $\to$ 2 sec. vtxs];
- B ref. sys: $\tau(B^{\pm,0}) \approx 1.5\times10^{-12}$ s $\to \ell^* = c\,\tau_B \approx 500\ \mu$m;
- $\beta_B \approx 1 \to\ \ell\ (= \ell_B) = \ell^*\ \beta_B\gamma_B \approx c\ \tau_B\gamma_B \approx$ few mm;
  ℓ_T (= ℓ tanθ) is invariant wrt a $\mathbb{L}$-transform along $\vec{p}_B$
  $\to \ell_T = \ell^*_T = \ell^*\ \sin\theta^* \approx 100 \div 500\ \mu$m
  (θ^* is the angle B/π in the B ref. sys., **NOT** small);
- ℓ_T can be approximated by ℓ'_T, the <u>impact parameter</u> (extrapolation of a track) $\leftrightarrow$ (primary vtx):
  $\theta \sim m_B/E_B \approx 1/\gamma_B$ = small $\to \sin\theta \approx \tan\theta \to \ell'_T \approx \ell_T$;
- [call both ℓ'_T and ℓ_T "impact parameter ℓ_T"];

➢ need a detector with an accuracy <u><< 100 μm</u> in ℓ_T (i.e. in the extrapolation of the line of flight of a charged particle after 20÷30 mm from the last meas;

➢ **i.e. a very precise microvertex detector may identify and reconstruct b, c, τ decays.**

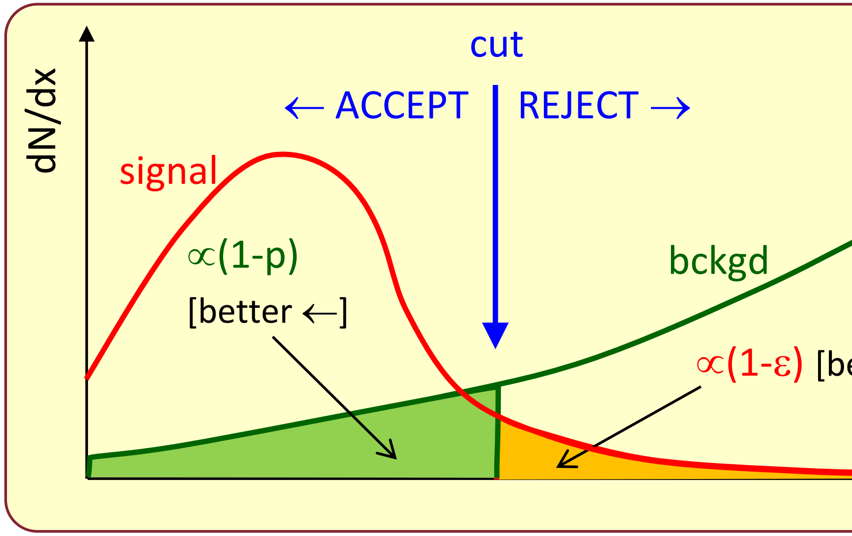




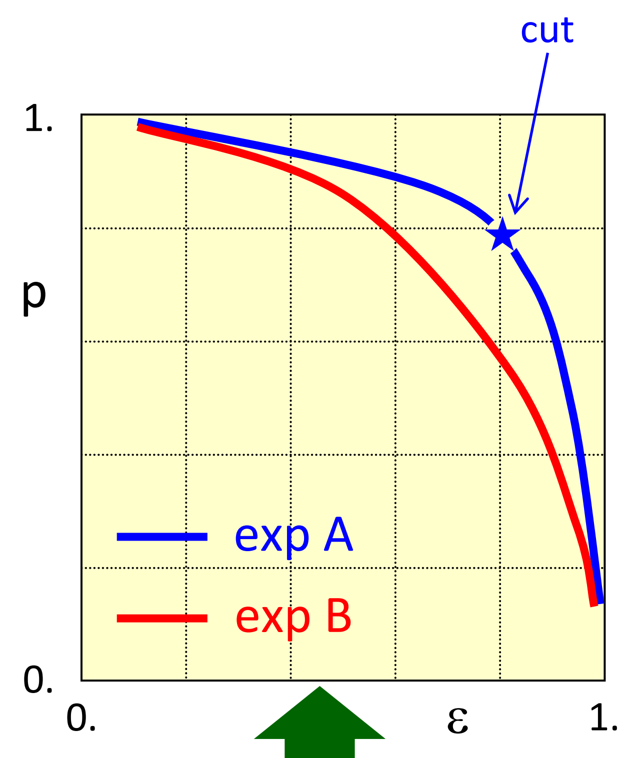
a real B^0 decay in Delphi (only one B vtx shown]

efficiency and purity

- No selection method is fully "pure" and "efficient", i.e. in a selected sample of events of type "i", there are some events "j" ($j \neq i$), while some events "i" have been rejected;
- if N_i^{sel} is the number of events of the sample, define :
  - efficiency : $\varepsilon_i = N_i^{sel,true} / N_i^{true,all} < 1$ [ideally = 1];
  - purity : $p_i = N_i^{sel,true} / N_i^{sel,all} < 1$ [ideally = 1];
  - [contamination : $k_i = N_i^{sel,false} / N_i^{sel,all} = 1 - p_i$] ;
- in general, ε_i and p_i are anti-correlated (see below);
- an algorithm (e.g. a cut in a kin. variable) produces $\varepsilon_i + p_i$;
- the "optimal" **choice** depends on the analysis and on $\mathscr{L}_{int}$.



an example of a variable "x" with a cut.



Example [no "$_i$" in the plots] :

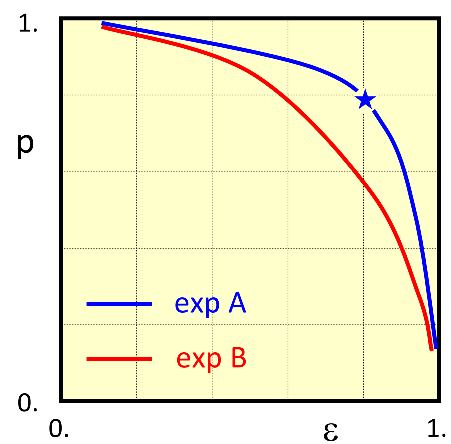
- two cases of p_i vs ε_i, when the cut varies.
- exp. A "is better" than B.
- "★" shows a possible choice for (p_i, ε_i) in A.

efficiency and purity: methods

$N_i^{sel,true}$ and $N_i^{true,all}$ are NOT directly measurable. Few methods to determine the relation ε / p, e.g. :

- Montecarlo (commonly used) :
  - 3 steps : "physics" [→ 4-mom.] + detector [→ pseudo-meas.] + analysis [exactly the same as in real data];
  - pros : large statistics, flexible, easy;
  - cons : (some) systematics cannot be studied;
- test-beam :
  - intrinsic purity + large statistics;
  - pros : less systematics;
  - cons : not flexible, difficult, expensive;
- "data themselves" [e.g. μ from $Z \to \mu\mu$ to study $b \to \mu X$] :
  - "tag and probe" [p ≈ 1 even if ε small] to force purity;
  - ok for systematics;
  - difficult reproduction of the required case [in the example isolated μ's 45 GeV instead of low-p_T μ in a jet].

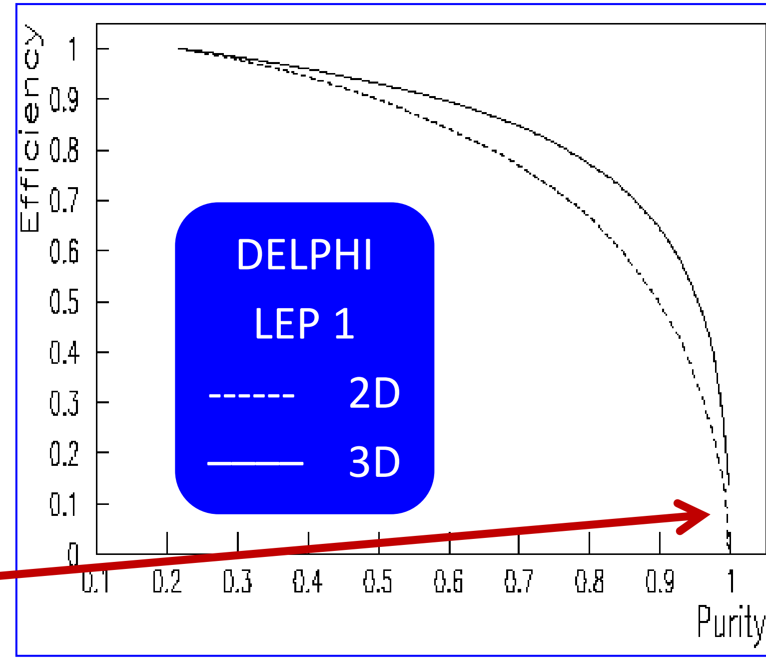
∴ Combination of the above, iterations, new ideas (i.e. **you** ☺)…

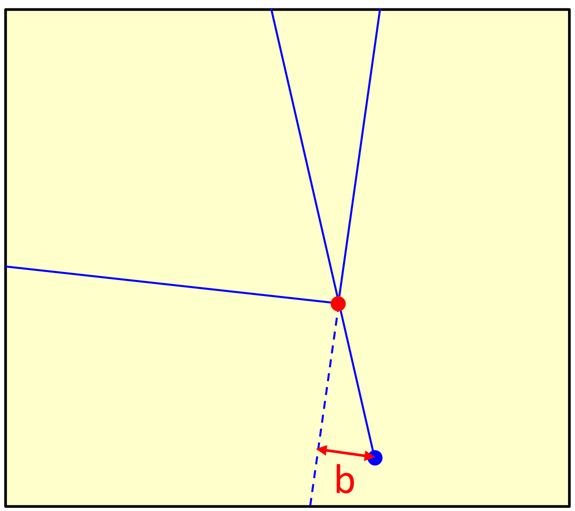


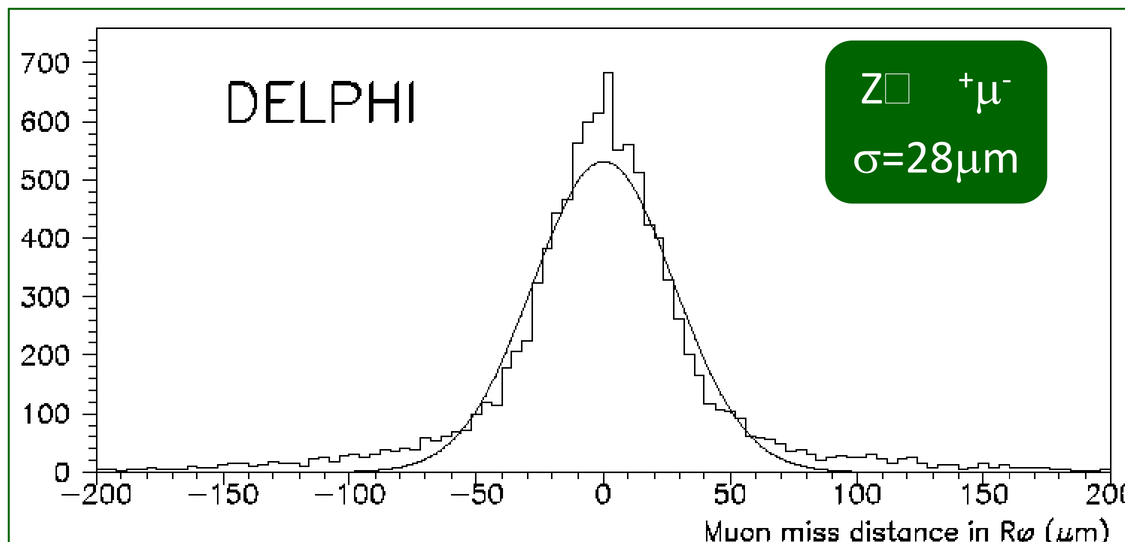
efficiency and purity: example

An example of the computation of ε vs p (secondary vtxs with impact parameter):

- use a mc (not shown) to define the distribution of impact parameter b in events with sec. vtxs;
- ➢ a cut on b $\rightarrow \varepsilon = \varepsilon(b_{cut})$;
- use a process without secondaries ($Z \rightarrow \mu^+\mu^-$) to define the distribution of the variable b;
- ➢ a cut on b $\rightarrow$ p = p(b_{cut});
- $\varepsilon = \varepsilon(b_{cut}) \oplus p = p(b_{cut})$ are parametric equations;
- repeat with more info → "3D" → better curve.







efficiency and purity: the bckgd

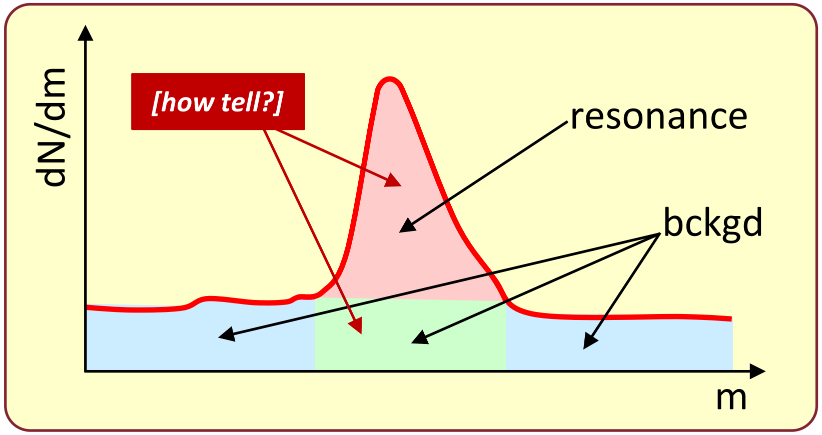
- The background ["bckgd"] may be conceptually divided into two categories :
  - irreducible bckgd(*): other processes with the same final state [e.g. $e^+e^- \to ZH$, Z□ $^+\mu^-$, $H \to b\bar{b}$ (signal) $\leftrightarrow$ $e^+e^- \to Z_1 Z_2$, Z_1□ $^+\mu^-$, $Z_2 \to b\bar{b}$ (bckgd)];
  - reducible bckgd :
    - badly-measured events,
    - detector mistakes,
    - physics processes which appear identical in the detector, because part of the event is undetected, e.g.

$$\begin{cases} e^+e^- \to \gamma Z \to \square\bar{\nu} \\ e^+e^- \to \gamma\,(e^+e^-)_{\text{beam-pipe}}; \end{cases}$$

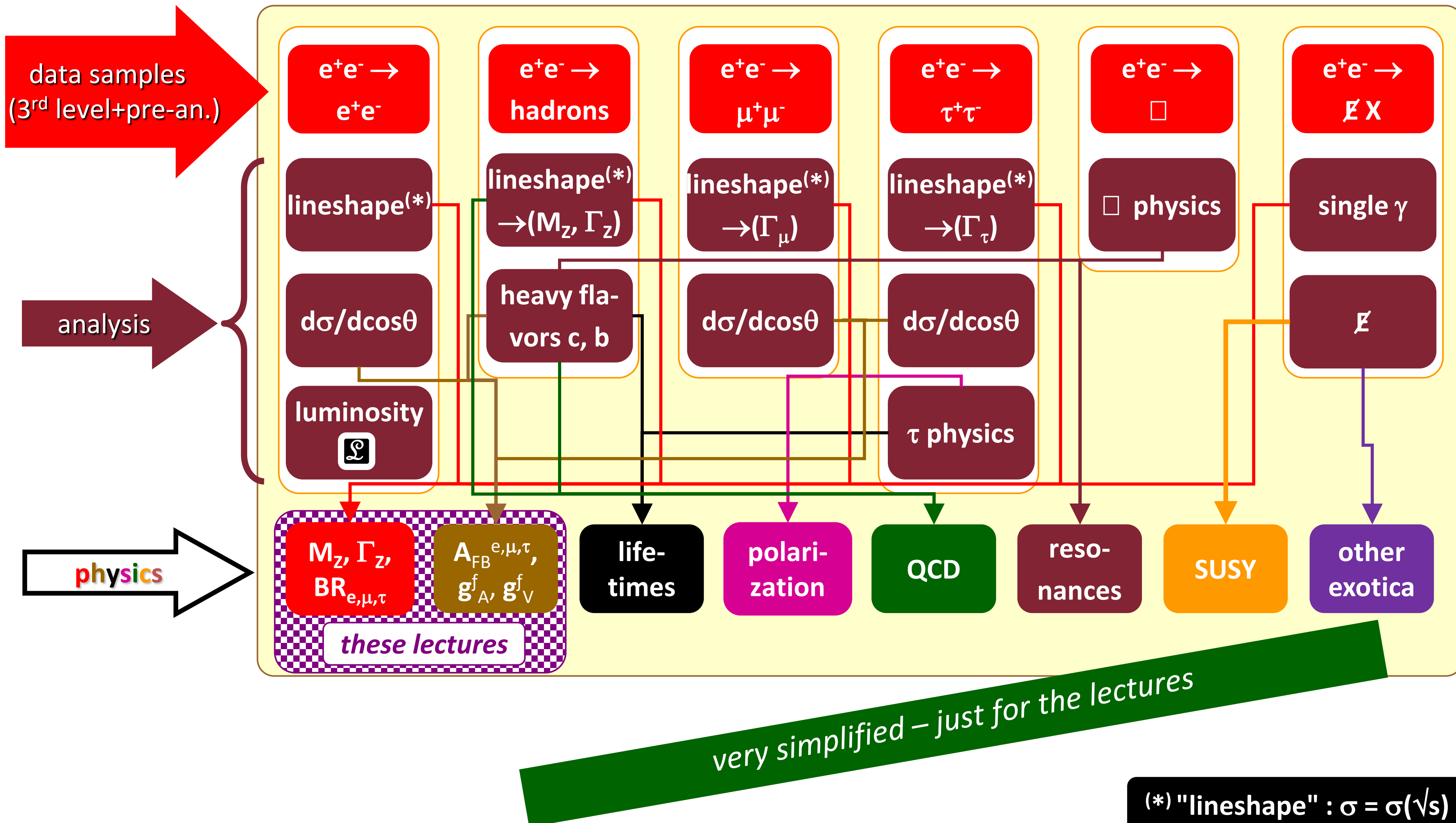
- the meaning of the distinction is that r.b. can be disposed with a better detector, or a more accurate selection (maybe with a loss in ε_s), while i.b. is intrinsic, and can only be subtracted statistically, by comparing [N^{exp} $\leftrightarrow$ (expected bckgd)] and [N^{exp} $\leftrightarrow$ (expected signal+bckgd)] ;

____

(*) Similar to the "resonances" of the strong interactions, where a mass distribution exhibits peaks, interpreted as short-lived particles. However, it is impossible to assign single events to the resonating peak or to the non-resonant bckgd.



data analysis: general scheme



(*) "lineshape" : $\sigma = \sigma(\sqrt{s})$

data analysis: events $\to \sigma$

- At LEP, as in any other experiment, a number of events N^{exp} has to be translated to a cross section σ_s ("signal");
- [also $dN^{exp}/d\Omega \to d\sigma_s/d\Omega$;]
- straightforward : $\sigma_s = N^{exp} / \mathcal{L}_{int}$;
- but (at least) two problems :
  - the **selection algorithm** loses true- and gains spurious-events:
    $N^{exp} = N_{true} - N_{lost} + N_{sp.}$;
  - the determination of $\mathcal{L}_{int}$, the **luminosity**.
- the experiment must measure/compute:
  - N^{exp} : number of selected events;
  - σ_b : cross-section of bckgd;
  - $\varepsilon_{s,b}$: efficiency (signal and bckgd);
  - $\Delta N^{exp} = \sqrt{N^{exp}}$ (statistical error);
  - $\Delta\varepsilon_{s,b}$ = "systematics";
  - $\mathcal{L}_{int}$ = int. luminosity (+ $\Delta\mathcal{L}_{int}$).

NB. In an ideal experiment, $N_{lost} = N_{sp.} = 0 \to \varepsilon_s = 1, \varepsilon_b = 0$.

- then (next slides) :
  - $N^{exp} = \mathcal{L}_{int} (\varepsilon_s \sigma_s + \varepsilon_b \sigma_b) \to$
    $\sigma_s = (N^{exp}/\mathcal{L}_{int} - \varepsilon_b\sigma_b) / \varepsilon_s$;
    $d\sigma_s/d... = [...]$;
- the luminosity $\mathcal{L}_{int}$ is equal for signal and bckgd and <u>must be measured</u>;
- LEP measures $\mathcal{L}_{int}$ from a process ("lumi process"), with a calculable cross section, triggered and acquired at the same time as other data ($\to$ so DAQ inefficiencies cancel out) :
  $$\mathcal{L}_{int} = N_{lumi} / (\varepsilon_{lumi}\, \sigma_{lumi} + \varepsilon_{b\text{-}lumi}\, \sigma_{b\text{-}lumi})$$
- therefore three new errors :
  (statistics) $\Delta N_{lumi} = \sqrt{N_{lumi}}$,
  (sistematics) $\Delta\varepsilon_{lumi,b\text{-}lumi}$, □ $_{b\text{-}lumi}$,
  ("theory") $\Delta\sigma_{lumi}^{theory}$.

data analysis: theory $\leftrightarrow$ exp. data

An example: $e^+e^- \to \mu^+\mu^-$:

- studies for efficiency and purity with MC simulation [*see later*].
- <u>signal</u>: true events $e^+e^- \to \mu^+\mu^-$; the yield depends on m_Z, Γ_Z, Γ_μ (unknown);
- <u>bckgd</u>: events from other sources, with similar final state (because really the same or similar in the detector), e.g. :

➢ $e^+e^- \to Z \to \tau^+\tau^- \to$
$\to (\mu^+\bar{\nu}_\tau\nu_\mu)\ (\mu^-\nu_\tau\bar{\nu}_\mu)$
$\to (\mu^+\mu^-)$ (+ *not-visible*);

➢ $e^+e^- \to e^+e^-\mu^+\mu^- \to$
$\to (e^+e^-)^{\text{beam chamber}}\ (\mu^+\mu^-)^{\text{detected}}$;
$\to (\mu^+\mu^-)$ (+ *not-detected*);

result !!!

$$N_{exp} = \mathscr{L}_{int}\,[\varepsilon_s\boxed{\sigma_s} + \varepsilon_b\sigma_b].$$

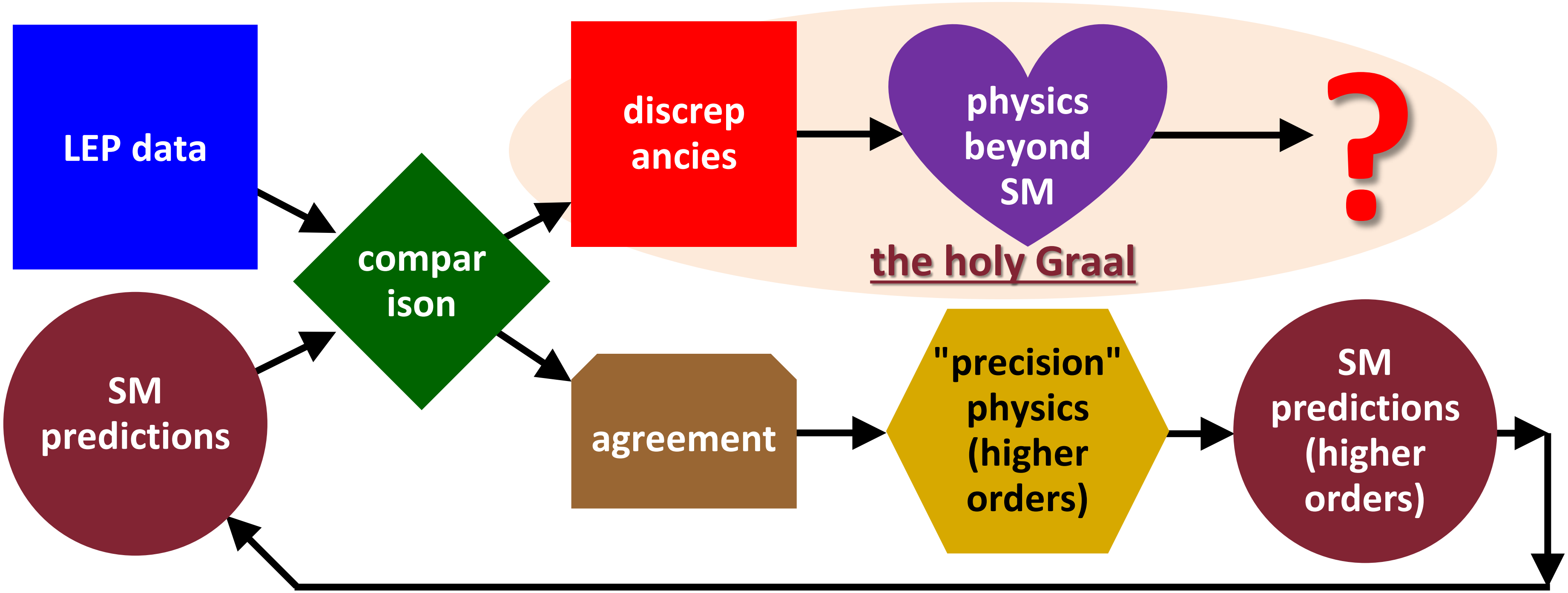
selected sample

lumi meas (see)

mc signal + mc bckgd

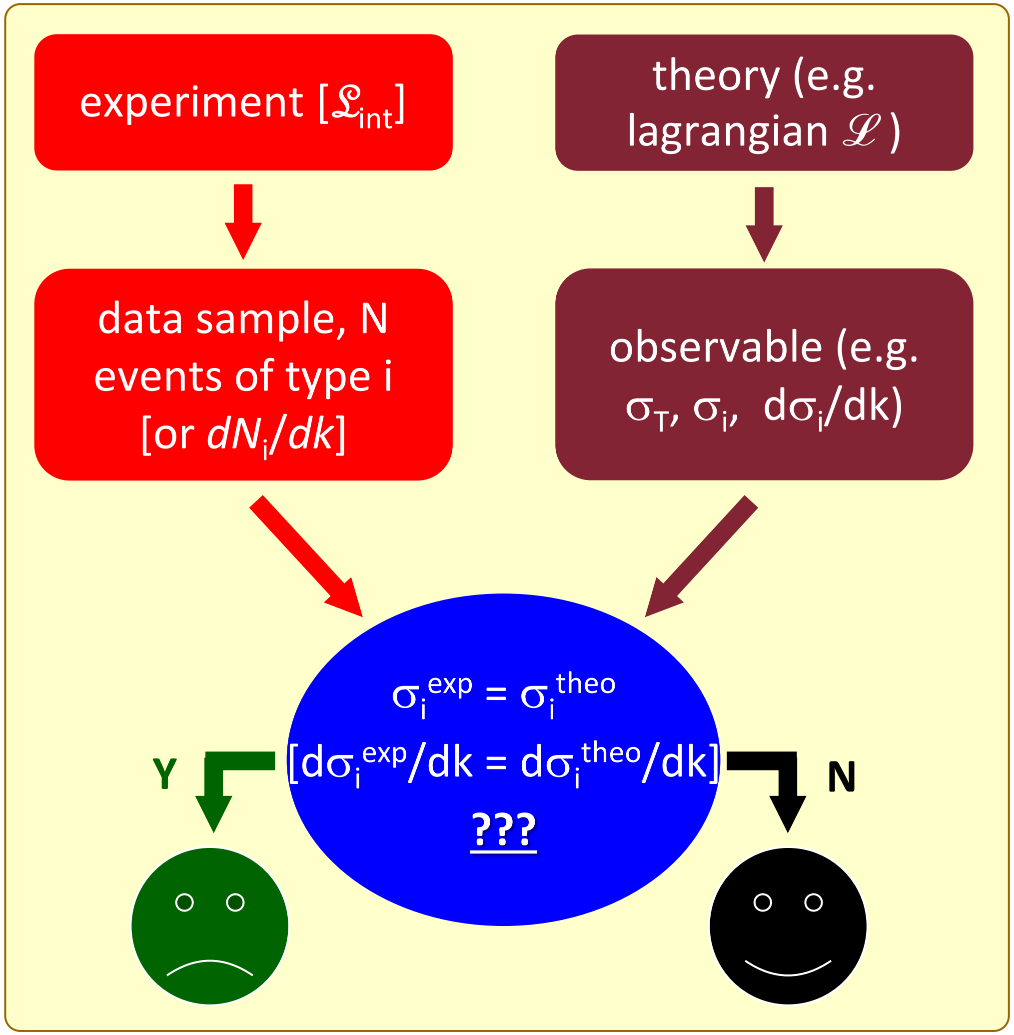
theory, other studies

data analysis: scheme



- In 1989, when LEP started, the SM was completely formulated and computed;
- the only missing pieces (at that time) were the top quark and the Higgs boson (both now discovered);
- the values of m_{top} and m_{Higgs} are such that they (in lowest order) have no role at LEP $\sqrt{s}$ [*but for H we did NOT know*];
- twelve years of LEP physics gave NO major surprise, but general agreement with SM predictions;
- tons of measurements, a superb unprecedented work of precision physics : the **number of light ν's** and the **predictions of m_{top} and m_{Higgs}** via higher orders are [*imho*] the LEP masterpieces.

data analysis: comparison theory ↔ data



Therefore, a ***measurement*** means :

- select a pure (as much as possible) sample of events N_i;
- measure the statistical significance of the experiment ($\rightarrow \mathscr{L}_{int}$);
- measure/compute the associated efficiency and purity ($\rightarrow$ ε,p);
- compute $\sigma_i \equiv \sigma_i^{exp}$ = [previous slide] [or $d\sigma_i^{exp}/dk$ = (…)];

→ finally **theory ↔ experiment**:

- compute σ_i^{theo} from theory;
- **compare** $\sigma_i^{theo} \leftrightarrow \sigma_i^{exp}$.

["**limits**" require a different method, see § limits].

data analysis: results

SM predictions :

- $\sigma(f\bar{f})$, $\sigma(e^+e^-)$, $d\sigma/d\cos\theta$ ... ("Born");
- radiative corrections;
- approximations;

experiment(s) (LEP, L3 as an example) :

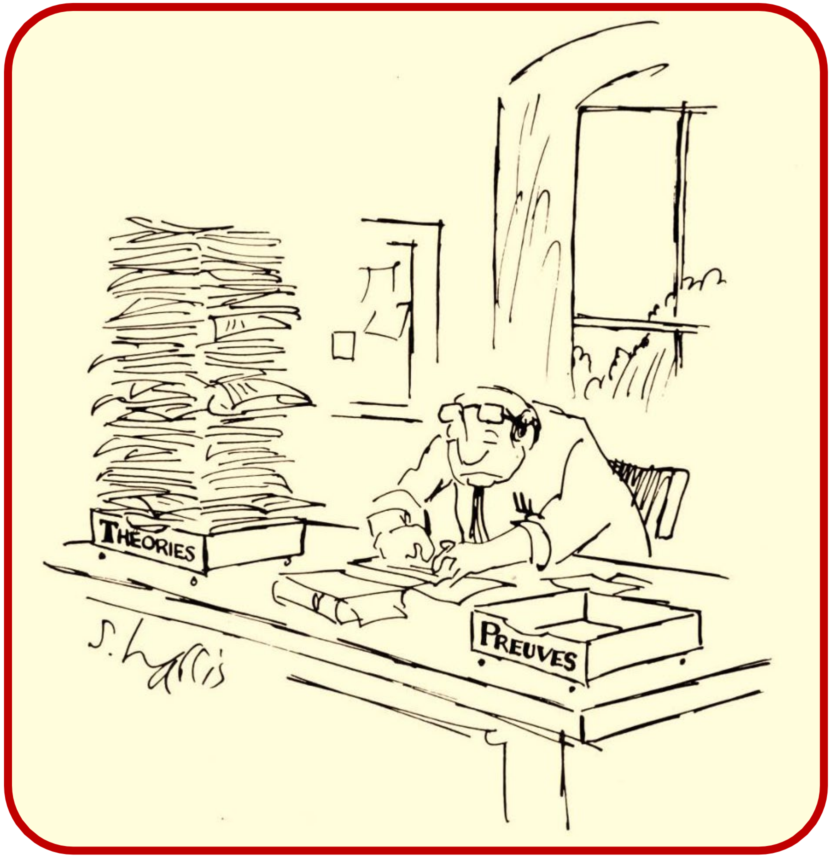
- cross sections $\sigma(e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$, hadrons, $\square^-)$;
- differential cross sections $d\sigma(e^+e^- \to ...) / d\cos\theta$;
- "lineshape" (i.e. $\sigma(e^+e^- \to ...)$ as a function of $\sqrt{s}$ [also $d\sigma(e^+e^- \to ...) / d\cos\theta$ vs $\sqrt{s}$].

data analysis and interpretations : <u>global fit (4 exp. data) $\leftrightarrow$ (SM)</u>:

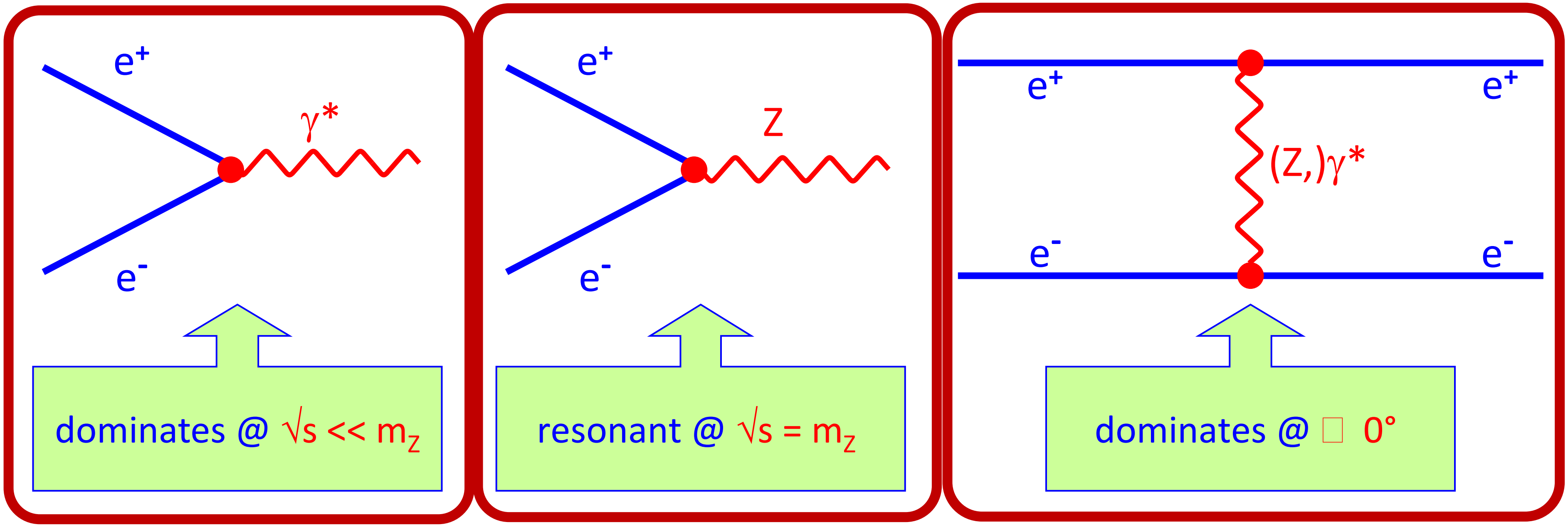
- Z mass, full and partial width (m_Z, Γ_Z, Γ_f);
- number of ν's from $\Gamma_{invisible}$ and from γ_{single};
- asymmetries $A_{forward\text{-}backward}$ for $e^+e^- \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-$, hadrons;
- global fit data $\leftrightarrow$ SM ($\to$ consistency);
- global fit data $\leftrightarrow$ SM ($\to$ predictions of m_{top}, m_{Higgs} from radiative corrections).

iii. Physics 1: Z & W

1. – 8. […]
9. $e^+e^- \to Z \to f\bar{f}$
10. $d\sigma(e^+e^- \to f\bar{f}) / d\Omega$
11. $e^+e^- \to Z \to e^+e^-$
12. Radiative corrections
13. LEP1 SM fit
14. Physics at LEP2
15. $e^+e^- \to W^+W^-$
16. $W^\pm$ properties
17. Global LEP(1+2) fit
18. […]



$e^+e^- \to Z \to f\bar{f}$



- Many possibility from e+e– initial state;
- similar couplings wrt already considered processes [PP §3, §4, §6, §7];
- at low energy, QED only (exchange of γ^* in the s-channel);

- at $\sqrt{s} \approx m_Z$:
  - $\sigma_{res}(e^+e^- \to f\bar{f}) \propto \Gamma_f \,/\, [\,(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2\,]$;
  - for each fermion pair, two (four for e^+e^-) diagrams + interferences);
  - at higher energy, new phenomena ($W^\pm$, exchange, IVB pairs in the final state, …).

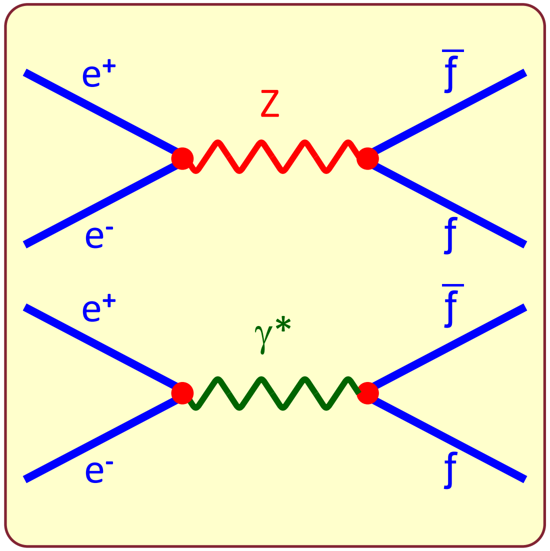
$e^+e^- \to Z \to f\bar{f}$: σ^{SM}_{Born}

In the SM, at lowest order, for $f \neq e^\pm$, $2m_f << m_z$:

- $\sigma_{Born}(e^+e^- \to f\bar{f}) = \sigma_{Zs} + \sigma_{\gamma s} + J_f$;
- $\sigma_{Zs} = \boxed{\dfrac{s\Gamma_Z^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2}} \times \dfrac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}$;
- $\sigma_{\gamma s} = \dfrac{4\pi\alpha^2(s)}{3s} c_f Q_f^2$; $\left[c_f = 1 \text{ (leptons), } 3 \text{ (quark)}\right]$;
- $J_f = -\dfrac{(s-m_Z^2)m_Z^2}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2} \dfrac{2\sqrt{2}\alpha(s)}{3} c_f Q_f G_F g_V^e g_V^f$;
- $\Gamma_Z = \Gamma_{tot} = \sum_f \Gamma(Z \to f\bar{f})$;
- $\Gamma_f \equiv \Gamma(Z \to f\bar{f}) = \dfrac{G_F m_Z^3 c_f}{6\sqrt{2}\,\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right]$;
- for $\sqrt{s} \approx m_z \to$ interference and γ^* negligible;
- $\sigma_{Born}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = \dfrac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}$.

i.e. neglect t-channel, both Z* and γ*

Z (s-channel)	= $\boxed{\text{bell-normalized-to-1}}$ $\times\, \sigma(\sqrt{s}=m_Z)$
γ* (s-channel)	[well known, see PP §3]
interference $Z_s \leftrightarrow \gamma^*_s$	new entry, possibly important for $\mathbb{P}$-violation



NB many parameterizations currently used in literature. With time, I tend to evolve [more sophisticated] → [simpler, more understandable]

$e^+e^- \to Z \to f\bar{f}$: g_V^f and g_A^f

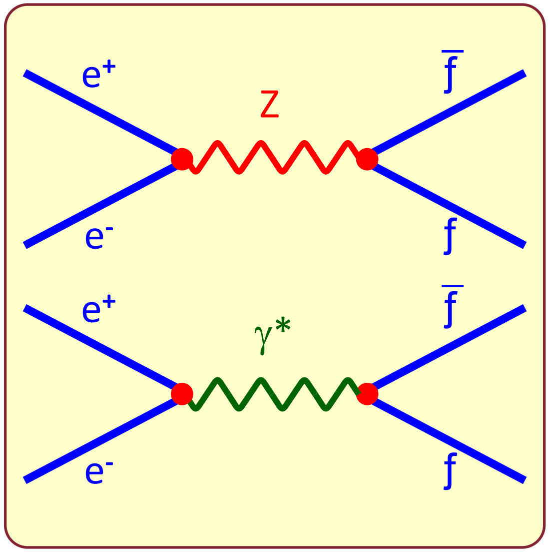
- the partial widths Γ_f (e.g. Γ_μ) are also easily computed in lowest order :

$$\Gamma_f = \frac{G_F m_Z^3 c_f}{6\sqrt{2}\,\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right] \;\to\; (f=\mu^\pm) \;\to\; \Gamma_\mu \approx \frac{1}{4}\frac{G_F m_Z^3}{6\sqrt{2}\,\pi} \approx 83\,\text{MeV};$$

[§ν] :
$g_A^f = t_{3L}^f$
$g_V^f = t_{3L}^f - 2Q^f \sin^2\theta_W$

- for the other Γ's it is found **[lowest order values, NOT "the best"]** :

f	Q_f	g_A^f	g_V^f	Γ_f (MeV)	Γ_f / Γ_μ	R_f (%)
$\nu_e\ \nu_\mu\ \nu_\tau$	0	+½	+½	166	1.99	6.8
$e^-\ \mu^-\ \tau^-$	−1	-½	−.038	83	[1]	3.4
u c [t]	⅔	+½	+.192	286	3.42	11.8
d s b	-⅓	-½	−.346	368	4.41	15.2



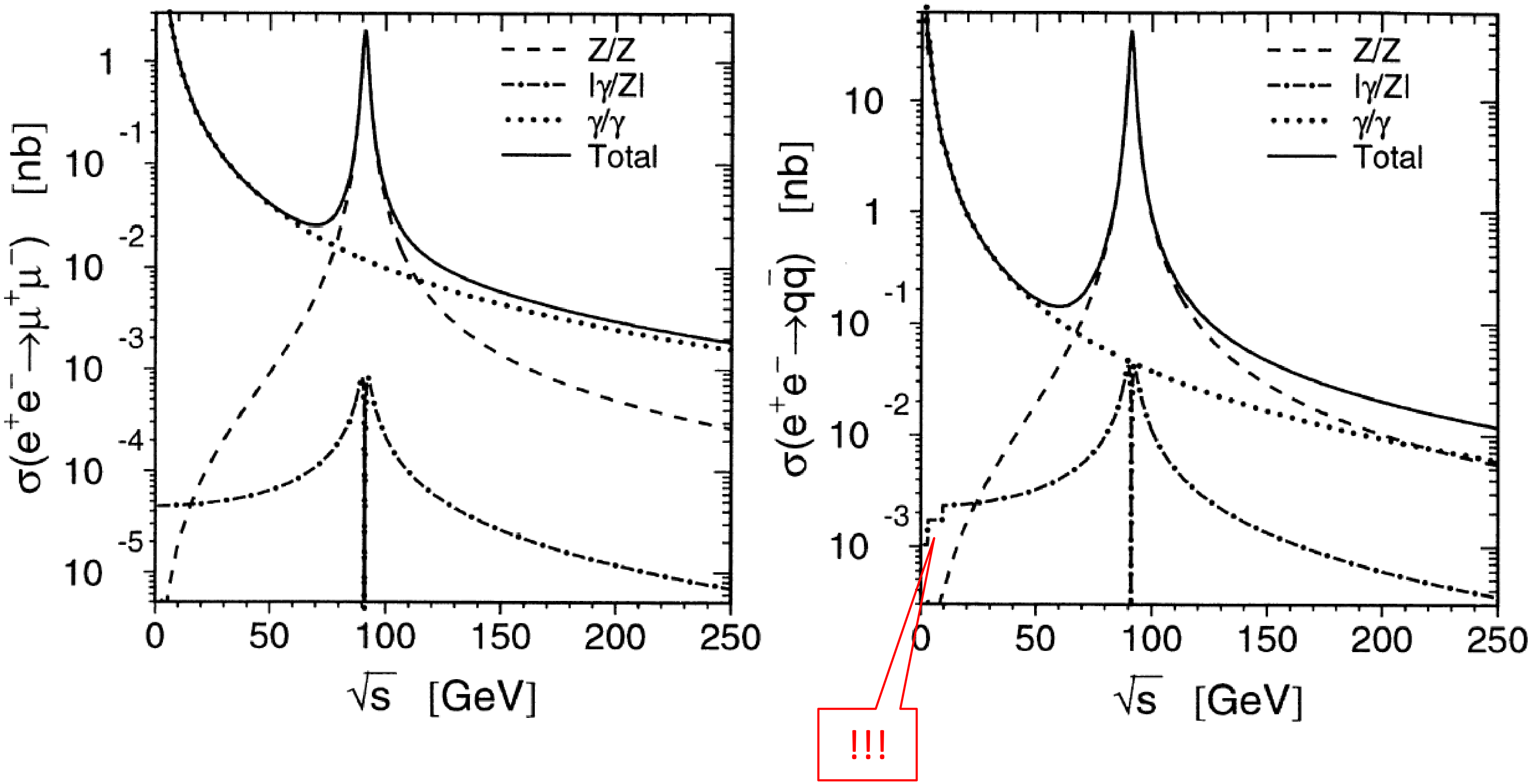
In Born approx. [B = "Born"] :

- Γ_Z^B = 2423 MeV, $\Gamma_{hadr.}^B$ = 1675 MeV, $\Gamma_{invis.}^B = \Gamma_\nu^B$ = 498 MeV;
- $R_{hadr.}^B$ = 69.1 %, $R_{lept\pm}^B$ = 10.2 %, $R_{invis.}^B = R_{\nu's}^B$ = 20.5 %,
- $R_{hadr.}^B / R_{vis.}^B$ = 87.0 %.

remember !

- $\Gamma_Z \approx$ 2.4 GeV, $\Gamma_\nu \approx$ 0.5 GeV,
- $\nu : \ell^\pm : u : d \approx 2 : 1 : 3.4 : 4.4$, hadr : $\ell^\pm : \nu \approx 70 : 10 : 20$.

$e^+e^- \to Z \to f\bar{f}$: predictions



Z/Z and γ^*/γ^* are +ve by definition,

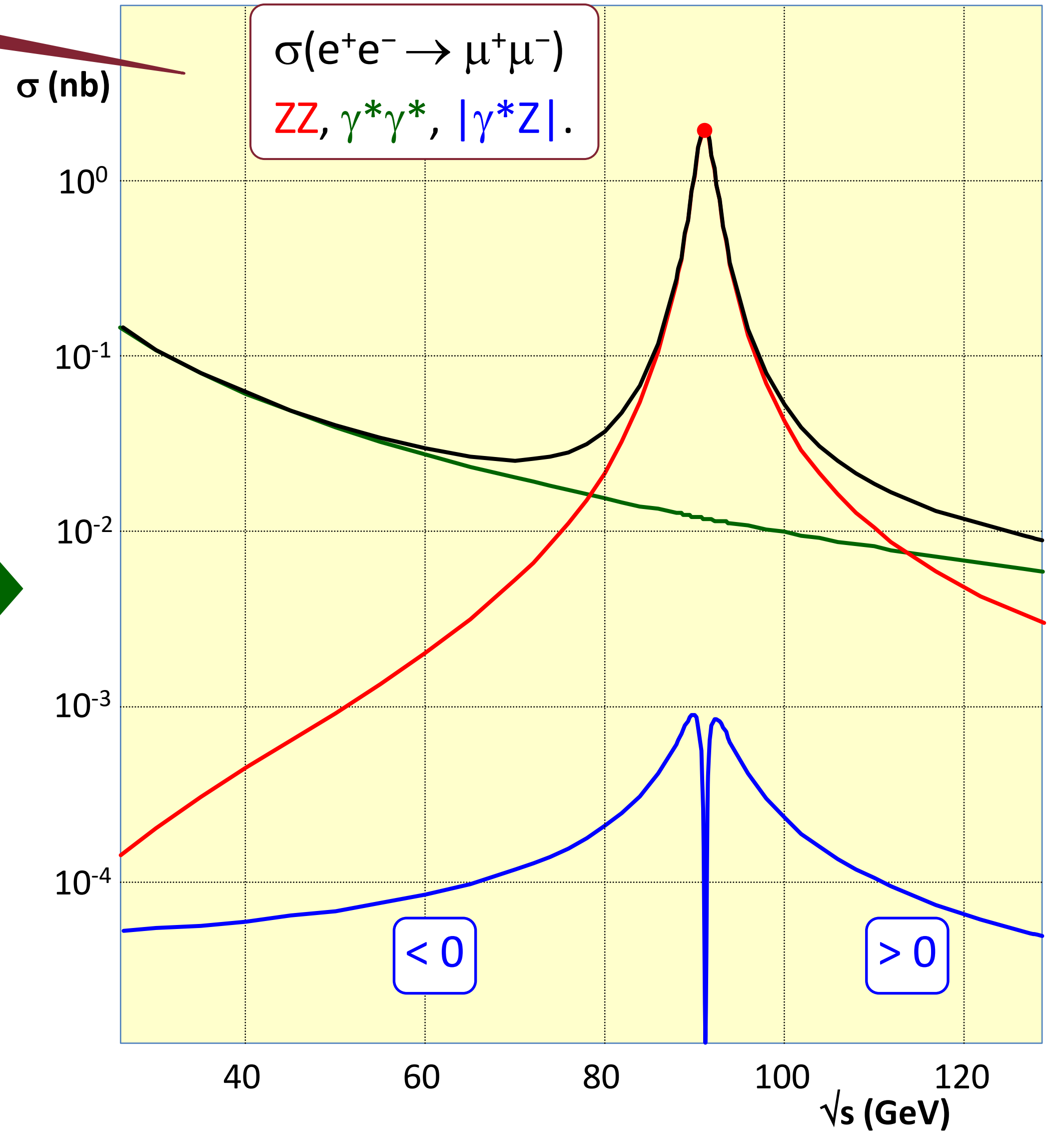
$|\gamma^*/Z|$ is plotted (**<0 @ $\sqrt{s}<m_Z$, >0 @ $\sqrt{s}>m_Z$**).

$e^+e^- \to Z \to f\bar{f}$: home-made predictions

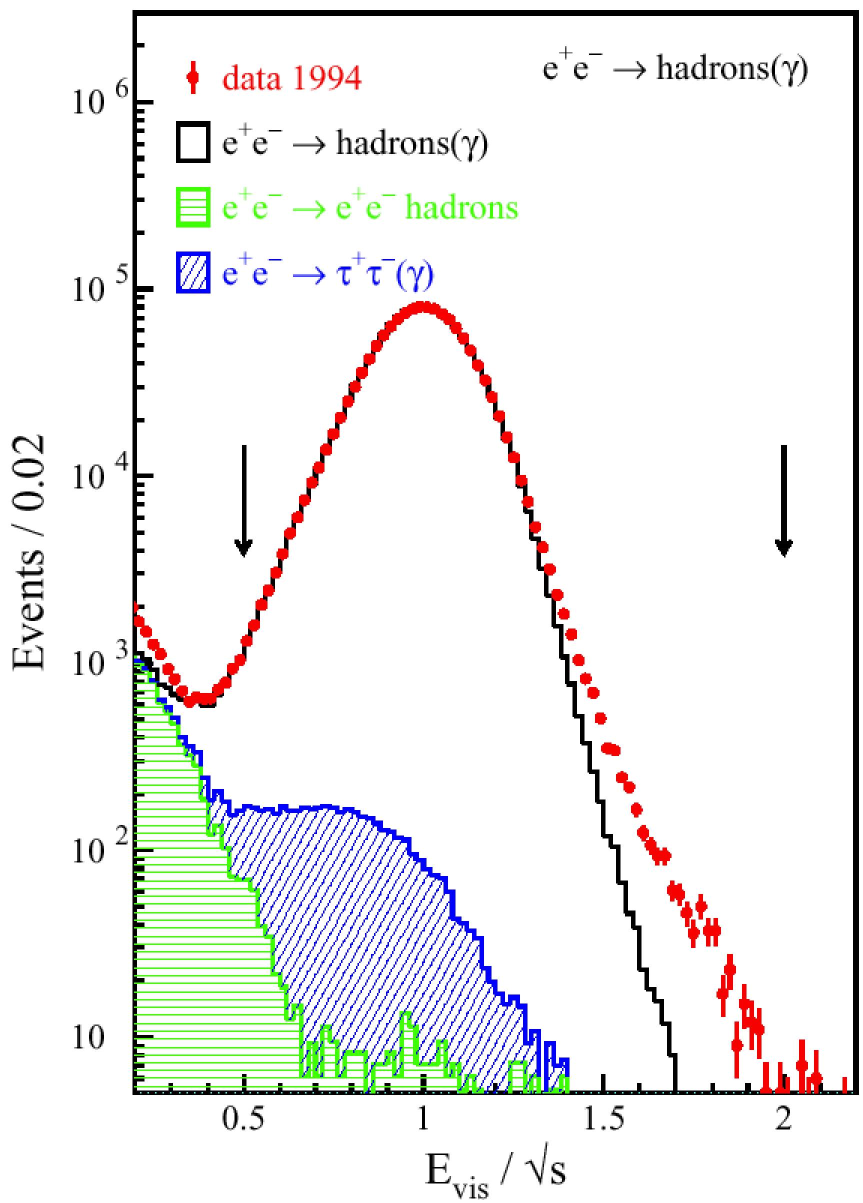
m_Z = 91.1876 GeV
Γ_Z = 2.4952 GeV
Γ_e = 0.083984 GeV
Γ_μ = 0.083984 GeV
$1/\alpha_{em}$ = 128.877
q_μ = -1
c_μ = 1
g_V^e = -0.03783
g_V^μ = -0.03783
G_F = 1.1664×10^{-5} GeV^{-2}
$(\hbar c)^2$ = 3.8938×10^5 GeV^2 nb

+ previous pages

just R®



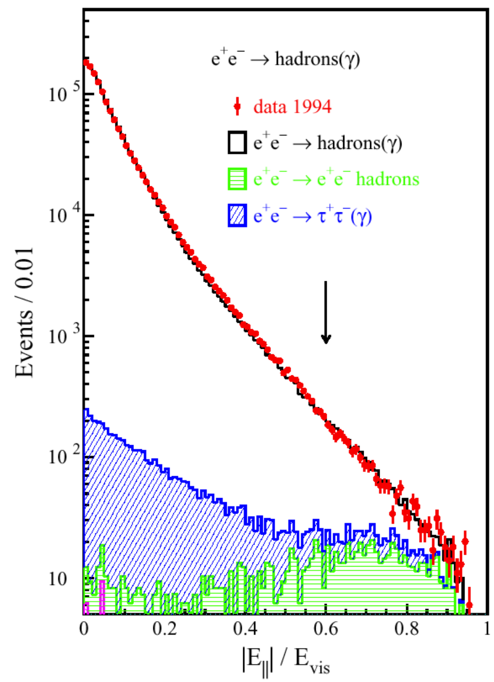
$e^+e^- \to Z \to f\bar{f}$: hadrons (1)

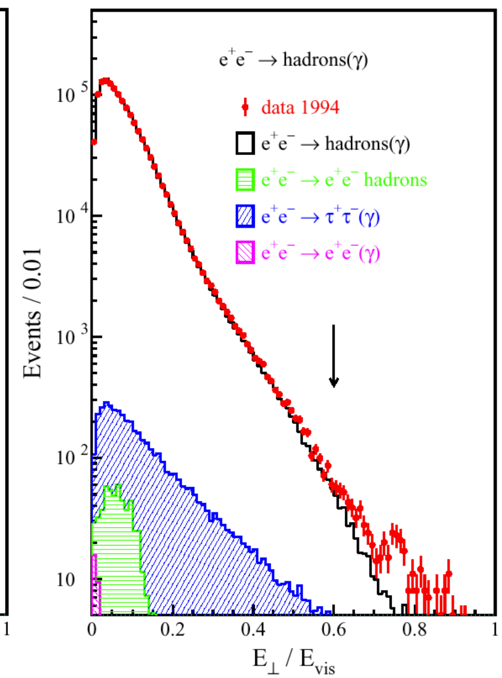


Example : $e^+e^- \to$ hadrons (i.e. $e^+e^- \to q\bar{q}$) in L3 1994 *(an old paper, chosen because well written)*. Selection :

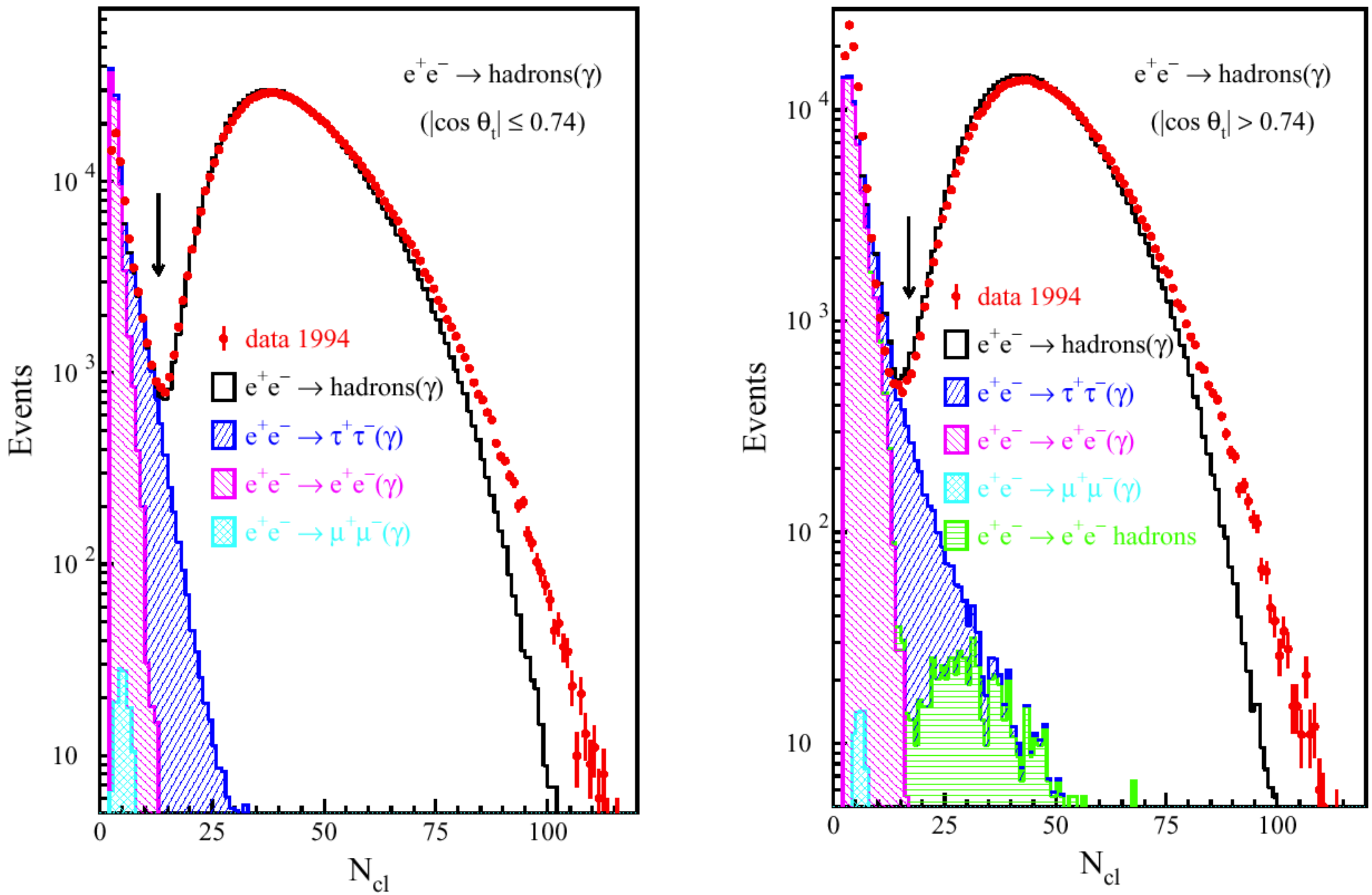
- $0.5 < E_{vis} / \sqrt{s} < 2.0$;
- $|E_{\parallel}| / E_{vis} < 0.6$;
- $E_\perp / E_{vis} < 0.6$;
- $N_{clusters} > 13$ (barrel), > 17 (endcap) [next]

$E_{VIS} = \sum_{seen} |\vec{p}_j|$;
$\vec{P} = \sum_{seen} \vec{p}_j$;
$|E_\parallel| = |P_z|$; $E_\perp = P_T$.





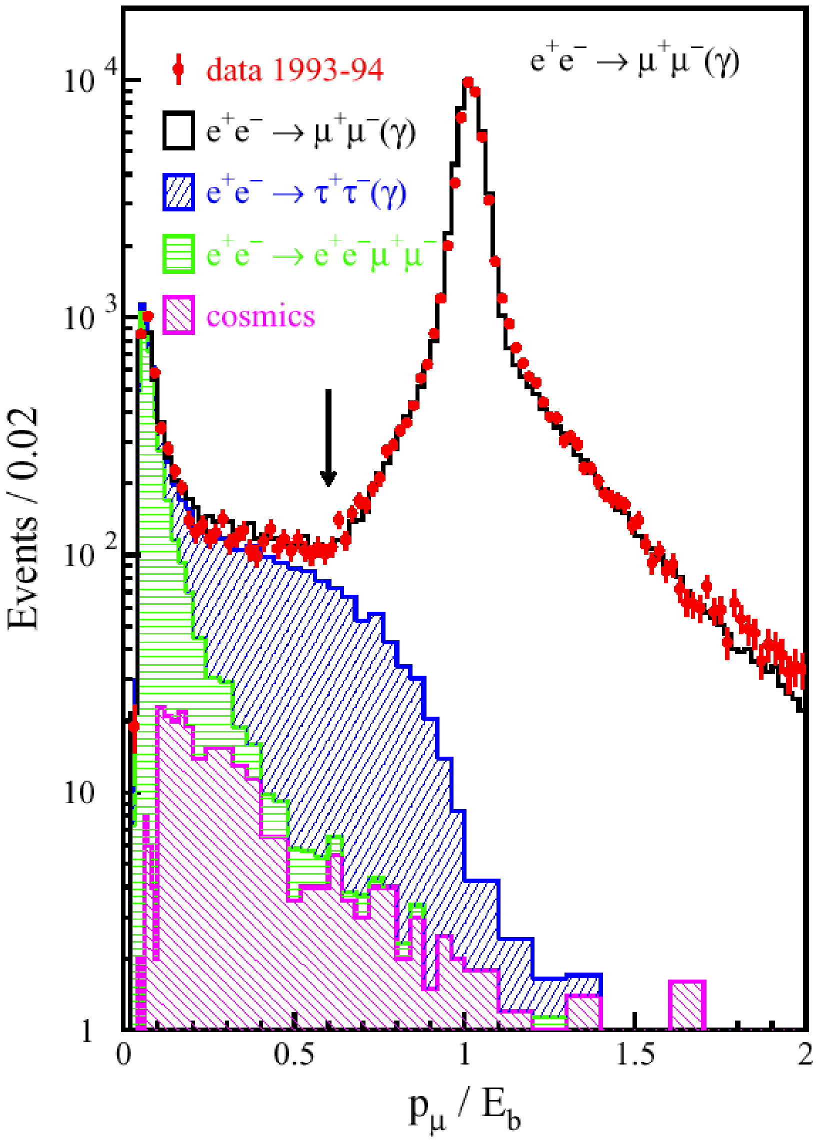
$e^+e^- \to Z \to f\bar{f}$: hadrons (2)

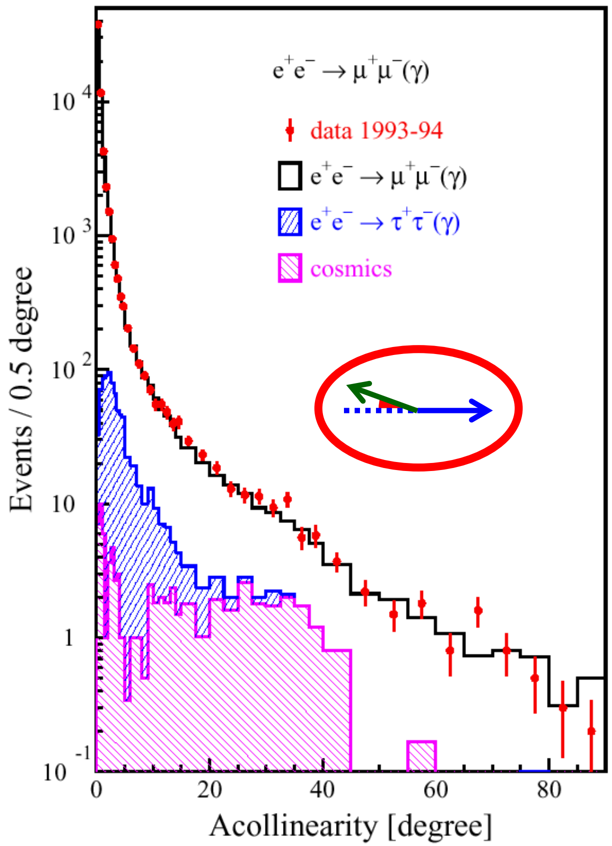


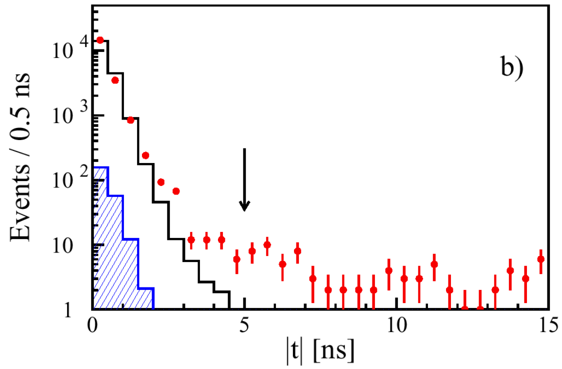
Example : $e^+e^- \to$ hadrons (i.e. $e^+e^- \to q\bar{q}$) in L3 1994 – pag. 2

[$N_{clusters}$ > 13 (barrel), > 17 (endcap)]

$e^+e^- \to Z \to f\bar{f}$: $\mu^+\mu^-$





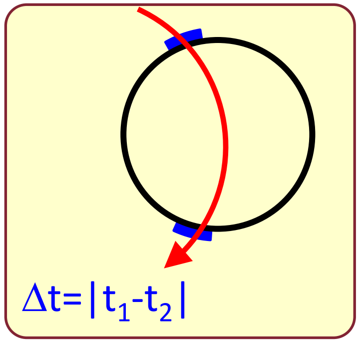


Other example (same paper) : $e^+e^- \to \mu^+\mu^-$

Selection :

- ≥ 1 μ identified;
- $|p_\mu| > 0.6\ (\sqrt{s}/2)$;
- $\alpha(\square\ )$ "small";
- $N_{clusters} < 15$;
- $time_{scintillators}$.

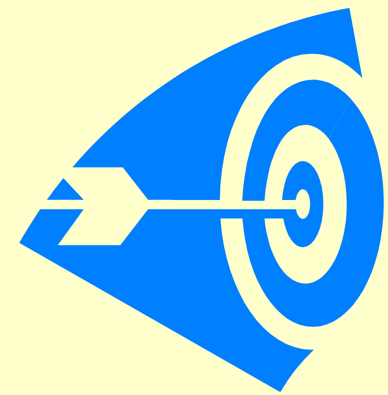
Q. : why μ's have smaller acollinearity than τ's ?



$e^+e^- \to Z \to f\bar{f}$: from W. Tell to LEP

Problem. Two variables (x, y) are normally (=Gauss) distributed with mean (m_x, m_y) and standard deviation $\sigma_x = \sigma_y = \sigma$. Find the distribution of the distance from the center

$$r = \sqrt{(x-m_x)^2 + (y-m_y)^2}.$$



- W. Tell's crossbow;
- the event $\not{E}_T$ at LEP/LHC;
- the sum of momenta of the charged particles wrt the jet axis, …

Solution:

$$\begin{cases} x - m_x = r\cos\theta \\ y - m_y = r\sin\theta \end{cases}; \quad f_{\text{Gauss}}(t|\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{t^2}{2\sigma^2}\right]; \qquad [t = x, y]$$

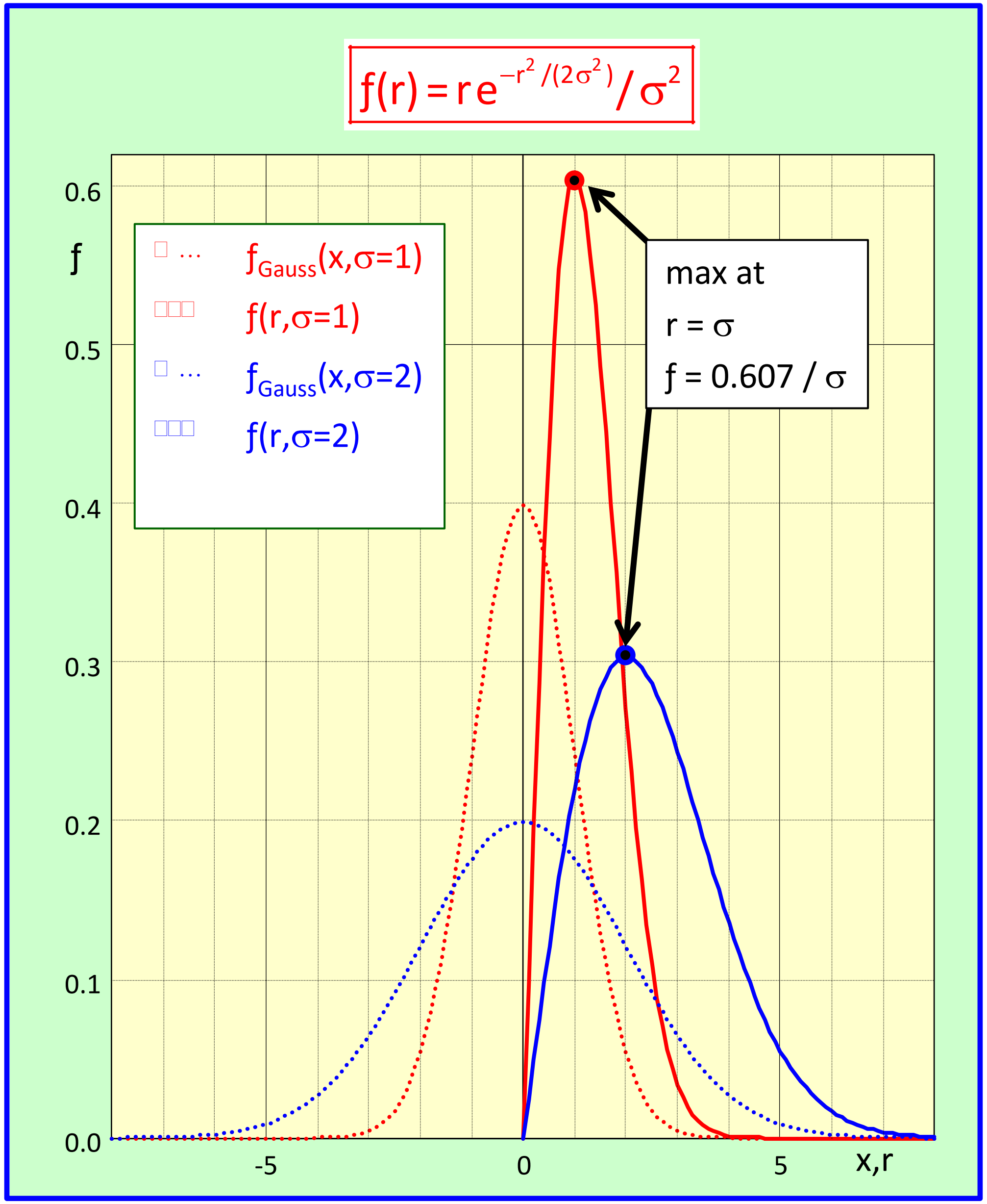
$$f(x,y) = f(x|\sigma)\times f(y|\sigma) = \frac{1}{2\pi\sigma^2}\exp\left[-\frac{x^2+y^2}{2\sigma^2}\right];$$

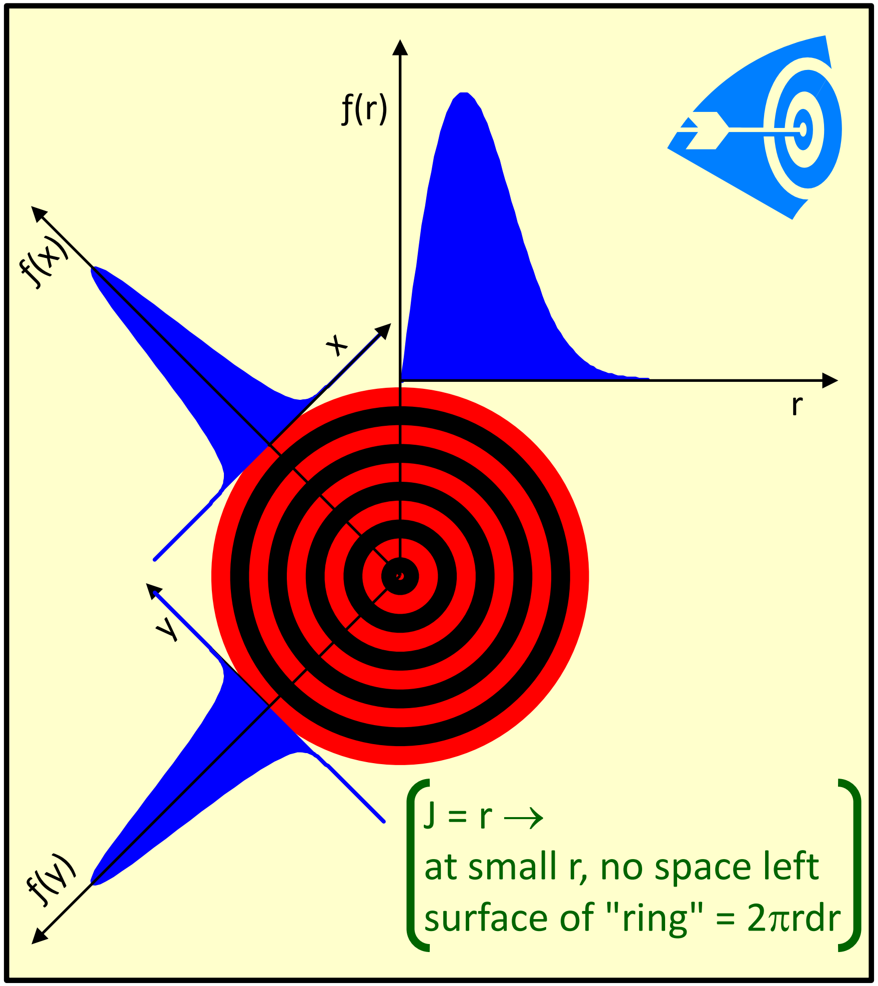
$$J\left(\frac{x,y}{r,\theta}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} = r;$$

m_x and m_y are translations wrt centre; they do NOT influence the result.

$$f(r,\theta) = f(x,y)\times\left|J\left(\frac{x,y}{r,\theta}\right)\right| = \frac{r}{2\pi\sigma^2}\exp\left[-\frac{r^2}{2\sigma^2}\right]; \qquad f(r) = \int_0^{2\pi} d\theta\ f(r,\theta) = 2\pi f(r,\theta) = \frac{r}{\sigma^2}\exp\left[-\frac{r^2}{2\sigma^2}\right].$$

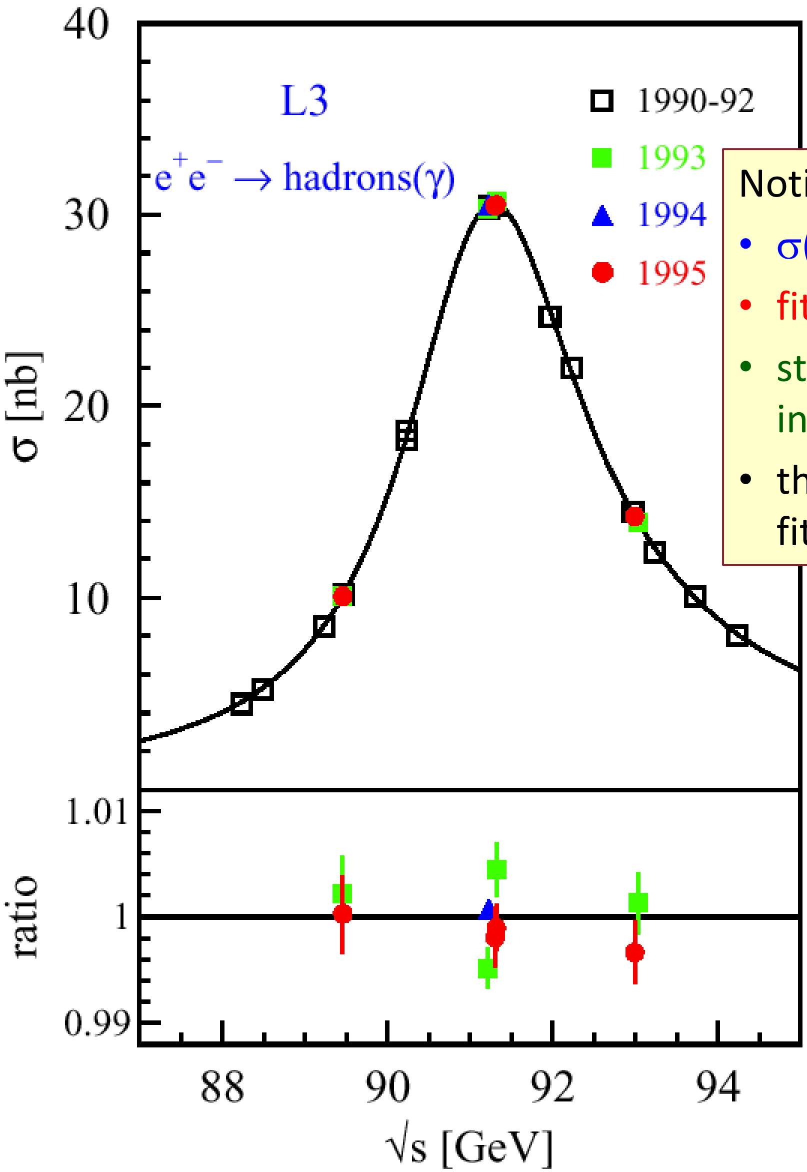
$e^+e^- \to Z \to f\bar{f}$: a W. Tell tale





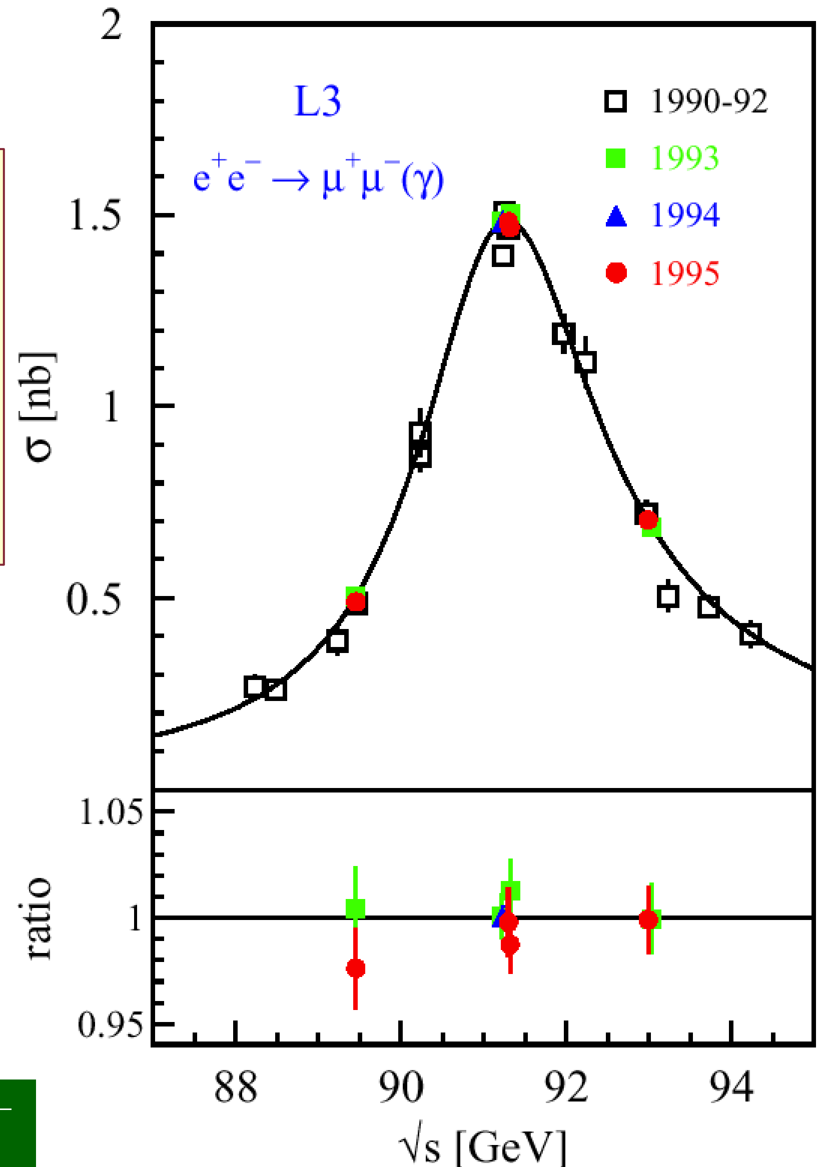
next question:
the case $\sigma_x \neq \sigma_y$
[easy, needs only one smart trick]

$e^+e^- \to Z \to f\bar{f}$: lineshape



Notice:

- σ(had) >> σ(□);
- fit quality;
- strategy change in 1993;
- the line is the SM fit (see later).



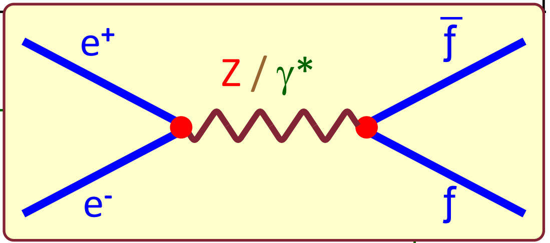
for $e^+e^- \to \tau^+\tau^-$ see later.

$d\sigma(e^+e^- \to f\bar{f})\ /\ d\Omega$

Differential cross-section in lowest (Born) order:

$$\frac{d\sigma_{Born}\left(e^+e^- \to f\bar{f}\right)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{ \begin{aligned} &(1+\cos^2\theta)\times\left[\begin{aligned} &Q_e^2Q_f^2 - 2\boxed{\chi}Q_fQ_e g_V^e g_V^f \cos\delta_R + \\ &+\boxed{\chi^2}\left[\left(g_A^e\right)^2+\left(g_V^e\right)^2\right]\left[\left(g_A^f\right)^2+\left(g_V^f\right)^2\right] \end{aligned}\right] + \\ &+2\cos\theta\times\left[-2\boxed{\chi}Q_eQ_f g_A^e g_A^f \cos\delta_R + 4\boxed{\chi^2} g_A^e g_A^f g_V^e g_V^f\right] \end{aligned}\right\};$$

$$\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)} \times \frac{s m_Z^2}{\sqrt{\left(m_Z^2 - s\right)^2 + m_Z^2\Gamma_Z^2}}; \qquad \tan\delta_R = \frac{m_Z\Gamma_Z}{m_Z^2 - s} \quad \left[\to \cos\delta_R(\sqrt{s} = m_Z) = 0\right];$$



$$A_f^{FB}\left(\sqrt{s}\right) \equiv \frac{\sigma\left(\cos\theta > 0, \sqrt{s}\right) - \sigma\left(\cos\theta < 0, \sqrt{s}\right)}{\sigma\left(\cos\theta > 0, \sqrt{s}\right) + \sigma\left(\cos\theta < 0, \right)};$$

$$A_f^{FB}\left(\sqrt{s} = m_Z, Z_{s-channel}\ \text{only}\right) =$$

$$= 3\frac{g_V^e g_A^e}{\left(g_V^e\right)^2 + \left(g_A^e\right)^2} \times \frac{g_V^f g_A^f}{\left(g_V^f\right)^2 + \left(g_A^f\right)^2};$$

$\boxed{A_f^{FB}}$ is the "forward-backward asymmetry" for $e^+e^- \to f\bar{f}$.

$d\sigma(e^+e^- \to f\bar{f})\ /\ d\Omega$: comments

$$\frac{d\sigma_{Born}\left(e^+e^- \to f\bar{f}\right)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s}\left\{ \begin{array}{l} \left(1+\cos^2\theta\right)\times\left[\begin{array}{l} Q_e^2Q_f^2 - 2\boxed{\chi}Q_fQ_e g_V^e g_V^f \cos\delta_R + \\ +\boxed{\chi^2}\left[\left(g_A^e\right)^2+\left(g_V^e\right)^2\right]\left[\left(g_A^f\right)^2+\left(g_V^f\right)^2\right] \end{array}\right] + \\ +2\cos\theta\times\left[-2\boxed{\chi}Q_eQ_f g_A^e g_A^f \cos\delta_R + 4\boxed{\chi^2} g_A^e g_A^f g_V^e g_V^f\right] \end{array} \right\};$$

$$A_f^{FB}\left(\sqrt{s}\right) \equiv \frac{\sigma\left(\cos\theta>0,\sqrt{s}\right)-\sigma\left(\cos\theta<0,\sqrt{s}\right)}{\sigma\left(\cos\theta>0,\sqrt{s}\right)+\sigma\left(\cos\theta<0,\sqrt{s}\right)} \xrightarrow{\sqrt{s}\to m_Z} 3\frac{g_V^e g_A^e}{\left(g_V^e\right)^2+\left(g_A^e\right)^2}\times\frac{g_V^f g_A^f}{\left(g_V^f\right)^2+\left(g_A^f\right)^2}.$$

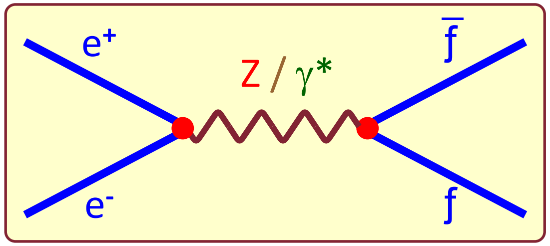
mediators : γ, Z [= $Z_A + Z_V$];

$\mathbb{P}$-cons : □ , γZ_V, ZZ [= $Z_A^2 + Z_V^2$];

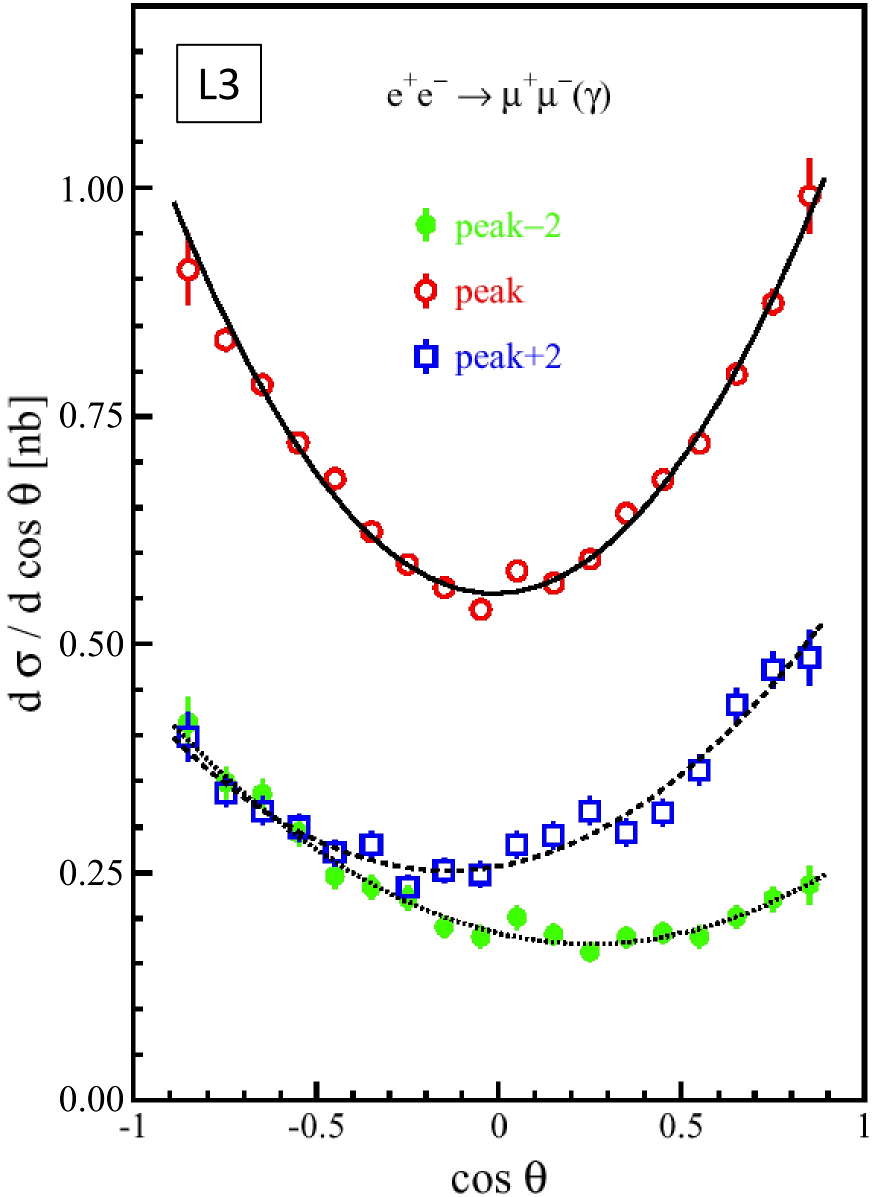
$\mathbb{P}$-viol. : γZ_A, $Z_A Z_V$.

- standard SM computation for $Z_s \oplus \gamma_s$ only (average on initial and sum on final polarization), then sum on φ:
- notice : the term $\propto$ ($\cos\theta$) is anti-symmetric; it does NOT contribute to σ_{tot} ($\int \cos\theta\, d\cos\theta = 0$), but only to the ($\mathbb{P}$-violating) forward-backward asymmetry;
- the $\mathbb{P}$-violation clearly comes from the interference between the vector (γ + Z_V) and axial (Z_A) terms.

- at the pole ($\sqrt{s}=m_Z$), only few terms :
  - $\cos\delta_R = 0$;
  - the asymmetry, i.e. the term $\propto \cos\theta$, is $\propto g_V^e$ (very small) for all fermions;
  - for the $\mu^+\mu^-$ case [easily measurable], it is even smaller ($\propto g_V^e g_V^\mu$).

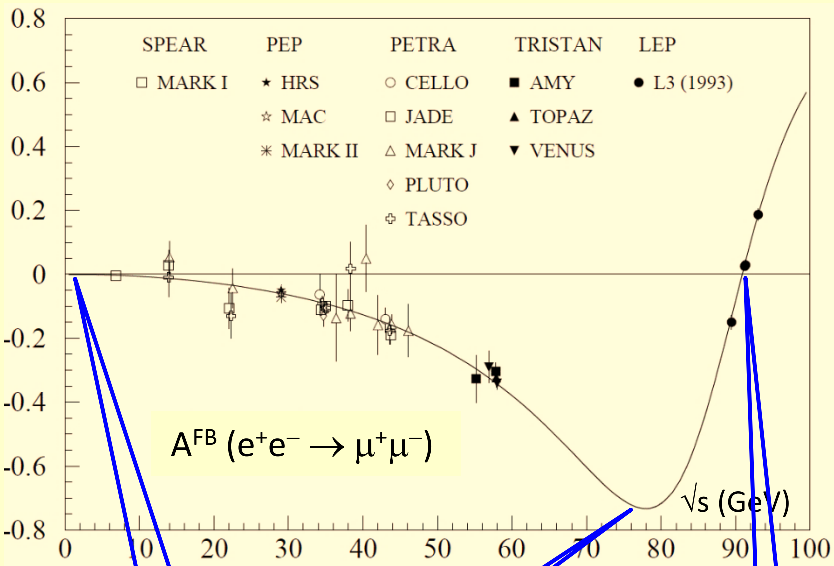


$d\sigma(e^+e^- \to f\bar{f})$ / $d\Omega$: data



- Experimentally, the main problem is the selection $f \leftrightarrow \bar{f}$ (i.e. $\theta \leftrightarrow$ □θ). This is
  - ➢ essentially impossible for light quarks $u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$ (despite heroic efforts based on charge counting);
  - ➢ difficult for heavy quarks c,b (based on lepton charge in semileptonic quark decays, e.g. $c \to s\ell^+\nu$, $\bar{c} \to \bar{s}\ell^-\bar{\nu}$);
  - ➢ "simple" for $\mu^\pm$ (only problem: wrong sagitta sign because of high momentum);
  - ➢ best channel for $d\sigma/d\cos\theta$ and A_{FB}: $e^+e^- \to \mu^+\mu^-(\gamma)$;
- unfortunately, $A_{FB}(\sqrt{s}=m_Z)$ is very small in the $\ell^+\ell^-$ channels, due to the extra small factor g_V^μ;
- notice the asymmetry change for peak ±2 GeV.

$d\sigma(e^+e^- \to f\bar{f}) / d\Omega$: $A_{fb}(\mu^+\mu^-)$

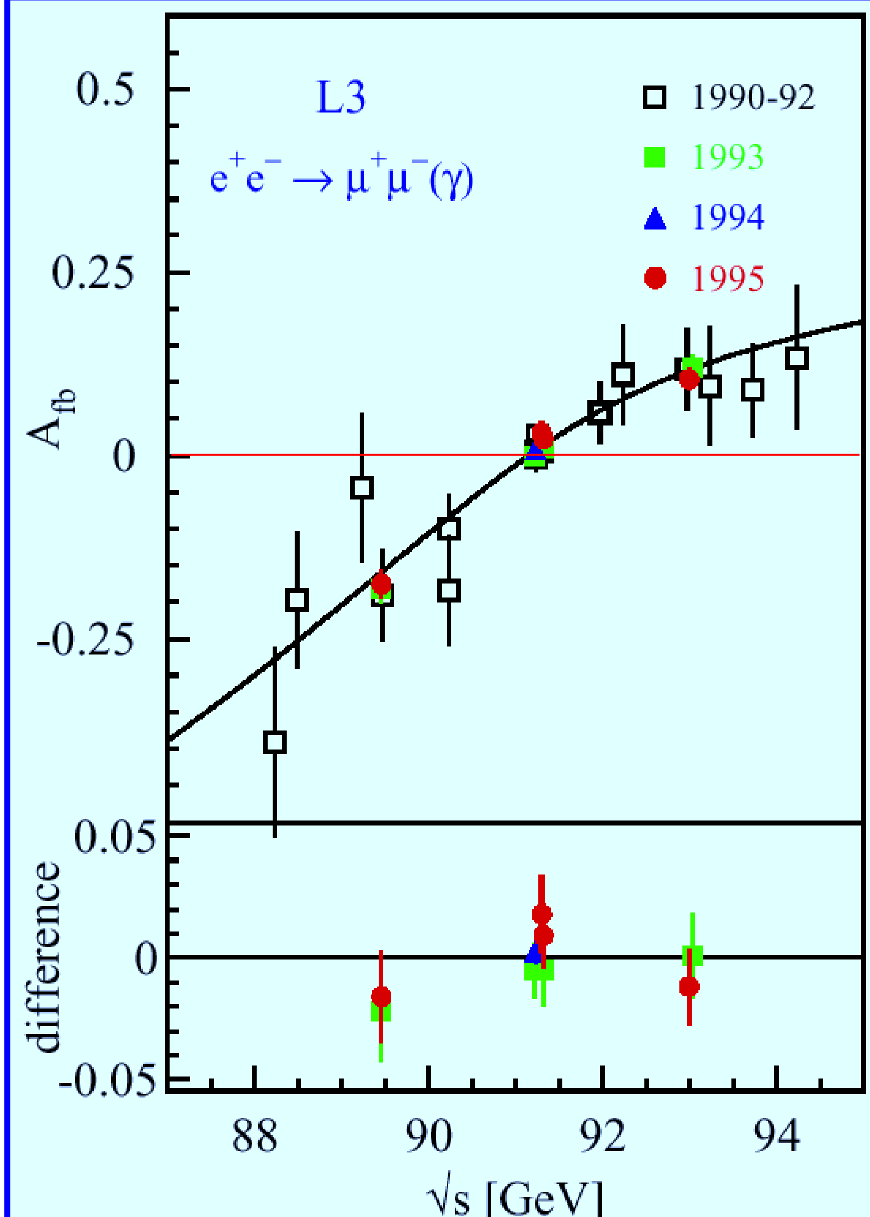


γ only $\to$ V only $\to A^{FB} = 0$

$Z \approx A$, $\gamma = V \to A^{FB}$ max @ max interference [no exp ever]

$\sqrt{s} \approx m_Z \to$ A dominates $\to A^{FB} \approx 0$

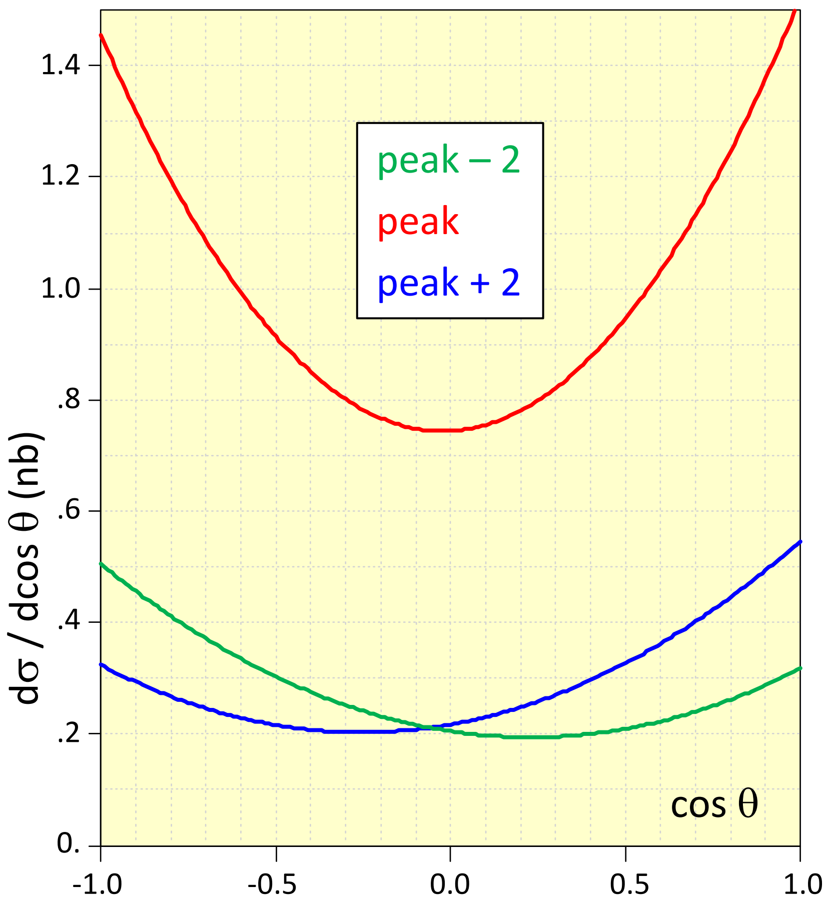
full $\sqrt{s}$ range + SM prediction

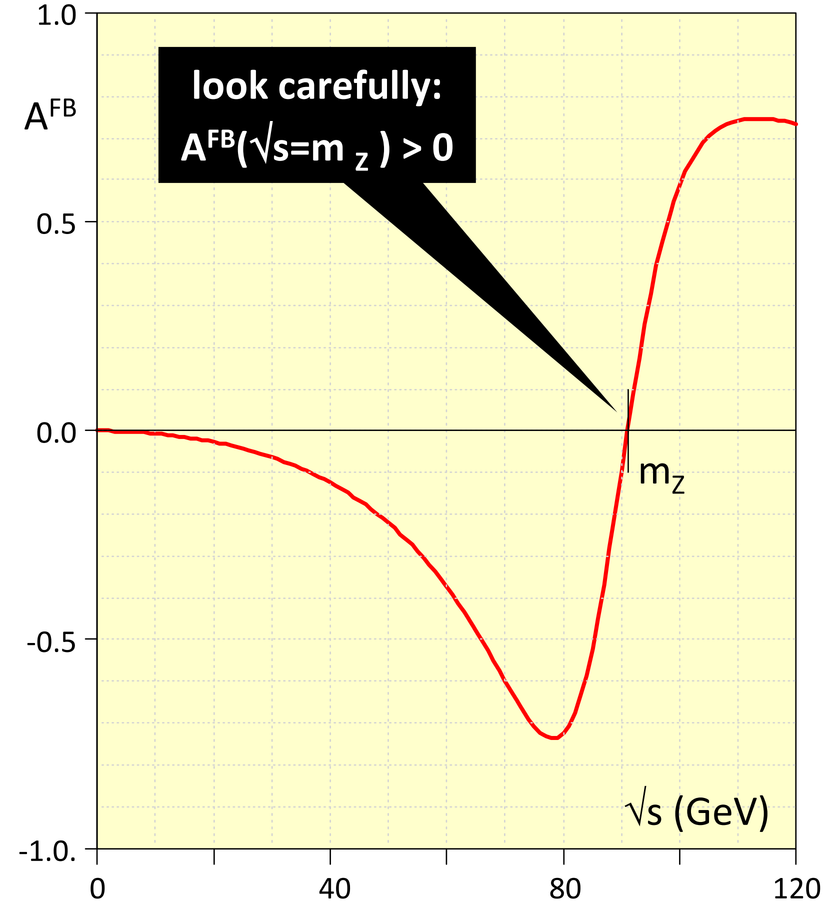


$d\sigma(e^+e^- \to f\bar{f})$ / $d\Omega$: problem

Problem. Compute $d\sigma/d\cos\theta$ and A^{FB} in lowest order from the formulæ. This is a case where the "tree approx." fails. Explain where and why.

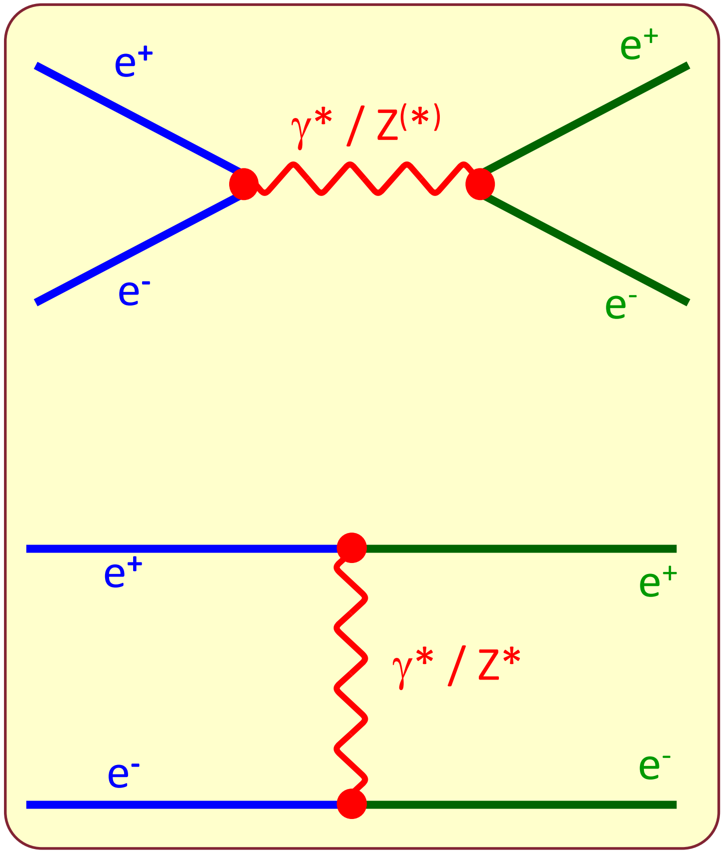
If no success, look to Grünewald, op. cit., pag. 230-232 [simplified explanation: higher orders and selection criteria are important, expecially for peak+2 (→ init. state brem). Necessary also for naïve understanding].



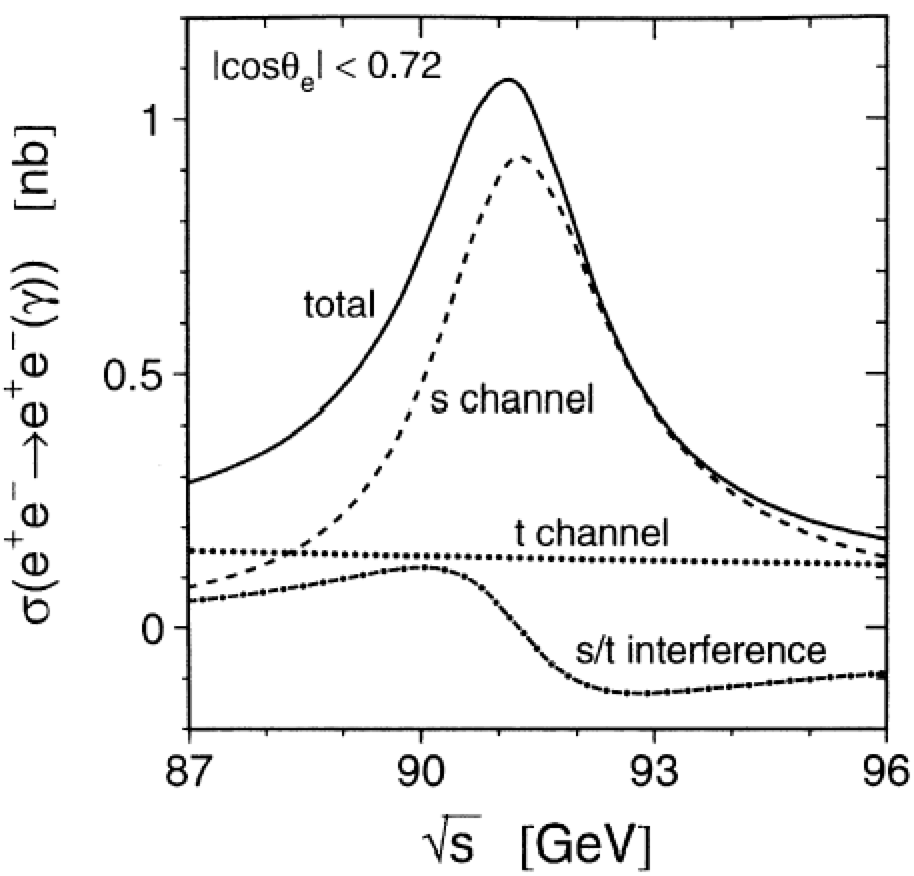


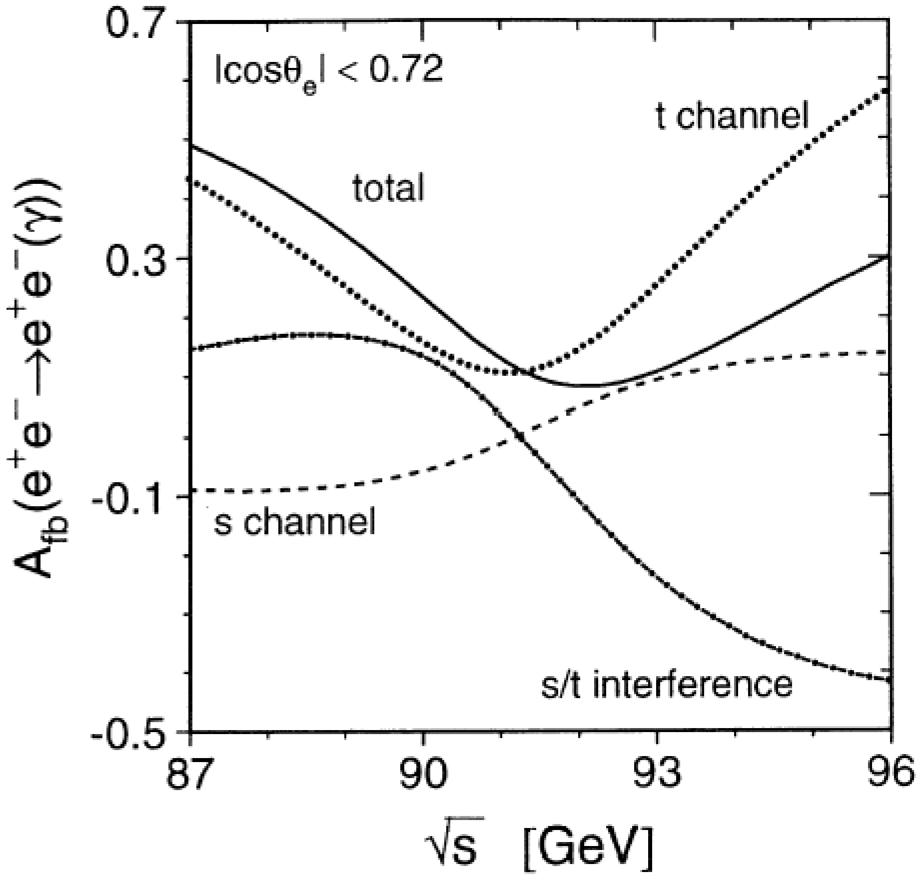
$e^+e^- \to Z \to e^+e^-$

- Bhabha scattering is more difficult, due to the presence of another Feynman diagram: the γ^* / Z exchange in the t-channel;
- 4 Feynman diagrams $\to$ 10 terms :
  - Z s-channel (Z_s);
  - γ^* s-channel (γ_s);
  - Z t-channel (Z_t);
  - γ^* t-channel (γ_t);
  - 6 interferences;
- qualitatively :
  - $\mathbf{Z_t}$ negligible;
  - @ $\sqrt{s} \approx m_Z$ and $\theta >> 0°$, $\mathbf{Z_s}$ dominates.
  - @ $\theta \approx 0°$, $\boldsymbol{\gamma_t}$ dominates for all $\sqrt{s}$;
  - @ $\sqrt{s} << m_Z$ and $\theta >> 0°$, $\boldsymbol{\gamma_s}$ and $\boldsymbol{\gamma_t}$ are both important, while $\mathbf{Z_s}$ is negligible.



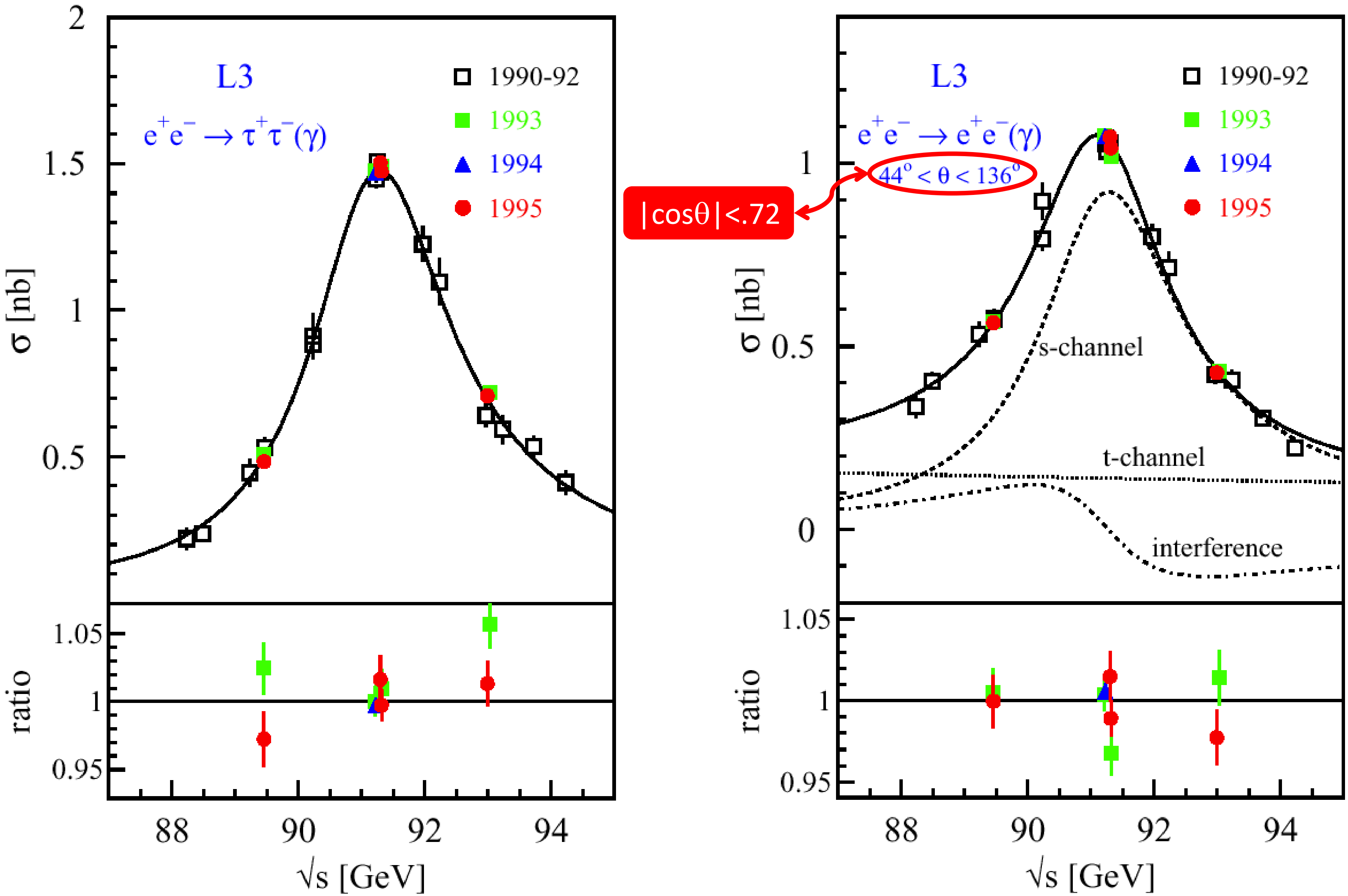
$e^+e^- \to Z \to e^+e^-$: σ_{SM}



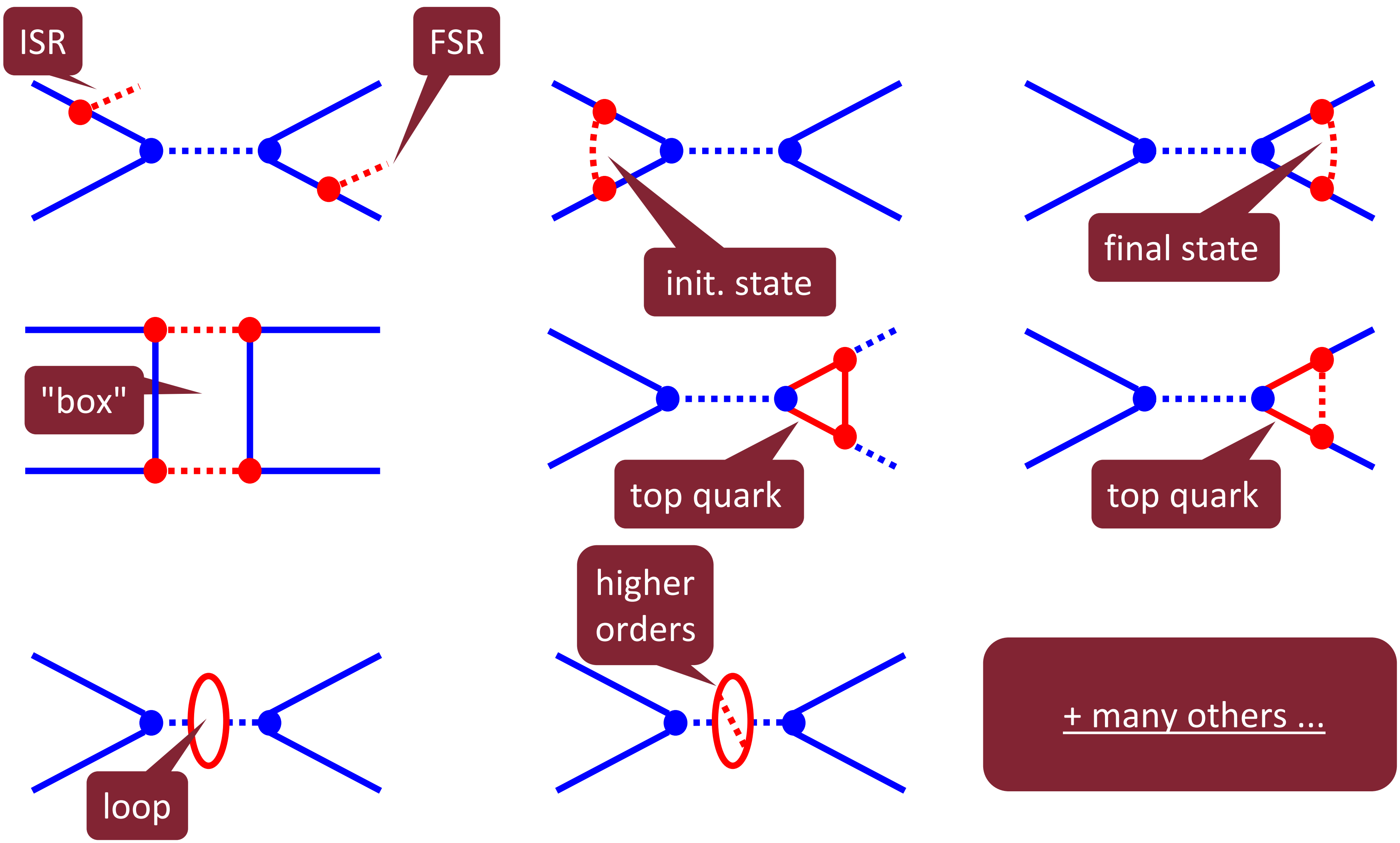


- s, t, interference s/t vs $\sqrt{s}$, with a θ cut ($|\cos\theta| < 0.72$, i.e. $44° < \theta < 136°$);
- data @ $|\cos\theta| > 0.72$ available, but not used here [used for lumi];
- notice : the cut on $\cos\theta$ is NOT instrumental, but used OFFLINE to enhance Z_s over γ_t, to increase signal/bckgd and decrease stat error.

$e^+e^- \to Z \to e^+e^-$: results



radiative corrections



radiative corrections: what ? why ?

what ?

- higher orders (both SM and bSM);
- dependent on full SM, QCD included;
- conventionally, classified into QED, weak, QCD, bSM (if any);
- … or initial and final state;
- also particles not kinematically allowed at lower $\sqrt{s}$ (e.g. top, Higgs);

computable ?

- in principle yes, if all parameters known;
- in practice, successive approximations ("order n");

necessary ?

- yes, because required by the measurement accuracy ($\sim 10^{-3}$);

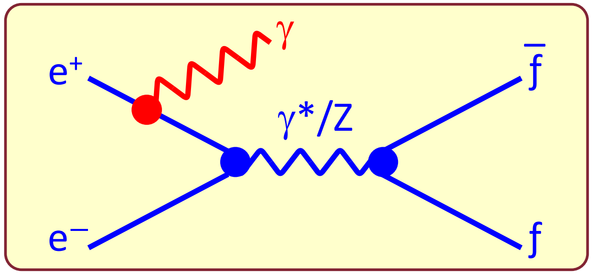
useful ?

- yes, because they give an indirect access to higher energy, by making lower energy observables (like m_z) dependent on higher energy parameters (like m_{top} or m_H);
- i.e., they "raise" the accessible $\sqrt{s}$;
- \+ more accurate and powerful test of the theory;
- [much work, theses, papers, …];

how to use the bSM part (e.g. SUSY), both tree-level and higher orders ?

- first, do not include it, and look for discrepancies;
- if disagreement (**εὔρηκα !!!**), include physics bSM and look for agreement;
- if not → put a **limit** on physics bSM.

radiative corrections: ISR kinematics



One of the simplest r.c. is the QED brem of a (real) γ from one of the initial state $e^\pm$: **ISR** (Initial State Rad.);

- the kinematics is :

$$
\begin{array}{llll}
e^+e^- (& \sqrt{s}, & 0, & 0 \quad); \\
\gamma \ (& E_\gamma, & E_\gamma \cos\alpha_\gamma, & E_\gamma \sin\alpha_\gamma \ ); \\
f\bar{f} \ (& \sqrt{s}-E_\gamma, & -E_\gamma \cos\alpha_\gamma, & -E_\gamma \sin\alpha_\gamma);
\end{array}
$$

$$s' \equiv m^2_{f\bar{f}} = \left(\sqrt{s}-E_\gamma\right)^2 - E^2_\gamma = s\left(1-2E_\gamma/\sqrt{s}\right);$$

$$z \equiv s'/s = 1-2E_\gamma/\sqrt{s}; \qquad [s'<s \rightarrow z<1]$$

$\rightarrow$ computing E_γ does NOT require α:

$$E_\gamma = \frac{\sqrt{s}}{2}\frac{s-s'}{s} = \frac{s-s'}{2\sqrt{s}} = \frac{s-m^2_{f\bar{f}}}{2\sqrt{s}}.$$

- **LEP 1** ($\sqrt{s} < m_z$ + few GeV) :
  - $\sqrt{s'} \approx m_z$, (but Γ_z) $\rightarrow$ large $\Delta E_\gamma/E_\gamma$;
  - α_γ small (brem. dynamics), γ's mostly in the beam pipe;
  - condition : $2m_f \leq \sqrt{s'} \leq \sqrt{s}$;
- **LEP 2** ($\sqrt{s} >> m_z$) :
  - $\sqrt{s'} \approx m_z$ (because of resonance), known as "**return to the Z**";
  - photon is really monochromatic ($\Gamma_z << E_\gamma$) and very energetic;
  - α_γ small (brem. dynamics), γ's mostly in the beam pipe, Z's with high longitudinal momentum, event very unbalanced;
  - events easily removed in the analysis, but it decreases the effective event yield.

radiative corrections: ISR results

Theoretical treatment :

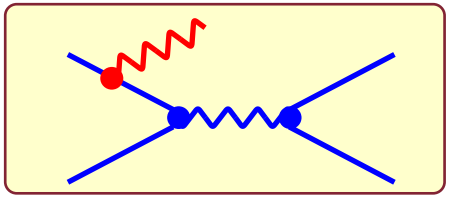
- assume factorization (ISR) ↔ (Z formation);
- the Z formation at √s' is equivalent to the standard process at √s, without ISR :

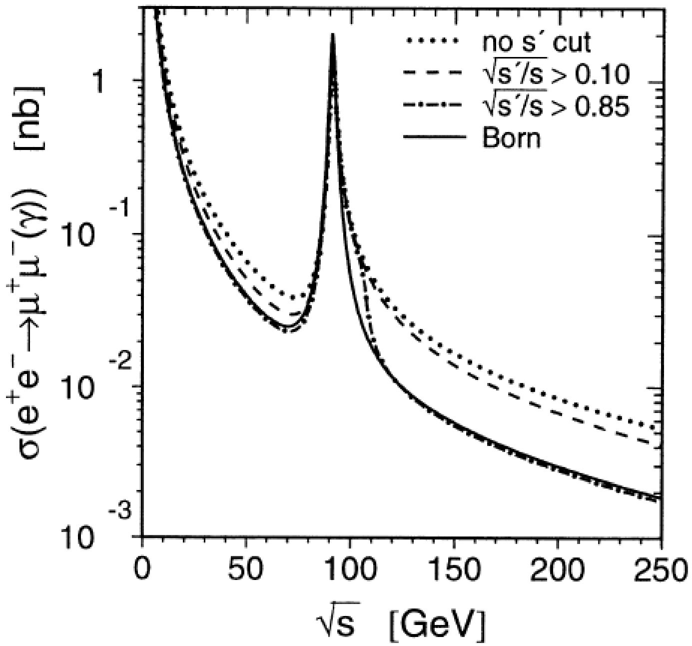
$$\sigma_{ISR}(e^+e^- \to f\bar{f}\gamma;\sqrt{s}) =$$

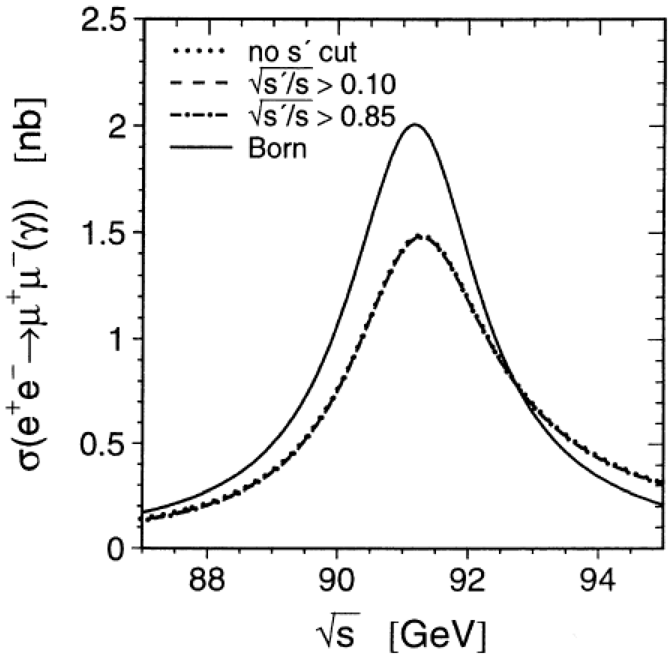
$$= \int_{4m_f^2/s}^{1} dz \begin{pmatrix} R(z,s,\alpha_\gamma) \times \\ \times\ \sigma_{Born}(e^+e^- \to f\bar{f};\sqrt{zs}) \end{pmatrix};$$

- $R(z,s,\alpha_\gamma)$ = ***radiator***, i.e. probability (function of √s, z, α_γ) for γ brem;
- R calculable in QED at a given order.

At LEP 2, cut on z (≈ E_{vis}/√s), tipically z<0.85).







radiative corrections: results for m_Z

The value of m_Z is measured at $\pm$ 2 MeV, so a **very precise** computation is required; these values are for the discussion, the used ones contains many more effects:

- $\sigma_0^f \equiv \sigma_{Born}(e^+e^- \to f\bar{f};\ \sqrt{s}=m_Z) =$
  $= 12\square\ {}_e\Gamma_f / (m_Z^2\Gamma_Z^2)$;
- $\sqrt{s}\,|^{max}_{Born} \approx m_Z\,(1 + \gamma^2)^{1/4} \approx m_Z\,(1+1/4\,\gamma^2) \approx$
  $\approx m_Z + 17$ MeV;
  [slightly larger]
- $\sigma(e^+e^-\to f\bar{f})\,|^{max}_{Born} \approx \sigma_0^f\,(1 + 1/4\gamma^2) \approx$
  $\approx \sigma_0^f\,(1 + .00019)$
  [slightly larger] ;
- $\sqrt{s}\,|^{max}_{ISR} \approx m_Z\,(1 - 1/4\,\gamma^2) + \square\Gamma_Z/8$
  $\approx m_Z + 89$ MeV;
  [slightly larger];
- $\sigma(e^+e^-\to f\bar{f})\,|^{max}_{ISR} \approx \sigma_0^f\,\gamma^\beta\,(1 + \delta_{sup}) \approx$
  $\approx 0.75\,\sigma_0^f$
  [<u>much</u> smaller];

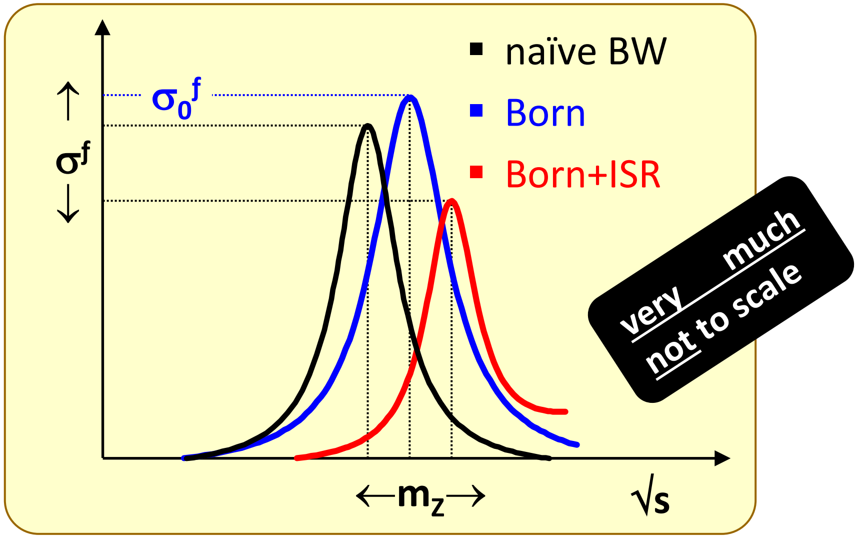
the most important effect

- similar method for Γ_Z :
  - Γ_Z s-dependent : $\Gamma_Z \to s\Gamma_Z / m_Z^2$;
  - (references);

$\gamma \equiv \Gamma_Z / m_Z \approx 0.027$;

$\beta \equiv 2\alpha[2\ell n\,(m_Z / m_e) - 1]/\pi \approx 0.108$;

$\delta_{sup} \equiv$ [soft- and virtual-γ's, calculable].



notice also that the lineshape is dependent on the type of the fermion (e.g., for $e^+e^-\square\ \nu\bar{\nu}$ no γ in final state).

radiative corrections: parameter Δr

[an example : radiative corrections for $W^\pm$ and Z mass]

- in the SM, m_W and m_Z are related by:

$$m_W^2 \sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}\,G_F}\ ;\ \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2};$$

- radiative corrections modify the formulæ;
- ***define*** the parameters Δr (*radiative correction parameter*), □ (*QED rad. corr.*), Δr_W (*weak rad. corr.*) :

$$m_W^2 \sin^2\theta_W \equiv \frac{\pi\alpha}{\sqrt{2}\,G_F} \times \frac{1}{1-\Delta r} \rightarrow$$

$$\Delta r = 1 - \frac{\pi\alpha}{\sqrt{2}\,G_F} \times \frac{m_Z^2}{m_W^2(m_Z^2 - m_W^2)};$$

$$\frac{1}{1-\Delta r} \equiv \frac{1}{1-\Delta\alpha} \times \frac{1}{1-\Delta r_W};$$

- □ is reabsorbed in $\alpha_{(s)}$, *running coupling constant* [*the* $_{(s)}$ *means "function of* $\sqrt{s}$*"*] :

□ $_{(s)} = (\alpha_{(s)} - \alpha_{(s=0)}) / \alpha_{(s)}$;

- from QED :

□ $_{(m^2_Z)} \approx 0.07 \rightarrow \alpha_{(m^2_Z)} \approx [128.89 \pm 0.09]^{-1}$;

[error from $\int \sigma(e^+e^- \rightarrow$ hadr.) @ $\sqrt{s} << m_Z$]

- the equation with $m_W + m_Z$ becomes :

$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha_{(s=m_Z^2)}}{\sqrt{2}\,G_F} \times \frac{1}{1-\Delta r_W};$$

- [to select top and Higgs terms] expand Δr_W into parts, dependent on m_t ($\propto m_t^2$) and m_H ($\propto \ell n\, m_H$), and the rest ($\Delta\bar{r}_W$) :

$$\Delta r_W = \Delta\bar{r}_W\Big|^{calc.}_{m_t=\hat{m}} + \left.\frac{\partial\Delta r_W}{\partial m_t}\right|_{m_t=\hat{m}} \delta m_t + \frac{\partial\Delta r_W}{\partial m_H}\delta m_H;$$

$$[\hat{m} = 175 \text{ GeV}].$$

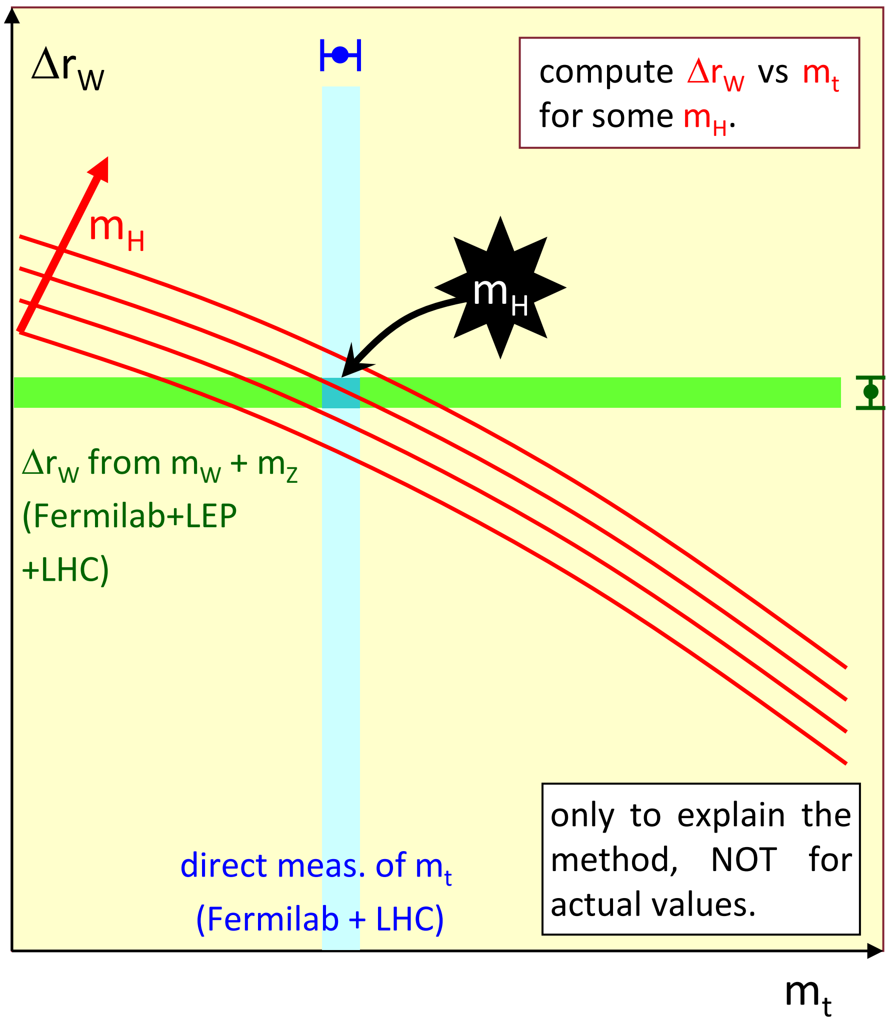
radiative corrections: method → discovery

- assume we are in the "post-top, pre-Higgs" era [i.e. 1995-2011] :
- numerically, the dependence is :

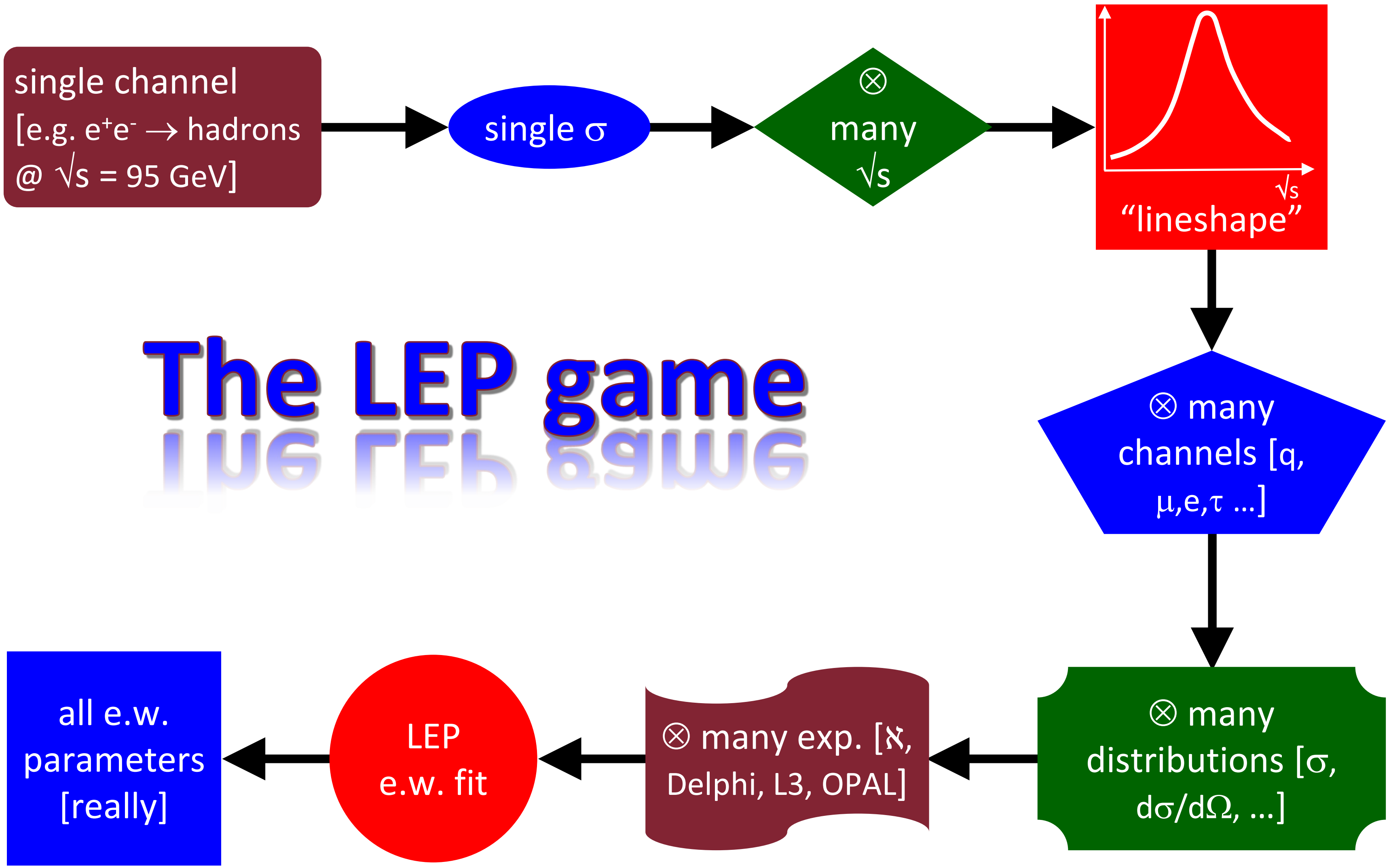
$$\Delta r_W \approx \Delta \bar{r}_W\big|_{calc.} + \\ -0.0019\left(\frac{m_t}{175\,\text{GeV}}\right)\left(\frac{\delta m_t}{5\,\text{GeV}}\right) + \\ +0.0050\left(\frac{\delta m_H}{m_H}\right);$$

[the two terms have <u>opposite sign</u> and <u>very different size</u>]

- <u>the meas. of</u> m_W, m_Z, m_t + the calculation of higher orders of SM allow for a "measurement" of m_H á la Hollik;
- in reality, many observables → global fit.



LEP1 SM fit


single channel
[e.g. e⁺e⁻ → hadrons
@ √s = 95 GeV]
single σ
⊗ many √s
√s
"lineshape"
The LEP game
⊗ many channels [q, μ,e,τ …]
⊗ many distributions [σ, dσ/dΩ, …]
⊗ many exp. [ℵ, Delphi, L3, OPAL]
LEP e.w. fit
all e.w. parameters [really]

LEP1 SM fit: explanation

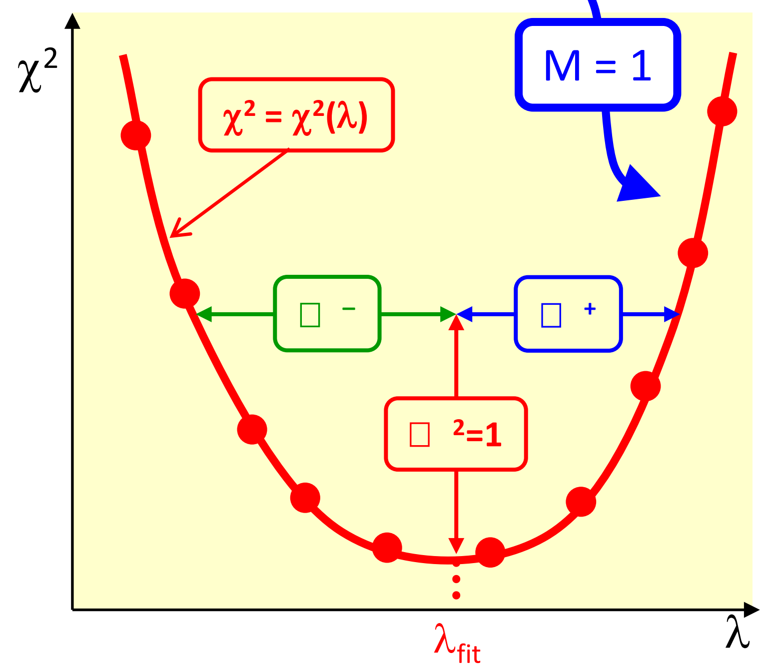
- in the SM, the observables [e.g. σ's, $d\sigma/d\cos\theta$'s, asymmetries, ...] are (functions of few) parameters like m_Z, Γ_Z, Γ_f, θ_W ...;
- in an experiment: **N** observables $\mathbf{t_i}$ (i = 1, ..., N) and **M** SM parameters $\boldsymbol{\lambda_k}$ (k=1,...,M);
- [at LEP 1, N = few×100, M ≤ 10, see later);
- [M is fixed, but the choice is free, e.g. one among m_Z, m_W and θ_W is redundant]
- the dependence of t_i from λ_k is known: $\mathbf{t_i = t_i(\lambda_k) \pm \Delta t_i}$ (Δt_i = the theoretical error);
- the N observables are measured : $\mathbf{m_i \pm \Delta m_i}$ (Δm_i = the convolution of stat. and sys.);
- a (difficult) numerical program computes the "best" λ_k's which <u>**fit**</u> the observations;
- then the <u>same</u> values of λ_k are used for <u>all</u> the computations (shown as the "SM fits").
- [*since N>>M, the dependence of any λ_k on the single i^{th} meas. is very small.*]
- [*also test the agreement SM ↔ data.*]

[simplified example with χ^2 :

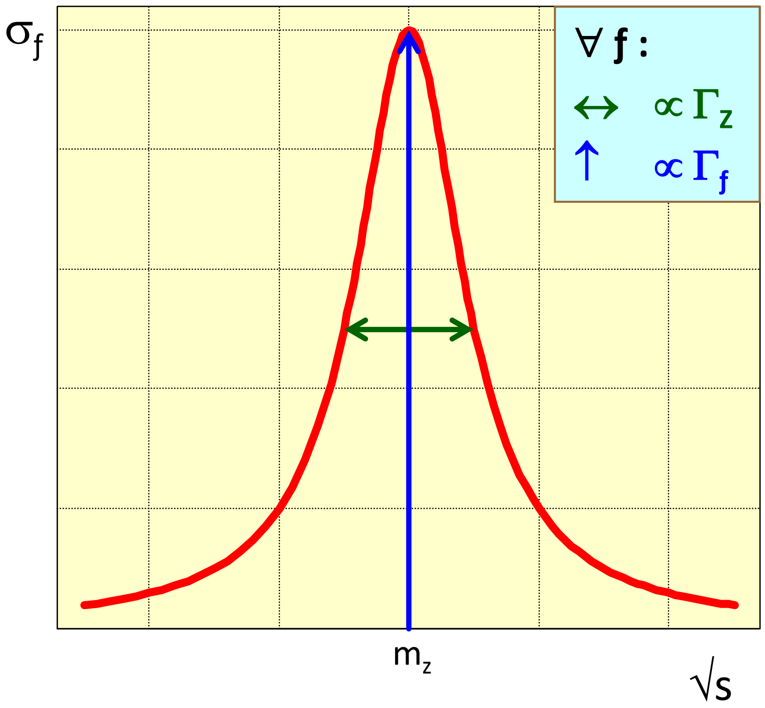
$$\chi^2 = \sum_i \frac{\left[t_i(\lambda_k) - m_i\right]^2}{\Delta t_i^2 + \Delta m_i^2}; \; i = 1,...,N; \; k = 1,...,M;$$

$$\frac{\partial\chi^2}{\partial\lambda_k} = 0 \text{ (M equations)} \xrightarrow{\text{solve the system}} \boxed{\text{all } \lambda_k\text{'s}}$$

+ errors, correlations, ...]



LEP1 SM fit: σ vs Γ



$$\sigma_{Born}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2}.$$

- in LEP jargon, "lineshape" means σ(e+e- $\to$ Z $\to$ $f\bar{f}$) vs $\sqrt{s}$ (*) for a given fermion pair of type f;
- the lineshape shows the characteristic "bell shape", due to the resonance;
- both the height and the width of the bell depend on the e.w. parameters;
- the strategy is
  - a) first, measure mass, full and partial widths of the Z;
  - b) then, fit :
    - number of light ν's (= fermion families);
    - electro-weak couplings.

(*) *warning : NOT "~~dσ/d$\sqrt{s}$~~", which is meaningless.*

LEP1 SM fit: m_Z, Γ_Z

without lepton universality		correlations								
$\chi^2/N_{df} = 32.6/27$		m_Z	Γ_Z	σ^0_h	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{FB}$	$A^{0,\mu}_{FB}$	$A^{0,\tau}_{FB}$
m_Z [GeV]	91.1876 ± 0.0021	1.00								
Γ_Z [GeV]	2.4952 ± 0.0023	−.024	1.00							
σ^0_h [nb]	41.541 ± 0.037	−.044	−.297	1.00						
R^0_e	20.804 ± 0.050	.078	−.011	.105	1.00					
R^0_μ	20.785 ± 0.033	.000	.008	.131	.069	1.00				
R^0_τ	20.764 ± 0.045	.002	.006	.092	.046	.069	1.00			
$A^{0,e}_{FB}$	0.0145 ± 0.0025	−.014	.007	.001	−.371	.001	.003	1.00		
$A^{0,\mu}_{FB}$	0.0169 ± 0.0013	.046	.002	.003	.020	.012	.001	−.024	1.00	
$A^{0,\tau}_{FB}$	0.0188 ± 0.0017	.035	.001	.002	.013	−.003	.009	−.020	.046	1.00

for updated values, check [pdg]

with lepton universality						
$\chi^2/N_{df} = 36.5/31$		m_Z	Γ_Z	σ^0_h	$\boxed{R^0_\ell}$	$A^{0,\ell}_{FB}$
m_Z [GeV]	91.1875 ± 0.0021	1.00				
Γ_Z [GeV]	2.4952 ± 0.0023	−.023	1.00			
σ^0_h [nb]	41.540 ± 0.037	−.045	−.297	1.00		
R^0_ℓ	20.767 ± 0.025	.033	.004	.183	1.00	
$A^{0,\ell}_{FB}$	0.0171 ± 0.0010	.055	.003	.006	−.056	1.00

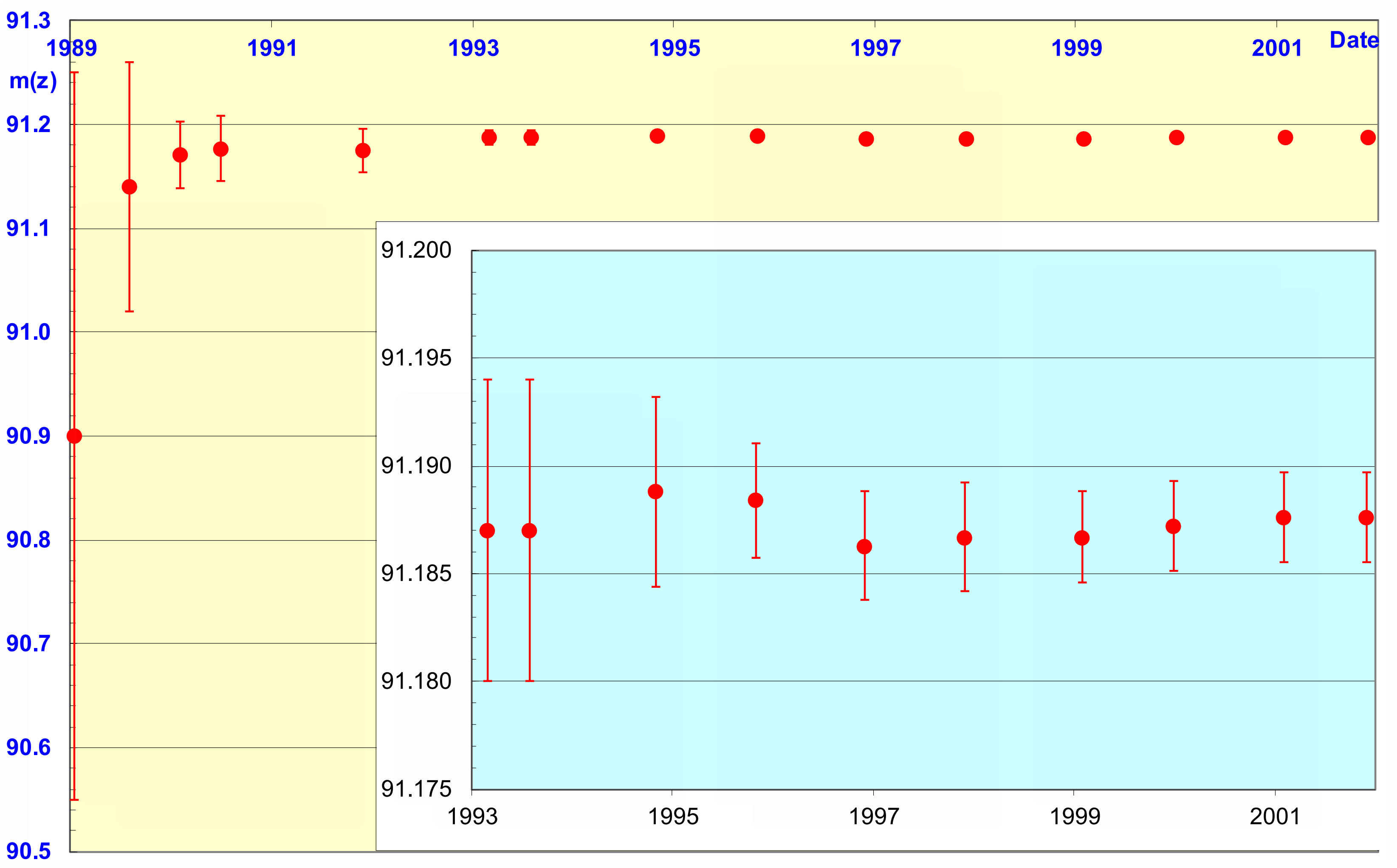
$R_x \equiv \Gamma_{hadr} / \Gamma_x = \sigma_{hadr} / \sigma_x$;
all values computed at the pole.

two fits :

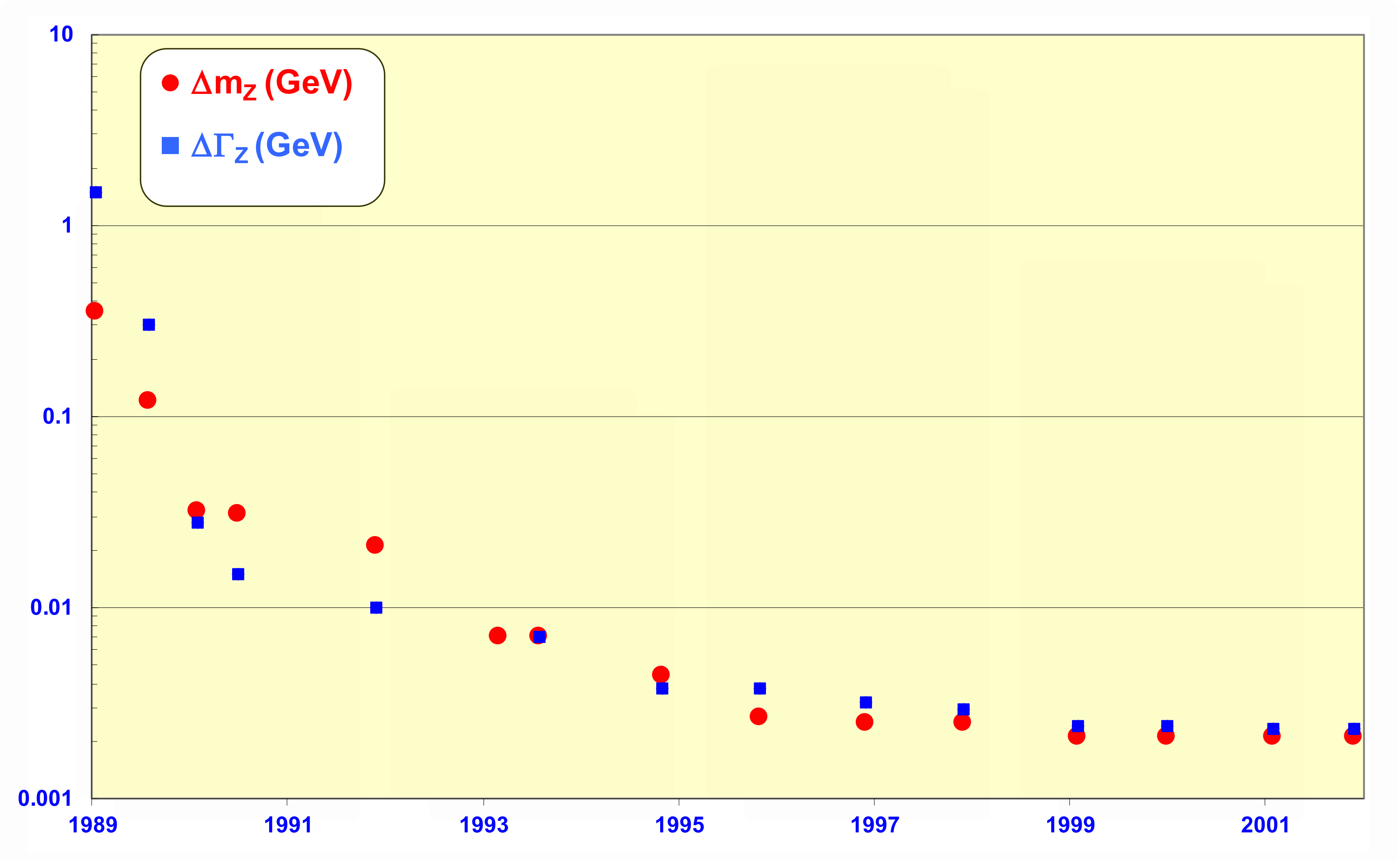
a) "without lepton universality", 9 parameters : larger errors, more general;

b) "with l. u.", 5 parameters, smaller errors, assume lepton universality.

LEP1 SM fit : m_z



LEP1 SM fit: Δm_Z, □ $_Z$



LEP1 SM fit: n_ν

- Neutrinos are the lightest component of the fermion families [in SM no theor. explanation, just matter of fact];
- assuming this case also for (hypothetical) further families, i.e. additional ν's lightest member of a family;
- the decay $Z \to \square^-$ is important (~20%), but not observable (but "single γ", not treated here);
- but it contributes to Γ_z (observable);
- indirect detection: measure Γ_z, subtract the contribution of observable decays ("Γ_{visible}"), get "$\Gamma_{\text{invisible}}$" and compute n_ν (more precisely the number of light ν, i.e. $m_\nu < m_z/2$) :

$$\Gamma_{\text{inv}} \equiv \Gamma_z - \sum_{j=q,\ell^\pm} \Gamma_j = \Gamma_z - \Gamma_{\text{hadr}} - 3\Gamma_{\ell^\pm};$$

$$n_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_\nu^{\text{SM}}} = \left(\frac{\Gamma_{\text{inv}}^{\text{exp}}}{\Gamma_Z^{\text{exp}}}\right)\left(\frac{\Gamma_z^{\text{SM}}}{\Gamma_\nu^{\text{SM}}}\right).$$

- [the last step to decrease stat and syst errors]
- it turns out :

$$n_\nu = 2.9840 \pm 0.0082$$

i.e. $n_\nu = 3$, no other families

[*probably the best, most known, most quoted LEP result, see* *fig on pag. 2*].

NB strictly speaking, n_ν *= width of invisible decays normalized to* Γ_ν*; i.e. it could get contributions from other invisible decays (physics bSM, e.g. neutralino); in such cases,* ***"n_ν" not an integer***.

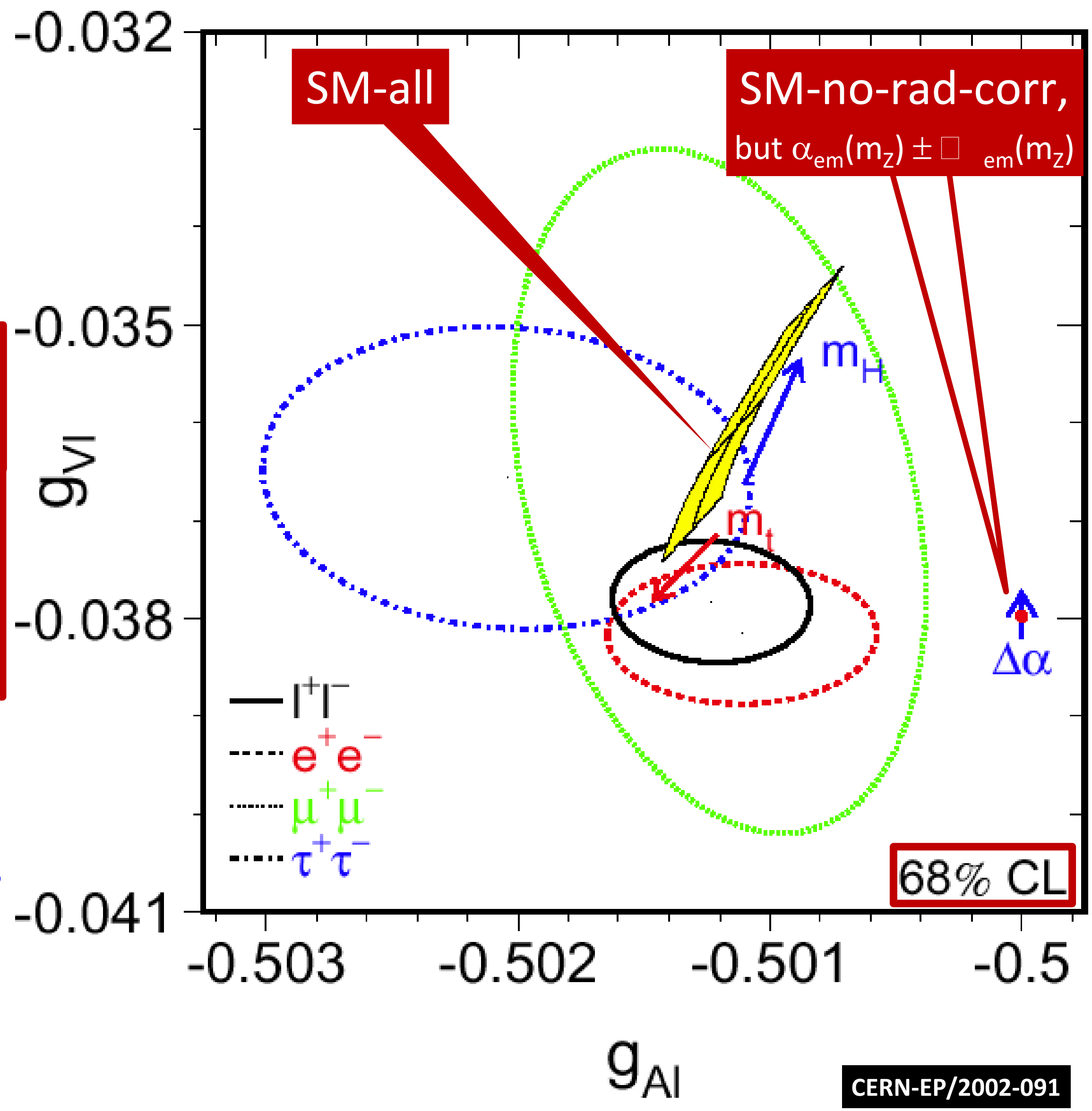
$$\sigma_{\text{Born}}(e^+e^- \to f\bar{f}, \sqrt{s} = m_Z) = \frac{12\pi\Gamma_e\Gamma_f}{m_Z^2\Gamma_Z^2};$$

$$\Gamma_\nu^{\text{SM}} = \frac{G_F m_Z^3 c_f}{12\sqrt{2}\pi}; \qquad \Gamma_z = \sum_i \Gamma_i.$$

LEP1 SM fit : g_A vs g_V for leptons

Example of global fit result : g_A vs g_V for leptons :

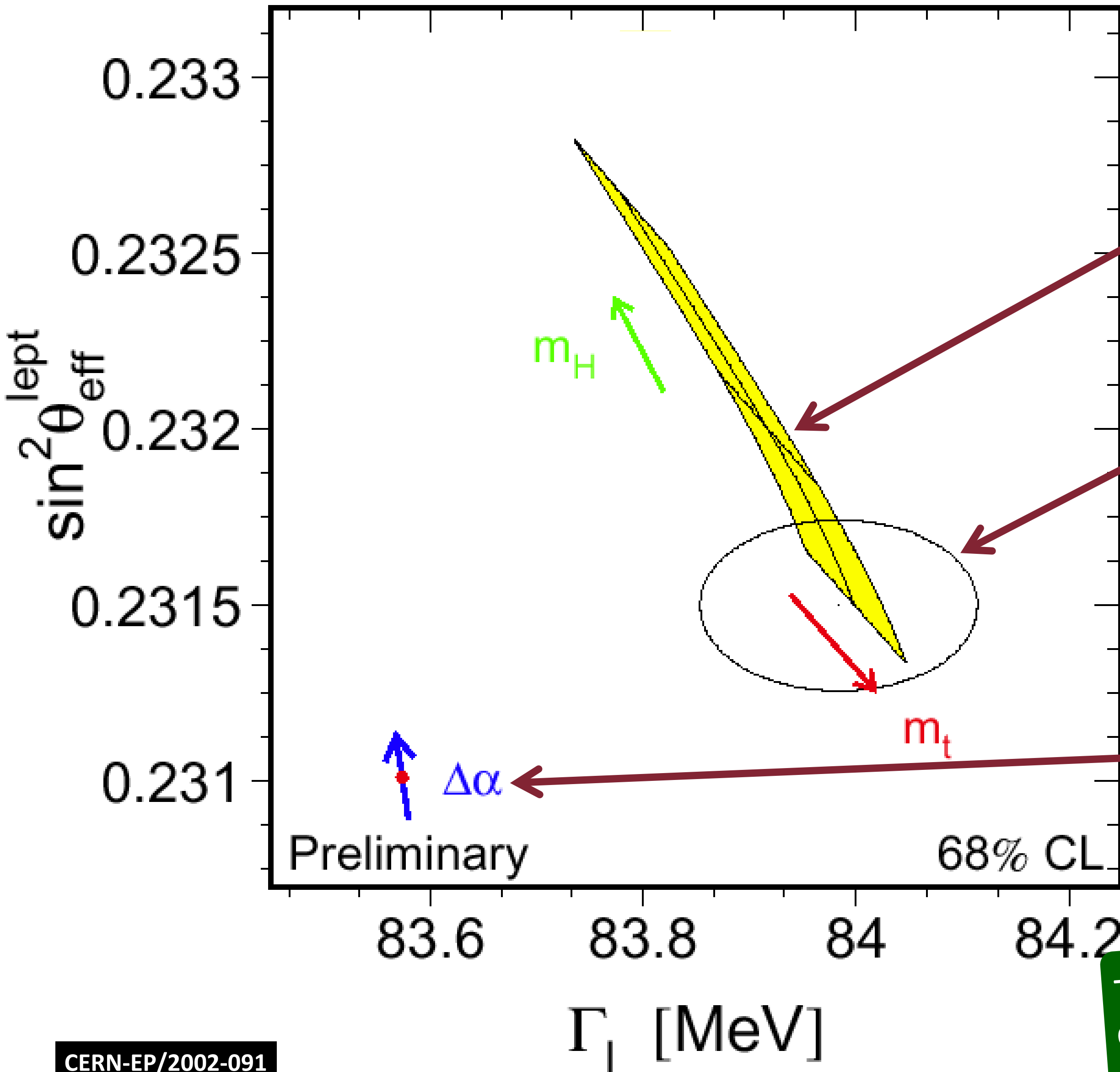
- 68% (i.e. 1 σ) contours;
- computed after top and before Higgs discovery;
- the "$\rightarrow$" shows $\pm$ 1 σ in α_{em}, m_t...
- ... and 114, 300, 1000 GeV for m_H.
- the **red** dot shows the SM Born point, with the QED corr. only (i.e. $\alpha_{em}(m_Z) \approx 1/128$ $\rightarrow$ **weak rad. corr.** are important.



Notice :

- good compatibility among leptons ($\rightarrow$ universality);
- preference for light Higgs (...wow)

LEP1 SM fit : $\sin^2\theta$ vs Γ_ℓ



SM-with-rad-corr:
m_H = 114, 300, 1000 GeV;
m_t = 174.3 ± 5.1 GeV.

meas. (68%)

SM-no-rad-corr,
but $\alpha_{em}(m_Z) \pm \Delta\alpha_{em}(m_Z)$

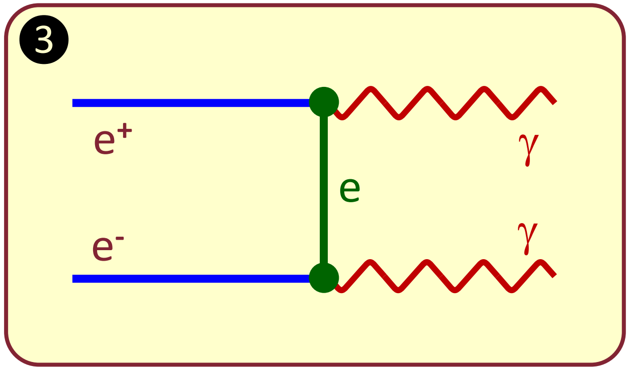
CERN-EP/2002-091

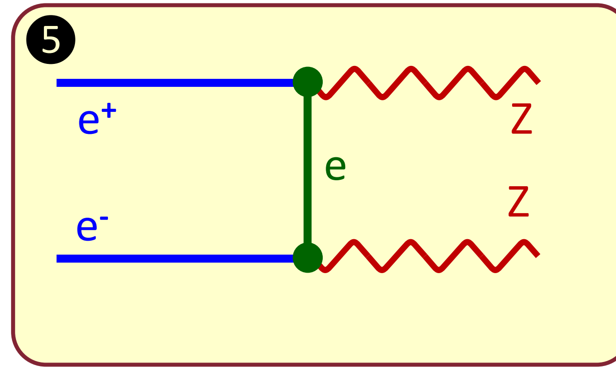
+ tons of similar plots, with all the e.w. parameters vs all others, compared with SM → agreement.

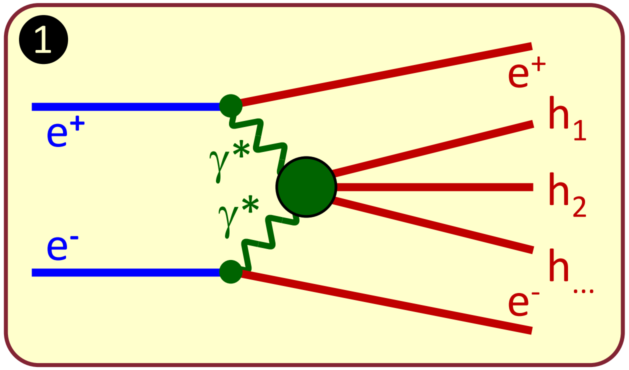
Physics at LEP2

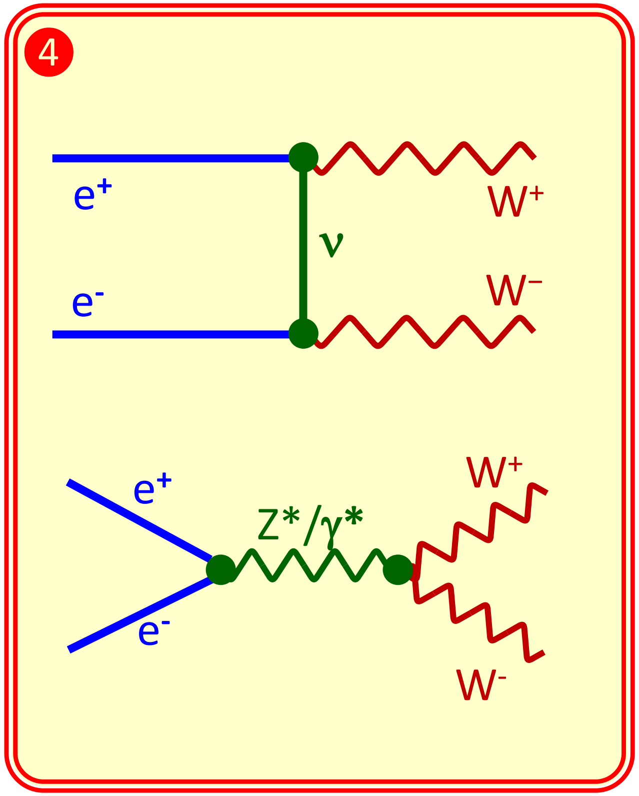
A "1st order" dictionary of some processes:

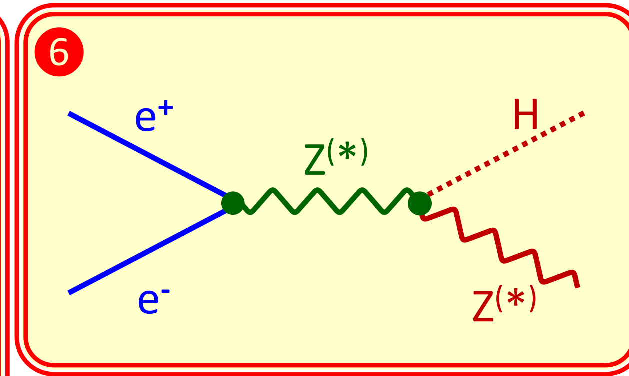
- processes ❷,❸ have been already studied;
- ❶, ❹, ❺, ❻ will be introduced soon.

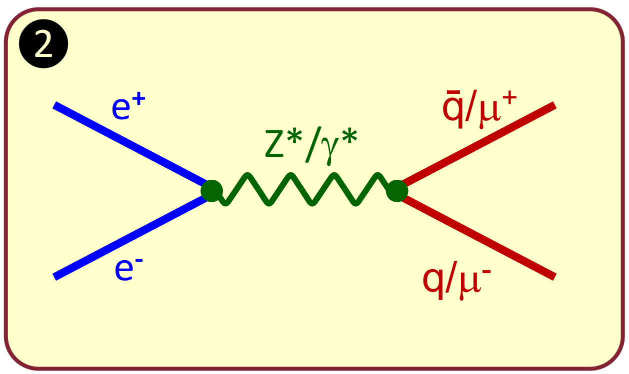








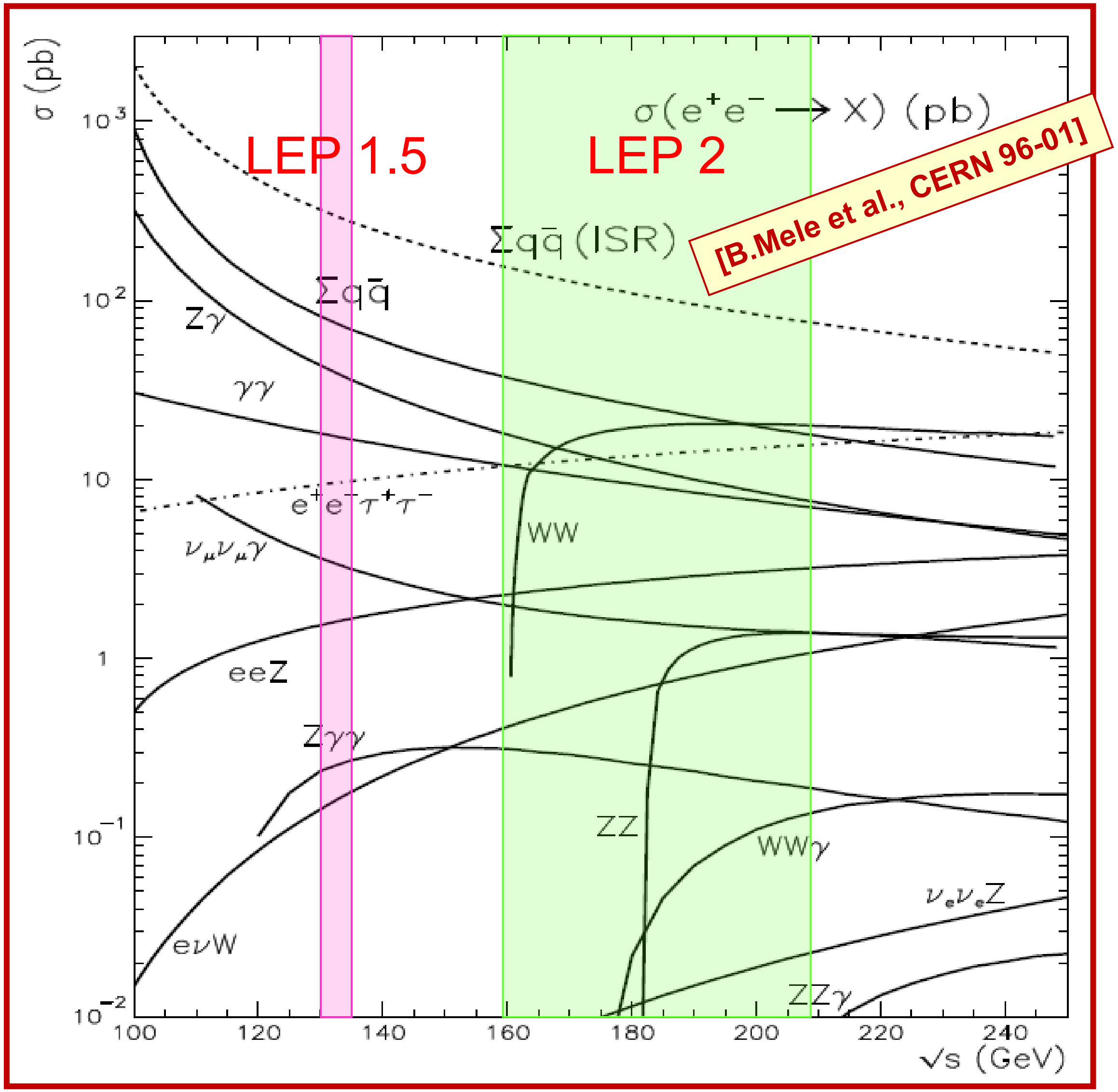




❷ is a "2-fermion final state";

❹, ❺, ❻ are "4-fermion f.s.", because $W/Z/H \to f\bar{f}$.

Physics at LEP2: expectations



In 1994-2000 LEP gradually changed $\sqrt{s}$: $m_Z \rightarrow$ 200 GeV:

- σ's vs √s, produced (BarbaraM. et al.) before the start of LEP2;
- notice:
  - the main processes were all well-known before the startup;
  - no "surprise" happened;
  - "$\Sigma q\bar{q}$" → "$\Sigma q\bar{q}$(ISR)" → cut on s'/s in the analysis;
  - [*surprisingly*] in 1996 they did NOT put the Higgs production in the plot;
- the color bands show the √s range actually used by LEP2;
- *why ?* [**physics** + availability of radio-frequencies].

Physics at LEP2: comments

Some important characteristics of the LEP2 physics:

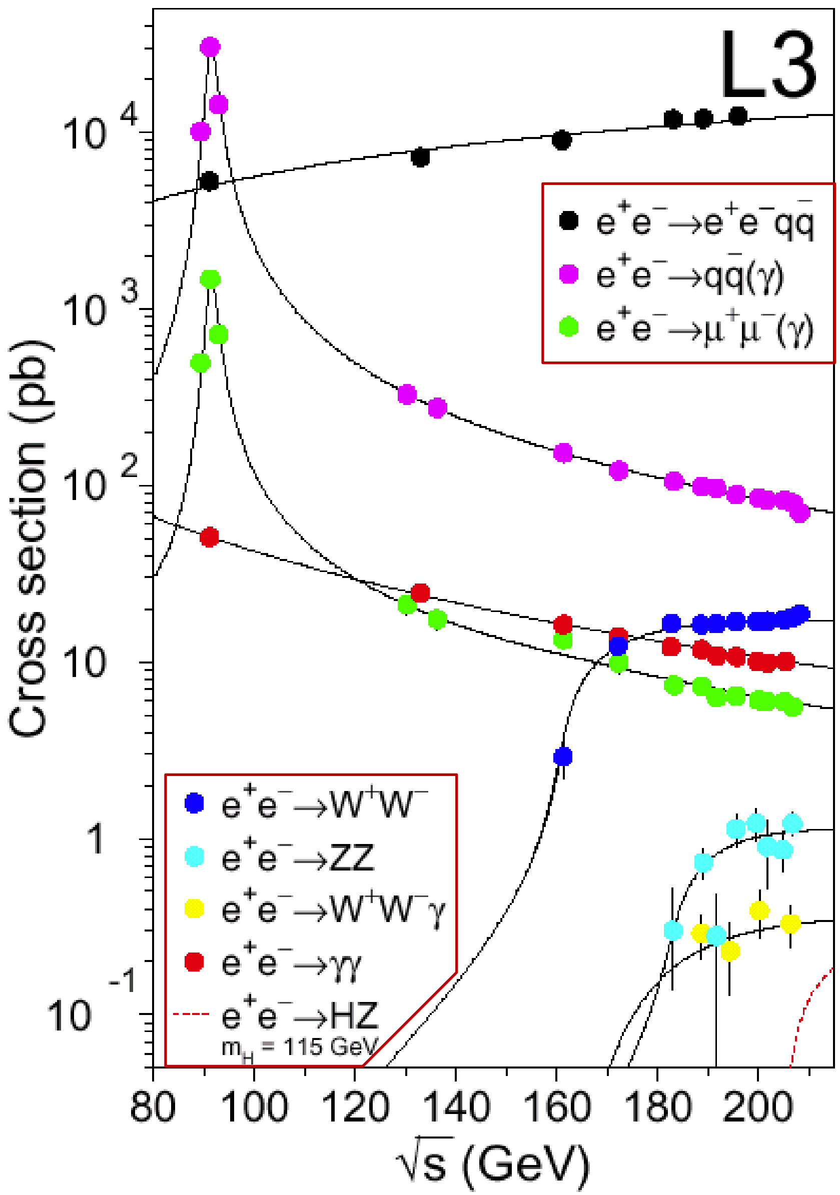
- larger luminosity (× 4, because [$\mathscr{L} \propto \gamma_{beam} \propto \sqrt{s}$] + [machine improvements]);
- much smaller cross sections (× 10^{-3}, because [no Z resonance] + [$\sigma_{ee} \propto 1/s$]) $\rightarrow$ few events;
- as a consequence, no "production factory" of interesting states, studied independently of the production (ex. b / c / τ a LEP 1); exception: $W^{\pm}$;
- errors dominated by the $1 / \sqrt{N}$ statistics; error on $\mathscr{L}$(uminosity) less important;
- no equivalent to m_Z measurement, so no E_{beam} calibration at MeV level necessary;
- not dominated by single Z formation, so many competing processes;

what is that ? (guess .....)

- offline computing dominated by the production of mc events (mostly production of background processes);
- physics interest (NOT event number) mainly in two channels:
  - in the first years, $e^+e^- \rightarrow W^+W^-$;
  - in the last years, $e^+e^- \rightarrow HX$ (search for, actually a limit on m_H).

Our real hero

Physics at LEP2: σ's vs √s



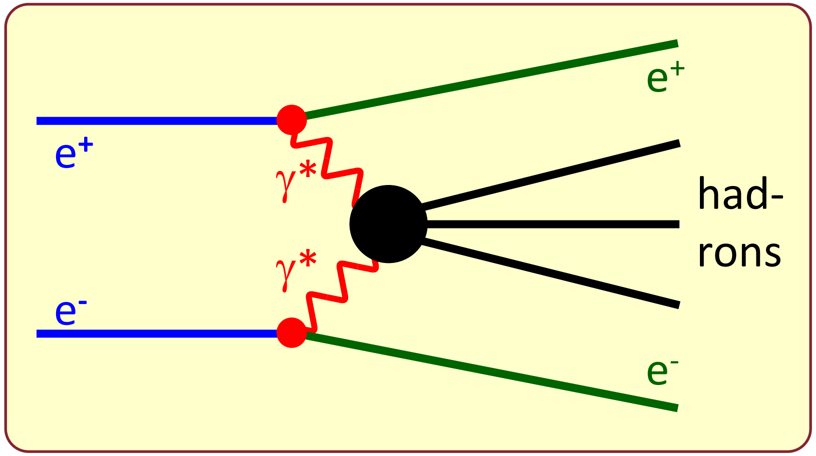
This plot is a summary of the results. Notice:

- LEP1 was dominated by the Z pole;
- on the contrary, LEP2 is "democratic";
- many final states :
  - "2 photons", e.g. $e^+e^- \to e^+e^- q\bar{q}$;
  - "2 fermions"[1], e.g. $e^+e^- \to Z^*/\gamma^* \to q\bar{q}$;
  - "4 fermions", e.g. $e^+e^- \to W^+W^- \to q\bar{q}\ q\bar{q}$;
  - $e^+e^- \to$ □ ;
  - Higgs searches (special case of 4 fermions).
- only W^+W^- and Higgs in these lectures.

---

[1] "2 fermions" physics is dominated by the *return to the Z* effect (see § radiative-corrections").

$e^+e^- \to Z \to f\bar{f}$: 2 γ physics



Introduce the process: "2 γ physics":

- it is so called because the initial state of the hard collision is given by two γ's;
- the two $e^\pm$ of the initial state retain much of the energy, and in most cases escape undetected in the beam chamber;
- classify events in "untagged", "single tag" and "double tag", depending on whether 0, 1, 2 and $e^\pm$ are detected;
- lot of nice kinematics [*try it*];

- events studied using two variables:
  - $\sqrt{s} = m_{ini}(e^+e^-)$;
  - $W = m(\gamma^*\gamma^*) = m(\text{hadrons})$;
- both prediction and detection require a cut (W_{cut}, here W_{cut} = 5 GeV) on W, i.e. define $\sigma_{\square} = \sigma_{\square}(W > W_{cut})$:
  - $\sigma_{\square} \sim \log(\sqrt{s})$ for fixed W_{cut} (~ constant);
  - $d\sigma_{\square} / dW \sim e^{-W}$ [very steep].

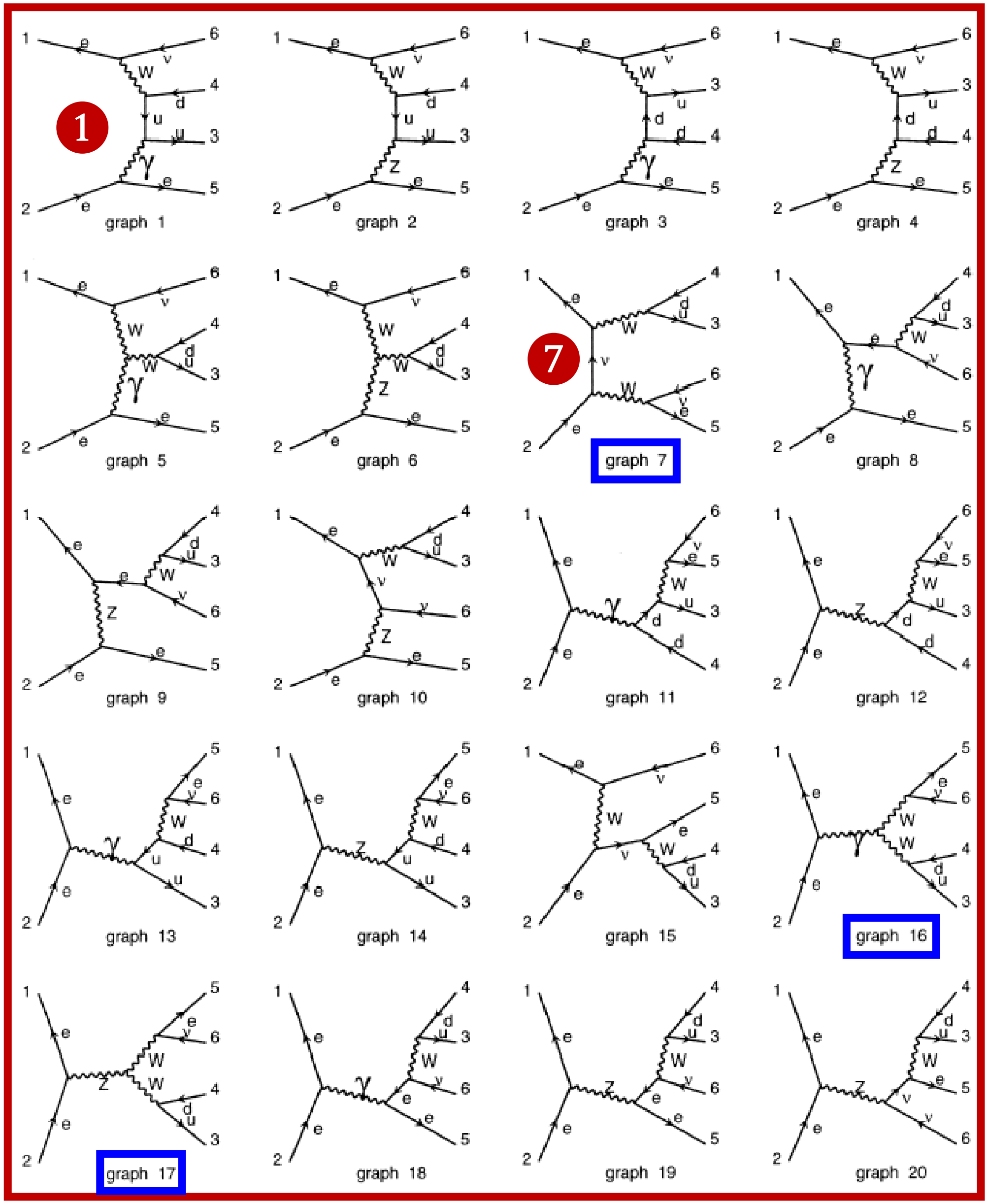
Why study "2 γ physics" ? Two main goals:

1. *intrinsic interest:*
   - *any process deserves a study;*
   - *rich "factory" of hadron resonances;*
   - *other low-energy processes;*
2. $\sigma_{\square}$ *is large:*
   - *LEP1: subtract from high precision meas.;*
   - *LEP2: other processes typically tiny σ's → an important background, especially if large $\not{E}$ required (this is why the discussion is here).*

Physics at LEP2: mc for 4-fermion processes

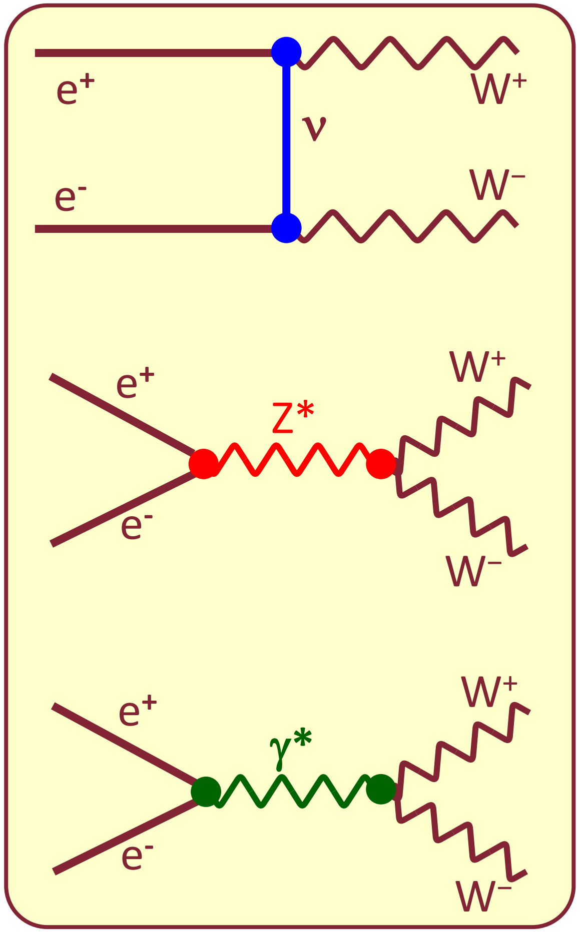
Four-fermion final states

- the process $e^+e^- \to f\bar{f}f\bar{f}$ is given in lowest order by the q.m. superposition of many diagrams with intermediate particles (+ interference) (*e.g. the final state* [e ν q $\bar{q}'$] *with 20 graphs, see box*);
- in q.m. it is impossible to assign a given ($e^-\bar{\nu}u\bar{d}$) event to the resonant production of two W's (e.g. ❼) or to a diagram without real W's (e.g. ❶);
- however, diagrams with s-channel W's, when $m(f\bar{f}) \approx m_W$, resonate and prevail;
- the mc calculations are divided between (a) no factorization, i.e. the full q.m. behavior and (b) factorization, i.e. only resonant diagrams;
- [*as predictable*] method (a) is heavy, slow and difficult to manage, while (b) is simpler and almost correct.

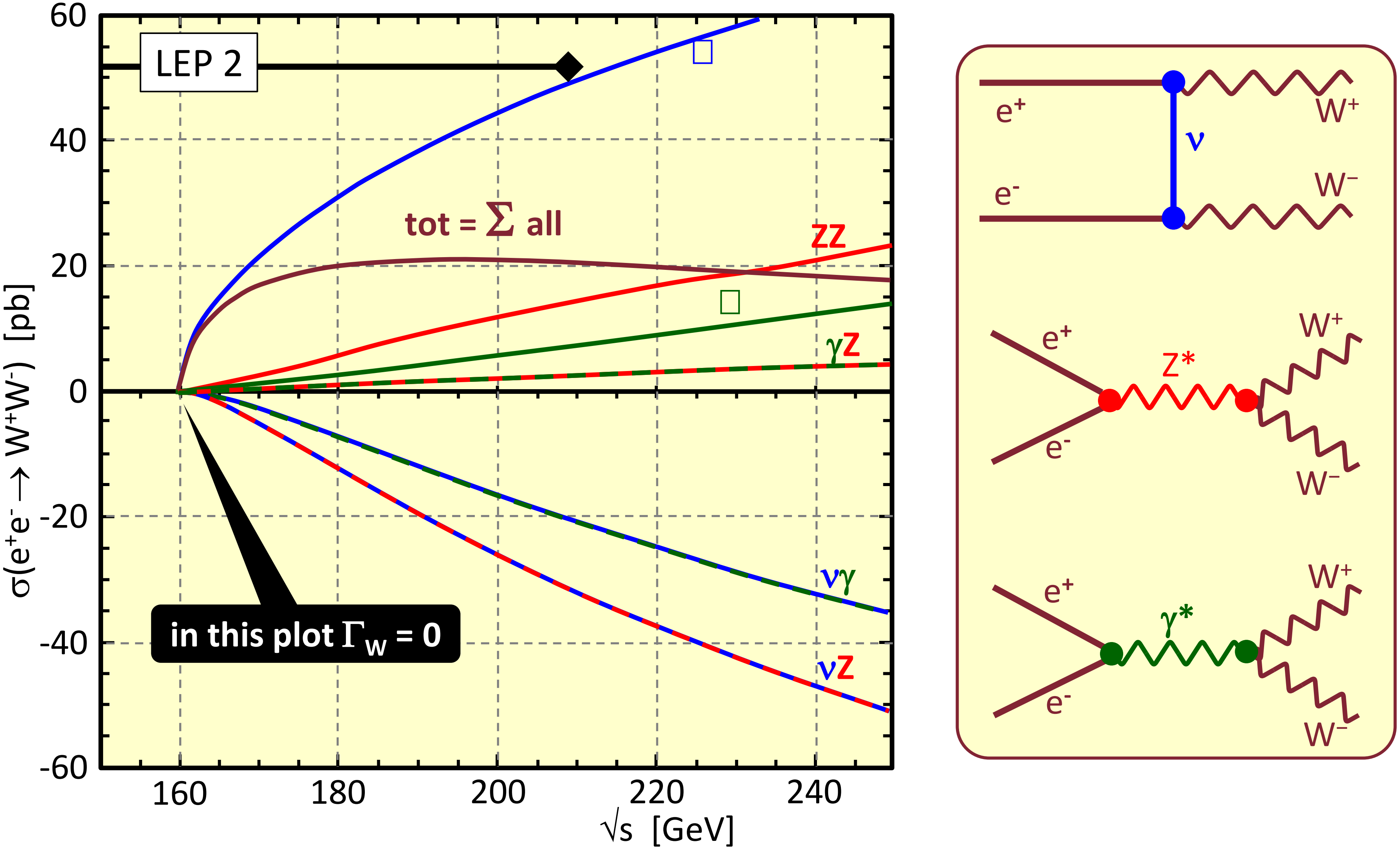


$e^+e^- \to W^+W^-$

- the process $e^+e^- \to W^+W^- \to f\bar{f}f\bar{f}$ dominates the 4 fermions sample;
- in lowest order, there are three Feynman diagrams;
- all the vertices of the e.w. theory: ffW, ffZ, ffγ, ZWW, γWW;
- the overall (finite) cross section results from delicate cancellations among the 6 terms (3 $|\text{module}|^2$ + 3 interferences) [*next slide*];
- therefore, almost any possible discrepancy wrt SM, (e.g. an anomaly in the couplings) would result in evident deviations from the predictions.

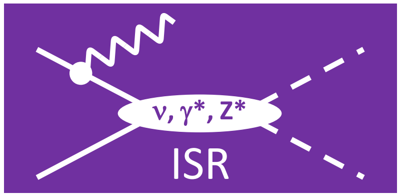


$e^+e^- \to W^+W^-$: cross section in SM



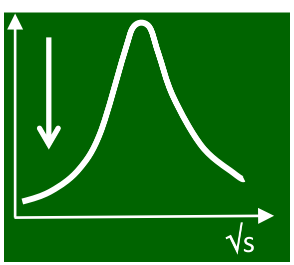
$e^+e^- \to W^+W^-$: effect of Γ_W + ISR on σ

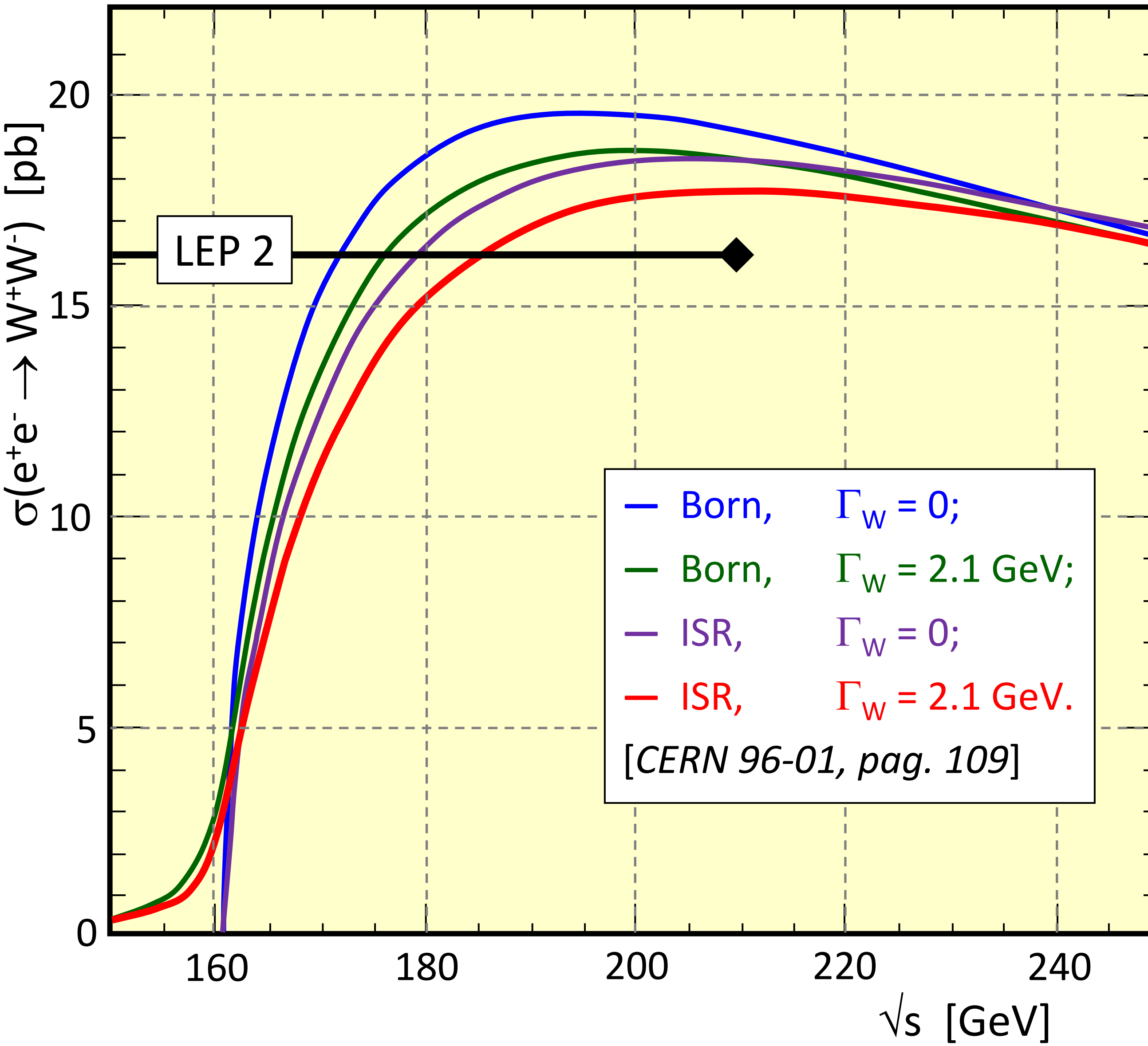
$\sigma(e^+e^- \to W^+W^-)$ vs $\sqrt{s}$



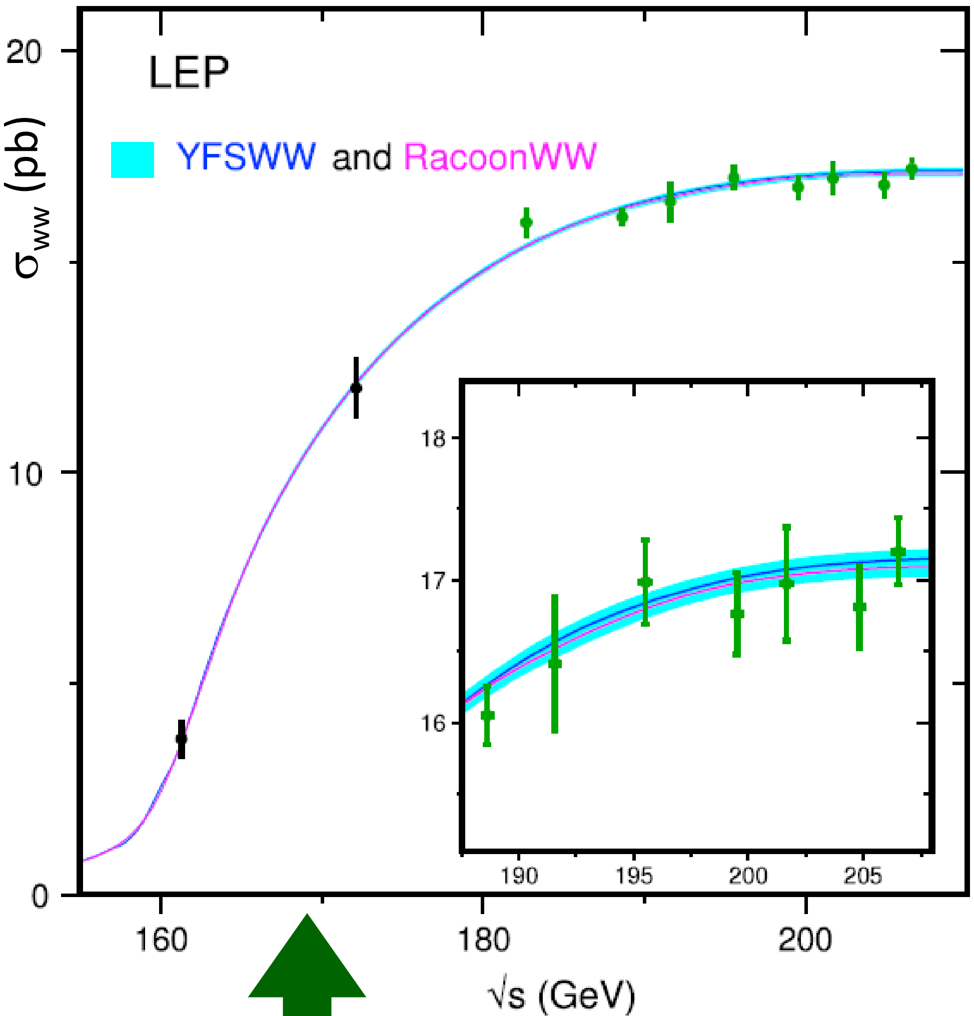
Notice :

- kin. threshold at $\sqrt{s} = 2\ m_W$;
- Γ_W (+ production of virtual W's);
- ISR (i.e. init. state γ's).





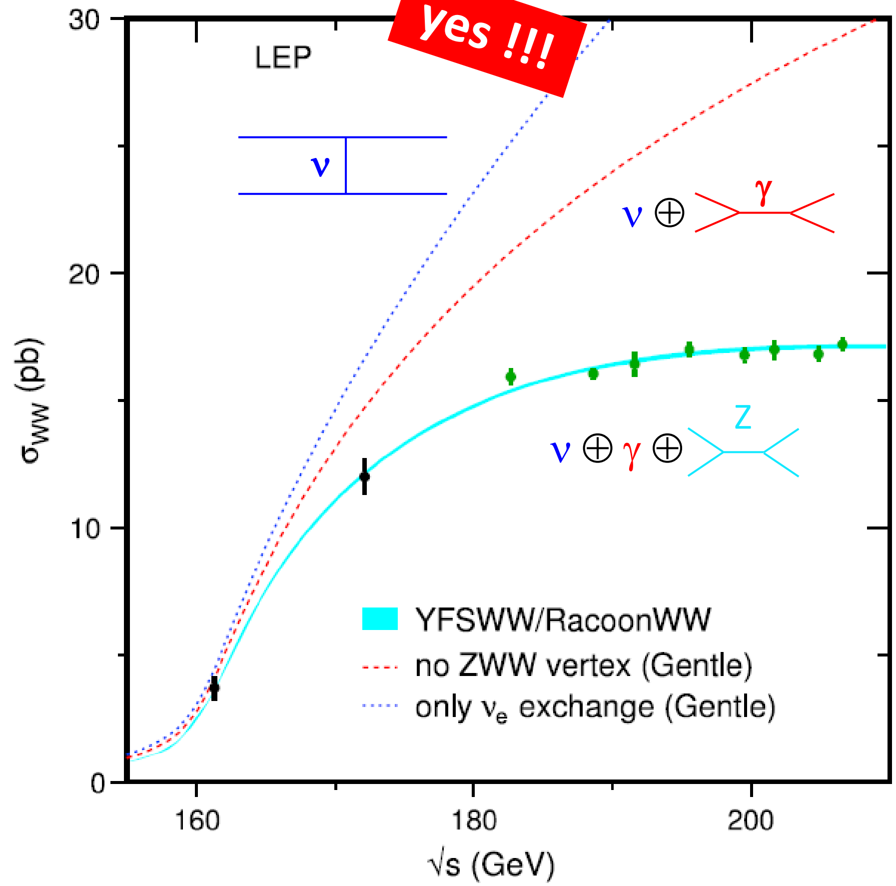
$e^+e^- \to W^+W^-$: cross section vs $\sqrt{s}$



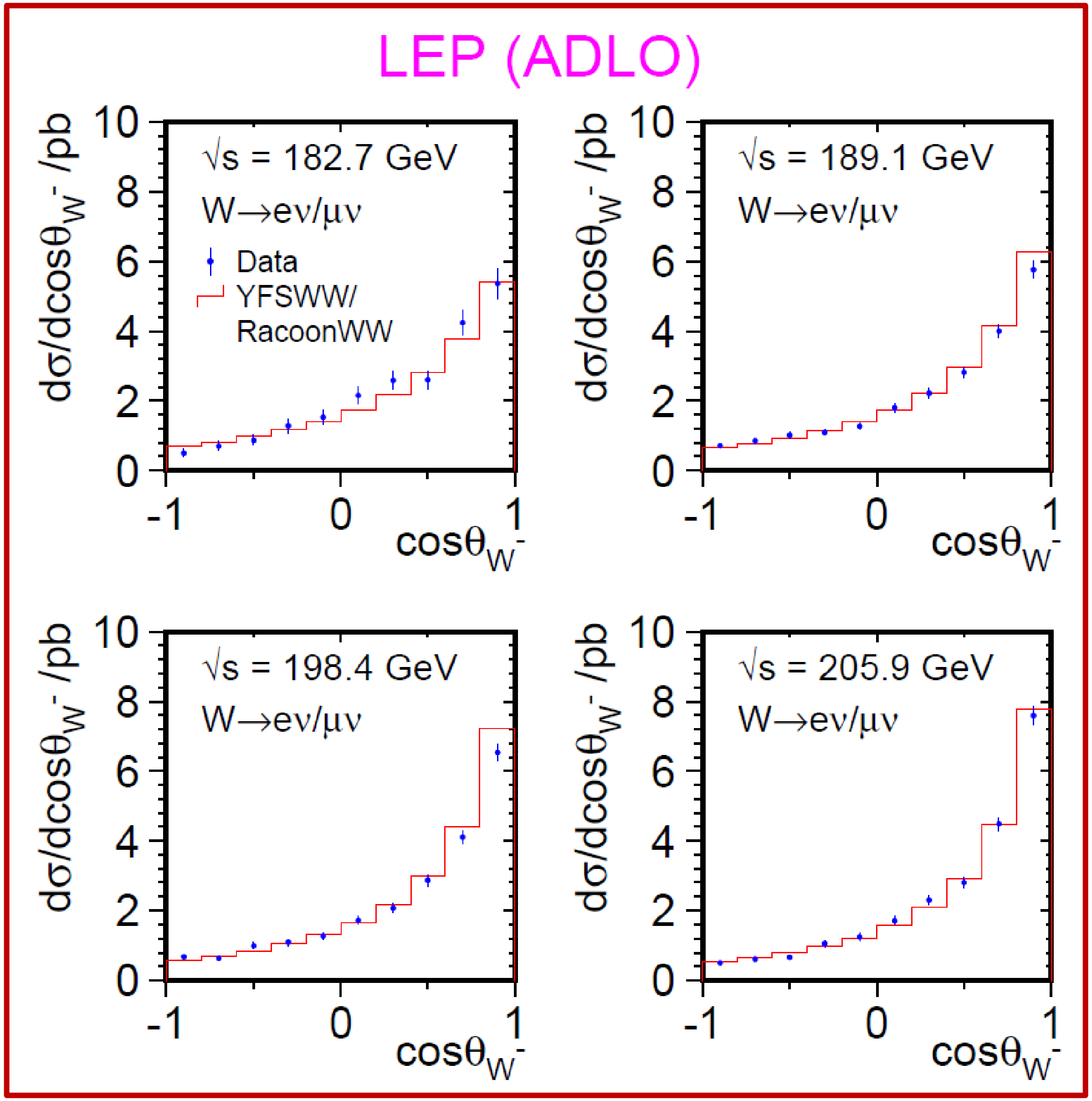
all data compared with best SM MC

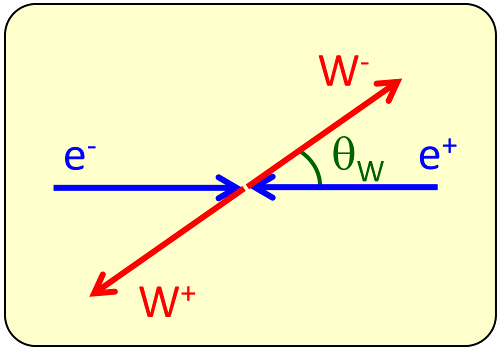
same data, similar question: are all SM couplings needed ?

yes !!!



$e^+e^- \to W^+W^-$: $d\sigma/d\cos\theta_W$ vs $\sqrt{s}$





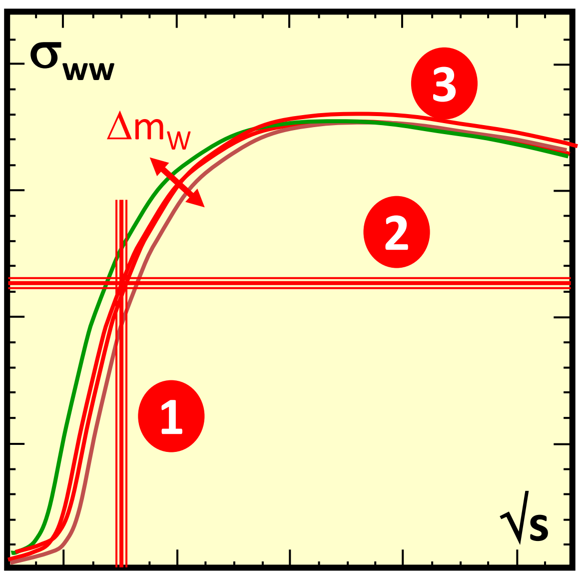
$d\sigma/d\cos\theta_W$ ($\sqrt{s}$) is forward-peaked (θ=0, $\cos\theta_W$=1), because of dominance of t-channel ν-exchange.

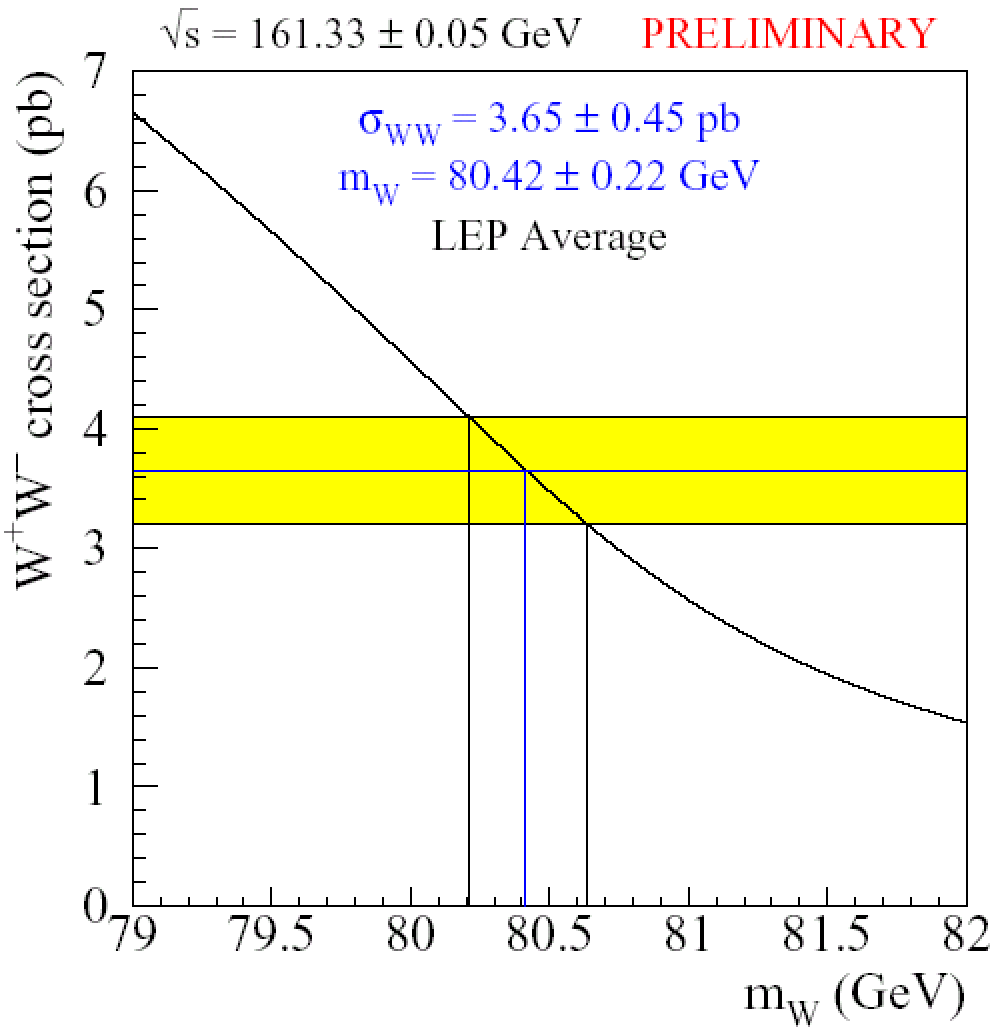
- data + SM MC ("best");
- W charge known if at least one lepton decay;
- well-known effect [*see CERN 96-01, pag. 94*];
- plot from Phys.Rep. 532 (2013), 173.

$W^{\pm}$ properties: W mass from σ

Technically clever and simple :

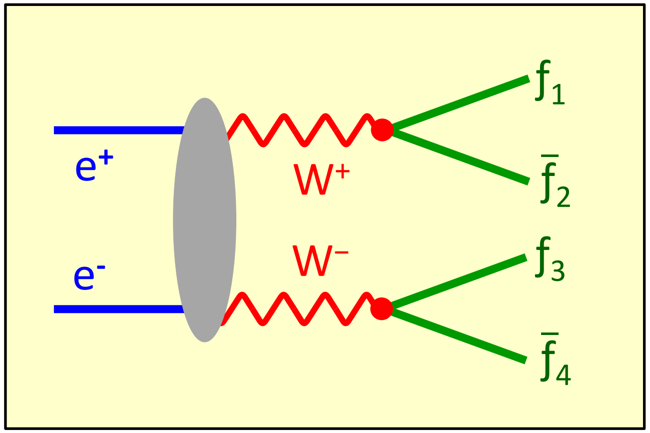
- compute $\sigma(e^+e^- \to W^+W^-) = \sigma(m_W)$;
- compute the "best" $\sqrt{s}$, by combining
  - sensitivity (□ $/\partial m_W$ = max) $\to \sqrt{s} \approx$ threshold;
  - (□ $_{stat}\downarrow$) $\to$ ($\sigma\uparrow$) $\to$ ($\sqrt{s}\uparrow$);
  - take into account Δ_{theory} and syst.;
- **measure**.





$W^{\pm}$ properties: constraints

- selection of WW events NOT difficult: little competition in 4-body final states (mainly $e^+e^- \to q\bar{q}gg$, with 2 QCD brem);
- kinematical constraints (e.g. 4-mom conservation) help in the analysis :
  - selection criterion (rejection of bad measurements or event from other processes);
  - resolution improvement [*see next*];
- discuss an example : likelihood fit to m_W, Γ_W;
- compare analysis/fit on real data wrt same procedure on "pseudo-events" (physics + detector mc);
- Γ_W strongly (anti-)correlated with experimental resolution ["pessimistic" detector mc $\to$ resolution too large $\to$ deconvolution $\to$ Γ_W too small !!!];
- systematics from:
  - ISR/FSR parameterization;
  - reconstruction algorithms (expecially jets, ex. color reconnection, Bose-Einstein correlations);
  - many other sources…
- consistency checks : in this case m_Z , Γ_Z from $e^+e^- \to ZZ$ (with smaller stat).

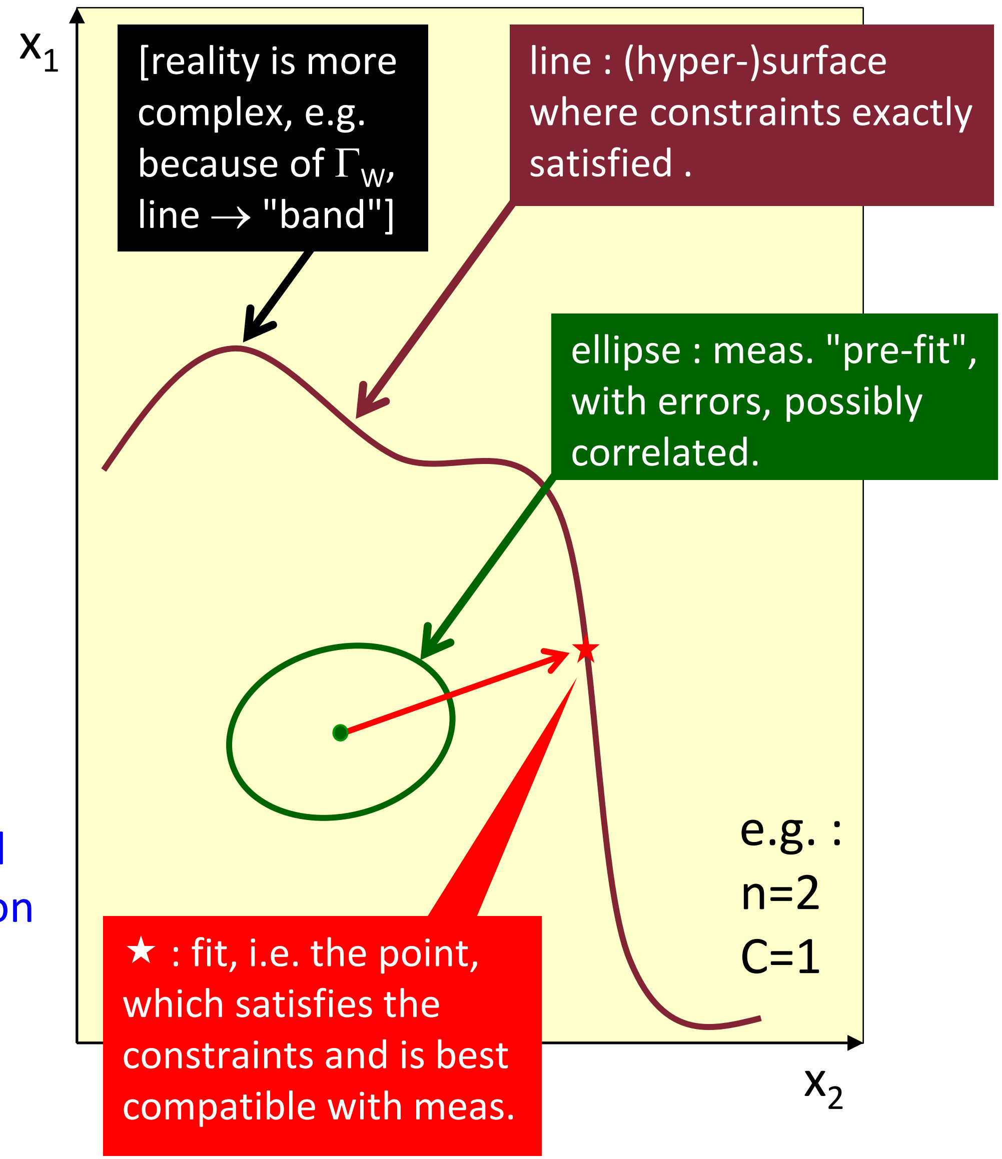


$W^{\pm}$ properties: mass fit

Energy-momentum conservation:

- n parameters = 4 * n_{body} = 16;
- N meas. [e.g. E, $\vec{p}$ for jets / $\ell^{\pm}$'s];
- K equations [= 4 mom + masses$^{(*)}$];
- C (=N+K-n) constraints;
- E.g. : $e^+e^- \to W^+W^- \to f_1 f_2 f_3 f_4$:
  - $q_1\bar{q}_2q_3\bar{q}_4$: N=16, K=4+1 → **C = 5**;
  - $\ell^{\pm}\nu q_1\bar{q}_2$: N=12, K=4+2 → **C = 2**;
  - $\ell^+\nu\ell^-\bar{\nu}$: N=8, K=4+? → **C ≤ 0**;
- If C > 0, a kinematical fit is possible (a simplified sketch in x_1, x_2, n=2, C=1)

[the red arrow "→" represents a statistical estimate (χ^2, likelihood) and a computation method (e.g. Lagrange multipliers)].

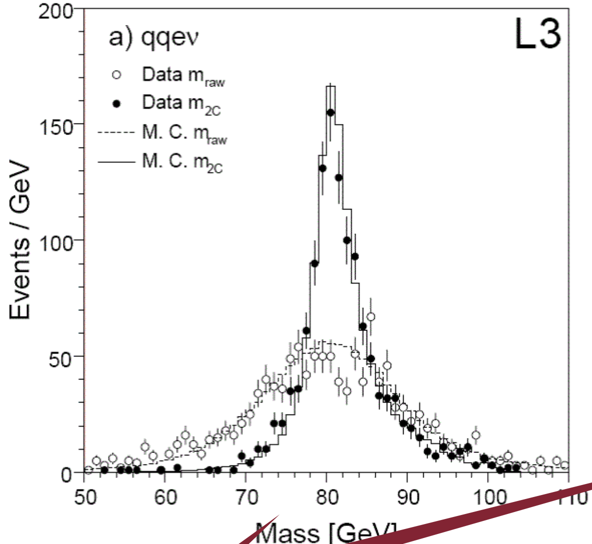


____

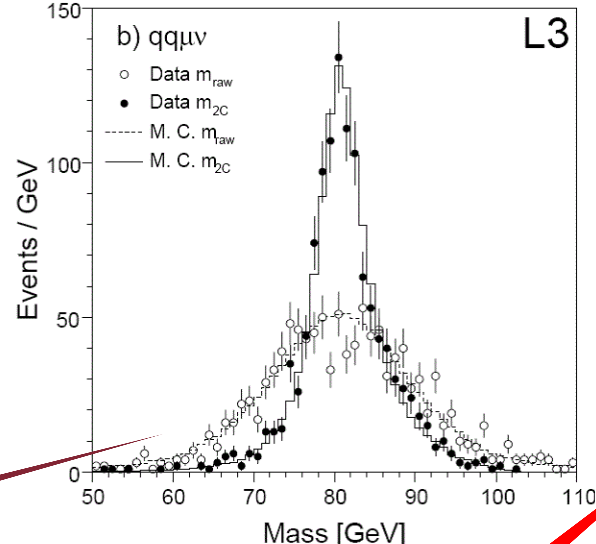
$^{(*)}$ $m_{W+} = m_{W-}$ and $m_\nu \approx 0$.

$W^{\pm}$ properties: mass plots

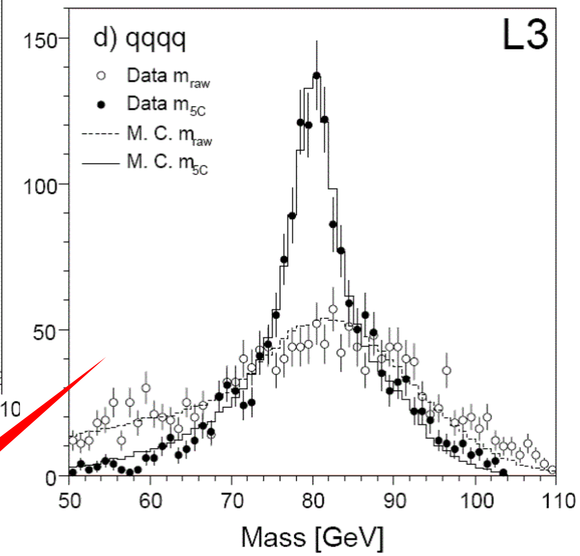
$e^+e^- \to W^+W^- \to q\bar{q}e\nu$



$e^+e^- \to W^+W^- \to q\bar{q}\mu\nu$

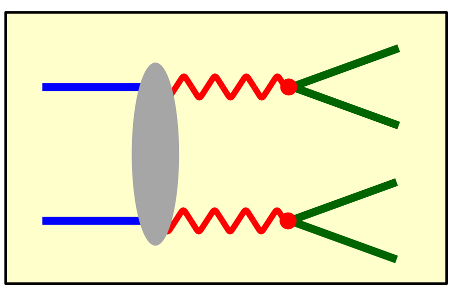


$e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$



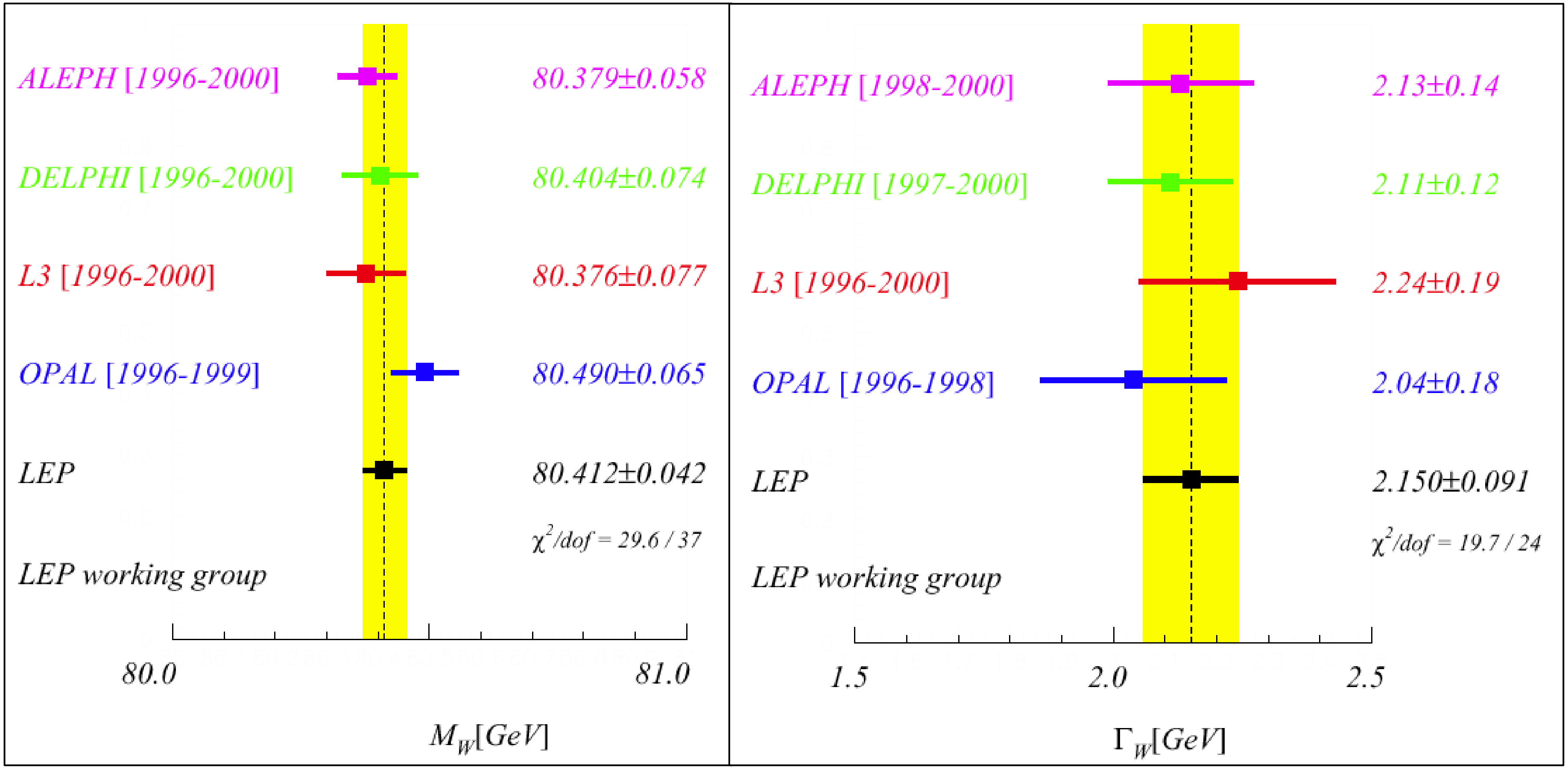
2C

5C



- the effects of kinematical fits :
- "C" (=constraints) from bubble chamber jargon;
- higher C, more constraints, more improvement;
- **"measurement" = m_W, Γ_W in MC with best agreement.**

$W^{\pm}$ properties: m_W, Γ_W results



$$m_W = 80.412 \pm 0.029 \pm 0.031 \text{ GeV};$$
$$\Gamma_W = 2.150 \pm 0.068 \pm 0.060 \text{ GeV}.$$

NB : 2003 values, WW events only
[no LEP global fit]

$W^\pm$ properties: $W^\pm$ decay

[NB : no higher orders in this page !!!]

- in the SM the $W^\pm$ boson decays through CC interactions (V-A);
- therefore the coupling is the same for all $f\bar{f}'$ pairs, providing :
  - $m(f\bar{f}') < m_W$ ($\rightarrow$ no t decays);
  - $q\bar{q}$ mixing (à la CKM) must be used;
- ASSUMING (*just for the discussion*) a diagonal CKM matrix, W^+ decays into $e^+\nu$, $\mu^+\nu$, $\tau^+\nu$, $u\bar{d}$, $c\bar{s}$, ($t\bar{b}$ forbidden);
- [if W^-, then corresponding antiparticles];
- ($m_f << m_W$ and CKM $\approx$ diagonal) $\rightarrow$ same BR for all channels (but color factor);
- the V-A theory gives in lowest order :

  $\Gamma(W\rightarrow f\bar{f}') = G_F\, m_W^3 / (6\sqrt{2}\pi) \approx 226$ MeV;
- (3 leptons + 2 quarks × 3 colors = 9) :

  $\Gamma_W = \Sigma\, \Gamma_i(W\rightarrow f\bar{f}') \approx 9 \times 226$ MeV =

  = 2.05 GeV;

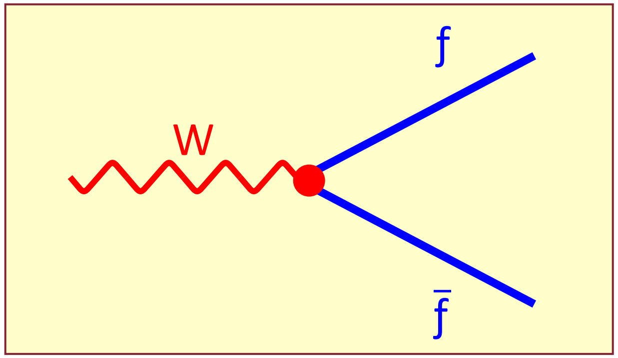
  $BR(W \rightarrow \ell^\pm\nu) \approx 1/9 \approx 0.11$;

  $BR(W^+ \rightarrow u\bar{d}) \approx BR(W^+ \rightarrow c\bar{s}) \approx 1/3 \approx 0.33$;
- if the correct quark mixing is used, the CKM matrix element $V_{qq'}$ must be considered :

  $\Gamma(W\rightarrow q\bar{q}') = |V_{qq'}|^2\, G_F\, m_W^3 / (6\sqrt{2}\pi)$;

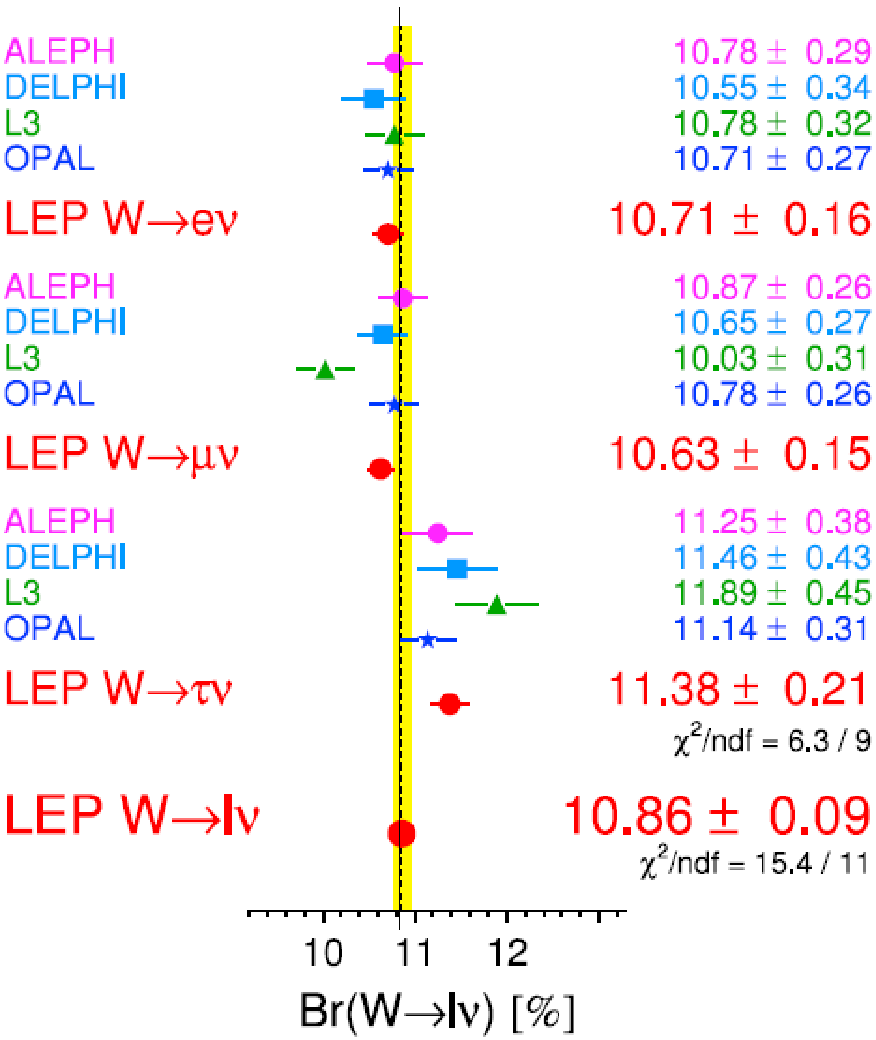
  $\Gamma_W = \Sigma\, \Gamma_i(W\rightarrow f\bar{f}')$ = unchanged;

  $BR(W \rightarrow q\bar{q}') \approx |V_{qq'}|^2 / 3$.

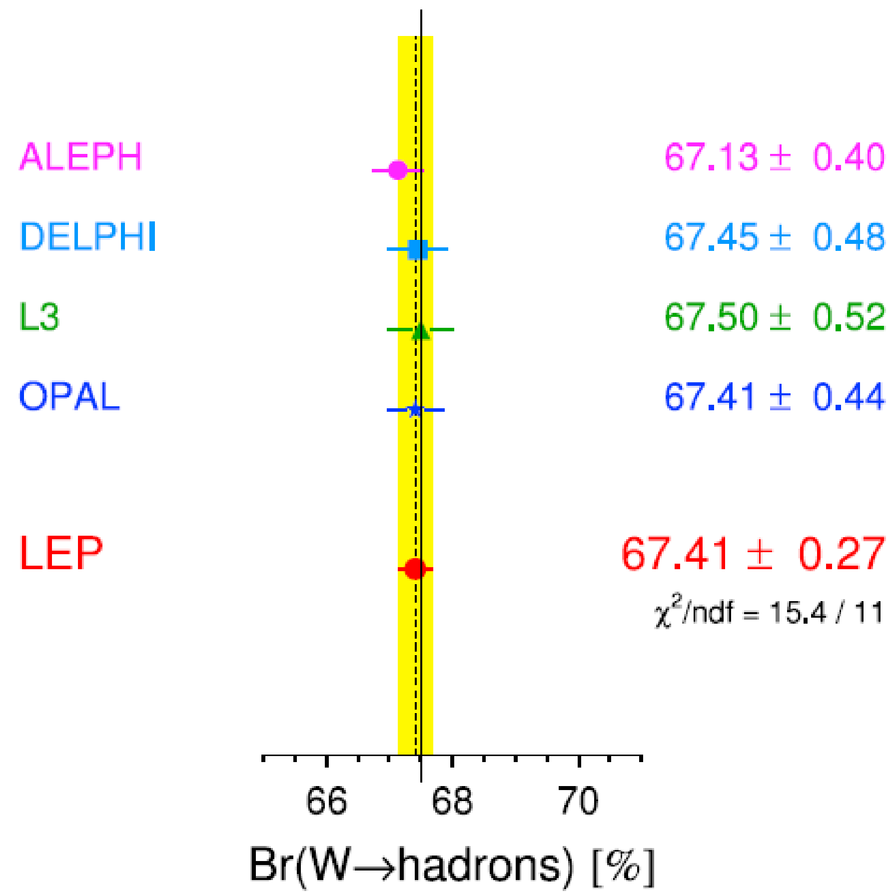


$W^\pm$ properties: $W^\pm$ decay results

W Leptonic Branching Ratios



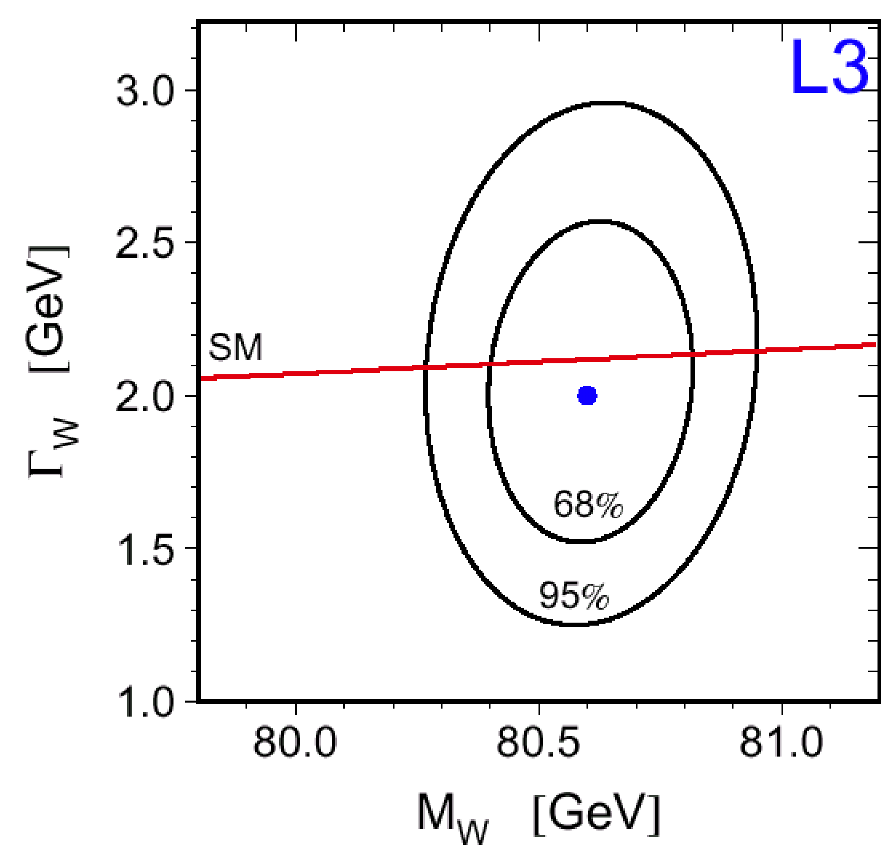
W Hadronic Branching Ratio



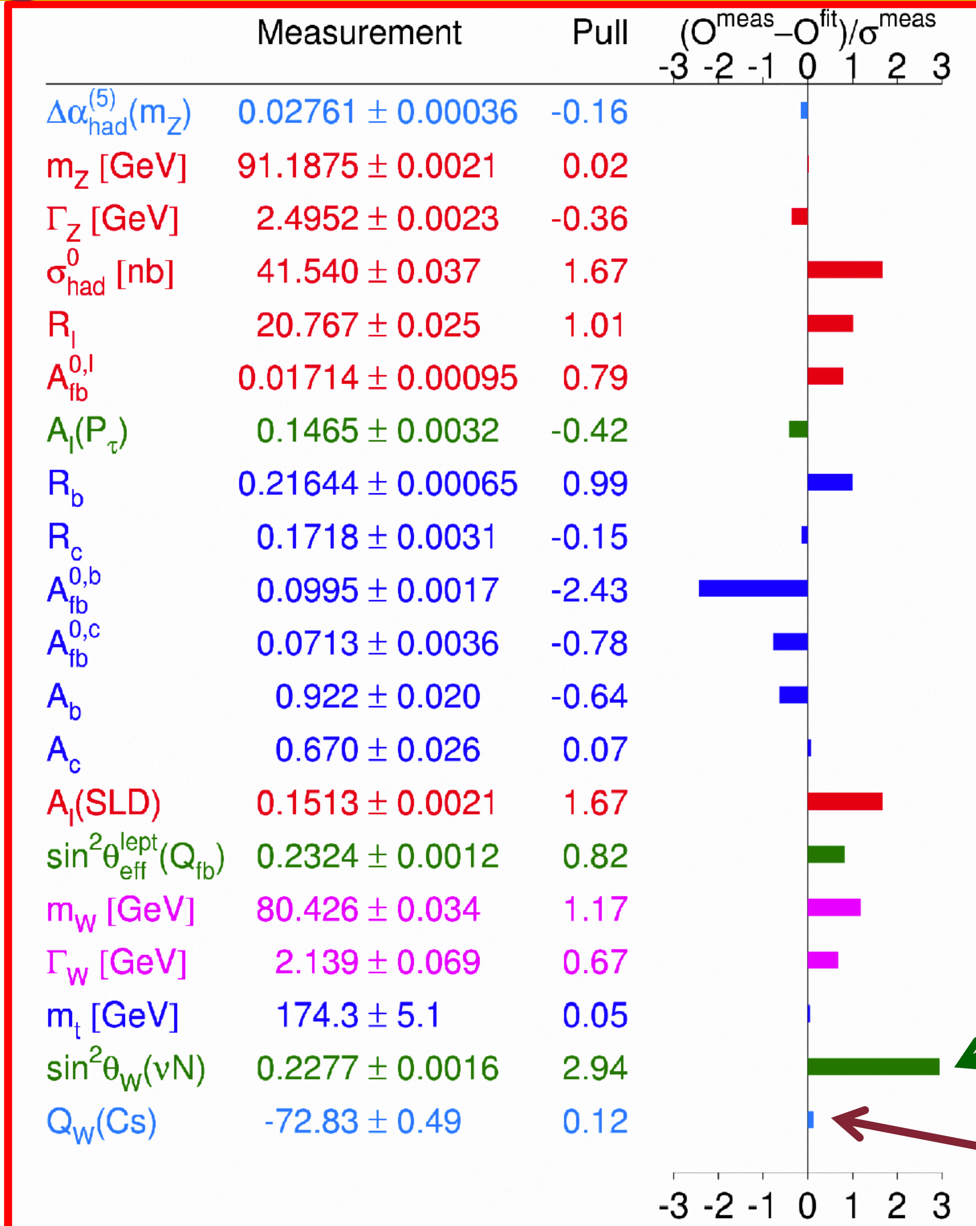
$W^{\pm}$ properties: m_W vs Γ_W

In the SM, m_W and Γ_W are correlated:

- are the previous measurements consistent ?
  - yes, see the plot;
- can do better ? i.e. check the SM with all the LEP measurement ?
  - yes;
- even better ? i.e. add also the other SM non-LEP measurement, i.e. ν's and low-energy ?
  - yes, see next slide;
- is the fit producing a value for the (still) unknown parameters, e.g. m_H ?
  - yes.



global LEP(1+2) fit



circa A.D. 2000, at the end of LEP era.

$$\text{pull} \equiv \frac{\text{experiment - theory}}{\text{error}};$$

expected gaussian, $\mu=0$, $\sigma=1$;

$\chi^2 = \Sigma_i\,(\text{pull}_i)^2$;

χ^2 / dof = 25.5 / 15 $\rightarrow \mathcal{P}(\chi^2)=4.4\%$.

NuTeV $\sigma_{CC,NC}(\nu N)$

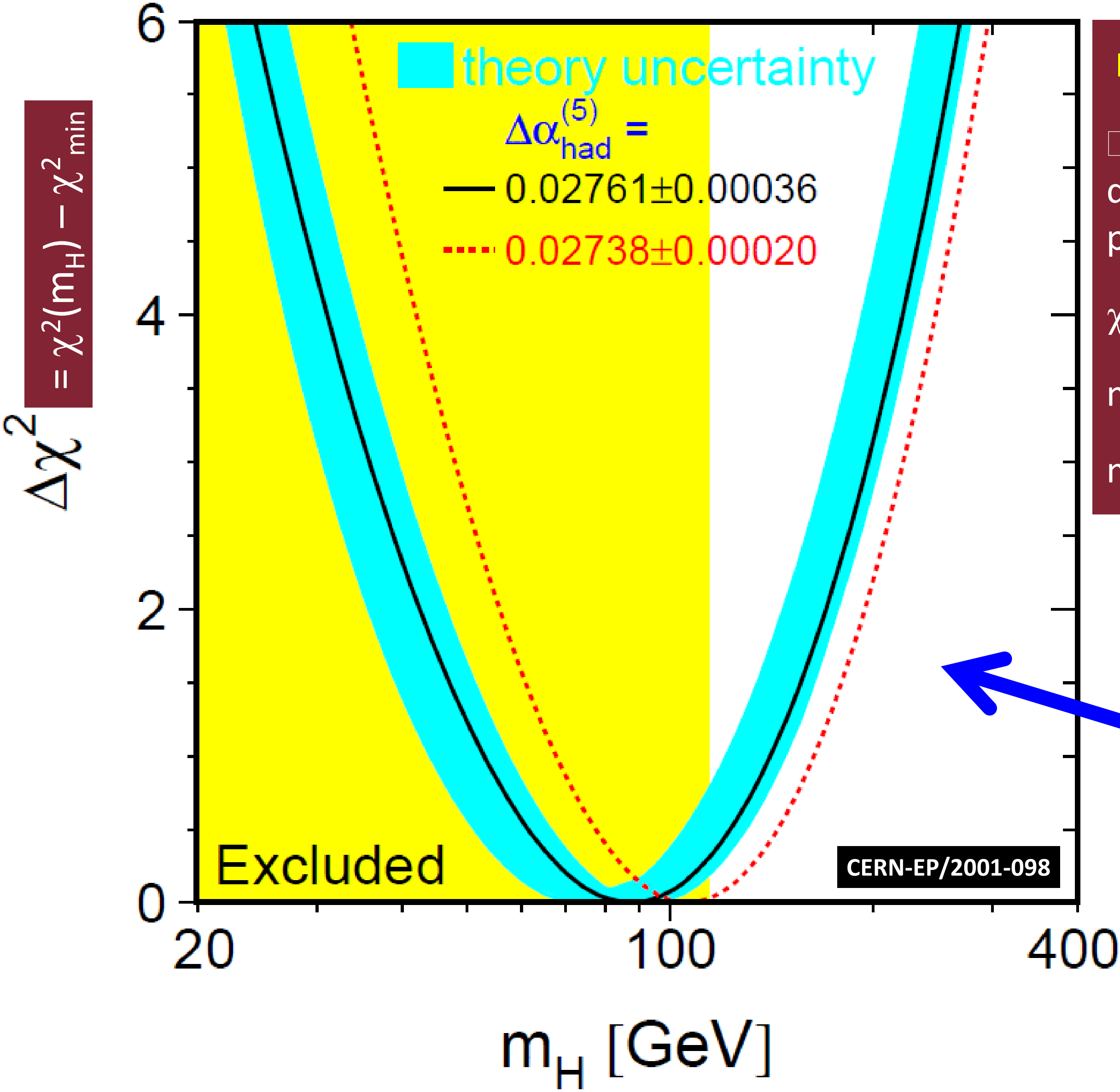
parity violation in Cs

This nice agreement was mainly used to:

- claim the quality of the SM (and exp.'s);
- predict the (unknown) mass of the Higgs.

global LEP(1+2) fit : m_H prediction

$\mathcal{H}$



excl. by direct search [*see*];

□ $^{(5)}_{had}$: contribution of light quarks to photon vacuum polarization (two computations).

χ^2_{min} / dof = 22.9 / 15

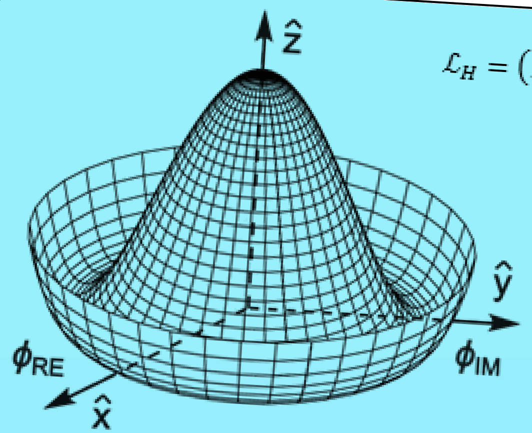
$m_H = 88^{+53}_{-35}$ GeV

m_H < 196 GeV @ 95% CL

Just an example, often remade with small variations before LHC. The 2nd most quoted LEP plot (after n_ν) will disappear soon.

"$\mathfrak{H}$" (="$\mathcal{H}$istory") (here and in the following) means "now obsolete" .

iv. Physics 2 : Higgs



$$\mathcal{L}_H = (D_\mu\phi)^* D_\mu\phi - V(\phi) - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu\phi = \partial\phi + ieA_\mu\phi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = \alpha\phi^*\phi + \beta(\phi^*\phi)^2$$

$$\alpha < 0, \qquad \beta > 0$$

- The Higgs boson has been (*very likely*) discovered at LHC, definitely not at LEP.
- Why remember an old and not-so-nice story, like the LEP search of the Higgs ?
- Because it is very instructive – almost all searches are unsuccessful → in practice limits and exclusions are much more frequent than discoveries;
- [*in the past, fluctuations/mistakes have been rare, but not null*]

- go → § searches, then come back;
- Higgs properties are treated in § LHC [+ RQM + EWI];
- here only an incomplete discussion for Higgs production in e⁺e⁻ at LEP1 & LEP2 energies.

Higgs search @ LEP1

- In the SM the Higgs boson is at the origin of fermion masses;
- at least one H, neutral, spin-0;
- only 1 H → "minimal SM" (MSM, the case discussed in these lectures);
- m_H free parameter of SM (but $m_H < 1$ TeV);
- in the MSM, if m_H is given, the dynamics is completely determined and calculable (couplings, cross sections, BR's, angular distributions, …);
- properties :
  - charge : 0; spin : 0; $J^P = 0^+$;
  - coupling with fermions f :

$$\Gamma(H \to f\bar{f}) = \frac{c_f}{4\pi\sqrt{2}} G_F m_H m_f^2 \beta_f^3;$$

$$\beta_f = \sqrt{1 - 4m_f^2/m_H^2}; \qquad c_f = \begin{cases} 1 & [\text{leptons}] \\ 3 & [\text{quarks}] \end{cases};$$

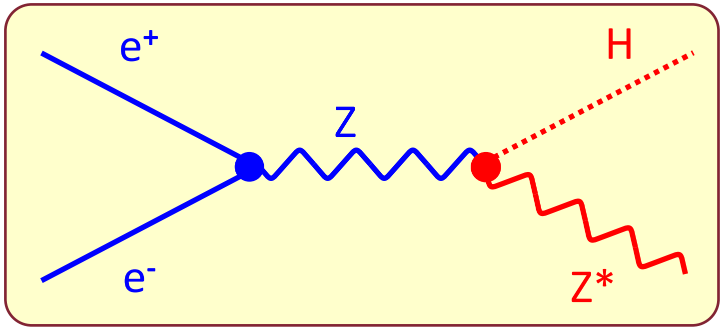
  - [notice: $\Gamma_f \propto m_f^2$);
  - therefore, H decays mainly in the fermion pair of highest mass kinematically allowed;
  - therefore, if $m_H > 2m_b$ (i.e. > 10 GeV), mainly $H \to b\bar{b}$.

- $Z \not\to HH$ (spin-statistics, *like* $\rho^0 \not\to \pi^0\pi^0$);
- in lowest order only:
  - $Z \not\to H\gamma$ (Z, H neutral !!!) [or $H \not\to Z\gamma$];
  - $H \not\to \gamma\gamma$) (H neutral !!!)

    $\boxed{\text{however, (H} \to \square\text{) essential for the discovery (see § LHC).}}$
  - $H \not\to gg$ (no strong interactions);
  - $\boxed{\text{but H} \to Z\gamma, \square\text{, gg through higher order processes.}}$

more complete discussion in § LHC, e.g. discussion of H → Z, W decays.

Higgs search @ LEP1: Bjorken process



$e^+e^- \to Z \to HZ^*$
[Bjorken process]

- LEP 1 ($\sqrt{s} \approx m_z$) : $e^+e^- \to Z \to HZ^* \to \left(f\bar{f}\right)\left(f\bar{f}\right)$;
  i.e. the Higgs production is one of the possible Z decays :

$$\frac{1}{\Gamma(Z \to f\bar{f})} \frac{d\Gamma(Z \to Hf\bar{f})}{dx} =$$

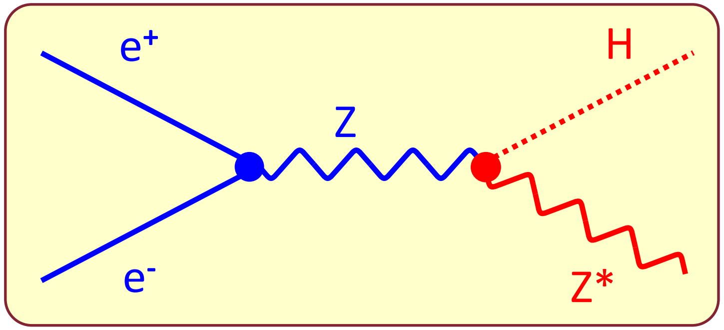
$$= \frac{G_F m_Z^2}{24\sqrt{2}\pi^2} \frac{(12 - 12x + x^2 + 8y^2)\sqrt{x^2 - 4y^2}}{(x - y^2)^2};$$

$$x = \frac{2E_H}{m_Z} = \frac{m_Z^2 + m_H^2 - m_{Z*}^2}{m_Z^2}; \quad y = \frac{m_H}{m_Z}; \quad 2y < x < 1 - y^2.$$

- best observable when
  $Z^* \to \ell^+\ell^-$ (no bckgd),
  $H \to b\,\bar{b}$ (BR $\geq$ 80%, if m_H>$2m_b$);
- BR($Z \to H\ell^+\ell^-$) $\approx 10^{-4}$ @ m_H= 8 GeV
  $\approx 10^{-7}$ @ m_H=70 GeV;
- kinematical constraint :
  $\sqrt{s} \approx m_Z > m_{Z*} + m_H \to m_H < m_Z$
- kinematics not difficult, e.g. $Z^* \to \mu^+\mu^-$,
  $m(Z^*) = m_{\mu\mu}$, $E(Z^*) = E_{\mu\mu}$,
  $m_H^2 = s + m_{\mu\mu}^2 - 2\sqrt{s}E_{\mu\mu}$.

ok ?

Higgs search @ LEP1: Bjorken process (2)



$e^+e^- \to Z \to HZ^*$
[Bjorken process]

kinematics not difficult, e.g. $Z^* \to \mu^+\mu^-$,

$m(Z^*) = m_{\mu\mu}$, $E(Z^*) = E_{\mu\mu}$,

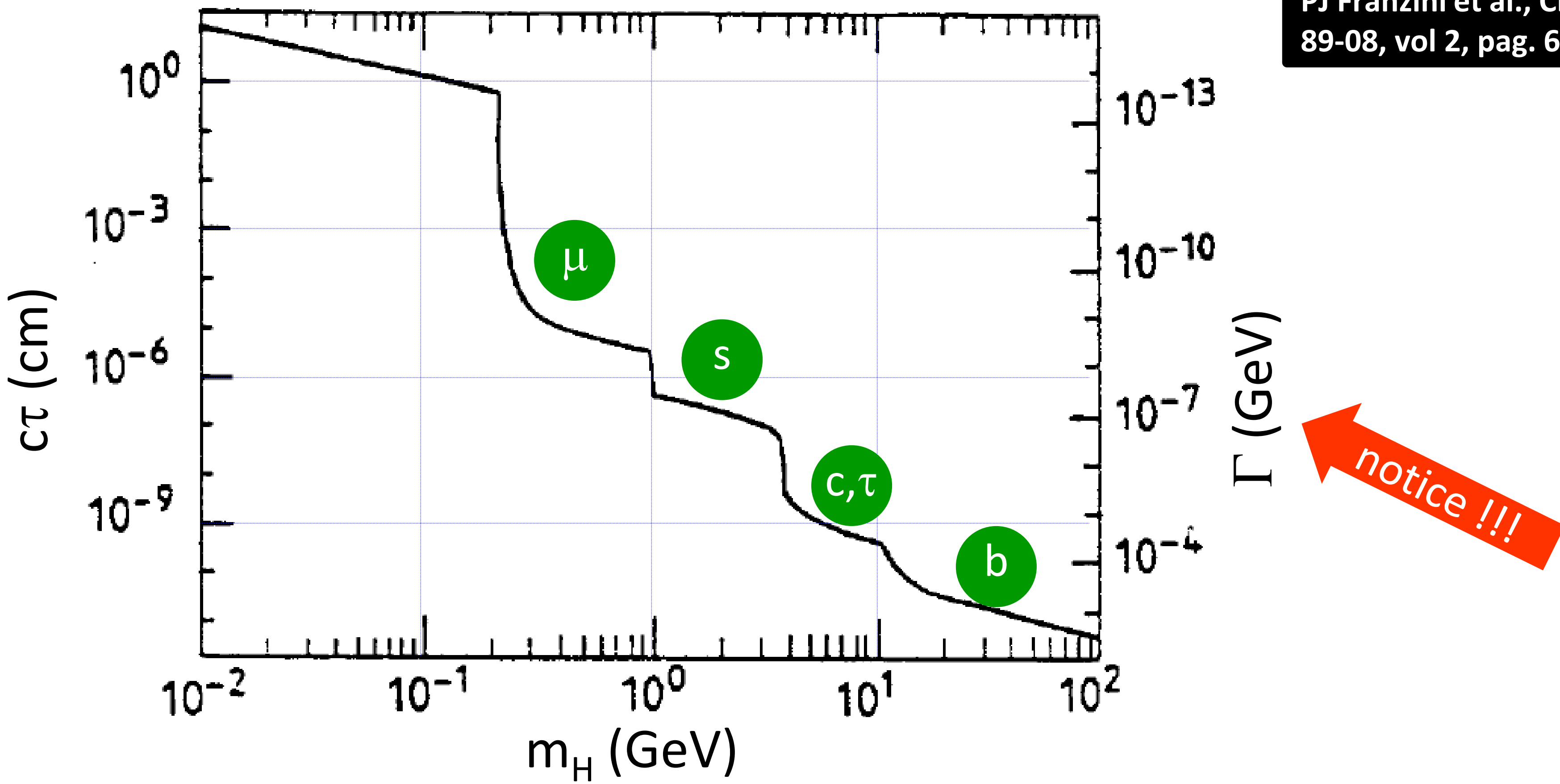
$m_H^2 = s + m_{\mu\mu}^2 - 2\sqrt{s}E_{\mu\mu}$.

i.e. the meas. of m_H does NOT

require the meas. of the H decay.

Higgs search @ LEP1: decay predictions

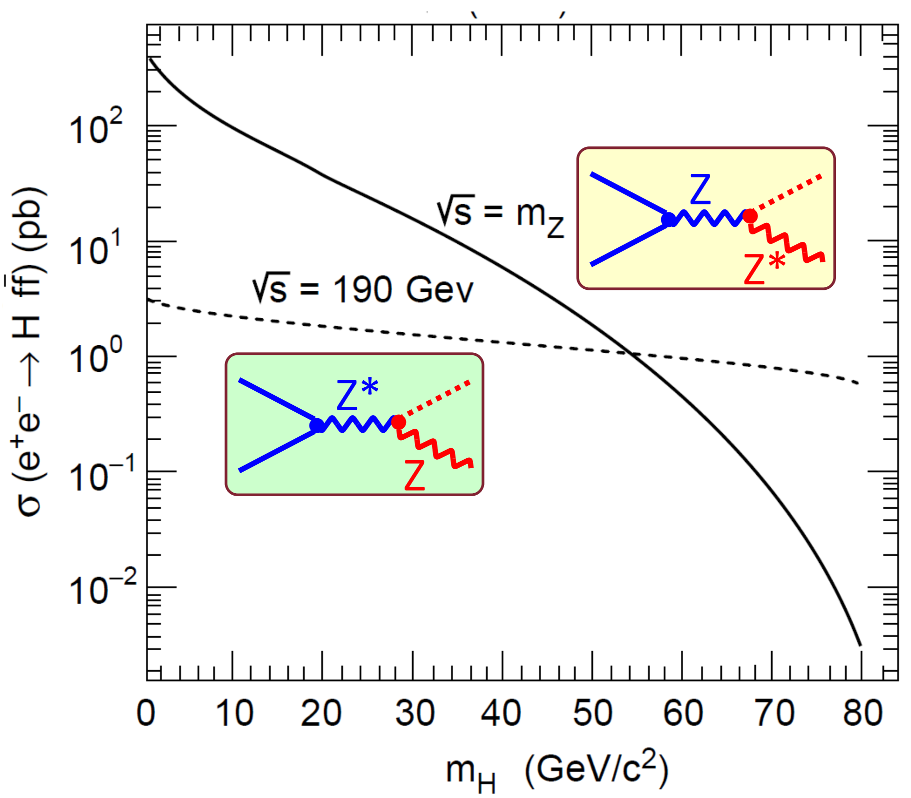
PJ Franzini et al., CERN 89-08, vol 2, pag. 65'.

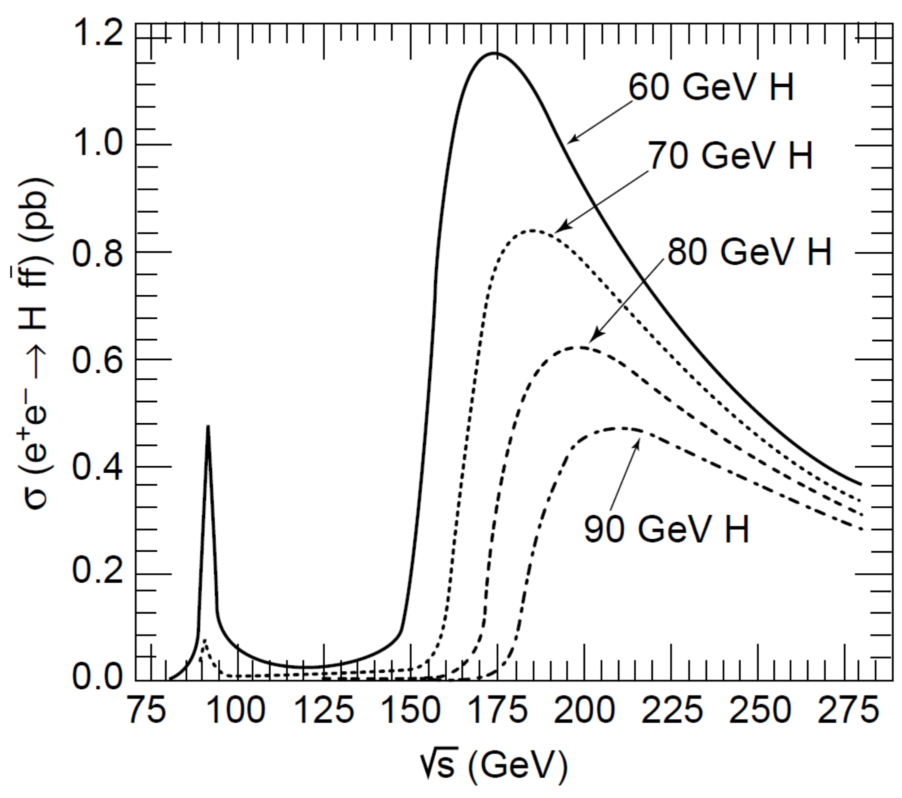


The main decay product of H is the $f\bar{f}$ of largest mass compatible with m_H: e.g. s means $H \to s\bar{s}$.

When a new threshold opens up, there is a "step" in cτ (~1/Γ), rounded by phase space (clearly not done in the calculation).

Higgs search @ LEP1: predictions





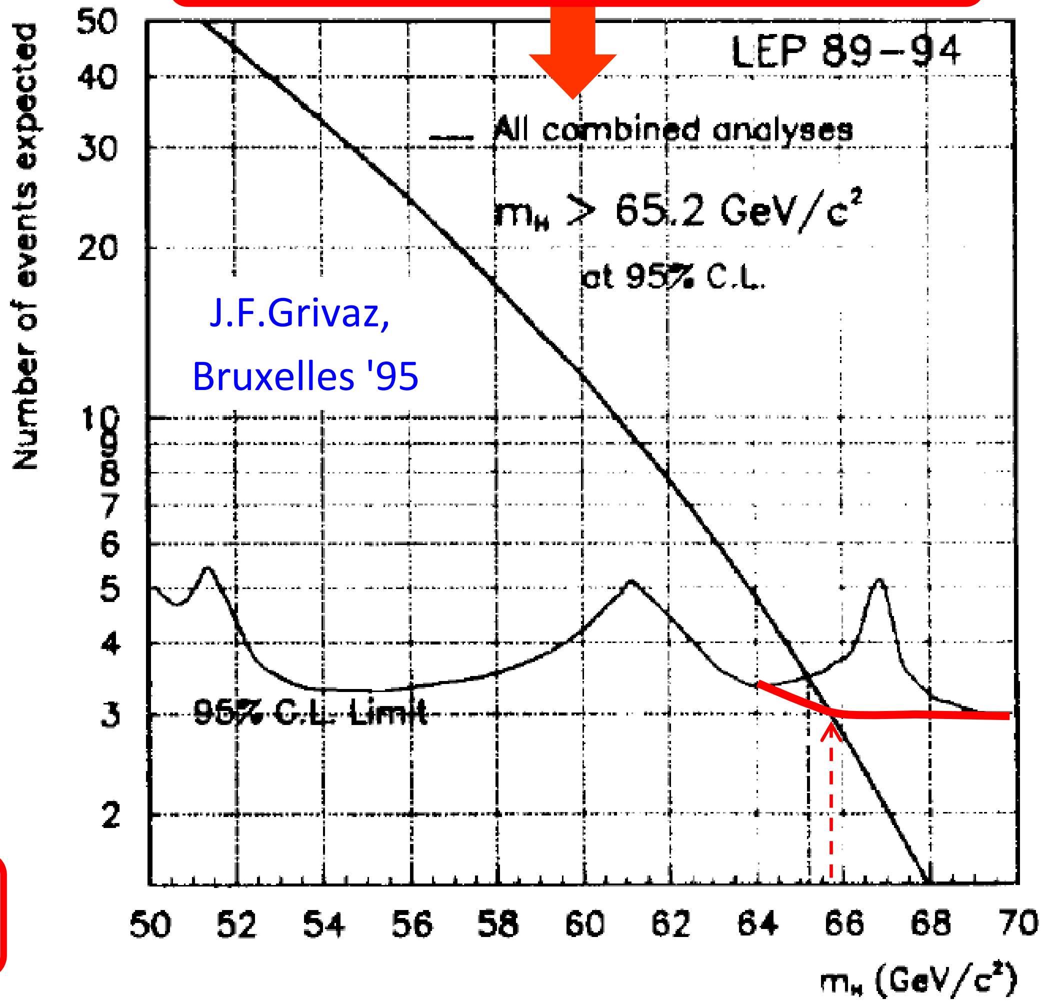
For $\sqrt{s} \approx m_Z$ (real Z) and $m_H << m_Z$, the Bjorken process ($e^+e^- \to Z \to HZ^*$) has a sizeable cross section, but at larger m_H it essentially disappears $\to$ go to larger $\sqrt{s}$.

The predictions at $\sqrt{s} >> m_Z$ come from a similar process ($e^+e^- \to Z^* \to HZ$, virtual Z^*), known as "higgs-strahlung" [*next slides*].

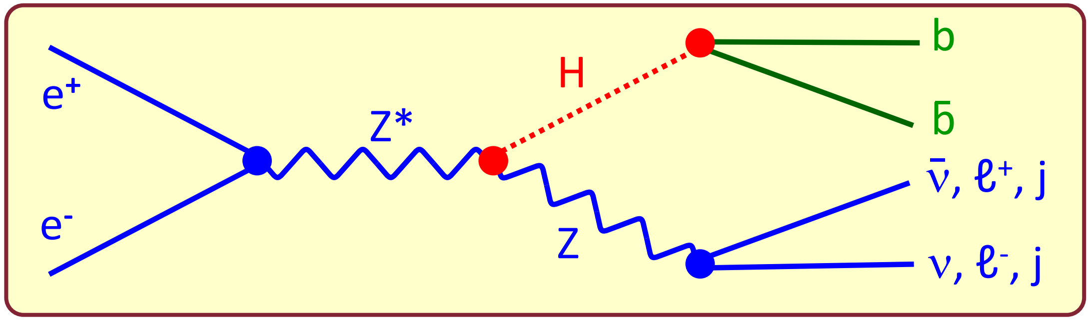
Higgs search @ LEP1: results

- this plot summarizes the limits of the four experiments :
  - A : 63.1 GeV
  - D : 55.4 "
  - L : 60.2 "
  - O : 59.1 ";
- the candidate @ m_H = 67 GeV (OPAL) reduces the limit by few × 100 MeV;
- a test case for the method, discussed in § limits; notice :
  - ➢ the combined limit is "better" than any single exp.;
  - ➢ the "worst" observed limit does not come necessarily from the "worst" exp.;
  - ➢ … because it is a random variable;
- conclusion: move to higher $\sqrt{s}$, i.e. Bjorken process → higgs-strahlung.

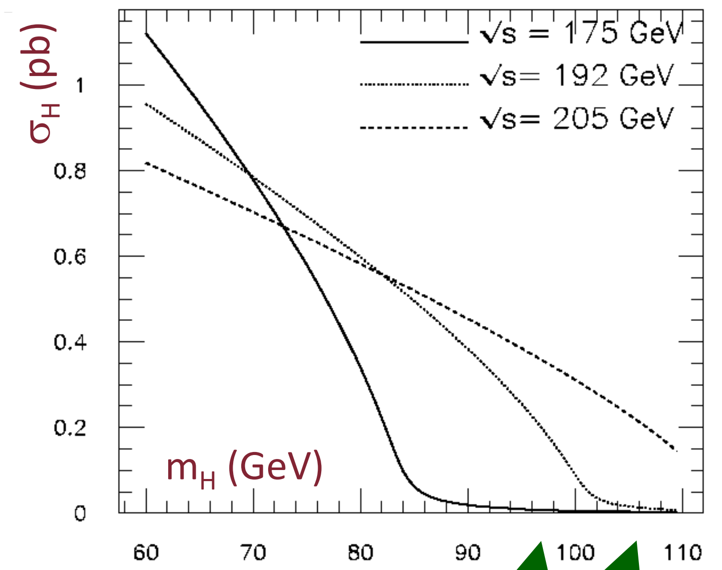
LEP 1, $\sqrt{s} \approx m_Z$:
~3.7 M [Z→ hadrons] / exp in 1989-94;
m_H > 65.2 GeV @ 95% CL



Higgs search @ LEP2



$$e^+e^- \rightarrow Z^* \rightarrow HZ$$
[higgs-strahlung]



- LEP 2 : process of "higgs-strahlung" (= radiative emission of a Higgs boson from a Z*);
- i.e. the higgs production is a 4-fermion final state, mediated by a virtual Z* [like $e^+e^- \rightarrow W^+ W^- \rightarrow 4f$];
- kinematical constraint :

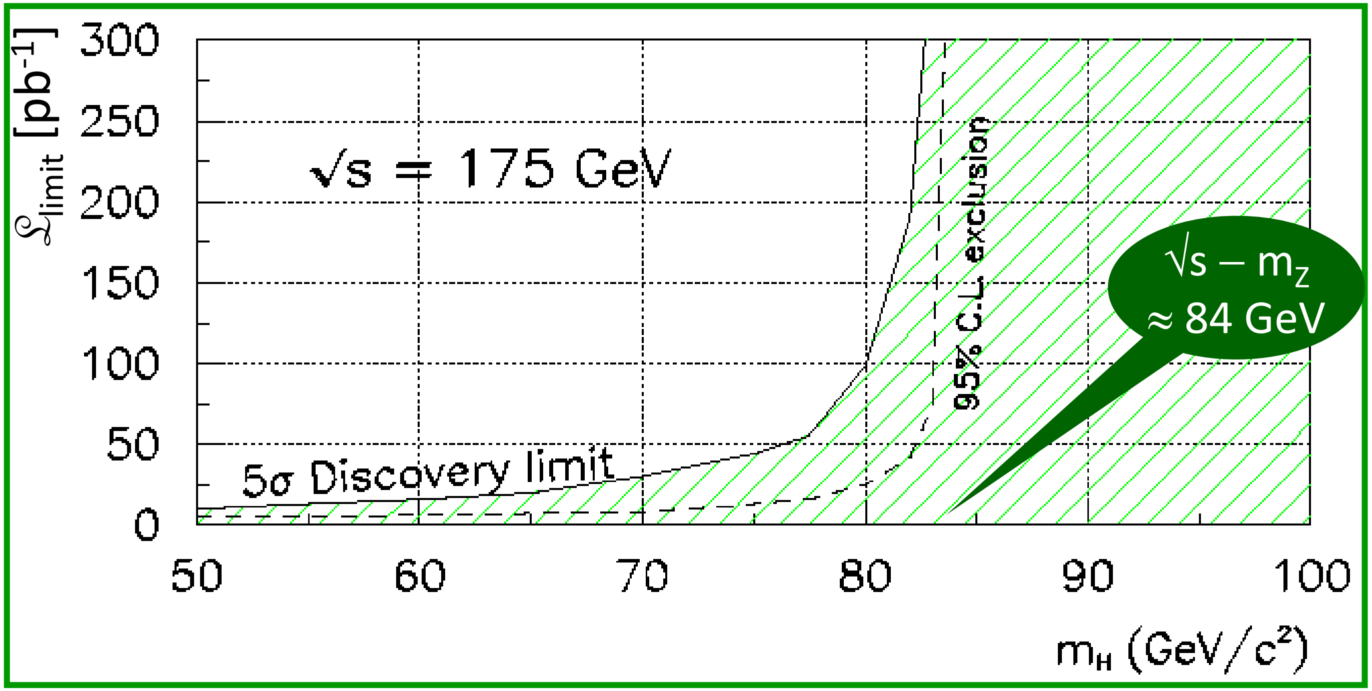
  $\sqrt{s} = m_{Z*} > m_Z + m_H$
- [*no* $\mathcal{H}$ *here, because of possible future colliders, see later*].

$$\sigma_0(e^+e^- \rightarrow Z^* \rightarrow ZH) = \frac{G_F^2 m_Z^4}{24\pi s}\left[\left(g_V^\ell\right)^2 + \left(g_A^\ell\right)^2\right]\sqrt{\lambda}\,\frac{\lambda + 12 m_Z^2/s}{\left(1 - m_Z^2/s\right)^2};$$

$$\left[\lambda = \left(1 - m_H^2/s - m_Z^2/s\right)^2 - 4 m_H^2 m_Z^2/s^2;\right]$$

$$\frac{1}{\sigma_0}\frac{d\sigma_0}{d\cos\theta} = \frac{\lambda^2\sin^2\theta + 8m_Z^2/s}{4\lambda^2/3 + 16 m_Z^2/s} \xrightarrow{s\ >>\ 8m_Z} \frac{3}{4}\sin^2\theta.$$

Higgs search @ LEP2: E vs $\mathscr{L}$

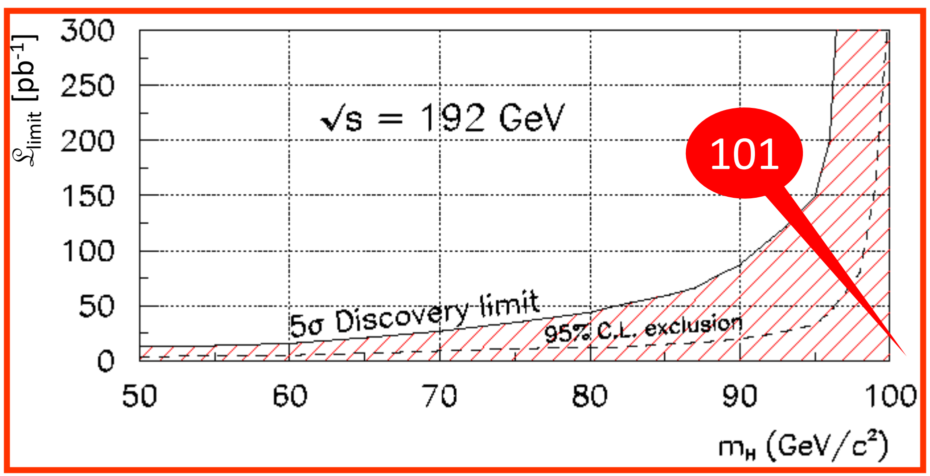


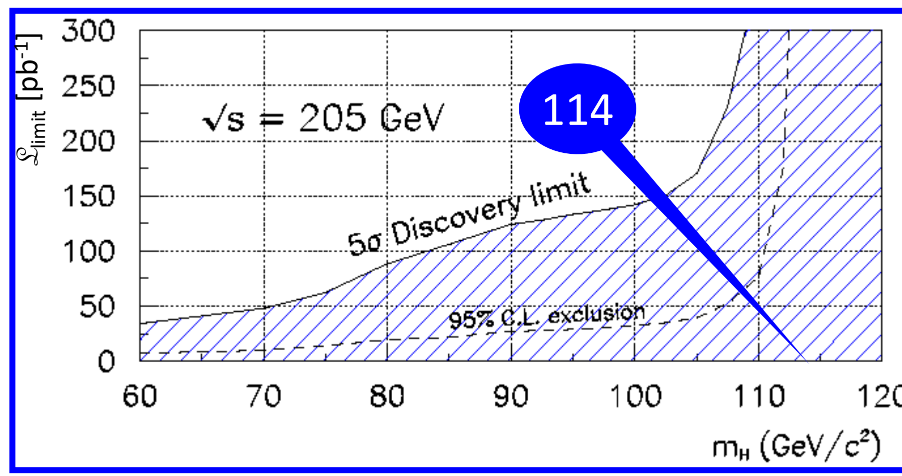
An old study by PB et al in 1995, before the start of LEP2.

Notice the shape of $\mathscr{L}_{disc}$ and $\mathscr{L}_{excl}$.

Conclusion:

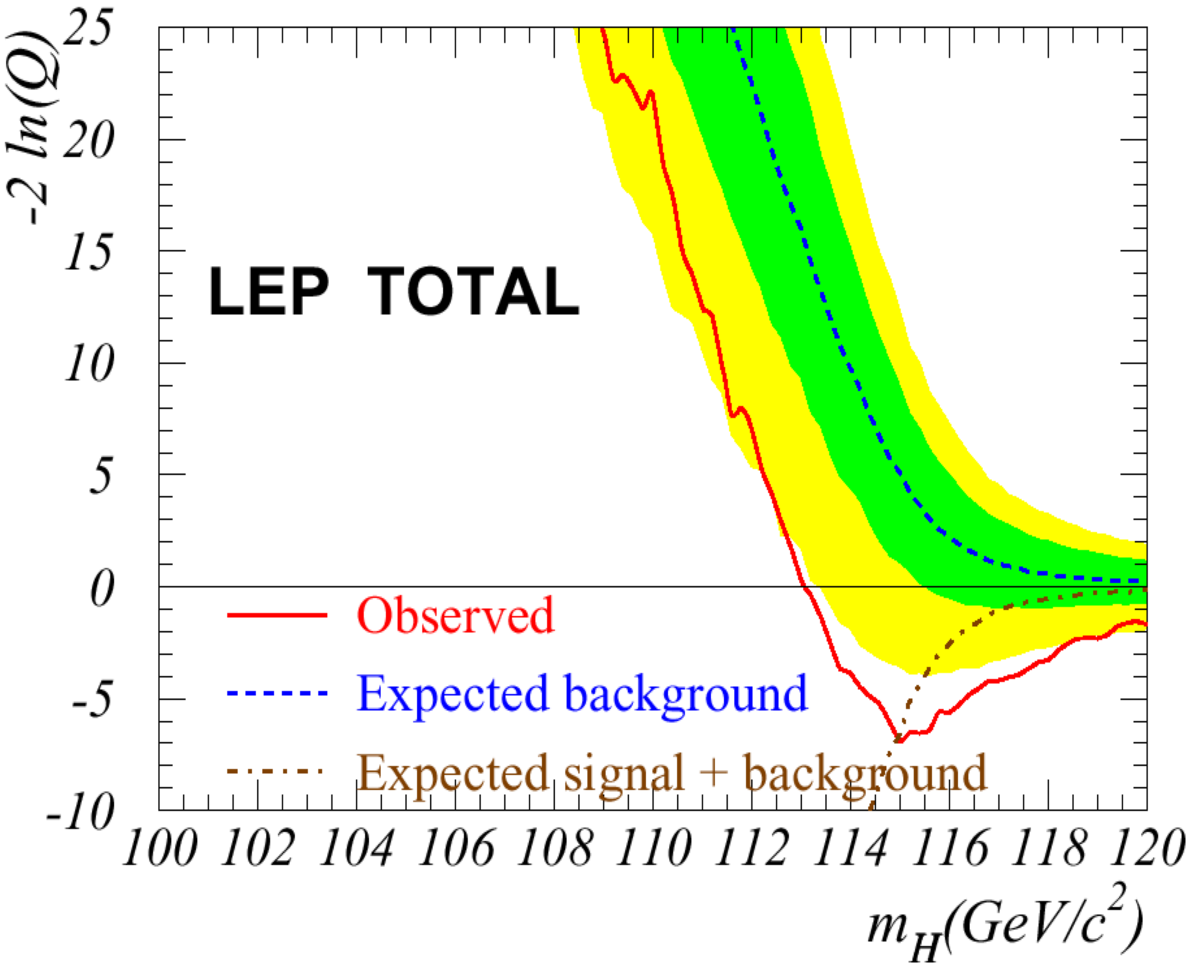
Energy is very very much better than **luminosity** !!!





Higgs search @ LEP2: LEPC 3/11/2000

- $-2\ell nQ = -2\ell n(\Lambda_s/\Lambda_b)$;
- $-2\ell nQ(m_H{=}115) = -7$;
- if interpreted as a discovery
  - $m_H = 115^{+1.3}_{-0.9}$ GeV;
  - $1\text{-}CL_b = 4.2\times10^{-3}$;
  - i.e. "2.9 σ";
- if interpreted as a limit :
  - $m_H > 113.5$ GeV @ 95%CL.



???

Higgs search @ LEP2: LEPC 3/11/2000

RECOMMENDATION

Given the consistency for the combined results with the hypothesis of the production of a SM Higgs boson with a mass of 115 GeV, and an observed excess in the combined data set of 2.9σ, a further run with 200 pb^{-1} per experiment at 208 GeV would enable the four experiments to establish a 5σ discovery.

The four experiments consider the search for the SM Higgs boson to be of the highest importance, and CERN should not miss such a unique opportunity for a discovery.

Therefore, we request to run LEP in 2001 to collect $\mathcal{O}$(200 pb^{-1}) at $\sqrt{s} \geq$ 208 GeV.

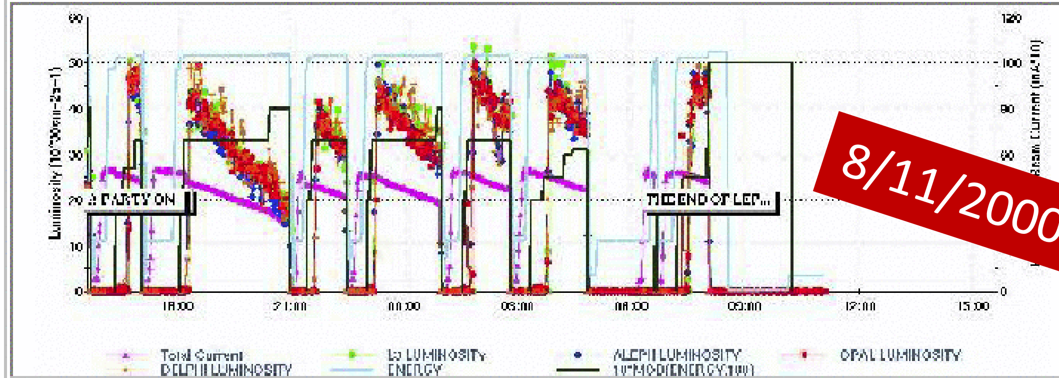
ALEPH, DELPHI, L3, OPAL

The LEP Higgs Working Group

3/11/2000

P. Igo-Kemenes - LEP Seminar - Nov. 3, 2000

LEP shuts down after eleven years of forefront research



8/11/2000

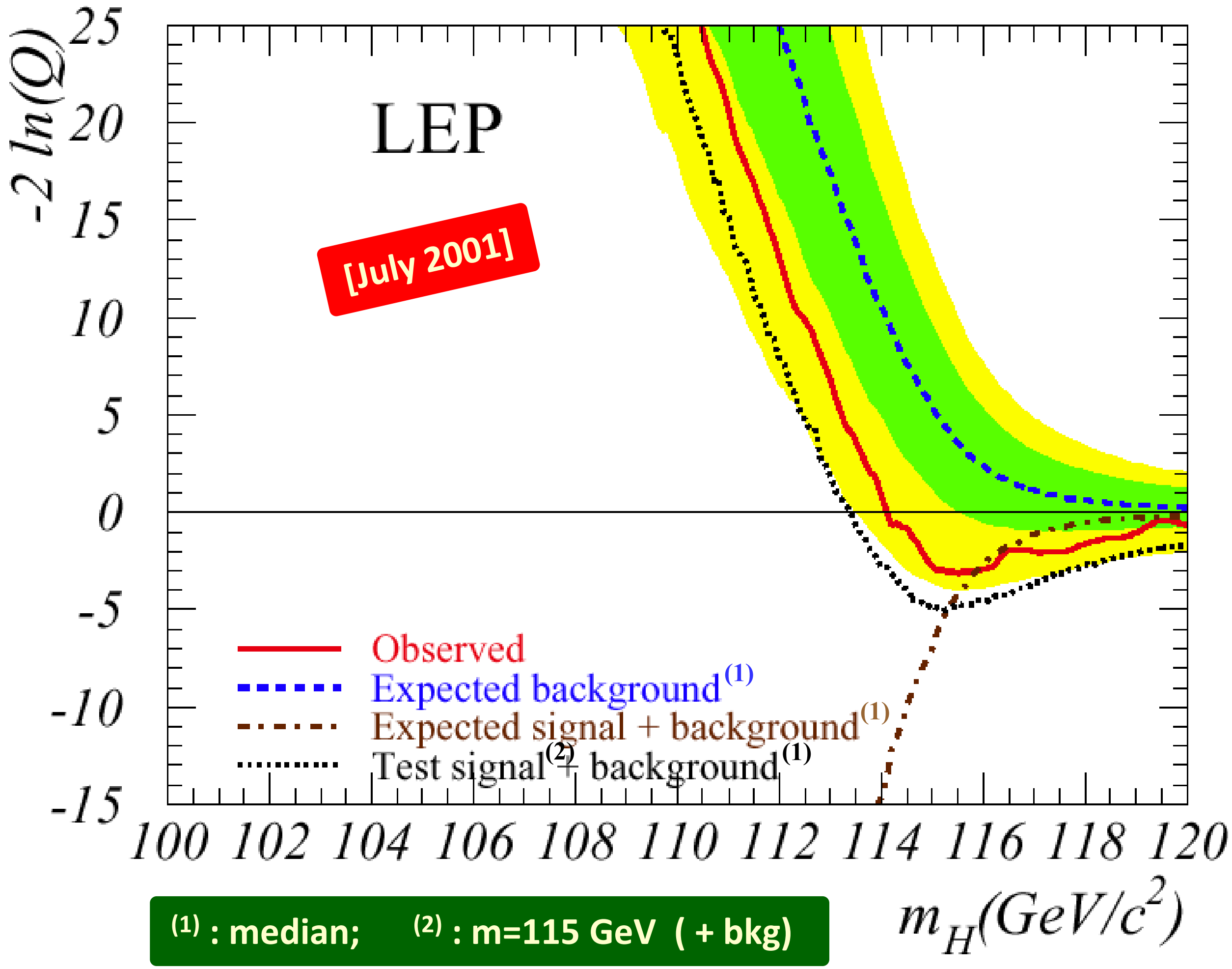
These are the measurements taken of LEP's final beam. The accelerator was switched off for the last time at 8:00 am on 2 November. (Click on photo for enlargement)

After extended consultation with the appropriate scientific committees, CERN 's Director-General Luciano Maiani announced today that the LEP accelerator had been switched off for the last time. LEP was scheduled to close at the end of September 2000 but tantalising signs of possible new physics led to LEP's run being extended until 2 November. At the end of this extra period, the four LEP experiments had produced a number of collisions compatible with the production of Higgs particles with a mass of around 115 GeV. These events were also compatible with other known processes. The new data was not sufficiently conclusive to justify running LEP in 2001, which would have inevitable impact on LHC construction and CERN's scientific programme. The CERN Management decided that the best

a difficult decision

Higgs search @ LEP2: July 2001

- if intepreted as a discovery:
  - m_H=115.6 GeV:
  - $-2\ell nQ|_{\text{actual events}}$ = -2.9;
  - $1\text{-}CL_b = 3.5\times10^{-2}$;
  - "2.1 σ";
- if interpreted as a limit:
  - $m_H > 114.1$ GeV @ 95%CL.

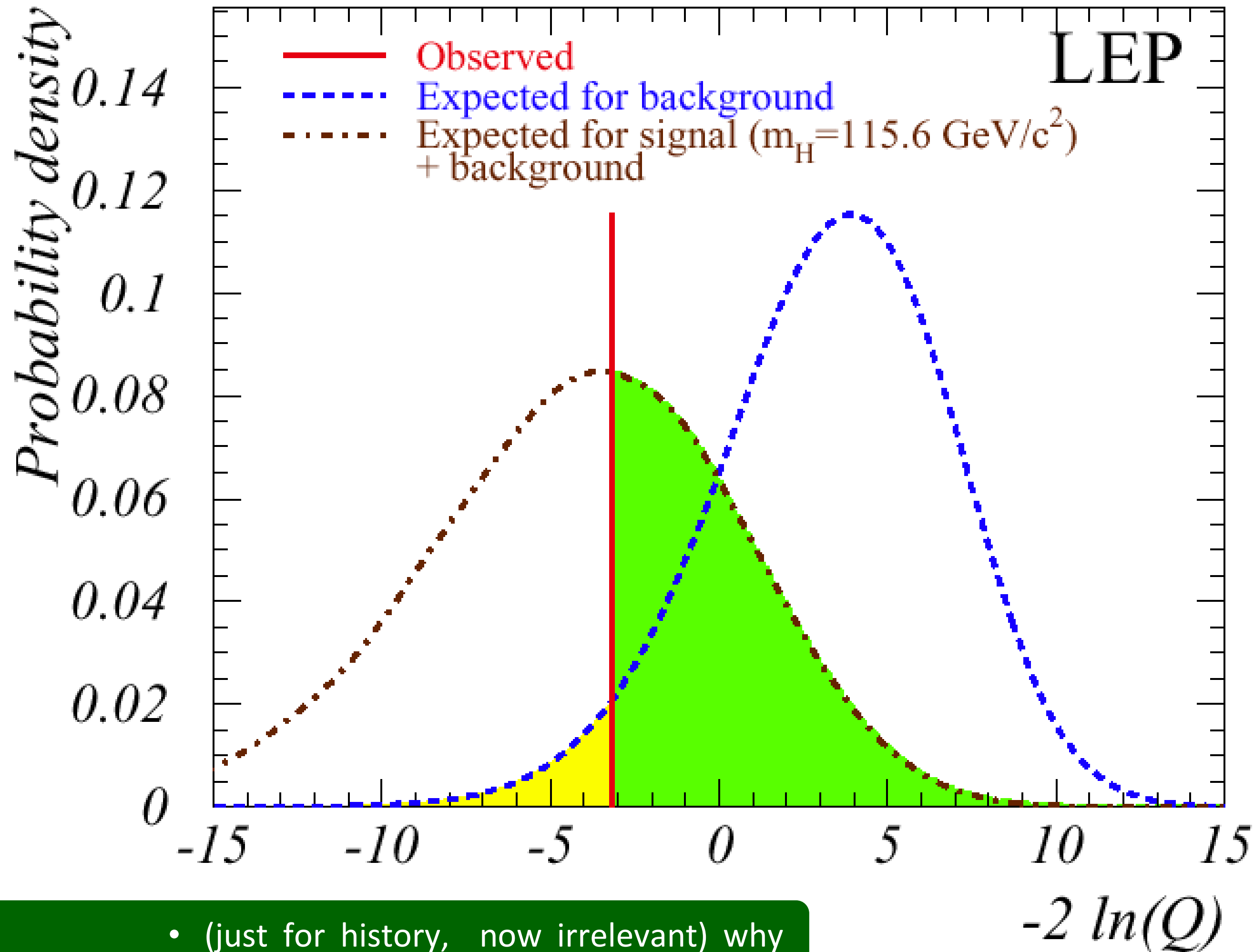


(1) : median; (2) : m=115 GeV (+ bkg)

????

Higgs search @ LEP2: the end

- method "gedanken-experiment" [i.e. produce via mc many experiments, with the same quality and $\mathcal{L}_{int}$ of the present one] :
- m_H^{test} = 115.6 GeV;
- $\int f_{b,s}\, d(-2\ell nQ) = 1$;
- " ◆ " = 1-CL_b= 3.5%;
- " ◆ " = CL_{s+b}= 43%.



Comments/questions (**imho**):

- if this result had been presented in November 2000, there would have been no problem: no one would have claimed the need for further studies.
- (just for history, now irrelevant) why was the first analysis wrong ? well, ... **?**
- why to show it to students ? because it is very instructive, normal classes see only the standard (discovery vs limits).

Higgs search @ LEP2: conclusion

- the "LEPC result" is difficult to explain (NOT only to students) : stat. fluctuations, mistakes, systematics out-of-control, …
- the CERN management (L. Maiani) took the right decision at a high risk;
- the real threat was a delay of LHC, a huge human and economic price;
- instead, the final results are relatively simple to explain: a honest fluctuation at 3.5% does not deserve a discussion;
- the Higgs boson search crossed the ocean, but the TeVatron did not really enter in the game;
- and finally LHC … [*you know*].

Other more personal comments:

- unlike theoretical physics, statistics (and human behavior) require risk evaluation;
- experimental physics lies in the middle;
- you should understand and judge the decisions of the experiments and the management (often they did NOT agree);
- … while the landscape was changing (November '00, July '01, post-LEP, now);
- you might conclude that the "right decision" is a function of role and time (???);
- *… and that searches are risky, not for gutless people.*

the Higgs boson @ LEP : $\sigma(e^+e^- \to HZ)$

AFTER the LHC discovery:

Q: could LEP see a 126 GeV Higgs ?

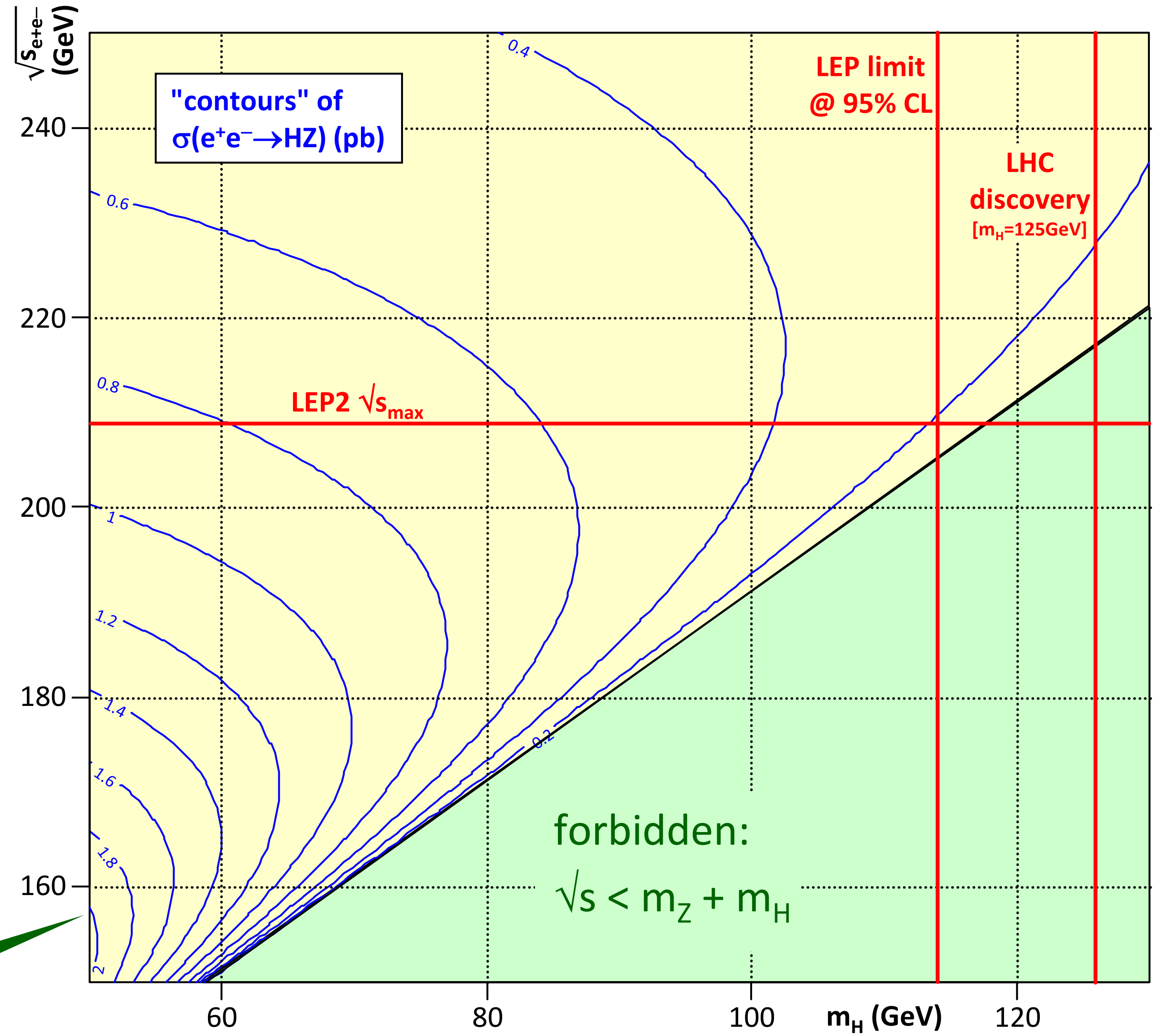
plot the cross section:

- $\sigma = 0.2 \div 1.8$ pb;
- strongly m_H dependent;
- $\mathcal{L}_{int} \approx 200\ \text{pb}^{-1}$/year;
- i.e. n = 40÷200 events/y, shared among many decay channels (some undetectable).

A: the plot is very clear: you should be able to judge yourself !

warning:
- tree level,
- $\Gamma_H = \Gamma_Z = 0$;

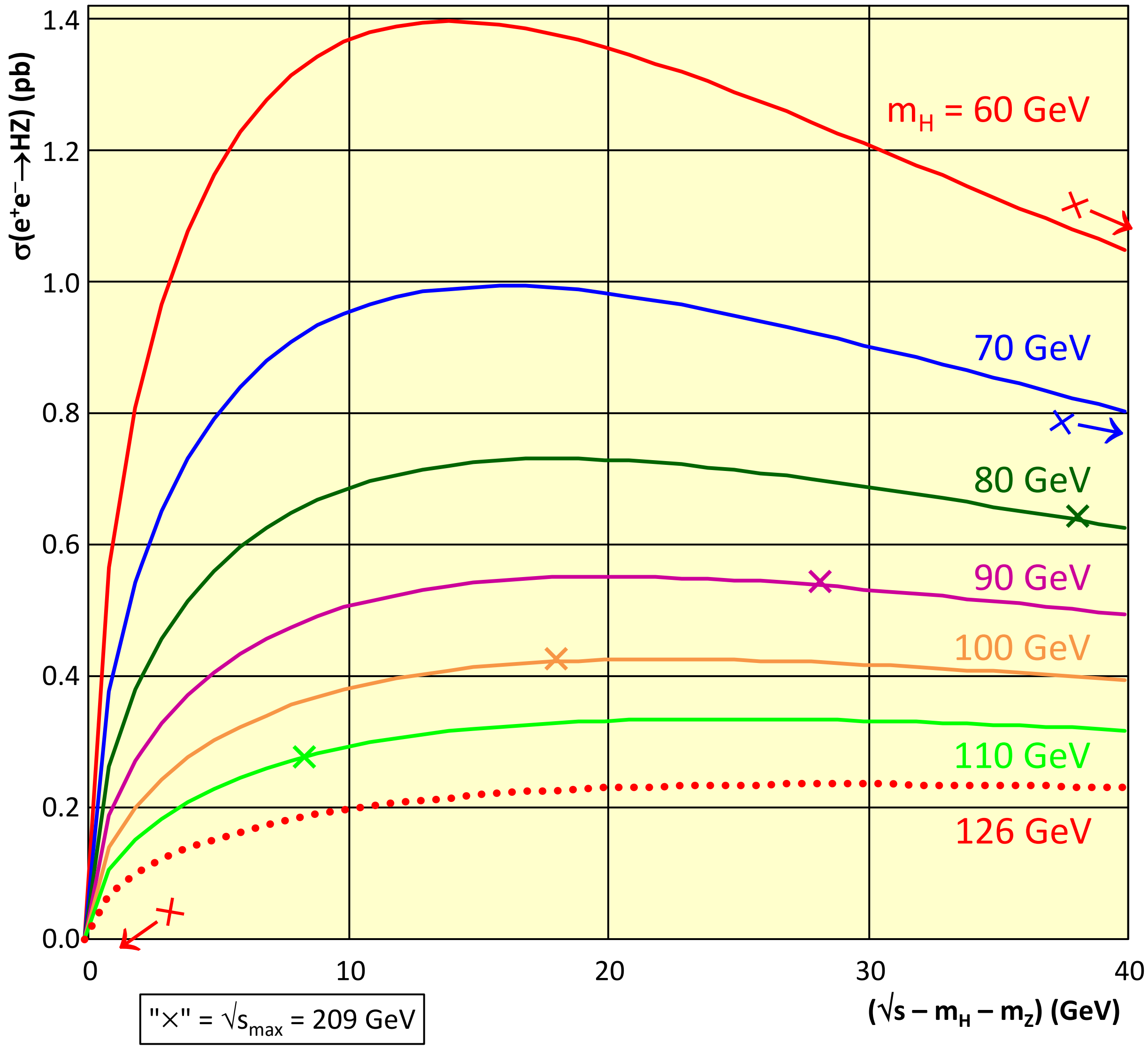
but ok for discussion.



the Higgs boson @ LEP : higgs-strahlung

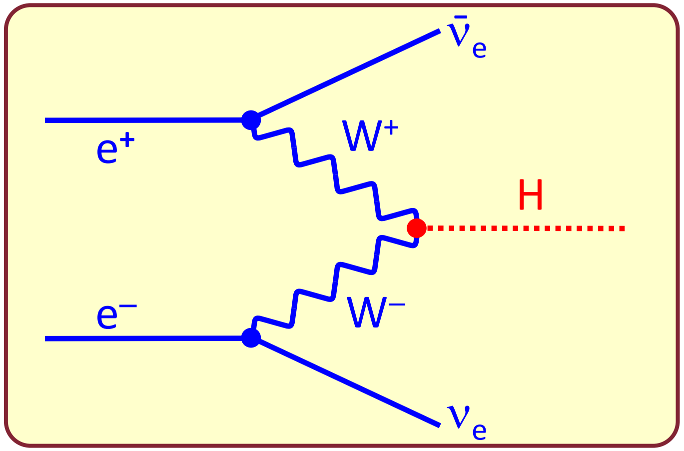
Plot $\sigma(e^+e^- \to Z^* \to HZ)$ vs the "kinetic" energy, i.e. ($T = \sqrt{s} - m_H - m_Z$), in the approx. $\Gamma_Z = \Gamma_H = 0$:

- $T \leq 0 \to \sigma = 0$ (obvious);
- the ×'s show $\sqrt{s}$ = 209 GeV;
- $\sigma_{max}(T)$ at $T \approx 15 \div 20$ GeV, slightly increasing with m_H;
- $\sigma_{max}(m_H)$ decreases a lot when m_H increases;
- for $\sqrt{s} >> m_H+m_Z$, $\sigma \propto s^{-1}$ (obvious);
- for m_H > 110 GeV, other processes (not shown), other than higgsstrahlung;
- if m_H = 126 GeV (LHC), H not produced at LEP 2.



the Higgs boson @ LEP : the future in e^+e^-

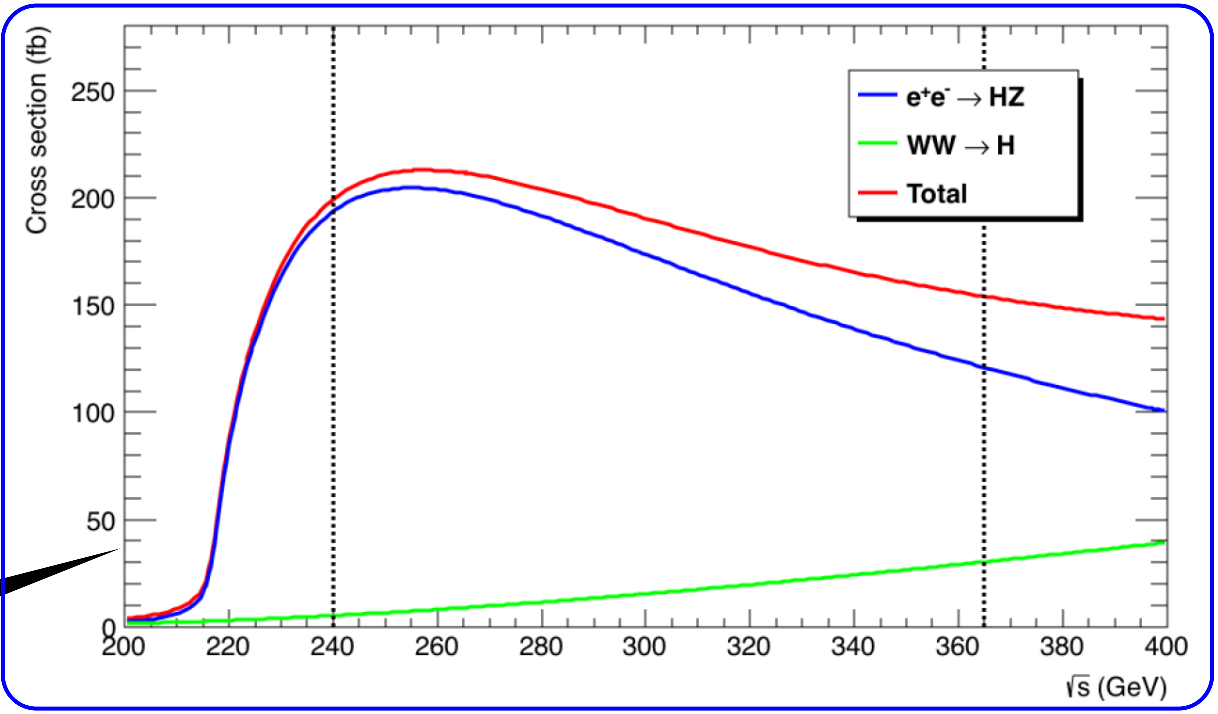
In the post-LEP (and post-H-discovery) era, the interest has shifted to a possible higher energy e^+e^- collider (circular or linear).



In this case:

- consider also other processes (e.g. the so called "WW-fusion" $e^+e^- \to H\bar{\nu}_e\nu_e$ [*see*]);
- compute the cross-section for m_H=126 GeV, as a function of $\sqrt{s}$ [*see*];
- study the physics of (say) ~1 million H:
  - ➢ measure Γ_H *à la* J/ψ;
  - ➢ measure all H couplings;
- [*obviously no ℋ here*].

Future Circular Collider Study, CERN 2018



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Jan Brueghel the Elder and Hendrick de Clerck – Abundance and the Four Elements – 1606 – Prado Museum

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End of chapter 3