Particle Physics - Chapter 7
High energy ν interactions

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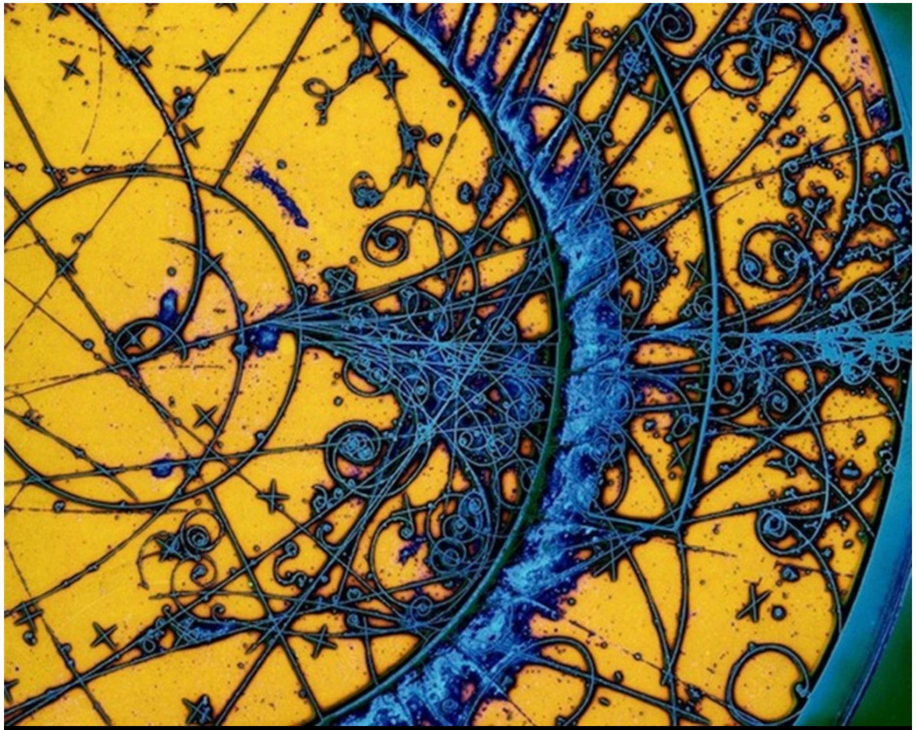
7 – High energy ν interactions

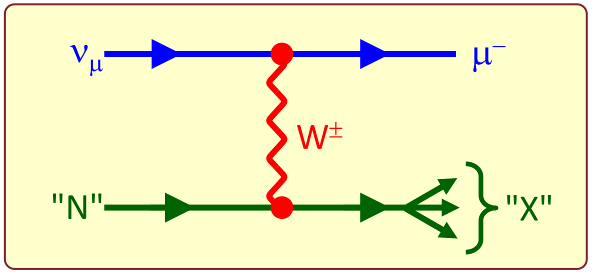


A ν interaction in BEBC

[*original in bw, the colors are an artistic invention*]

High energy ν interactions

After 1960, the accelerator production of ν-beams of high intensity and high energy has led to a dramatic development of our understanding of weak interactions.

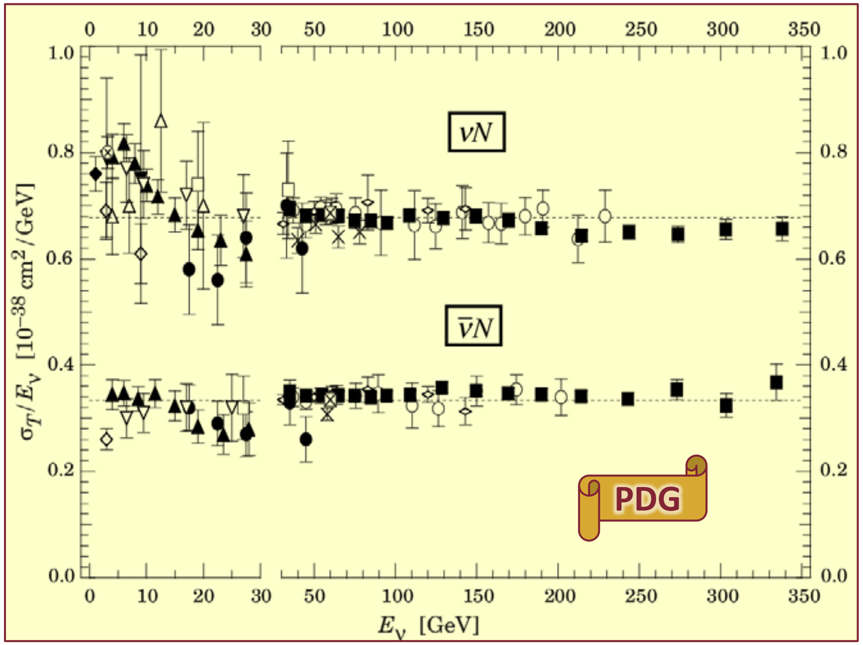


"N" and "X" are all the relevant hadrons/quarks /systems [many different cases]

It is important to explain, albeit in a schematic way, what are the key points to realize a scattering experiment ν-hadrons :

$\sigma(\nu N) = kE_\nu$; $k \approx 0.67 \times 10^{-38}$ cm^2/GeV;
$\sigma(\bar{\nu}N) = k'E_\nu$; $k' \approx 0.34 \times 10^{-38}$ cm^2/GeV.

- The neutrino cross-sections are very small (for E_ν = 1 GeV, $\sigma(\nu N) \sim 10^{-38}$ cm^2, while for the same energy $\sigma(pp) \sim 10^{-26}$ cm^2.
- Beams, detectors, experimental setups have to compensate (bulky, intense, expensive …)



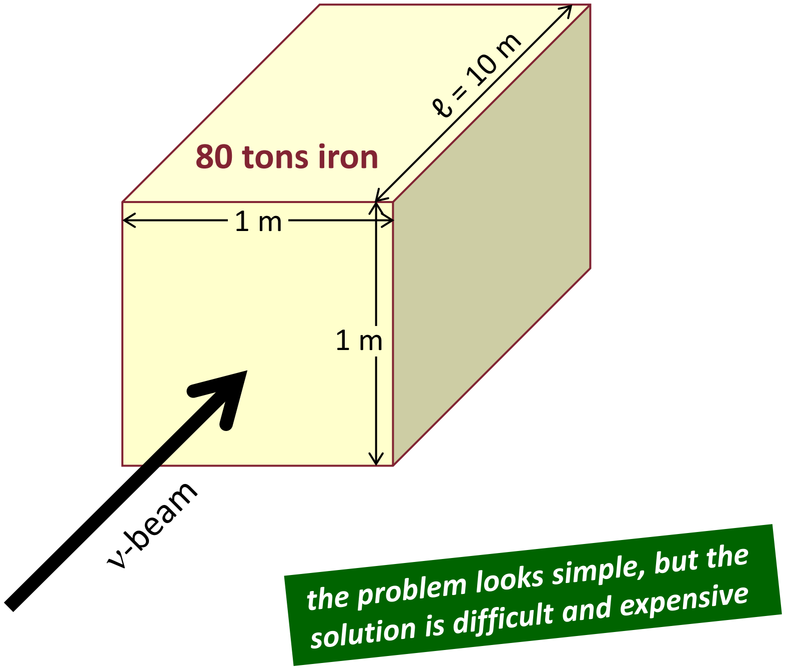
Q. : from the plot, it seems that ($\sigma_{cc} \propto E_\nu$); why ? it looks ugly (actually impossible, because of high energy divergences ("unitarity violations"). [*Wait and see …*]

High energy ν interactions: problem

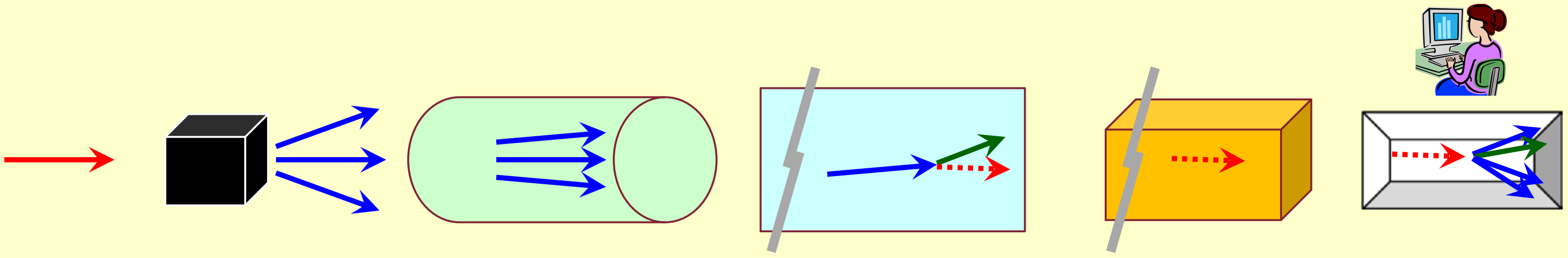
Problem. How many 1-GeV ν's are necessary to produce 100 interactions in a detector of "reasonable" size and material (e.g. iron, $1 \times 1 \times 10$ m^3) ?

- Interaction probability $\mathcal{P}$ for 1 ν :
  - σ = cross section @ 1 GeV,
  - ℓ = length of traversed material,
  - M = nucleon mass,
  - n = [$N_{nucleons}$ per unit volume] = $= m_{detector} / (M\ V_{detector}) = \rho_{Fe} / M$;
  - $\mathcal{P} = \sigma\, n\, \ell = \sigma\, \rho_{Fe}\, \ell / M$ = [MKS] $\approx (0.7 \times 10^{-42}) \times (7.9 \times 10^3) \times (10) / (1.7 \times 10^{-27}) =$ $\approx 4 \times 10^{-13} \times (\rho_{Fe}/\rho_{H2O}) \times (\ell / 1\ \text{m}) =$ $\approx 3.2 \times 10^{-11}$.

- i.e. we need 30 billions ν's, in order to get one interaction in 10 meters of iron !
- Other used quantities : $\lambda_{int} = M / (\rho\sigma)$ = interaction length, the length of material to be traversed by a beam, to have a reduction 1/e of its intensity [*compute it in our case*].



The ν beam : schema



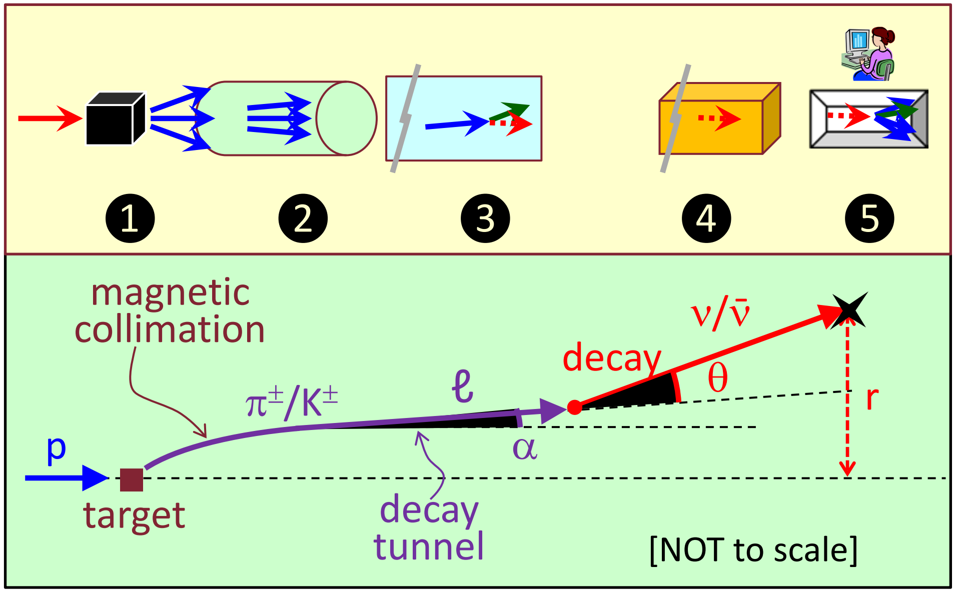
p	target	π,K,…	collection	decay tunnel	absorber	detector(s)
450 GeV	(Cu, Be)		+ magnetic selection			
~10^{13} /cycle		~10 /p	(WB / NB)	$\pi^\pm/K^\pm \to \mu^\pm\nu_\mu$	no $\mu^\pm$	10^{11} $\nu_\mu/\bar{\nu}_\mu$ (WB) /cycle

<u>Schematic view of a ν beam (especially CERN WBB/NBB).</u>

[*not to scale*]

[NB a) in all the beam discussion, *mutatis mutandis* "ν" means both "ν" and "$\bar{\nu}$";
b) in this presentation the focus is on beams from CERN SPS: similar beams from PS, Fermilab, Serpukhov]

The ν beam: computation method



The relevant observable is the cross-section σ (or dσ/dΩ). In order to measure it, the experiments need **the flux of incoming ν/ν̄**.

A ν/ν̄ cannot be observed before its interaction ❺. Therefore the flux can only be computed statistically, together with its stat. and syst. uncertainties. The ingredients are:

❶ the inclusive differential cross sections of the $\pi^\pm$ and $K^\pm$ production in the target;

❷ the collection and collimation of $\pi^\pm/K^\pm$;

❸ the distribution of the decay length $f(\ell)$;

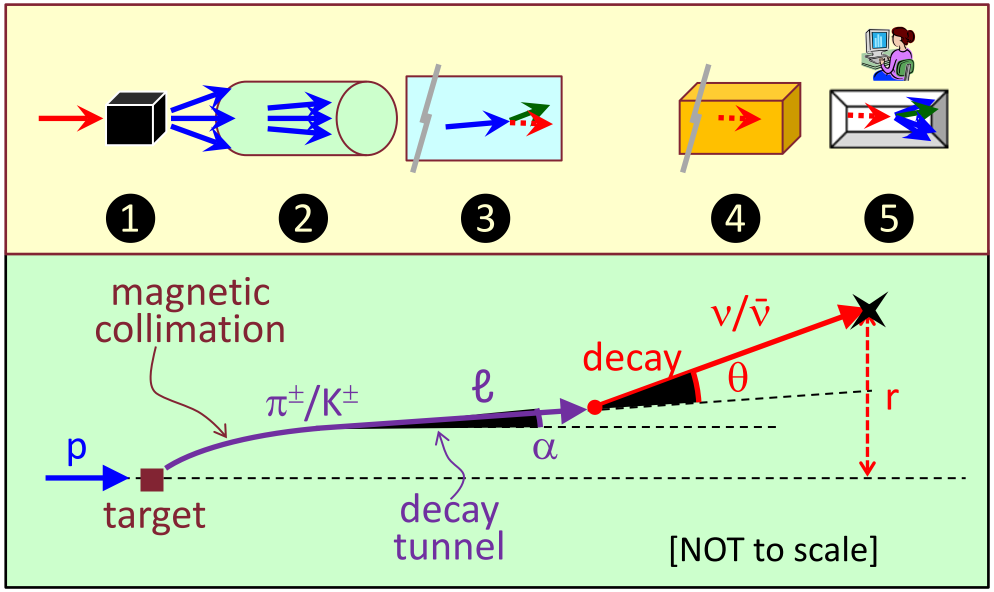
❹ the distribution of the ν/ν̄ decay angle $f(\theta^*)$ [boost $\pi^\pm/K^\pm$ CM system → lab];

Using all these distributions, the flux, as a function of the ν/ν̄ angles, energy and positions, is numerically computed, usually with a MC, and used in the analysis.

In the next slides some of these features will be examined.

despite all the efforts, in ν data analysis the beam is "the" problem. (Almost) all the systematics, mistakes, discussions, fights, come from the wrong control of the beam.

The ν beam : details of the method



Some details:

- the statistical distribution of ❶ and ❷ can be directly measured;
- the momentum distribution of $\mu^\pm$ from $\pi^\pm/K^\pm$ decay can be computed and checked using their measurement in the decay and absorber tunnels; the $\nu/\bar\nu$ flux is then inferred;
- the collection and collimation system ❷ may use different stategies (an option for the user):
  - "**wide band beam**" (WBB): more intense beam, but not "monochromatic" (π/K collection with high acceptance, e.g. van der Meer horn);
  - "**narrow band beam**" (NBB): more monochromatic and higher energy, but much less intense (standard $\pi^\pm/K^\pm$ selection);

  in practice, both beams are optimized for different physics measurements;
- $f(\ell)$ and $f(\theta^*)$ can be analytically calculated and boosted to the LAB system, using β,γ $[\beta=|p_{\pi/K}|/E_{\pi/K}$, $\gamma=E_{\pi/K}/m_{\pi/K}]$ and the lifetimes $\tau_{\pi/K}$;
- many more subtleties, e.g. rare $\pi^\pm/K^\pm$ decays, punch-throughs, … are included in the computations.

The ν beam : $\pi^\pm/K^\pm$ decays

- Only beams of ν_μ (or $\bar{\nu}_\mu$) can be created: ν_e (or $\bar{\nu}_e$) are small contaminations (e.g. from K^+_{e3} decays);
- the ν's are not directly measurable → some info about their 4-momentum comes from the kinematics of the decay of the $\pi^\pm$'s and $K^\pm$'s ($\pi^\pm$ / $K^\pm \to \mu^\pm\nu_\mu$);
- the $\pi^\pm$ ($K^\pm$) has spin 0 → in its CM-frame isotropic decay (φ*, cos θ* flat);
- boost it (β_π, γ_π) to get the longitudinal momentum $p^{//}_\nu$ and its distribution;
- no boost for the transverse momentum $p^\perp_\nu$ distribution.

Results [*see next slides*] :

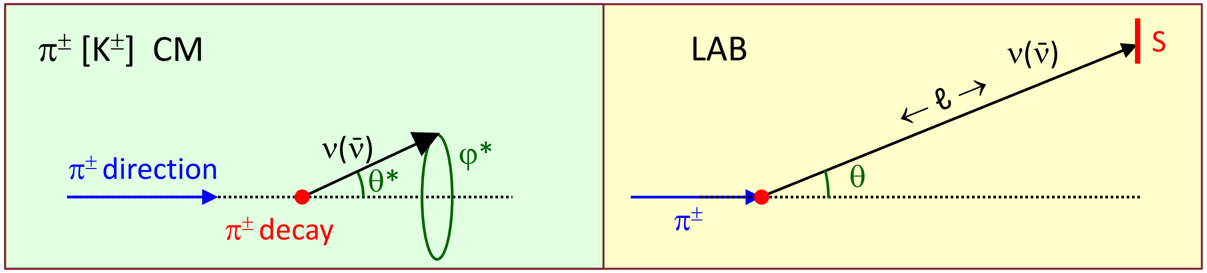
- the angular distribution for a ν, respect to a $\pi^\pm$ of energy $E_\pi = m_\pi\gamma$, is

$$\frac{dn}{d\Omega} \approx \frac{1}{4\pi}\frac{4\gamma^2\left[1+\tan^2\theta\right]^{3/2}}{\left(1+\gamma^2\tan^2\theta\right)^2};$$

[Kopp, Phys. Rep. 439, 101]

- therefore, a detector of surface S, positioned at distance ℓ and angle θ, sees a flux ϕ of ν's :

$$\phi \approx \frac{S}{4\pi\ell^2}\left(\frac{2\gamma}{1+\gamma^2\theta^2}\right)^2.$$



The ν beam : decay kinematics

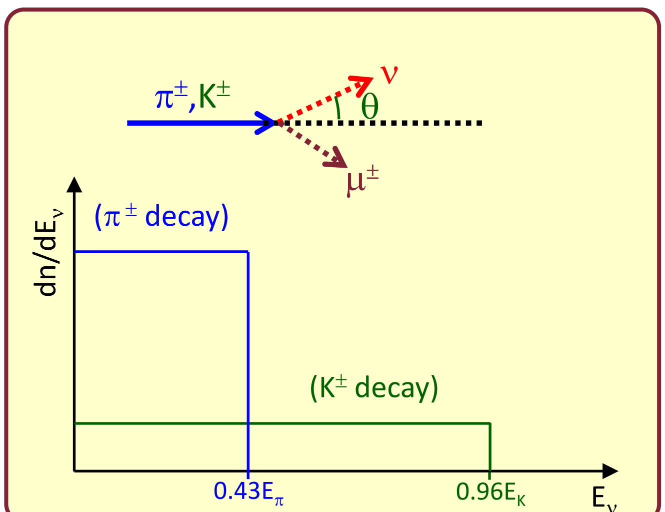
Kinematics is simple :

- since the $\pi^{\pm}$ have spin 0, the $(\nu\mu)$ distribution in the CM system is flat;

→ the momentum of the ν's in the LAB has a (roughly) flat distribution;

→ the distribution ranges between $E_\nu^{min} \approx 0$ and $E_\nu^{max} = 0.43\ E_\pi$.

- [for $K^{\pm}$ decay, the same formula gives a higher maximum : $E_\nu^{max} = 0.96\ E_K$]



$$\text{CM:}\begin{cases}\pi: & (\ m_\pi, & 0, & 0\ )\\ \nu: & (\ p^*, & p^*\cos\theta^*, & p^*\sin\theta^*)\\ \mu: & (m_\pi - p^*, & -p^*\cos\theta^*, & -p^*\sin\theta^*)\end{cases}$$

$$m_\mu^2 = m_\pi^2 + p^{*2} - 2m_\pi p^* - p^{*2} = m_\pi^2 - 2m_\pi p^* \quad \rightarrow$$

$$p^* = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}; \qquad E^*_\mu = m_\pi - p^* = \frac{m_\pi^2 + m_\mu^2}{2m_\pi};$$

$$p_\nu^{//}\Big|_{LAB} = p\cos\theta = \gamma p^*\cos\theta^* + \beta\gamma p^*;$$

$$\frac{dn}{dp_\nu^{//}\Big|_{LAB}} = \left|\frac{dn}{d\cos\theta^*}\right|\left|\frac{d\cos\theta^*}{dp_\nu^{//}\Big|_{LAB}}\right| = \frac{\text{const}}{\gamma p^*} = \text{const};$$

$$p_\nu^{//}\Big|_{LAB}^{max} = p_\nu^{//}\Big|_{LAB}(\cos\theta^* = 1) = \gamma p^*(1+\beta) \approx 2\gamma p^* =$$

$$= 2\frac{E_\pi}{m_\pi}\frac{m_\pi^2 - m_\mu^2}{2m_\pi} = E_\pi\frac{m_\pi^2 - m_\mu^2}{m_\pi^2} = E_\pi\left(1 - \frac{m_\mu^2}{m_\pi^2}\right).$$

$$p_\nu^{//}\Big|_{LAB}^{min} = p_\nu^{//}\Big|_{LAB}(\cos\theta^* = -1) = \gamma p^*(\beta - 1) \approx 0.$$

$$p_\nu^{\perp} = p^*\sin\theta^* = \mathcal{O}(m_\pi) \ll p_\nu^{//}\Big|_{LAB}^{max} \approx E_\nu\Big|_{LAB}^{max} = \mathcal{O}(E_\pi).$$

The ν beam : dn/dΩ

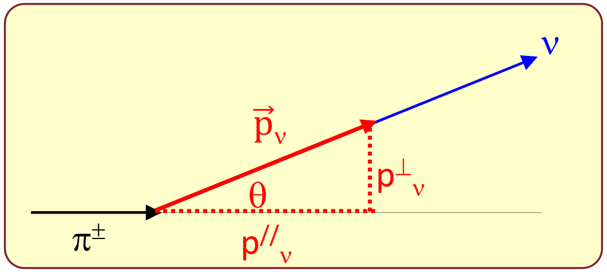
Moreover :

- 2-body decay;
- in the CM (p^*, Ω^*, θ^*), the angular distribution is flat (=1/4π);
- in the LAB (p, Ω, θ), boost β,γ;
- long, but simple (see box) :

$$\frac{dn}{d\Omega} = \frac{dn}{d\Omega^*}\left|\frac{d\Omega^*}{d\Omega}\right| = \frac{dn}{d\Omega^*}\left|\frac{d\cos\theta^*}{d\cos\theta}\right| =$$

$$= \frac{dn}{d\Omega^*}\left|\frac{d\cos\theta^*}{d\tan^2\theta}\right|\left|\frac{d\tan^2\theta}{d\cos\theta}\right| =$$

$$\approx \boxed{\frac{1}{4\pi}\frac{4\gamma^2\left[1+\tan^2\theta\right]^{3/2}}{\left(1+\gamma^2\tan^2\theta\right)^2}}.$$



$$p_\nu^\perp = p_\nu \sin\theta = p^*\sin\theta^*;$$

$$p_\nu^{//} = p_\nu\cos\theta = \gamma\left(p^*\cos\theta^* + \beta E^*\right) \approx \gamma p^*\left(\cos\theta^* + 1\right);$$

$$p_\nu^\perp / p_\nu^{//} = \tan\theta = \sin\theta^* / \left[\gamma\left(1+\cos\theta^*\right)\right];$$

$$\gamma^2\tan^2\theta = \left(\frac{\sin\theta^*}{1+\cos\theta^*}\right)^2 = \frac{1-\cos^2\theta^*}{\left(1+\cos\theta^*\right)^2} = \frac{1-\cos\theta^*}{1+\cos\theta^*};$$

$$\boxed{b = \frac{1-a}{1+a} \to b+ab = 1-a \to a = \frac{1-b}{1+b}} \to \cos\theta^* = \frac{1-\gamma^2\tan^2\theta}{1+\gamma^2\tan^2\theta};$$

$$\frac{d\cos\theta^*}{d\tan^2\theta} = \frac{-\gamma^2}{1+\gamma^2\tan^2\theta} - \frac{\gamma^2\left(1-\gamma^2\tan^2\theta\right)}{\left(1+\gamma^2\tan^2\theta\right)^2} =$$

$$= \boxed{\frac{-2\gamma^2}{\left(1+\gamma^2\tan^2\theta\right)^2}};$$

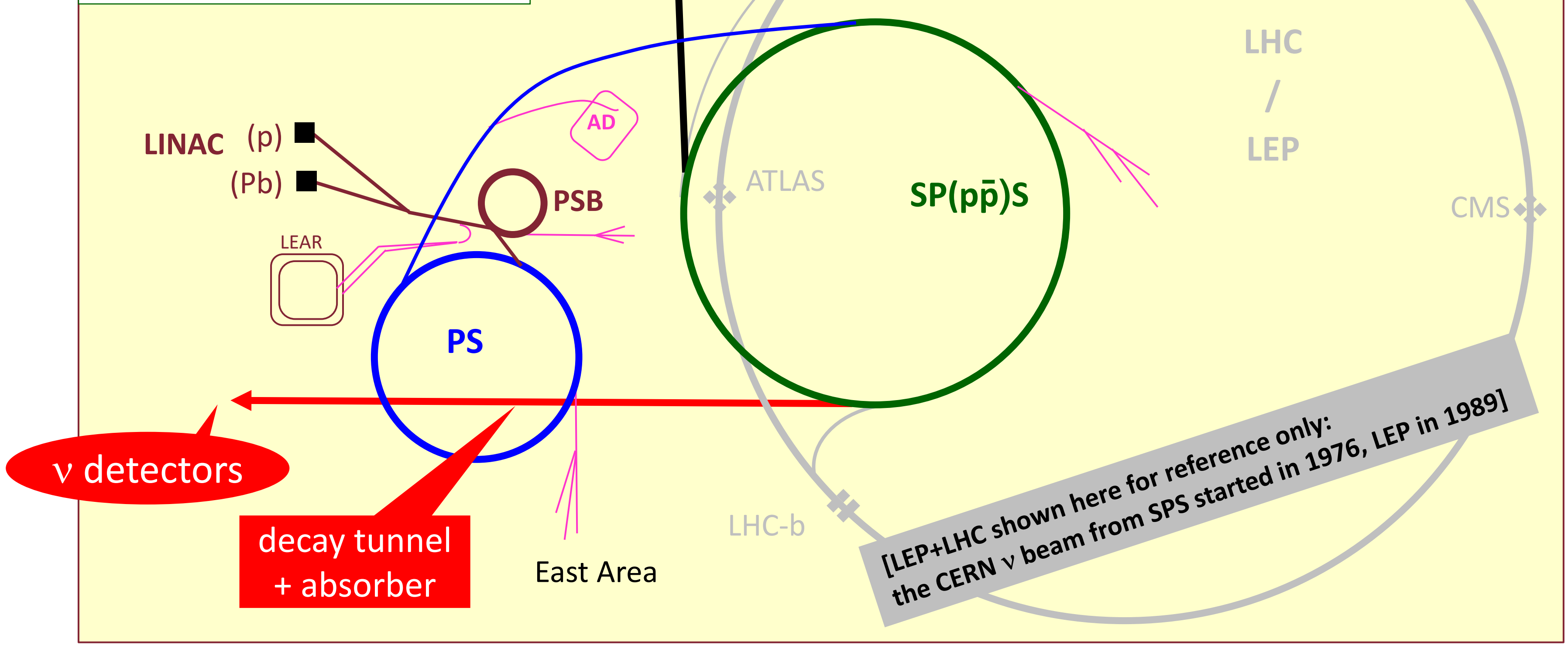
$$\boxed{\frac{1}{\cos^2} = \frac{\cos^2+\sin^2}{\cos^2} = 1+\tan^2}$$

$$\frac{d\tan^2\theta}{d\cos\theta} = \frac{d}{d\cos\theta}\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) = \frac{d}{d\cos\theta}\left(\frac{1}{\cos^2\theta} - 1\right) =$$

$$= -2/\cos^3\theta = \boxed{-2\left(1+\tan^2\theta\right)^{3/2}};$$

The ν beam : CERN accelerators

protons to SPS/LHC :
- Linac2 (0-50 MeV);
- PSB (→ 1.4 GeV);
- PS (→ 28 GeV);
- SPS (→ 450 GeV);
- LHC (→ 7 TeV).



The ν beam : CERN SPS

The accelerator : as an example, the Super Proton Synchrotron (SPS) at CERN, which (today) accelerates ~5 × 10^{13} protons per cycle to an energy E_p = 450 GeV.

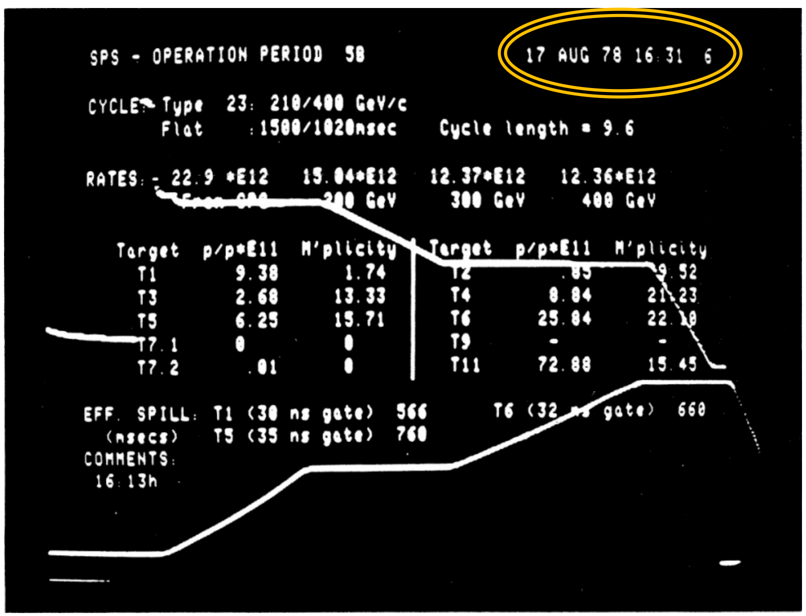
The proton beam is extracted and sent to a target (copper, beryllium, graphite). The average secondary multiplicity is ~10 charged, with energies from 10 to 100 GeV.

The secondaries ($\pi^\pm$, $K^\pm$) are focused and let decay.

The focusing is a compromise: resolution [ideally a monochromatic ν beam] vs intensity [as many ν's as possible].

A good solution is the WBB beam, where a "Van der Meer horn" selects with good acceptance $\pi^\pm$ and $K^\pm$, with given sign :

- +ve for a ν beam from $\pi^+/K^+ \rightarrow \mu^+\nu_\mu$,
- −ve for a $\bar{\nu}$ beam.



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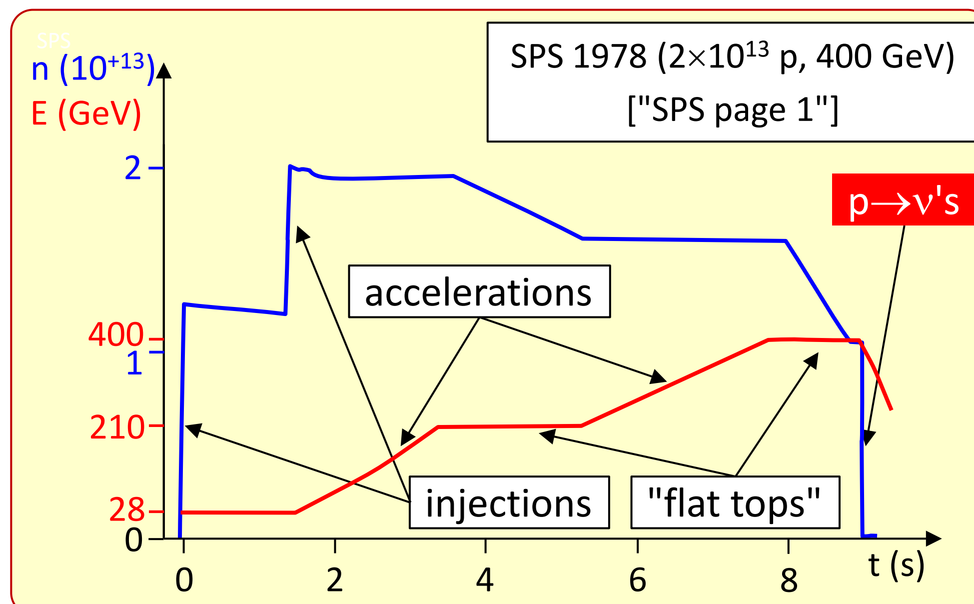
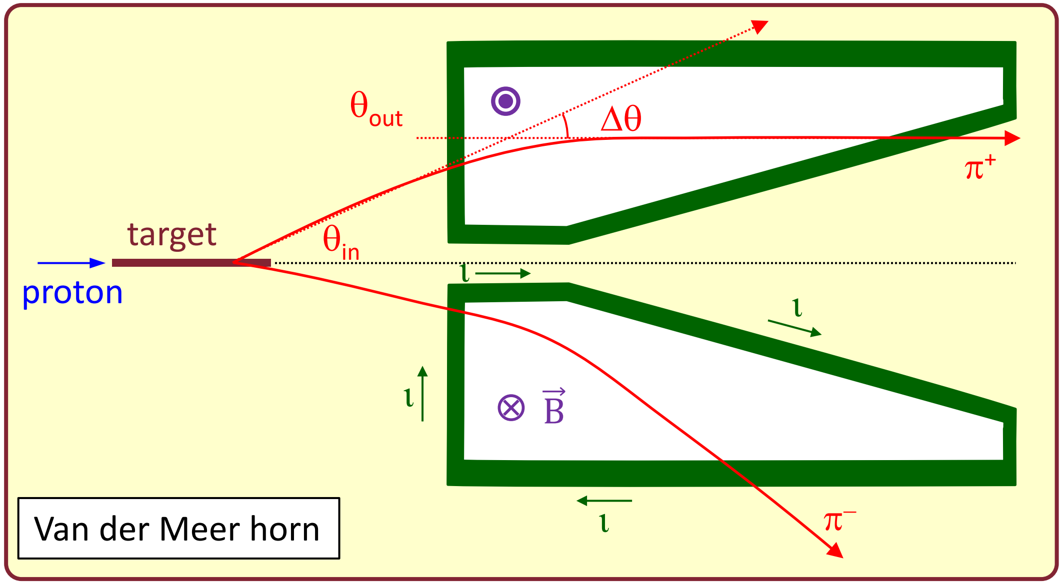


The ν beam : the horn in the WBB

- The Van der Meer horn consists in a magnet, pulsed with currents (up to 100 kA), positioned just after the target.
- It collimates particles of a given sign (π^+,K^+ in the scheme) and sweeps away the opposite charge (π^-, K^-). Multi-horn setups have also been built.
- The direction of the current in the horn(s) **selects a beam of $\nu_\mu \leftrightarrow \bar{\nu}_\mu$** : $(\pi^+ \to \mu^+ \nu)$ vs $(\pi^- \to \mu^- \bar{\nu})$.



Imho, one of the two great contributions of SVdM to particle physics (he got the Nobel prize for the other).

The ν beam : decay tunnel

In the decay tunnel $\pi^{\pm}$'s and $K^{\pm}$'s decay.

The length of the tunnel is a compromise between cost and intensity : it should be about the average decay length.

→ In the laboratory frame :

$$\ell = \beta\gamma c\tau = p\,c\,\tau\,/\,m.$$

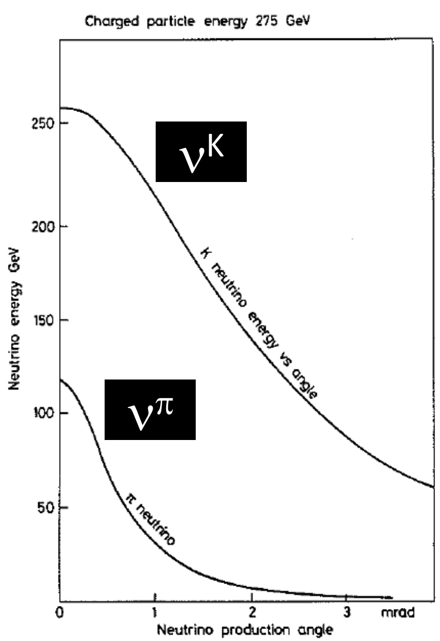
E.g. for 50 GeV π^+, [$c\tau(\pi^+)$ = 7.8 m] :

$$\ell = 50 \times 7.8\,/\,.140 = 2800 \text{ m}.$$

(in reality the tunnels are only few×100 m).

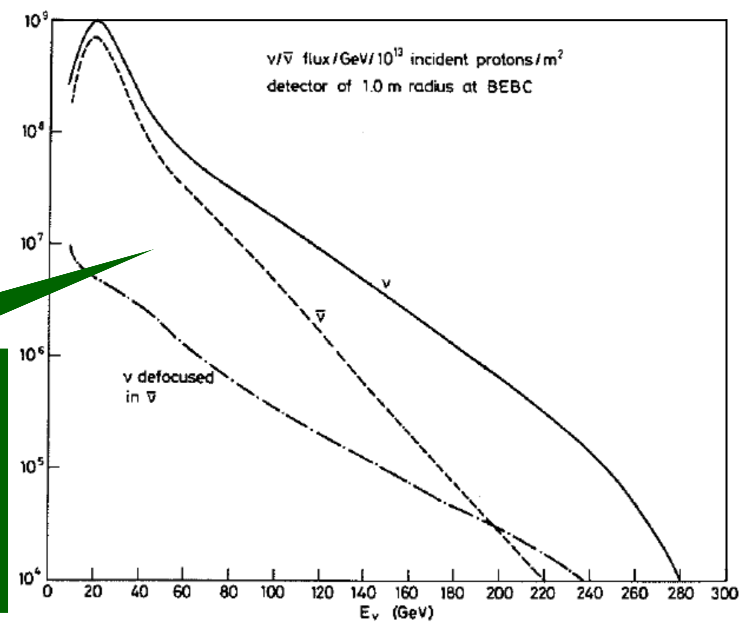
The figures show :

- the angle between the ν and its parent (i.e. the additional angular spread of the beam due to the decay), for ν originating from K or π (ν^K and ν^π);
- the energy distribution of the ν and $\bar\nu$ beams for 10^{13} protons on target.



Q. Why more ν than $\bar\nu$?

A. No exotic motivation, but the initial state pN is +ve, so π^+ are more abundant than π^-.



The ν beam : the μ's absorber

The Absorber : the detectors must obviously get ONLY ν's and NOT the μ's (initially as many as ν's), π's and K's (few, but not zero).

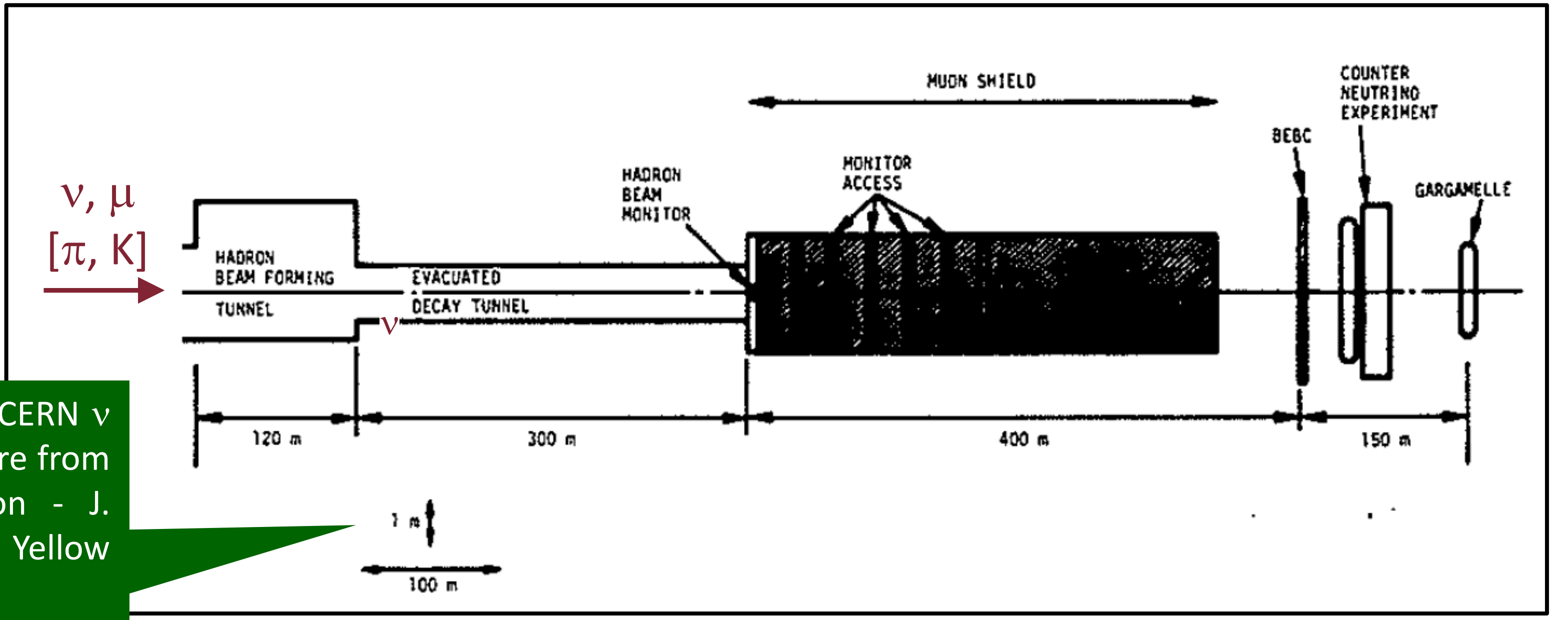
Therefore a thick absorber is positioned at the end of the decay tunnel.

At the CERN SPS it was made with 185 m iron + 220 m rock.

As an exercise, compute the range in iron for a high energy μ. From the numerical integration of the function

$$E = \int_0^{range} (dE/dx)dx :$$

E_μ (GeV)	range(Fe)	range(rock)
100 GeV	56 m	156 m
500 GeV	180 m	583 m



The setup of the CERN ν beam [a dark figure from a clear discussion - J. Steinberger, CERN Yellow Report 76-20].

The ν beam : conclusions

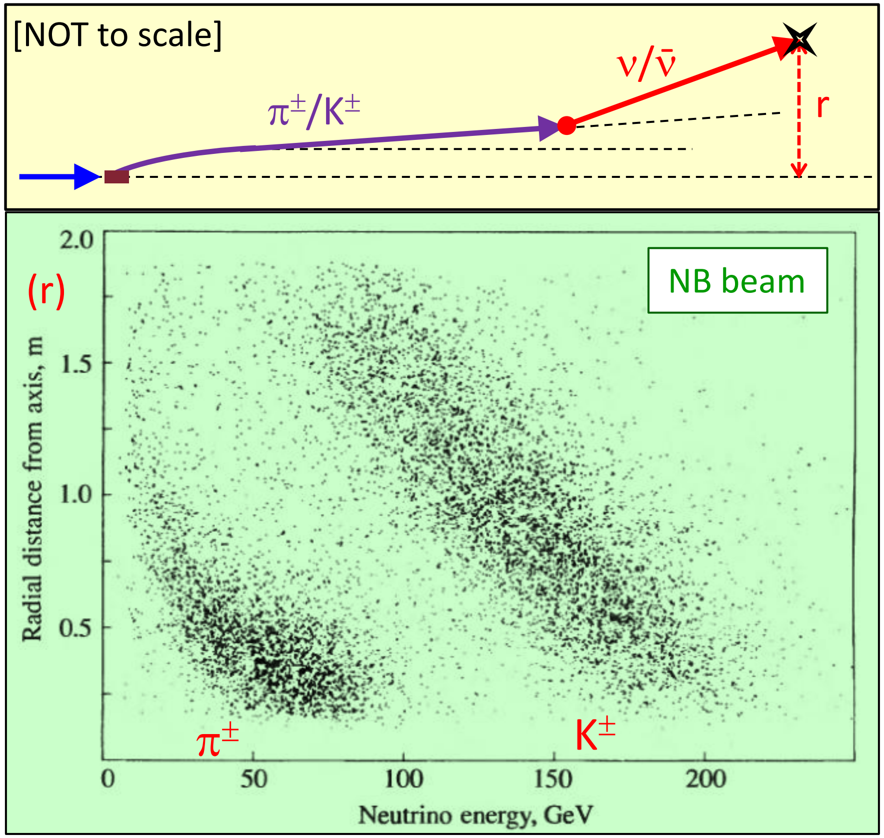
The table and the plot summarize the main performances of the two CERN beams :

- for WBB the relative contaminations:

	WBB beam	
	ν_μ	$\bar{\nu}_\mu$
ν_μ	91%	15%
$\bar{\nu}_\mu$	8%	84%
ν_e	1%	0.4%
$\bar{\nu}_e$	0.1%	0.7%

E.g., it means : you think you have built a WBB $\bar{\nu}_\mu$ beam, but actually you have only 84% $\bar{\nu}_\mu$, plus 15% ν_μ, 0.40% ν_e, 0.70% $\bar{\nu}_e$.

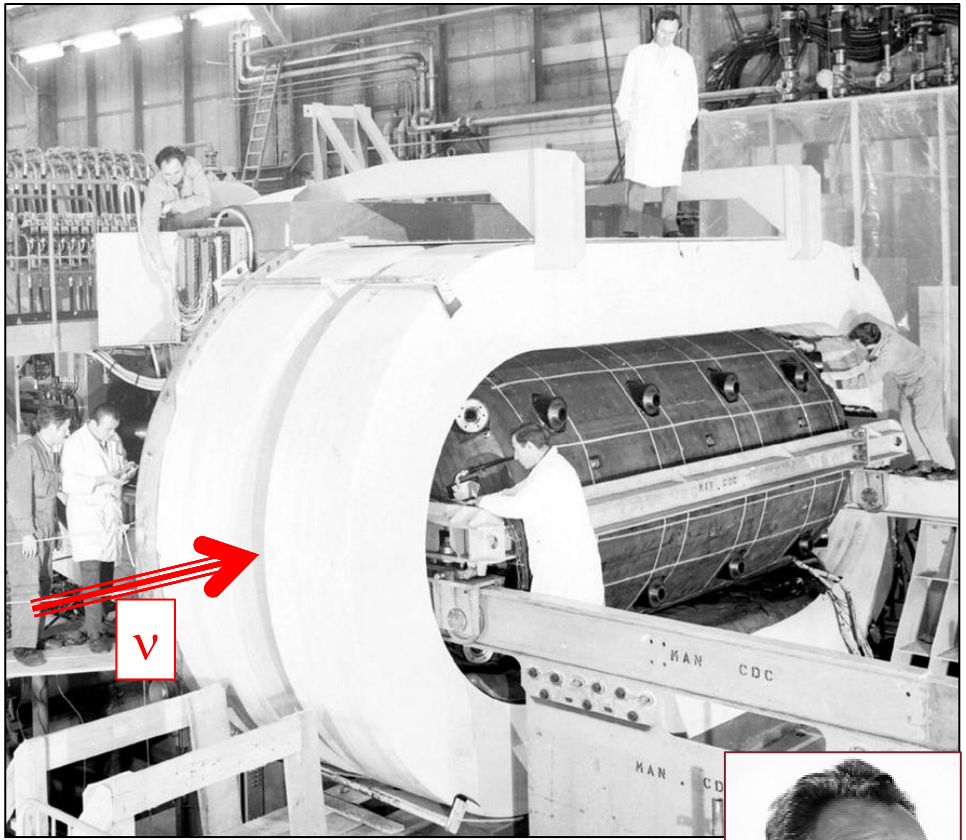
- for NBB the relation between the radial distance (r) of the impact point in the detector (P) and the ν energy allows for a determination of the ν energy with a certain resolution, and little π/K ambiguity.



The ν detectors: Gargamelle

The ν detectors are of different types, but have to share common characteristics :

- big size (detect small cross sections);
- good lepton identification (CC vs NC);
- meas. of the hadronic shower (NC);
- rate capability is NOT a bonus, due to the small number of events.
- traditionally, the best ν detectors were heavy liquid bubble chamber, filled with (**freon CF_3BR**, Ne, propane), and embedded in a strong magnetic field.
- Gargamelle is one of the first and most glorious of them : "she" discovered the neutral currents [*many thanks to her "father" A. Lagarrigue*].

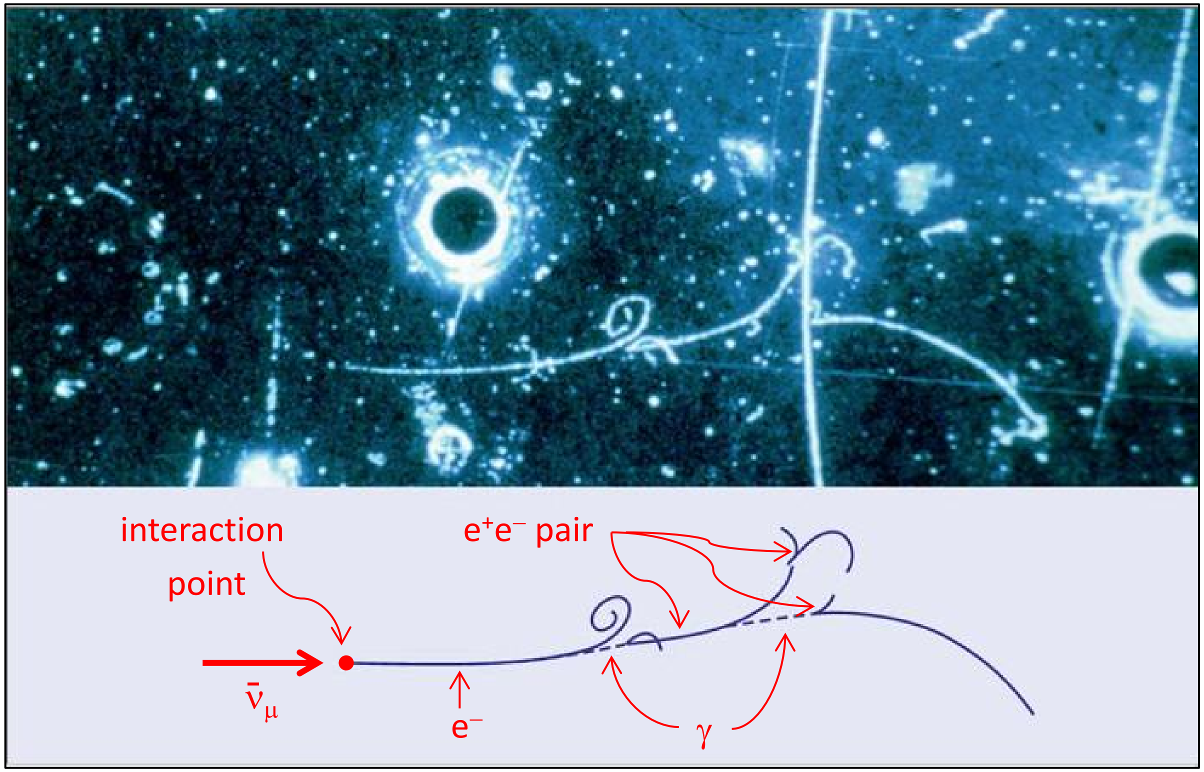


André Lagarrigue (1924-1975)

Notice :

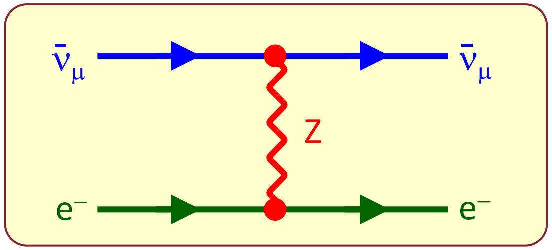
- coils for mag. field generation;
- holes for the cameras;
- big size (for the 70's);
- absence of cryostat;
- ν's enter from the left.

The ν detectors: Gargamelle



Gargamelle discovery of NC [1973] - the famous event:

- the key point is the e⁻ identification, via its brem(s);
- ... and the <u>absence</u> of anything else (especially a $\mu^\pm$ candidate);
- the event was interpreted as a purely leptonic NC process [$\bar{\nu}_\mu\, e^- \rightarrow \bar{\nu}_\mu\, e^-$].



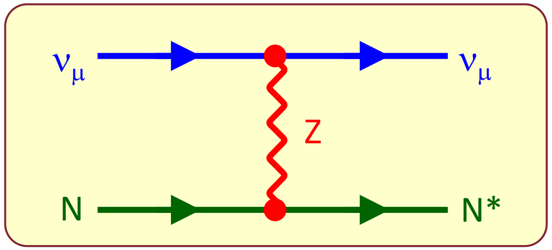
The ν detectors: Gargamelle

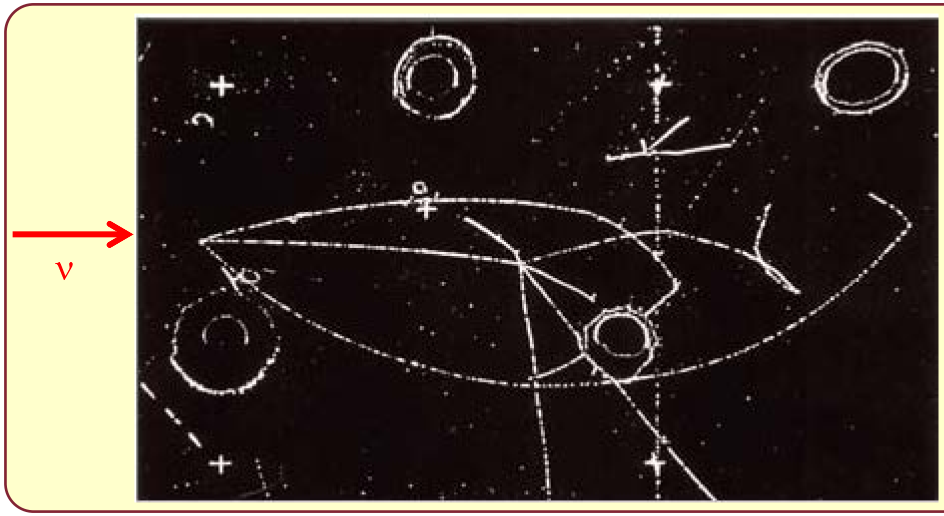
Gargamelle discovery of NC.

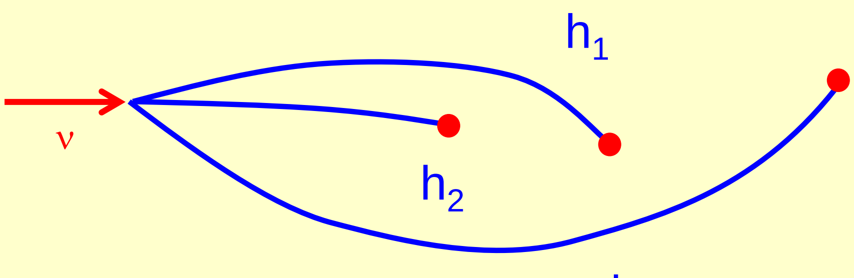
A beautiful hadronic neutral current event, where the interaction of the neutrino coming from the left produces three secondary particles, all clearly identifiable as hadrons, as they interact with other nuclei in the liquid. There is no charged lepton (muon or electron).

(D.Cundy, CERN Courier)

this is the key point







$\nu\, N \rightarrow [\nu]\, h_1\, h_2\, h_3\, [N^*]$

The ν detectors: BEBC

In ≥ 1976 the CERN SPS was operational : new ν beam, higher energy, new detectors.

BEBC (Big European Bubble Chamber) :

- cryostatic (H_2, D_2, Ne, mixtures) [*cryo not shown*];
- giant solenoid around (not shown); at the time the largest superconducting coil in the world;
- millions of frames : extensive studies of exclusive processes (see next slide)

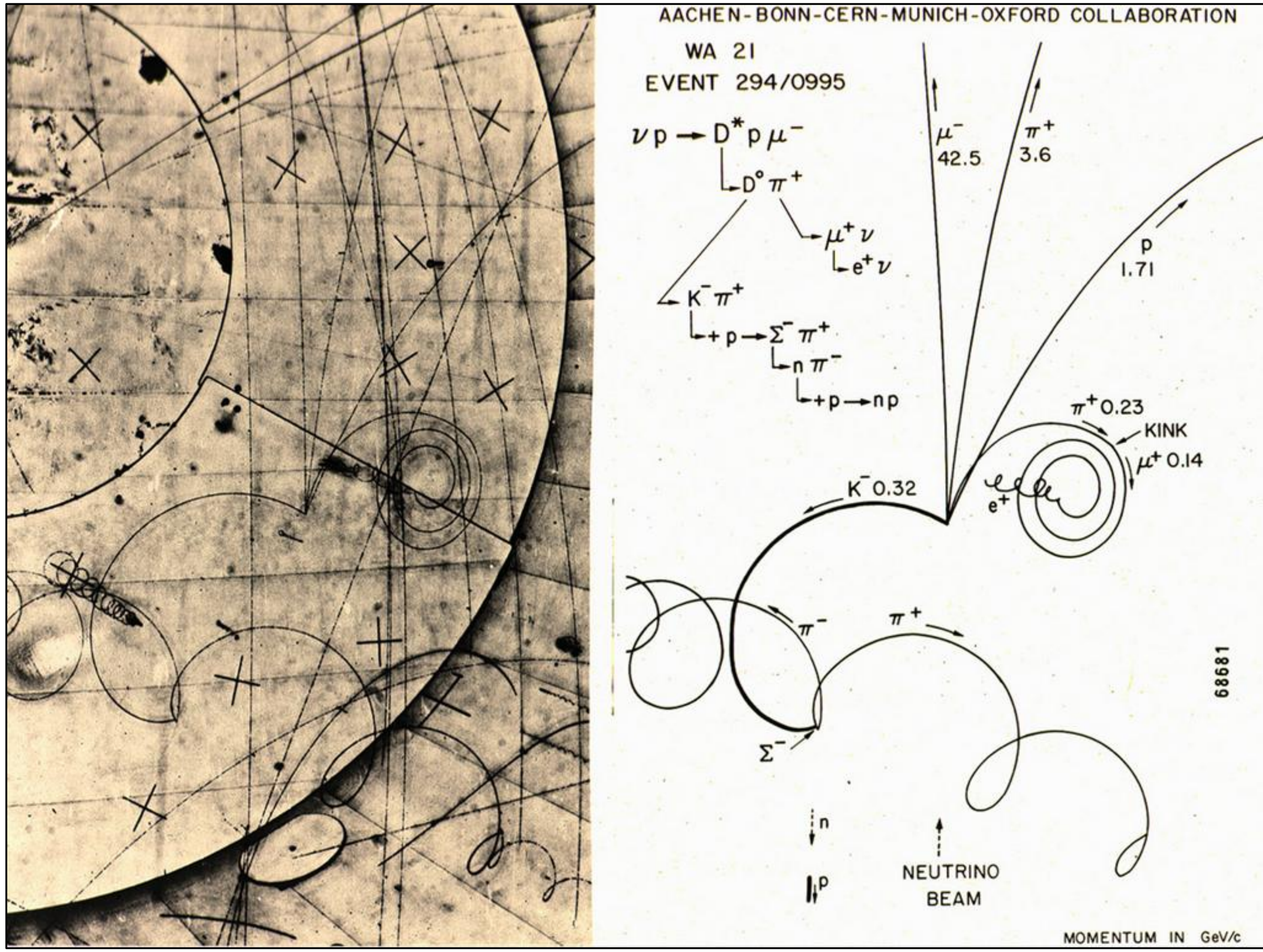
Curiosity : in 1977, an emulsion stack in front, to act as a target; aim : select and measure charm production in ν interactions, and subsequent decays, by identifying the decay vertex;

- first direct identification of charmed mesons and baryons; first measurement of their lifetime;
- Spokesman : Marcello Conversi [*believe me, it was a lot of fun*].

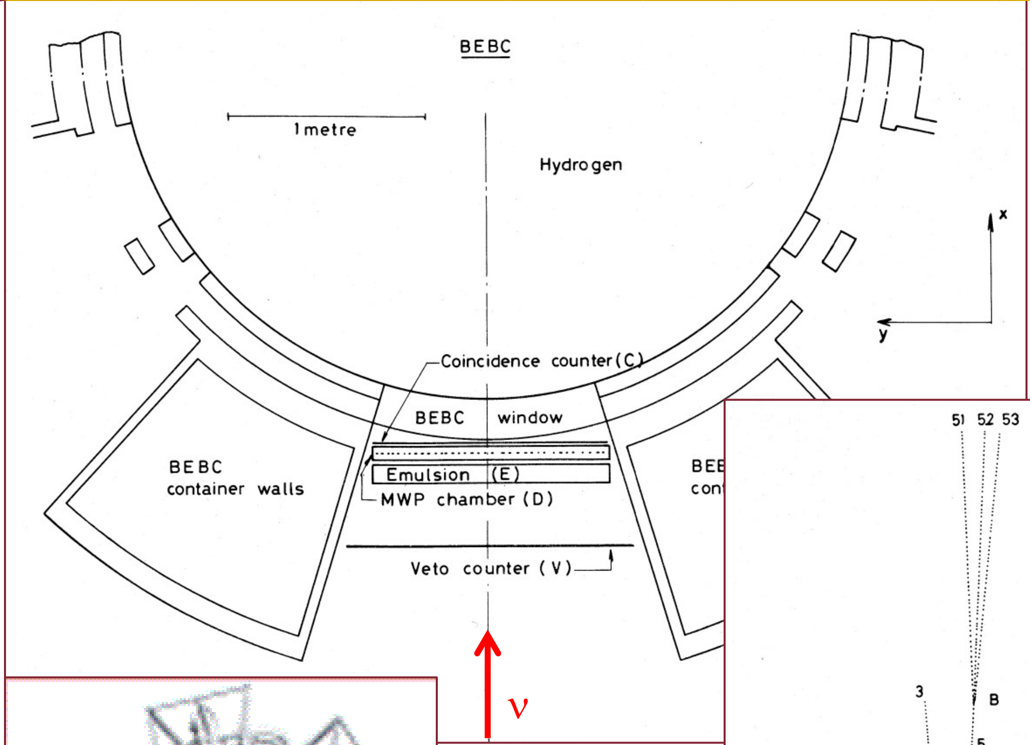
The ν detectors: BEBC

A beautiful charm event inside BEBC :

- very clear;
- 4 photo / event (at different angles → 3D reconstruction);
- momenta / charges measured by the mag. deflections;
- $e^{\pm}$ via energy loss;
- $\mu^{\pm}$ by external device (EMI);
- then, combined masses, kinematical fits, … fun.



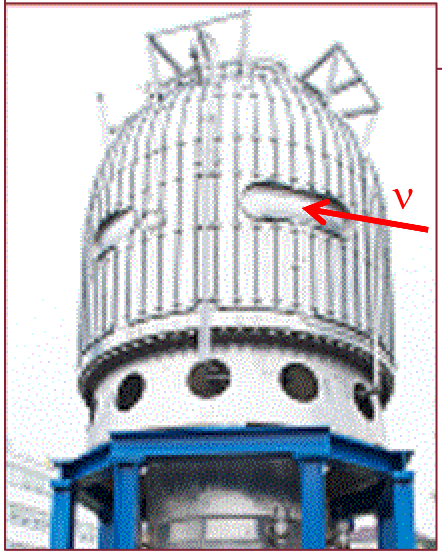
The ν detectors: BEBC + emulsions

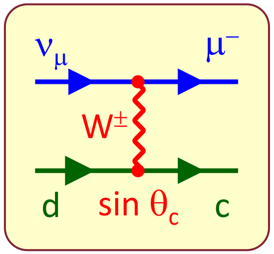


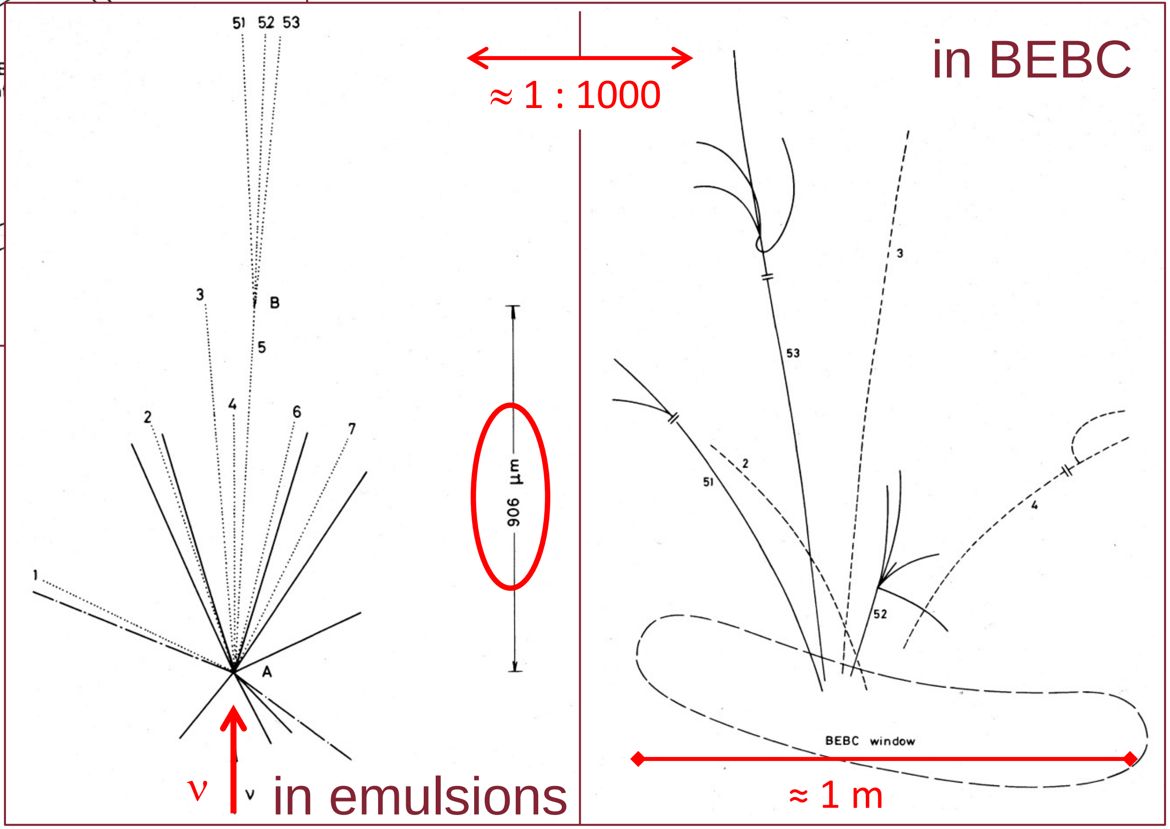
An event in the Conversi experiment;

interpreted as $D^+ \to \pi^+\pi^+\pi^-\bar{K}^0$;

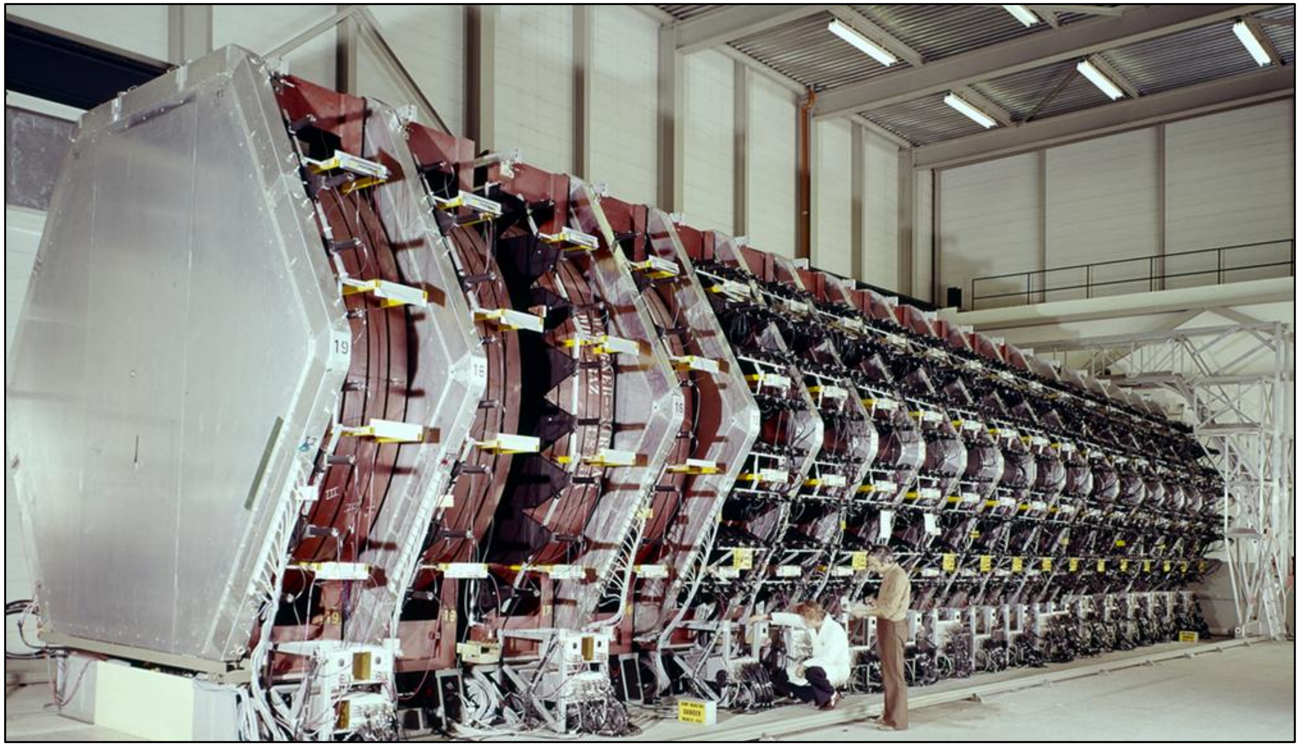
$t(D^+)$ = 2 (or 4) × 10^{-13} s [two kin. solutions].







The ν detectors: CDHS



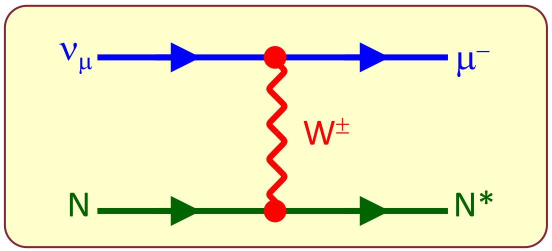
The lion share went to two electronic calorimeters :

- **CDHS** (J. Steinberger et al.), a sandwich of magnetized iron disks and scintillator planes;
- [ν's from the left];
- huge mass, great $\mu^{\pm}$ identification via the iron absorbers;
- almost all the measurement which we will discuss in the next slides are from it.

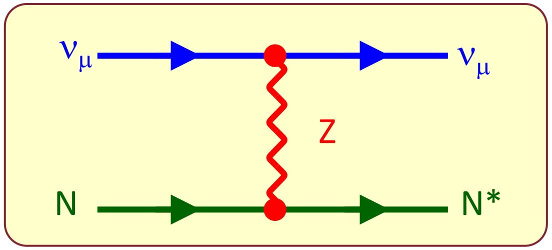
The ν detectors: CDHS events

Display of two events in CDHS :

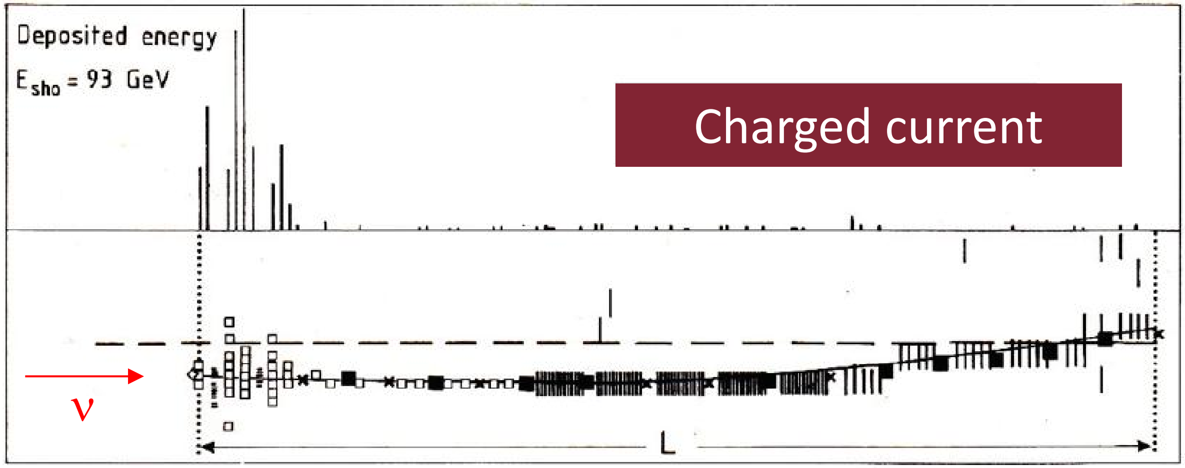
- ν ($\bar{\nu}$) from the left;
- upper event, interpreted as CC (early hadronic shower + penetrating μ^-);

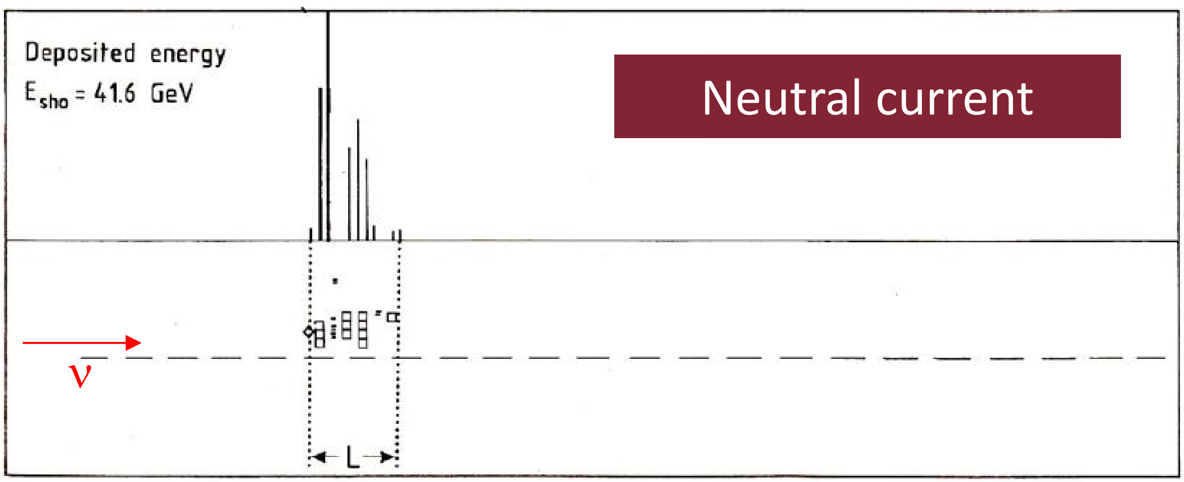


- lower event is a NC (no μ);



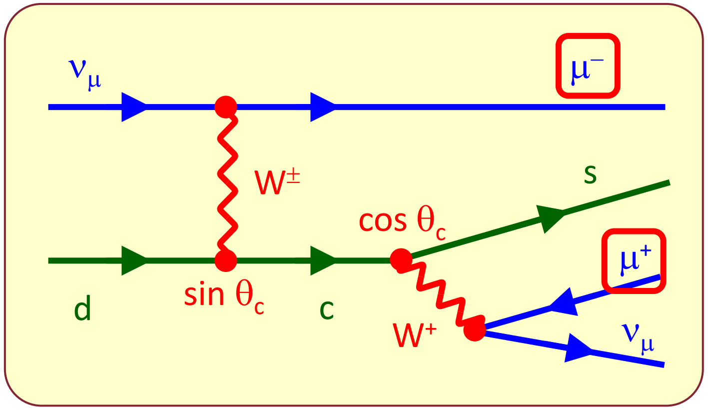
- notice the $E_{sho[wer]}$ measurement.



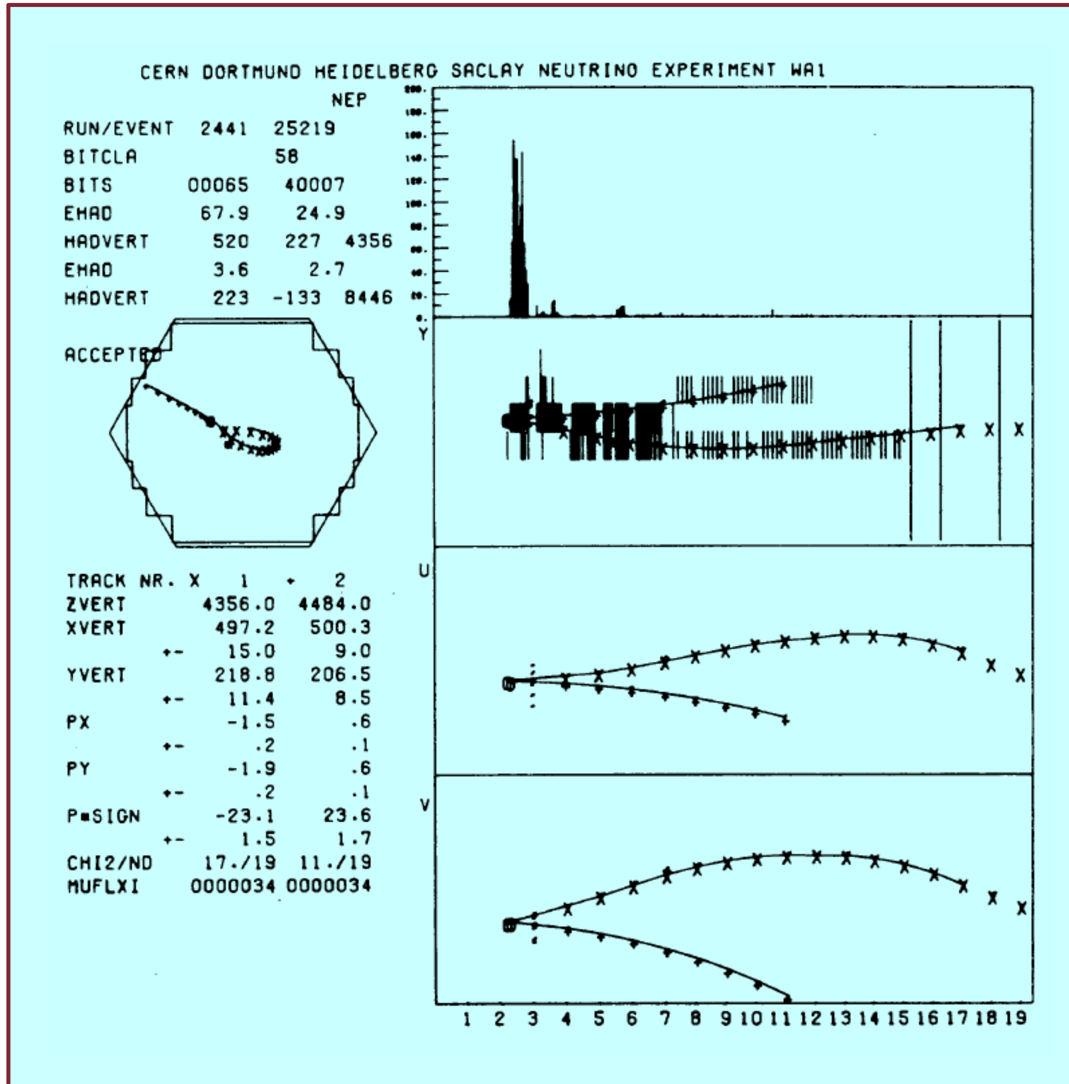


The ν detectors: CDHS 2μ

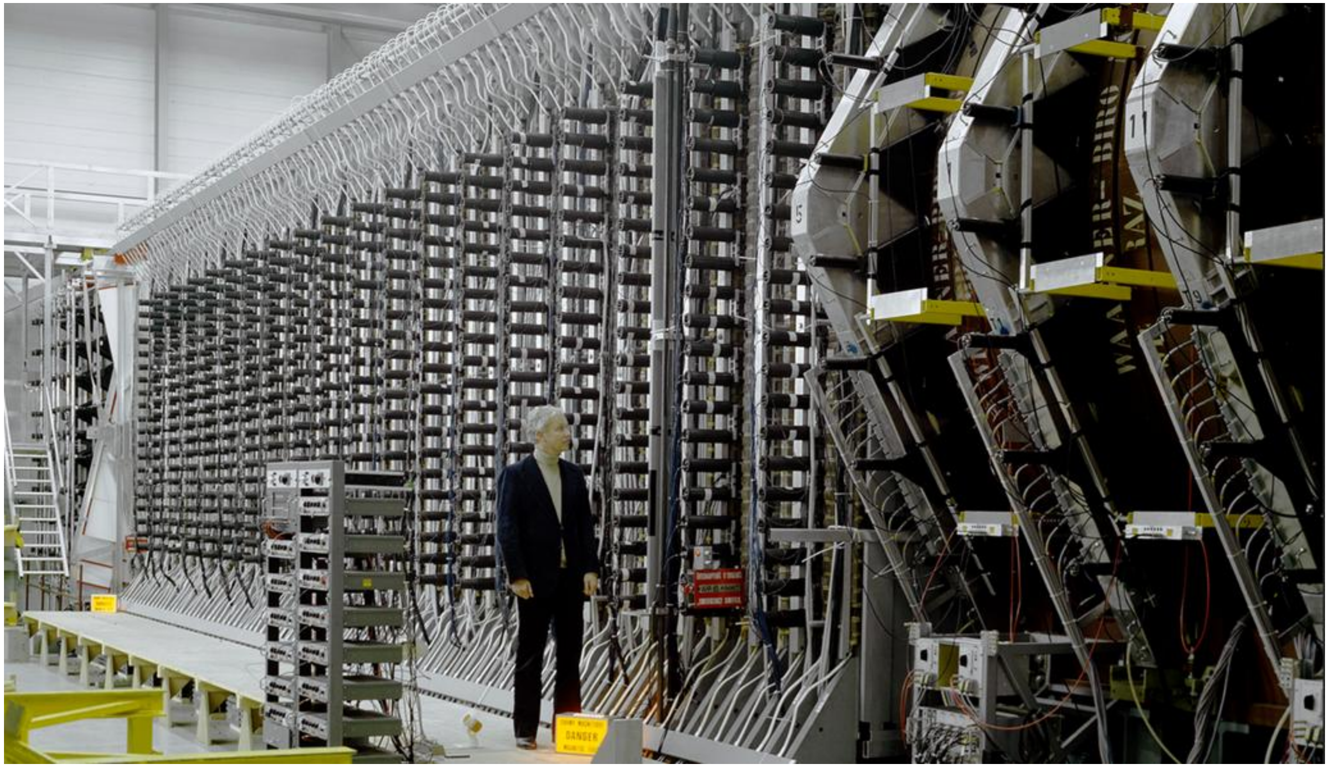
An "**opposite sign dimuon**" event in CDHS:



- *today this explanation looks almost trivial;*
- *but many years ago the origin of the "dimuons" was hardly understood, because of the lack of knowledge / confidence in the quark model and Cabibbo theory;*
- *they had an important role in convincing the physics community.*



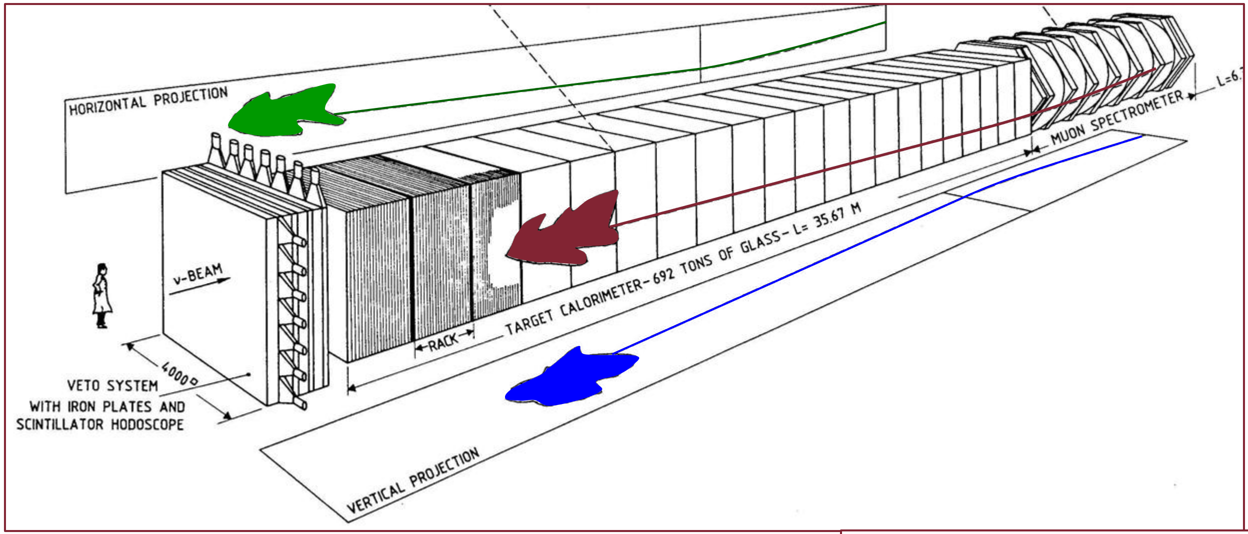
The ν detectors: CHARM



… and this is **CHARM** (CERN-Hamburg-Amsterdam-**Roma**-Moscow) :

- less massive, more granular;
- sandwich of 78 marble planes (1 X_0) + scintillators, drift and streamer tubes;
- almost 100 tonnes in total;
- designed to measure Energy and direction of the hadronic shower;
- ideal for NC.

The ν detectors: CHARM detector



Data taking : 1987-1991 :

2.5×10^{19} p on target →

~ 10^8 ν and $\bar{\nu}$ interactions.

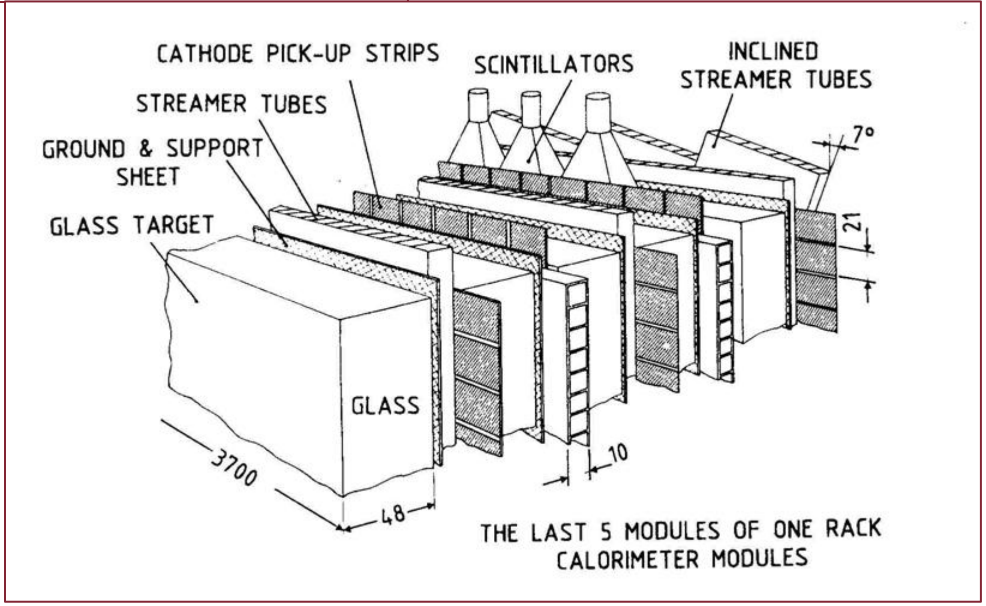
$\langle E(\nu)\rangle$ = 23.8 GeV;

$\langle E(\bar{\nu})\rangle$ = 19.3 GeV.

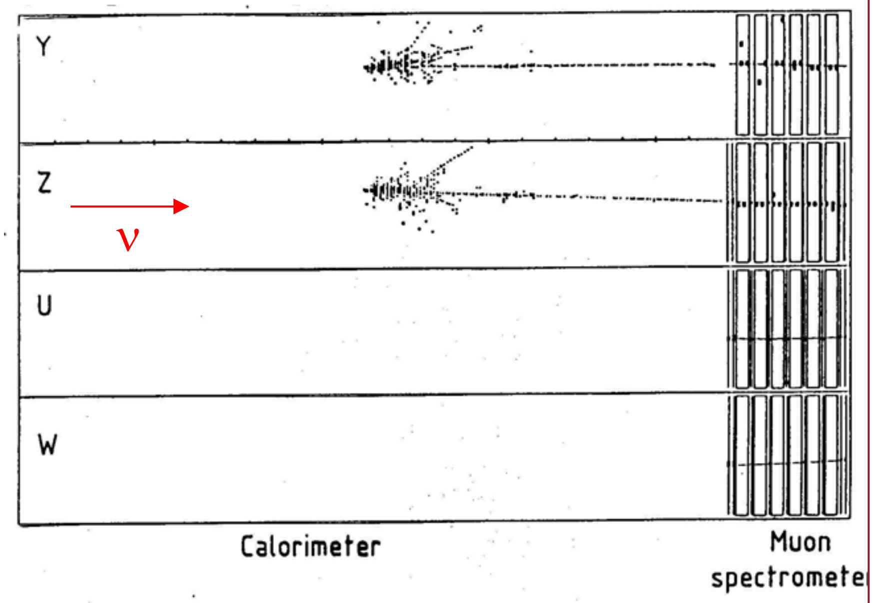
1. large mass: 692 t;
2. good angular resolution, because of low-Z absorber (glass) :

   $\sigma(\theta) / \theta \propto Z \sqrt{E}$
3. granularity for vertex definition (e/π^0 separation) : fine-grained trackers, Iarocci tubes with cells of 1 cm.

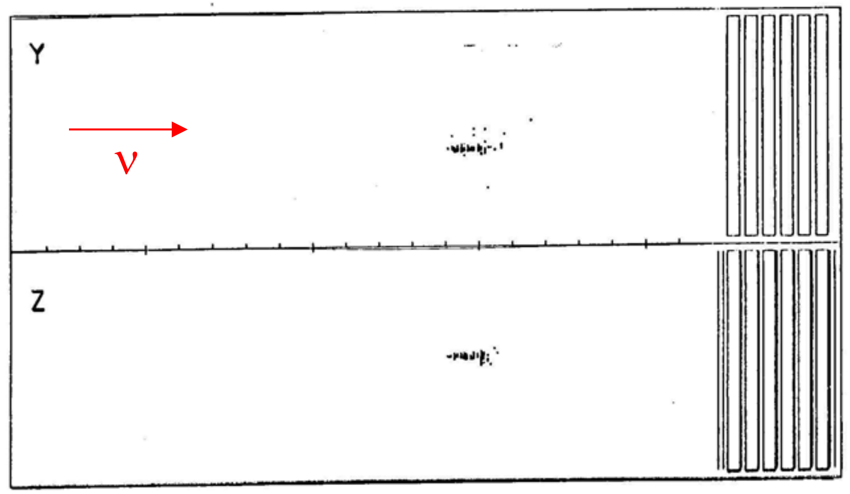
[*tech. detail: in previous page CHARM-1 (marble, ca 1978), while in this page CHARM-2 (glass, ca 1987)*]



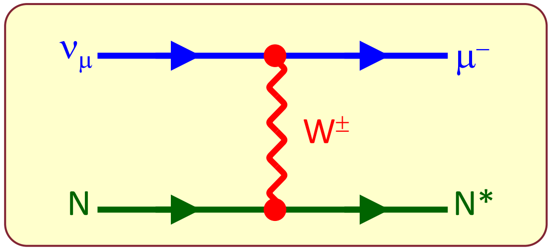
The ν detectors: CHARM event

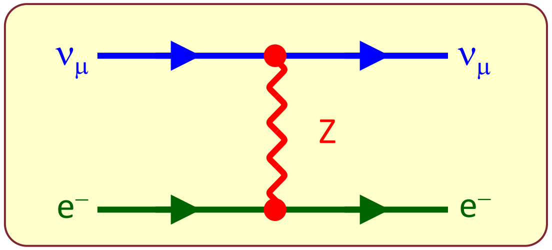


Charged current : $\nu_\mu N \rightarrow \mu^- N^*$

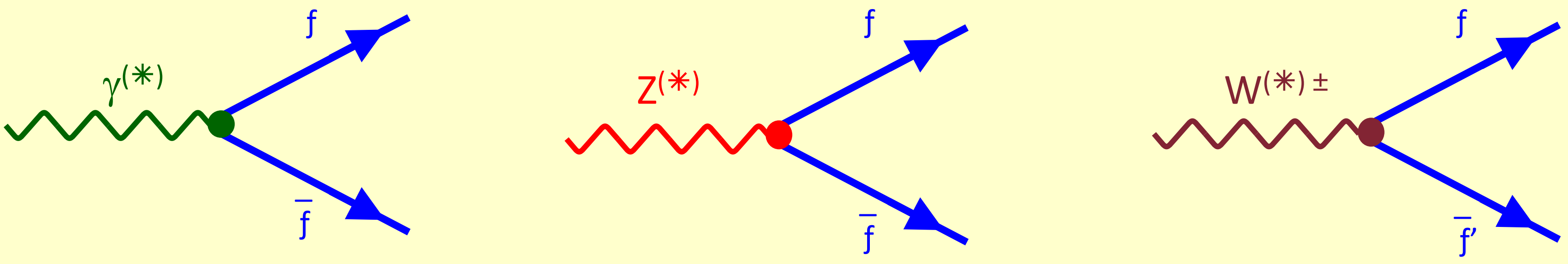


Neutral leptonic current : $\nu_\mu e^- \rightarrow \nu_\mu e^-$





[remember : summary : e.m., NC, CC]



photon (γ) (electromagnetism)

$$\mathcal{L}_F = -eJ^\mu_{e.m.}A_\mu;$$

$$J^\mu_{e.m.} = Q_f\overline{\Psi}_f\gamma^\mu\Psi.$$

[V]

neutral IVB (Z) (neutral current)

$$\mathcal{L}_F = \frac{-e}{\sin\theta_W\cos\theta_W}J^\mu_{nc}Z_\mu;$$

$$J^\mu_{nc} = \overline{\Psi}_f\gamma^\mu\frac{g^f_V - g^f_A\gamma^5}{2}\Psi_f.$$

[combination g^f_V V + g^f_A A]

charged IVB (W±) (charged current)

$$\mathcal{L}_F = \frac{-e}{\sqrt{2}\sin\theta_W}J^\mu_{cc}\tau^\pm W^\pm_\mu;$$

$$J^\mu_{cc} = \overline{\Psi}_f\gamma^\mu\frac{1-\gamma^5}{2}\Psi_f.$$

[V - A]

from § 4

ν interactions : the landscape

How many types of $\nu/\bar{\nu}$ processes exist ?

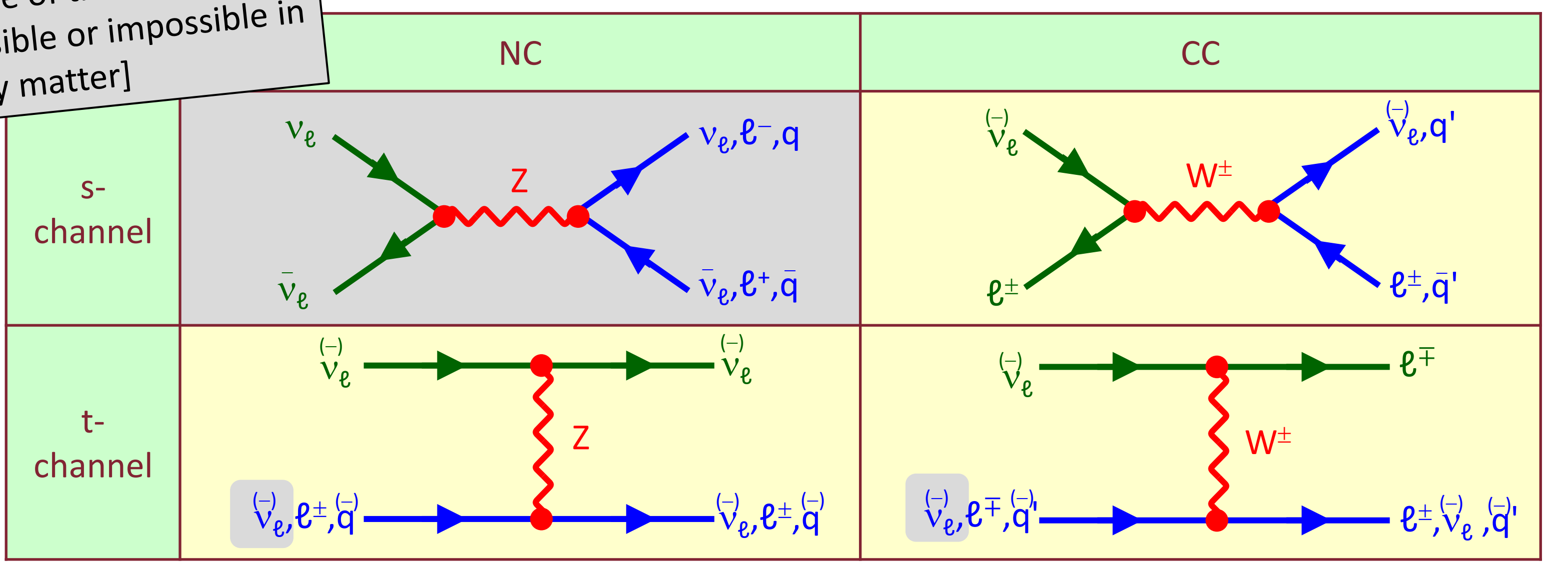
A lot, even in lowest order :

- (NC + CC) × (s-, t-channel);
- for each of them, many lepton replica ($\ell^\pm = e^\pm, \mu^\pm, \tau^\pm$);
- the semi-leptonic case : change only one fermion pair to quarks, i.e. $q\bar{q}$ for NC and $q'\bar{q}'$ for CC (q' is a CKM-rotated quark);
- each q' line counts for three (e.g. a d' is a mixture of dsb, with coefficients given by the CKM matrix).

The key feature of the SM is that all these hundreds of processes reduce to a handful number of coupling constants and charges, which allow to quantify all of them.

E.g.: $\nu_e e^+ \to \nu_\mu \mu^+$ is CC-s;
$\nu_\mu e^\pm \to \nu_\mu e^\pm$ and $\nu_e e^- \to \nu_e e^-$ are NC-t;
$\nu_e e^+ \to \nu_e e^+$ is NC-t ⊕ CC-s.

[NB some of these processes are invisible or impossible in ordinary matter]

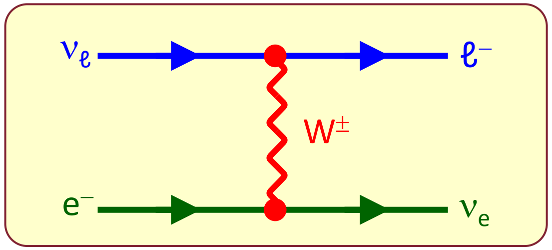


ν interactions : kinematics

An important kinematical constraint.

The threshold energy computation (§ K^0), applied to this case, puts limits on two CC processes :

- the creation of a $\mu^\pm$ requires high energy ν_μ's;
- with present accelerators, no τ's are created, even if the beam contains a ν_τ contamination.



In a process (ab → cd), with b at rest :

$$E_a^{min} = \frac{(m_c + m_d)^2 - m_a^2 - m_b^2}{2m_b}.$$

For $\nu_\mu e^- \to \nu_e \mu^-$:

$m_a \approx m_c \approx 0;\quad m_b = m_e;\quad m_d = m_\mu \quad \to$

$$E_\nu^{min} = \frac{m_\mu^2 - m_e^2}{2m_e} \approx \frac{m_\mu^2}{2m_e} \approx 11 \text{ GeV}.$$

For $\nu_\tau e^- \to \nu_e \tau^-$:

$$E_\nu^{min} \approx \frac{m_\tau^2}{2m_e} \approx 3 \text{ TeV (!!!)}.$$

So, $\nu_\tau e^- \to \nu_e \tau^-$ is NOT possible with present accelerators, even if there is a small number of ν_τ's in a ν_μ beam (from D_s decays) .

CC ν processes

A very simple (possibly the simplest) CC process is the pure lepton scattering ($\nu_\mu e^- \to \mu^- \nu_e$); no hadron garbage, only CC, only one Feynman diagram in l.o. ($\hbar = c = 1$) :

- in Fermi theory (see), when the energy $E_\nu \gg m_{e,\mu}$, since $\sqrt{s}$ is the only energy scale, for dimensional considerations :

  $\sigma \propto G_F^2 s \approx G_F^2 (2 m_e E_\nu) \propto G_F^2 E_\nu$;

  or, with a more refined computation:

  $d\sigma/d\Omega = G_F^2 s/(4\pi^2) = G_F^2 m_e E_\nu / (2\pi^2)$;

  $\sigma = G_F^2 s/\pi = 2\, G_F^2\, m_e E_\nu / \pi$;

  the space isotropy of the cross section is explained by the conservation of the total angular momentum (= 0 both in initial and final state).

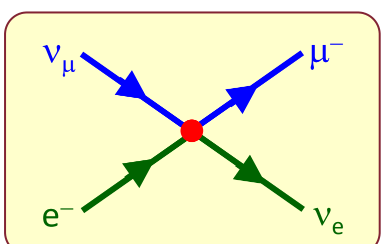
- the above equation reproduces well the data ($\sigma \propto E_\nu$), but becomes "impossible" at high energy, because σ would diverge ("violate unitarity").

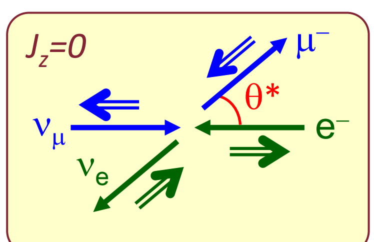
- In the SM, the process is mediated by a $W^\pm \to$ use the W propagator :

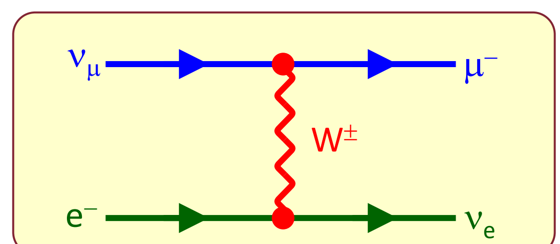
$$\frac{d\sigma}{d\Omega} = \frac{g^4 \alpha^2 m_e E_\nu}{2\pi^2 (m_w^2 + Q^2)^2} \xrightarrow{Q^2 << m_w^2} \frac{g^4 \alpha^2 m_e E_\nu}{2\pi^2 m_w^4};$$

$$\sigma_{Q^2 << m_w^2} = \frac{g^4 \alpha^2}{m_w^4} \frac{2 m_e E_\nu}{\pi} = G_F^2 \frac{2 m_e E_\nu}{\pi} = \sigma_{\text{Fermi}}.$$

- instead, for $Q^2 >> m_W^2$, the cross-section has the (well-understood) $1/s$ behavior.







CC ν processes: quasi-elastic

However, the purely lepton process is so rare, that it is hard to compare it with data.

A more common process is $\nu_\mu\ n \rightarrow \mu^-\ p$, "the *quasi-elastic* scattering", where nucleons interacts coherently :

- in Fermi theory :

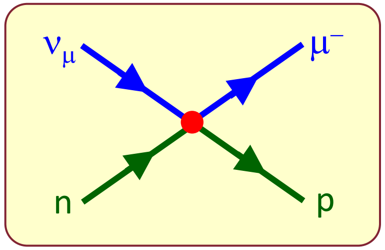
  $d\sigma/d\Omega = G_F^2 s / (4\pi^2) = G_F^2\, m_N E_\nu / (2\pi^2);$

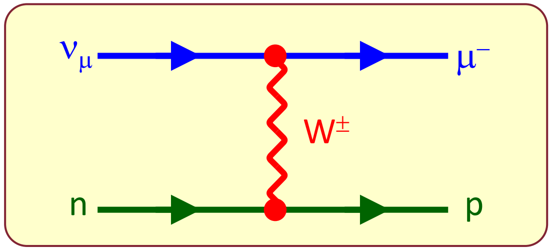
  $\sigma = G_F^2 s / \pi = 2\, G_F^2\, m_N E_\nu / \pi;$

  actually the results agree pretty well with the prediction, as shown in the fig.
- In the SM, the same considerations :

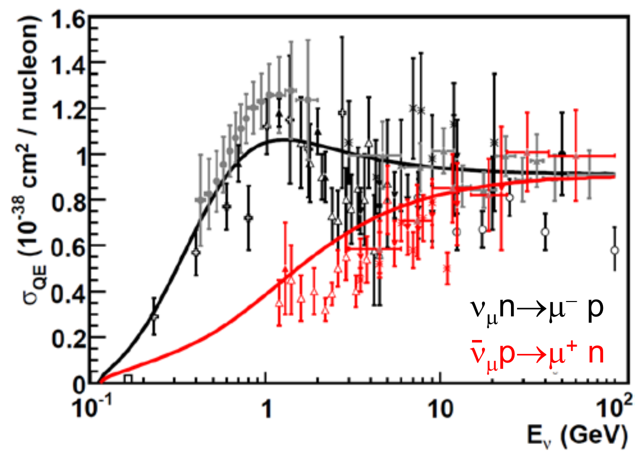
$$d\sigma/d\Omega = g^4\alpha^2\, m_N E_\nu / [2\pi^2\, m_W{}^4] = d\sigma/d\Omega|_{Fermi};$$

$$\sigma = 2\, g^4\alpha^2\, m_N E_\nu / [\pi\, m_W{}^4] = \sigma_{Fermi}.$$





- Advantage of the nucleon process over the purely lepton one : the factor m_N/m_e, [≈ 2,000] → yield measurable with the present experiments.
- …, but paid by the theoretical approximation (the demand of "coherence") and the less clean experimental condition.
- Also valid for $\bar{\nu}_\mu\ p \rightarrow \mu^+\ n$, which has a similar cross section [*Problem : discuss the spin structure for angular momentum conservation*].



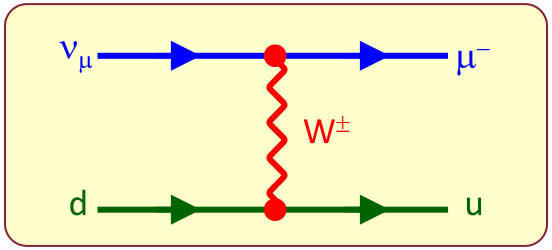
CC ν processes: parton level

- Individual hadronic or semileptonic processes happen at parton level (*at high Q^2 "coherence" becomes meaningless*).
- Partons (=quarks) are :
  - elementary;
  - spin ½;
  - (almost) massless.
- Consider the process :

$$\nu_\mu\, d \to \mu^- u.$$

- Do some simple kinematics at parton level, using the DIS variables.
- *The variables y ("inelasticity") and θ* will be used a lot:*

cos θ* = 1 – 2y
dcosθ* = – 2 dy



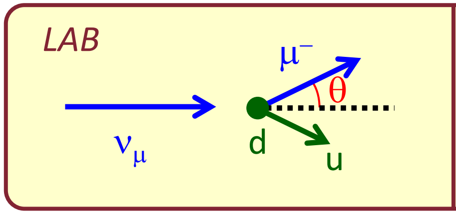
$$\text{LAB sys} \begin{cases} \nu_\mu & (E, \quad E, \quad 0 \quad) \\ d & (m_d, \; 0, \quad 0 \quad) \\ \mu^- & (E', \; E'\cos\theta, \; E'\sin\theta) \\ u & (\,\ldots, \; \ldots, \quad \ldots \quad) \end{cases}$$

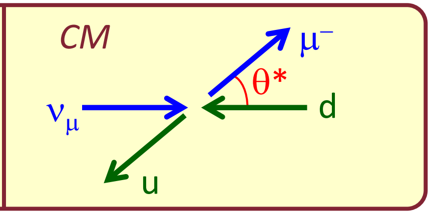
$$\text{CM sys} \begin{cases} \nu_\mu & (E^*, E^*, \quad 0 \quad) \\ d & (E^*, -E^*, \quad 0 \quad) \\ \mu^- & (E^*, E^*\cos\theta^*, \; E^*\sin\theta^*) \\ u & (\,\ldots, \; \ldots, \quad \ldots \quad) \end{cases}$$

$$p_\mu \cdot p_d\big|_{LAB} = E'm_d = p_\mu \cdot p_d\big|_{CM} = E^{*2}\left(1+\cos\theta^*\right);$$

$$p_\nu \cdot p_d\big|_{LAB} = Em_d = p_\nu \cdot p_d\big|_{CM} = 2E^{*2};$$

$$y = \frac{q\cdot P}{k\cdot P} = \frac{\nu}{E} = \frac{E-E'}{E} = 1 - \frac{E'}{E} = \frac{1-\cos\theta^*}{2}.$$

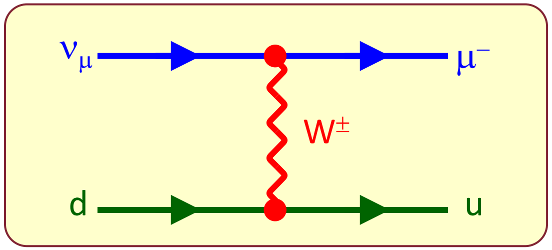


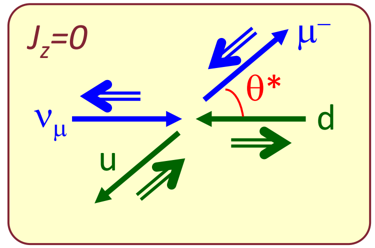


CC ν processes: helicity

Using a "quasi-Fermi" approximation, it is possible to compute angular cross sections for the CC semileptonic processes.

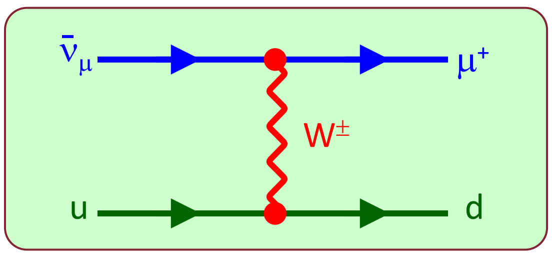
"Quasi-Fermi" means "Fermi-style" total cross-section × angular dependence from V–A, i.e. CC current $\propto (1-\gamma_5)$.

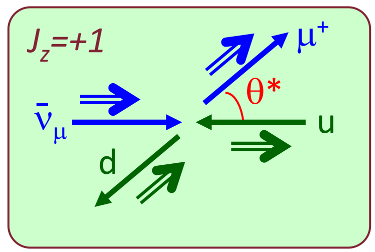




$\nu_\mu d \to \mu^- u$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2}; \quad \frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi}.$$





$\bar{\nu}_\mu u \to \mu^+ d$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2} \times \left(\frac{1+\cos\theta^*}{2}\right)^2;$$

$$\frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi} \times (1-y)^2.$$

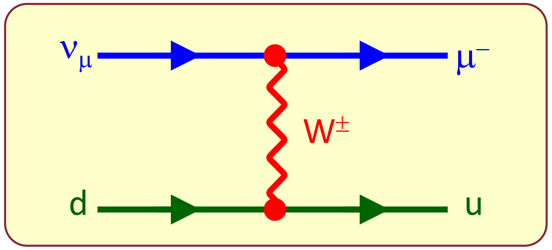
In the $(\bar{\nu}_\mu\ u)$ case, $\theta^*=180°$ clearly violates angular momentum conservation, while $\theta^*=0°$ is allowed : hence the $(1-y)^2$ factor [*next slide*].

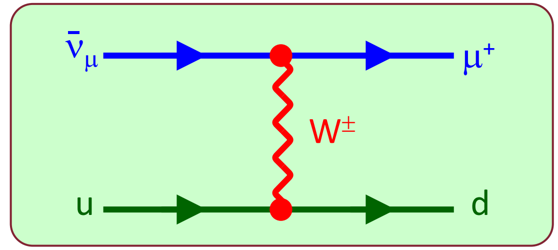
[*notice : θ^* and $\hat{s}$ are the CM variables at parton level, very useful for understanding, but y=(E-E')/E is the experimental variable, which is really measured; in fact, it is independent from the "hadronic garbage"*].

CC ν processes: dσ/dy

all $f\bar{f} \to f\bar{f}$

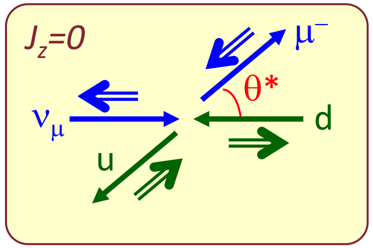
$\bar{f}\bar{f} \to \bar{f}\bar{f}$

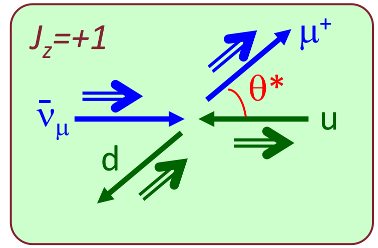




all $f\bar{f} \to \bar{f}f$

constant, NOT cosθ*-dependent





suppressed for θ*=180°

Some simple kinematics :

$$y = 1 - \frac{E'}{E} = \frac{1-\cos\theta^*}{2};$$

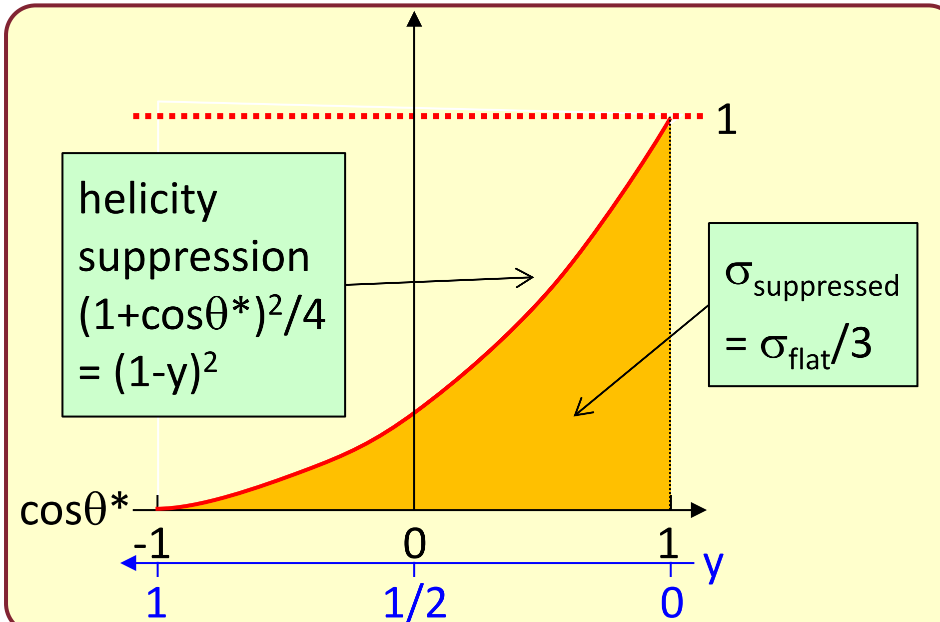
$$\cos\theta^* = 1 - 2y;$$

$$(1+\cos\theta^*)/2 = 1 - y;$$

$$(1+\cos\theta^*)^2/4 = (1-y)^2;$$

$$|d\cos\theta^*| = 2dy;$$

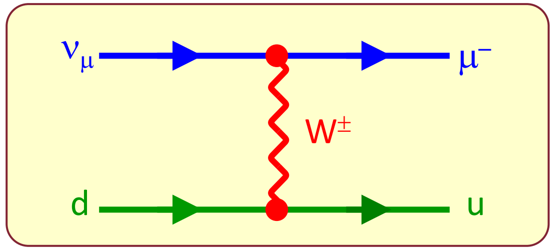
$$d\Omega = d\phi\, d\cos\theta^* = 4\pi dy.$$

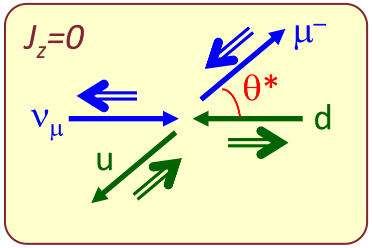


CC ν processes: score

$\nu_\mu d \to \mu^- u$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2}; \quad \frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi}.$$

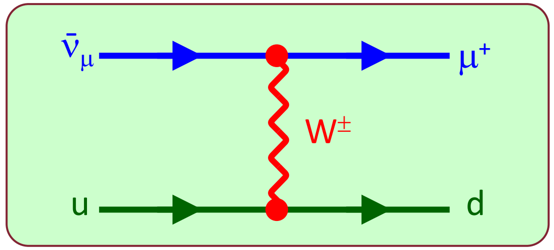


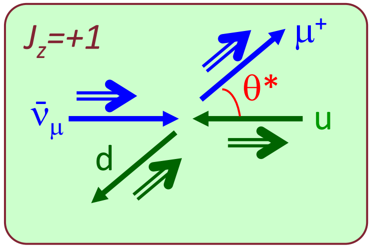


$\bar{\nu}_\mu u \to \mu^+ d$:

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \hat{s}}{4\pi^2} \times \left(\frac{1 + \cos\theta^*}{2} \right)^2;$$

$$\frac{d\sigma}{dy} = \frac{G_F^2 \hat{s}}{\pi} \times (1-y)^2.$$





score	process	J_z	$d\sigma/d\cos\theta^*$	$d\sigma/dy$	σ
	$\nu_\mu u \to \mu^-?$, $\bar{\nu}_\mu \bar{u} \to \mu^+?$	impossible			
score	$\nu_\mu d \to \mu^- u$, $\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}$	0	flat	flat	~1
	$\nu_\mu \bar{u} \to \mu^- \bar{d}$, $\bar{\nu}_\mu u \to \mu^+ d$	1	$\sim(1+\cos\theta^*)^2/4$	$\sim(1-y)^2$	~1/3
	$\nu_\mu \bar{d} \to \mu^-?$, $\bar{\nu}_\mu d \to \mu^+?$	impossible			

→ isoscalar target
$\sigma(\nu N) > \sigma(\bar{\nu} N)$!!!

Structure functions

Goal : describe the νN ($\bar{\nu}N$) scattering.

All the building blocks have been studied; put everything together :

- the elementary cross section $d\sigma/d\Omega$ (better, $d\sigma/dy$) for individual ν-parton scattering;
- the parton distribution in the nucleon [$f(x)$; x is the fraction of the nucleon momentum, carried by a single parton];
- the "factorization" hypothesis of DIS [*i.e. the interaction regards only one single parton; the other partons do NOT participate*].

For both ν and $\bar{\nu}$, and each final state F:

$$\left.\frac{d^2\sigma(\nu N \to F)}{dxdy}\right|_{s=2E_\nu M} = \sum_j f_j(x) \left.\frac{d\sigma(\nu p_j \to F)}{dy}\right|_{\hat{s}=sx} ;$$

$\hat{s} = sx = 2E_\nu Mx$ = energy^2 at parton level; the sum runs on all interacting partons p_j (q_j, $\bar{q}_j$, both valence and sea).

Connect this picture with the studies of the nucleon structure in eN DIS :

- the quark distributions (**pdf**) have already been defined; [*e.g. u(x)dx is the number of u quarks in the proton with fractional momentum between x and x+dx* ($0 \leq x \leq 1$)];
- the same for $d(x)$, $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$, $\bar{s}(x)$ ...;
- a general formula for ($d^2\sigma / d\Omega dE'$) has been developed, which includes two structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$;
- the transformation (Ω, E') $\to$ (x,y) is pure (trivial) kinematics [see §2];
- a third function $W_3(Q^2, \nu)$ [$\to F_3(x, Q^2)$] has to be defined, because of terms, like the interference between V and A, which were absent in the ep case;
- if Bjorken scaling holds, the functions F_1 F_2 F_3 are functions of x and not of Q^2.
- the next slides contain the math.

Structure functions : $d^2\sigma/dxdy$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{ep\\DIS}} = \frac{4\pi\alpha^2\left(s-M^2\right)}{Q^4}\left[xy^2F_1(x,Q^2)+\left(1-y-\frac{M^2xy}{s-M^2}\right)F_2(x,Q^2)\right] =$$

$$\xrightarrow{s>>M^2}\frac{4\pi\alpha^2 s}{Q^4}\left[xy^2F_1(x,Q^2)+(1-y)F_2(x,Q^2)\right];$$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{\nu p\\DIS}} = \frac{G_F^2 s}{2\pi}\left[xy^2F_1^{\nu p}(x,Q^2)+(1-y)F_2^{\nu p}(x,Q^2)+xy\left(1-\frac{y}{2}\right)F_3^{\nu p}(x,Q^2)\right];$$

$$\left.\frac{d^2\sigma}{dxdy}\right|_{\substack{\bar{\nu} p\\DIS}} = \frac{G_F^2 s}{2\pi}\left[xy^2F_1^{\bar{\nu} p}(x,Q^2)+(1-y)F_2^{\bar{\nu} p}(x,Q^2)-xy\left(1-\frac{y}{2}\right)F_3^{\bar{\nu} p}(x,Q^2)\right].$$

For the νn scattering, $\left(F_1^{\nu p},F_2^{\nu p},F_3^{\nu p}\right)\rightarrow\left(F_1^{\nu n},F_2^{\nu n},F_3^{\nu n}\right)$, and so on.

Structure functions : $d^2\sigma/dxdy$

- Define u(x), d(x), ū(x), đ(x) the x-distribution of quarks u, d, ū, đ in the proton;
- then, some simple consistency relations between p and n follows :
- [*first* ❶ *the algebra on the right, then* ❷ *the case* νp *fully computed in the next slide, finally* ❸ *the results, equating the coefficients with same power of y*];
- notice that the Callan-Gross equation (see next slide) comes out again, together with other "rules".

❶

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[d(x) + (1-y)^2\,\bar{u}(x)\right];$$

$$\frac{d^2\sigma(\bar{\nu} p)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\bar{d}(x) + (1-y)^2\,u(x)\right];$$

$$\frac{d^2\sigma(\nu n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[d^n(x) + (1-y)^2\,\bar{u}^n(x)\right];$$

$$\frac{d^2\sigma(\bar{\nu} n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\bar{d}^n(x) + (1-y)^2\,u^n(x)\right];$$

$$u^n(x) = d(x); \quad \bar{u}^n(x) = \bar{d}(x);$$

$$d^n(x) = u(x); \quad \bar{d}^n(x) = \bar{u}(x);$$

$$\frac{d^2\sigma(\nu n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[u(x) + (1-y)^2\,\bar{d}(x)\right];$$

$$\frac{d^2\sigma(\bar{\nu} n)}{dxdy} = \frac{G_F^2 sx}{\pi}\left[\bar{u}(x) + (1-y)^2\,d(x)\right];$$

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 s}{2\pi}\left[\begin{array}{l} xy^2 F_1^{\nu p}(x) + (1-y) F_2^{\nu p}(x) + \\ +xy(1-y/2) F_3^{\nu p}(x) \end{array}\right].$$

Structure functions: $d^2\sigma/dxdy$

- math for the νp case shown in ❷;
- *neglect heavy quarks, i.e.* $s(x) = \bar{s}(x) = 0$;
- νn, $\bar{\nu} p$, $\bar{\nu} n$ left as an exercise; results for νn shown in ❸ together with νp.

❷

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 s x}{\pi}\frac{2}{2}\left[d(x) + (1-y)^2\,\bar{u}(x)\right];$$

$$\frac{d^2\sigma(\nu p)}{dxdy} = \frac{G_F^2 s}{2\pi}\left[\begin{array}{l} xy^2 F_1^{\nu p}(x) + (1-y)F_2^{\nu p}(x) + \\ + xy(1-y/2)F_3^{\nu p}(x)\end{array}\right];$$

a) $y^0 \rightarrow \boxed{2x(d+\bar{u}) = F_2}$;

b) $y^1 \rightarrow 2x(-2)\bar{u} = -4x\bar{u} = -F_2 + xF_3 =$

$$= -[2xd + 2x\bar{u}] + xF_3;$$

$$xF_3 = 2xd - 2x\bar{u} = 2x(d - \bar{u});$$

$$\boxed{F_3 = 2(d - \bar{u})};$$

c) $y^2 \rightarrow 2x\bar{u} = xF_1 - xF_3/2;$

$$xF_1 = 2x\bar{u} + x[2(d-\bar{u})]/2 = x(d+\bar{u});$$

$$\boxed{F_1 = d + \bar{u}};$$

$\boxed{2xF_1 = F_2}$ (Callan – Gross).

❸

$$F_2^{\nu p}(x) = 2xF_1^{\nu p}(x) = 2x[d(x) + \bar{u}(x)];$$
$$xF_3^{\nu p}(x) = 2x[d(x) - \bar{u}(x)];$$
$$F_2^{\nu n}(x) = 2xF_1^{\nu n}(x) = 2x[u(x) + \bar{d}(x)];$$
$$xF_3^{\nu n}(x) = 2x[u(x) - \bar{d}(x)].$$

Structure functions: results

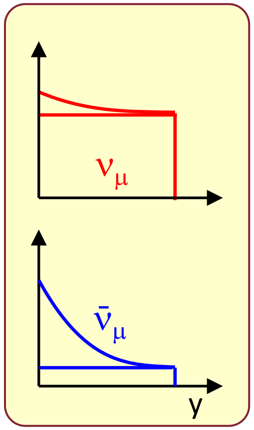
For CC process (ν_μ N) and ($\bar{\nu}_\mu$ N), expect [target "isoscalar", i.e. composed by same number of p / n (all heavy materials] :

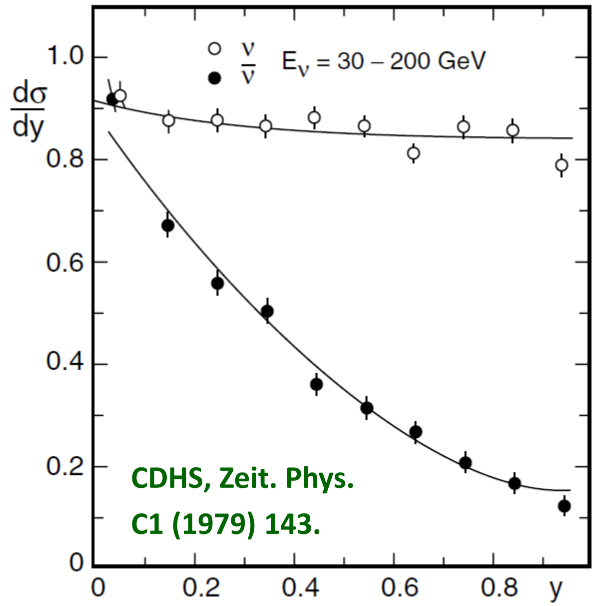
- same number of u and d (valence), and much smaller amount of ū đ (sea); s and s̄ are negligible;
- for ν_μ a mixture of (ν_μ d) and (ν_μ ū), because (ν_μ u) and (ν_μ đ) do NOT interact in CC;
- for $\bar{\nu}_\mu$ a mixture of ($\bar{\nu}_\mu$ u) and ($\bar{\nu}_\mu$ đ);
- (ν_μ d), ($\bar{\nu}_\mu$ đ) have flat y distributions;
- (ν_μ ū), ($\bar{\nu}_\mu$ u) proportional to $(1-y)^2$;

➢ for ν_μ, expectation is large constant + some minor parabolic contribution;

➢ for $\bar{\nu}_\mu$, it is the opposite: a dominant parabola + a small constant;

- plot dσ/dy for ν and ν̄ after integrating over x and E_ν: ***great success !!!***





$$\frac{d^2\sigma(\nu N)}{dxdy} = \frac{1}{2}\left[\frac{d^2\sigma(\nu p)}{dxdy} + \frac{d^2\sigma(\nu n)}{dxdy}\right] = \frac{G_F^2 sx}{2\pi}\left\{\left[u(x)+d(x)\right] + (1-y)^2\left[\bar{u}(x)+\bar{d}(x)\right]\right\} = \frac{G_F^2 sx}{2\pi}\left[q(x) + (1-y)^2\bar{q}(x)\right];$$

$$\frac{d^2\sigma(\bar{\nu} N)}{dxdy} = \frac{1}{2}\left[\frac{d^2\sigma(\bar{\nu} p)}{dxdy} + \frac{d^2\sigma(\bar{\nu} n)}{dxdy}\right] = \frac{G_F^2 sx}{2\pi}\left\{\left[\bar{u}(x)+\bar{d}(x)\right] + (1-y)^2\left[u(x)+d(x)\right]\right\} = \frac{G_F^2 sx}{2\pi}\left[\bar{q}(x) + (1-y)^2 q(x)\right].$$

Structure functions: $\nu N \leftrightarrow eN$

- For an isoscalar target, we get

  $F_2^{\nu N} = (F_2^{\nu p}+F_2^{\nu n}) \;/\; 2 =$
  $= x\,[\,u(x) + d(x) + \bar{u}(x) + \bar{d}(x)];$

  $F_2^{eN} = (F_2^{ep}+F_2^{en}) \;/\; 2 =$
  $= 5x/18\,[\,u(x) + d(x) + \bar{u}(x) + \bar{d}(x)];$

  therefore :

  $F_2^{eN}(x) = 5/18\; F_2^{\nu N}(x).$

  [the value 5/18 is just the average of the quark charges squared : $[(⅓)^2 + (⅔)^2\,]/2$.]

  [*in other words, in e.m. processes the interactions are proportional to e², while in CC scattering they are normalized to 1; there is no relative normalization between e.m. e CC in the rule*].

  → eN

- For F_3, we get

  $F_3^{\nu N} = (F_3^{\nu p}+F_3^{\nu n}) \;/\; 2 =$
  $= [u(x) + d(x) - \bar{u}(x) - \bar{d}(x)];$

  the structure functions have contributions from valence and sea :

  - $u(x) = u_v(x) + u_s(x) = u_v(x) + Sea(x);$
  - $\bar{u}(x) = \bar{u}_s(x) = Sea(x);$
  - $\int_0^1 u_v(x)\,dx = 2;\qquad \int_0^1 d_v(x)\,dx = 1,$

  then

  $F_3^{\nu N} = [\,u(x) + d(x) - \bar{u}(x) - \bar{d}(x)] =$
  $= u_v(x) + d_v(x);$

  $\int_0^1 F_3^{\nu N}(x)\,dx = \int_0^1 [u_v(x) + d_v(x)]\,dx = 3;$

  known as the Gross – Llewellyn-Smith sum rule.

- Experimentally, the G.-L.S. rule is well verified = 3.0 ± 0.2.

Structure functions: $\nu N \leftrightarrow eN$

- In the same Q^2 range, F_2^{ν} from CDHS data shows a nice agreement with 18/5 $\times$ e.m. (μ^- from EMC, e^- from MIT).
- The figure shows also the contribution of F_3^{ν} and the antiquarks alone.
- Since $\int (1-y)^2\, dy = 1/3$, if there were no $\bar{q}$ in the nucleon, we would expect :

  $\sigma^{\nu N} / \sigma^{\bar{\nu} N} \approx 3.$
- If instead the cross-sections are written in terms of quarks and antiquarks :

  $\sigma^{\nu N} = G_F^2\, s / (2\pi)\ \ [f_q + ⅓\, f_{\bar{q}}];$

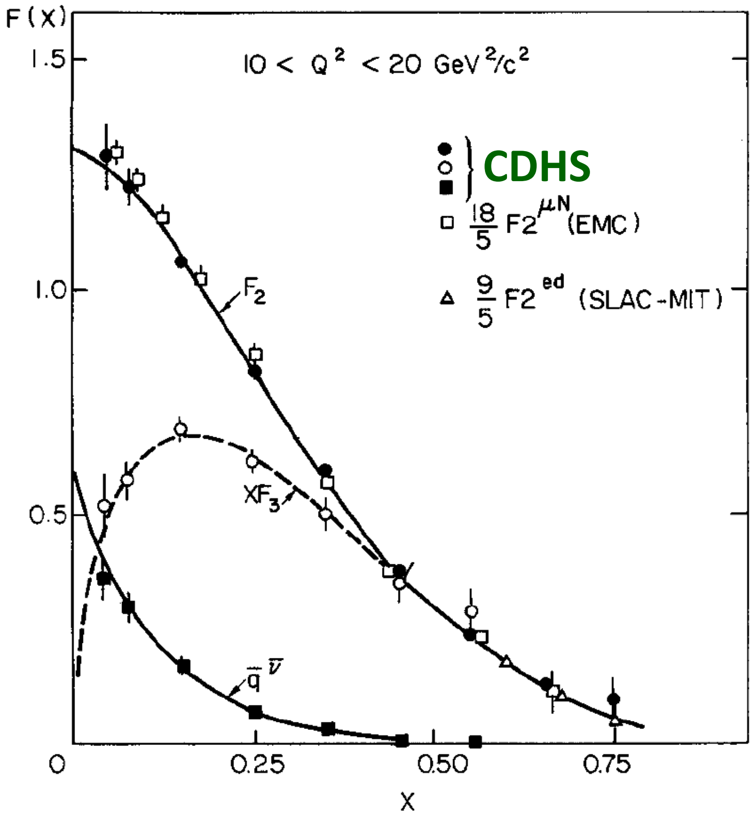
  $\sigma^{\bar{\nu} N} = G_F^2\, s / (2\pi)\ \ [⅓\, f_q + f_{\bar{q}}];$

  then, the value of f_q and $f_{\bar{q}}$ can be measured :

  $f_q \approx 0.41;\ \ f_{\bar{q}} \approx 0.08\ \rightarrow\ f_g \approx 0.50;$
- taking into account the $\bar{q}$ fraction, we expect

  $\sigma^{\nu N} / \sigma^{\bar{\nu} N} \approx [f_q + ⅓\, f_{\bar{q}}] / [⅓\, f_q + f_{\bar{q}}] \approx 2;$

  in reasonable agreement with the measurement [*see page 1 !!!*].



The discovery of neutral currents

- The search for NC events began in the early 1960s, when the e.w. theory of Glashow – Weinberg – Salam was still thought not to be "renormalizable".
- The searches were limited to FCNC: possible NC "non-FC" processes were thought to be obscured by e.m. currents [*in analogy with weak CC, which is visible only when flavor is violated*].
- Decays like $K^+ \to \pi^+ e^+ e^-$ and $K^0 \to \mu^+ \mu^-$ were searched and NOT found.
- The only escape from this difficulty is to make use of neutral particles, which do NOT sense e.m. interactions : the ν's.
- The signature for this process is given by the absence in the final state of a charged lepton, which is unavoidable in the CC coupling $\nu \ell^\pm W^\mp$.
- Motivated by the recent discovery of the renormalizability of the SM ('t Hooft and Veltman, 1971), the experimentalists from both sides of the Atlantic began a new "hunt" for neutral currents.

Historical Note: In 1960, experiments at CERN, by using a heavy liquid chamber and a ν beam, looked for NC. Unfortunately, they found that the ratio NC/CC is < 3%, a value much smaller than the correct one. This error was eventually corrected, but the new limit (12%) was published only in 1970.

The discovery of neutral currents

- The events [*see before*] were of the type
  - (a) $\nu_\mu + N \to \nu_\mu + X$;
  - (b) $\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X$;
  - (c) $\nu_\mu + e^- \to e^- + \nu_\mu$;
  - (d) $\bar{\nu}_\mu + e^- \to e^- + \bar{\nu}_\mu$;

  ["X" = hadronic system, without leptons].
- In 1973, the newly built Gargamelle was filled with 15 tons of Freon ($C\,F_3\,Br$).
- The first event interpreted as a pure leptonic NC.
- They had the following criteria :
  - fiducial volume 3 m^3;
  - events were defined as NC if :
    1. no visible $\mu^\pm$ is present;
    2. no charged track escapes the confidence volume;
  - Instead, events were CC if :
    1. a clearly visible $\mu^\pm$ is present;
    2. the $\mu^\pm$ has to exit out of the chamber.
- Results:
  - ν beam : 102 NC, 428 CC, 15 n(*);
  - $\bar{\nu}$ beam : 64 NC, 148 CC, 12 n(*).
- The result is then :
  - NC/CC (ν) = 0.21 $\pm$ 0.03;
  - NC/CC ($\bar{\nu}$) = 0.45 $\pm$ 0:09;
  - **inconsistent with the absence** of NC.

(*) *The main background was due to neutrons produced by* ν*'s in the chamber structure.*

There was also an American team, looking for NC. After an exciting race, they were unable to publish conclusive results before the Europeans.
Actually, the discovery of NC marks a clear turning point in high energy physics : after that, Europe was not anymore the expected looser in the game.

NC ν processes: couplings

The NC couplings do depend on the fermion type *f* :

symbol	formula	definition (physical meaning)
g		SU(2) coupling constant
g'		U(1) coupling constant
$\tan\theta_W$	$\equiv g' / g$	tangent (Weinberg angle)
e	$\equiv g \sin\theta_W$	e^+ charge (= − e^- charge)
g^f_V	$= I^f_{Wz} - 2\,Q^f \sin^2\theta_W$	NC vector coupling (also v_f, c_V)
g^f_A	$= I^f_{Wz}$	NC axial coupling (a_f, c_a)
g^f_L	$= ½\,(g^f_V + g^f_A) = I^f_{Wz} - Q^f \sin^2\theta_W$	"left-handed" NC coupling
g^f_R	$= ½\,(g^f_V - g^f_A) = -\,Q^f \sin^2\theta_W$	"right-handed" NC coupling
m^2_W	$\equiv \pi\alpha / (\sqrt{2}\, G_F \sin^2\theta_W)$	[$W^\pm$ mass]^2 *[careful : $m^{\mathbf{2}}_W$!!!]*
m_Z	$= m_W / \cos\theta_W$	Z mass

f	Q_f	g^f_V ($\sin^2\theta_W$=0.231)	$I^f_{Wz} = g^f_A$	g^f_L	g^f_R
$\nu_e\ \nu_\mu\ \nu_\tau$	0	+½+0 = +0.500	+½	+½	0
$e^-\ \mu^-\ \tau^-$	−1	−½ + 2 $\sin^2\theta_W$ = −0.038	−½	-½ + $\sin^2\theta_W$	+ $\sin^2\theta_W$
u c t	⅔	+½ − $^4/_3$ $\sin^2\theta_W$ = +0.192	+½	+½ −⅔ $\sin^2\theta_W$	− ⅔ $\sin^2\theta_W$
d s b	−⅓	−½ + ⅔ $\sin^2\theta_W$ = −0.346	-½	-½ + ⅓ $\sin^2\theta_W$	+ ⅓ $\sin^2\theta_W$

remember: $g^e_V \approx 0$

NC ν processes: σ

Some algebra, not really difficult, but quite tedious, produces for NC the analogous formulas already derived for CC :

ƒ : point-like fermions (ℓ^-, ν, q);

$\bar{f}$: point-like anti-fermions (ℓ^+, $\bar{\nu}$, $\bar{q}$);

N : "isoscalar" nucleon (p+n)/2;

N' : final state hadronic system.

$$\frac{d\sigma(\nu_\mu f \to \nu_\mu f)}{dy} = \frac{G_F^2 \hat{s}}{\pi}\left[\left\{g_L^f\right\}^2 + (1-y)^2\left\{g_R^f\right\}^2\right];$$

$$\frac{d\sigma(\bar{\nu}_\mu f \to \bar{\nu}_\mu f)}{dy} = \frac{G_F^2 \hat{s}}{\pi}\left[\left\{g_R^f\right\}^2 + (1-y)^2\left\{g_L^f\right\}^2\right];$$

e.g. Rev. Mod.Phys. 70, 1341 (1998)

$$\frac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = \frac{G_F^2 s x}{2\pi}\left\{\begin{array}{l}\left[\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)+(1-y)^2\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)\right]q(x)+ \\ +\left[\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)\right]\bar{q}(x)\end{array}\right\};$$

$$\frac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu N')}{dxdy} = \frac{G_F^2 s x}{2\pi}\left\{\begin{array}{l}\left[\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_L^u\right\}^2+\left\{g_L^d\right\}^2\right)\right]q(x)+ \\ +\left[\left(\left\{g_L^u\right\}^2+\left\{g_R^d\right\}^2\right)+(1-y)^2\left(\left\{g_R^u\right\}^2+\left\{g_R^d\right\}^2\right)\right]\bar{q}(x)\end{array}\right\}.$$

NC ν processes: R_ν and $R_{\bar\nu}$

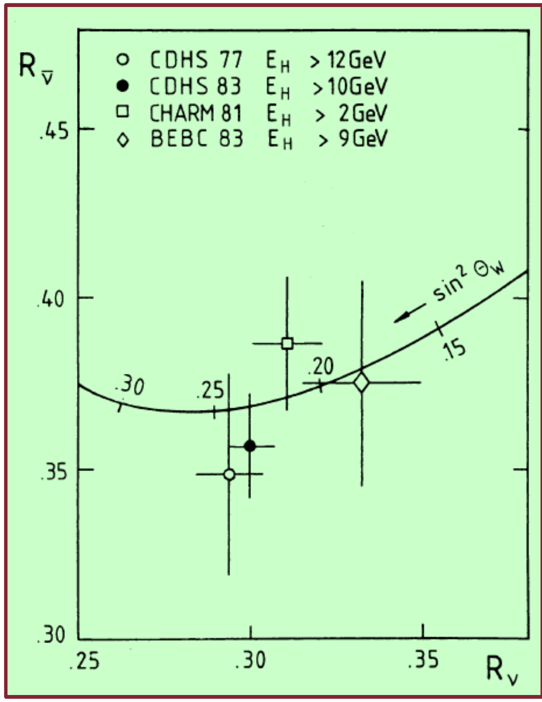
To measure $\sin^2\theta_w$:

- produce some algebra [*next slide, not for the exam*]:
  1. start with the CC and NC cross sections for isoscalar targets;
  2. neglect the sea contributions ū(x), đ(x);
  3. integrate over x and y ($\int(1-y)^2dy = 1/3$);
  4. divide the cross sections, to cancel the dependence of all the other parameters;
  5. use g_L and g_R for each f(ermion) :

$$R_\nu \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} \approx \frac{1}{2} - \sin^2\theta_w + \frac{20}{27}\sin^4\theta_w;$$

$$R_{\bar\nu} \equiv \frac{\sigma_{NC}(\bar\nu N)}{\sigma_{CC}(\bar\nu N)} \approx \frac{1}{2} - \sin^2\theta_w + \frac{20}{9}\sin^4\theta_w.$$

- The values of R_ν and $R_{\bar\nu}$ are well defined and, at least in principle, easy to measure :
  - unknown or difficult-to-measure parameters cancel out;
  - exp. systematics, beam effects, detector … (see next slides).



[Dieter Haidt, CERN school '84, CERN yellow report 85-11]

old data, but useful to explain the method :

$$\begin{cases} R_\nu = R_\nu(\sin^2\theta_w) \\ R_{\bar\nu} = R_{\bar\nu}(\sin^2\theta_w) \end{cases} \rightarrow$$

a point for each value of $\sin^2\theta_w$ → a curve in the plane R_ν / $R_{\bar\nu}$ → measure $\sin^2\theta_w$.

NC ν processes: $d^2\sigma/dxdy$

not difficult, but NOT for the exam.

1. Start with the CC and NC cross sections for isoscalar targets;
2. Neglect the sea contributions $\bar{u}(x)$, $\bar{d}(x)$;
3. Integrate over y $\left[\int_0^1 (1-y)^2 dy = 1/3\right]$;
4. Use g_L^f and g_R^f from the previous tables $\left[g_R^{u2} + g_R^{d2} = \frac{5}{9}\sin\theta_w^4,\quad g_L^{u2} + g_L^{d2} = \frac{1}{2} - \sin\theta_w^2 + \frac{5}{9}\sin\theta_w^4\right]$;
5. Divide NC/CC.

$$\text{CC}: \begin{bmatrix} \dfrac{d^2\sigma(\nu_\mu N \to \mu^- N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[q(x) + (1-y)^2 \bar{q}(x)\right]; \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\left[\bar{q}(x) + (1-y)^2 q(x)\right]; \end{bmatrix} \to \text{CC}: \begin{bmatrix} \dfrac{d^2\sigma(\nu_\mu N \to \mu^- N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi} q(x); \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \mu^+ N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}(1-y)^2 q(x); \end{bmatrix}$$

$$\text{NC}: \begin{bmatrix} \dfrac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = [\text{prev.slide}]; \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu N')}{dxdy} = [\text{prev.slide}]; \end{bmatrix} \to \text{NC}: \begin{bmatrix} \dfrac{d^2\sigma(\nu_\mu N \to \nu_\mu N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\begin{bmatrix} \left(g_L^{u2} + g_L^{d2}\right) + \\ +(1-y)^2\left(g_R^{u2} + g_R^{d2}\right) \end{bmatrix} q(x); \\ \dfrac{d^2\sigma(\bar{\nu}_\mu N \to \bar{\nu}_\mu N')}{dxdy} = \dfrac{G_F^2 sx}{2\pi}\begin{bmatrix} \left(g_R^{u2} + g_R^{d2}\right) + \\ +(1-y)^2\left(g_L^{u2} + g_L^{d2}\right) \end{bmatrix} q(x). \end{bmatrix}$$

NC ν processes: $\sin^2\theta_W$

Most recent results :

- $\sin^2\theta_W$ = 0.2356 ± .0050 CHARM
- = 0.2250 ± .0050 CDHS
- = 0.2332 ± .0015 (a)
- = 0.2251 ± .0039 (b).

Notes :

- (a) and (b) are "today's best" [PDG], for ν's on isoscalar target:
- they differ because of two different "definitions" of higher order parameters (see the radiative corrections in § LEP).

The quantities REALLY measured are R_ν ($R_{\bar{\nu}}$) :

$$R_\nu \equiv \frac{\sigma_{NC}(\nu N)}{\sigma_{CC}(\nu N)} = \frac{\varepsilon_{NC}\left[n_{NC}^{tot} - n_{NC}^{bckg}\right]}{\int \Phi(\nu)dE} \frac{\int \Phi(\nu)dE}{\varepsilon_{CC}\left[n_{CC}^{tot} - n_{CC}^{bckg}\right]} =$$

$$= \frac{\varepsilon_{NC}\left[n_{NC}^{tot} - n_{NC}^{bckg}\right]}{\varepsilon_{CC}\left[n_{CC}^{tot} - n_{CC}^{bckg}\right]}.$$

The flux cancels out; this is not a good news, because ε_{NC} and ε_{CC} DO depend on E_ν, and are very different for CC and NC, so better know the E_ν dependence on σ.

In fact :

- CC, due to the presence of a charged $\mu^\pm$, are "easy" to detect, and relatively background free (n^{bckg} small);
- NC, however, are hardly distinguishable from cosmics and CC-low-energy;
- at low y, $\mu^\pm$ id. is difficult → the selection algorithm gets confused : CC → NC .

Therefore :

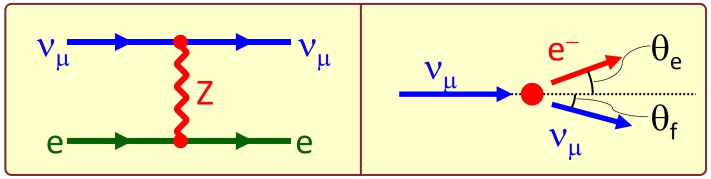
- accurate computation of the flux as a function of E_ν;
- accurate understanding of the systematics;
- reproduction via montecarlo, to study algorithms and systematics.

Pure leptonic ν processes : kinematics

- The cleanest NC process are

  $(\nu_\mu\, e^- \to \nu_\mu\, e^-)$ and $(\bar{\nu}_\mu\, e^- \to \bar{\nu}_\mu\, e^-)$.
- In fact, no hypothesis on "isoscalarity", no dependence on structure functions, on sea-content of the nucleon, …
- Only one problem : cross section ($\propto$ s = $2 m_e E_\nu$) VERY small :

  $s(\nu_\mu e^-) = 2\, m_e E_\nu \approx s(\nu_\mu N)$ / 2,000.
- However, the process has been extensively studied.
- The problem : select the tiny number of signal events from the overwhelming NC (hadronic) events.
- The key is the very particular kinematics (see box).



Lab sys. ($i = \nu_{initial}$, $f = \nu_{final}$, $p_i \approx E_i$, $p_f \approx E_f$, $p_e \approx E_e$) :

E) $E_i + m_e = E_e + E_f$;

x) $E_i = E_e \cos\theta_e + E_f \cos\theta_f$;

y) $0 = E_e \sin\theta_e + E_f \sin\theta_f$.

Subtract (x) from (E) and $\times$ 2 :

$2m_e = 2E_e\,(1 - \cos\theta_e) + 2E_f\,(1 - \cos\theta_f)$;

$0 \le 2\,E_e\,(1 - \cos\theta_e) \approx E_e\,\theta_e^2 \le 2\,m_e$;

i.e.

1. the value of E_e is (almost always) many GeV (think to the y distribution);
2. The angle θ_e must be very small : $\theta_e^2 \le 2\,m_e/E_e$;
3. the ν variables (E_i, E_f, θ_f) are not measured;
4. it is therefore compulsory to measure the e.m. shower (= E_e) very well;
5. … and (even more important) its direction θ_e;
6. and SELECT on ($E_e\,\theta_e^2$).

Pure leptonic ν processes : data selection

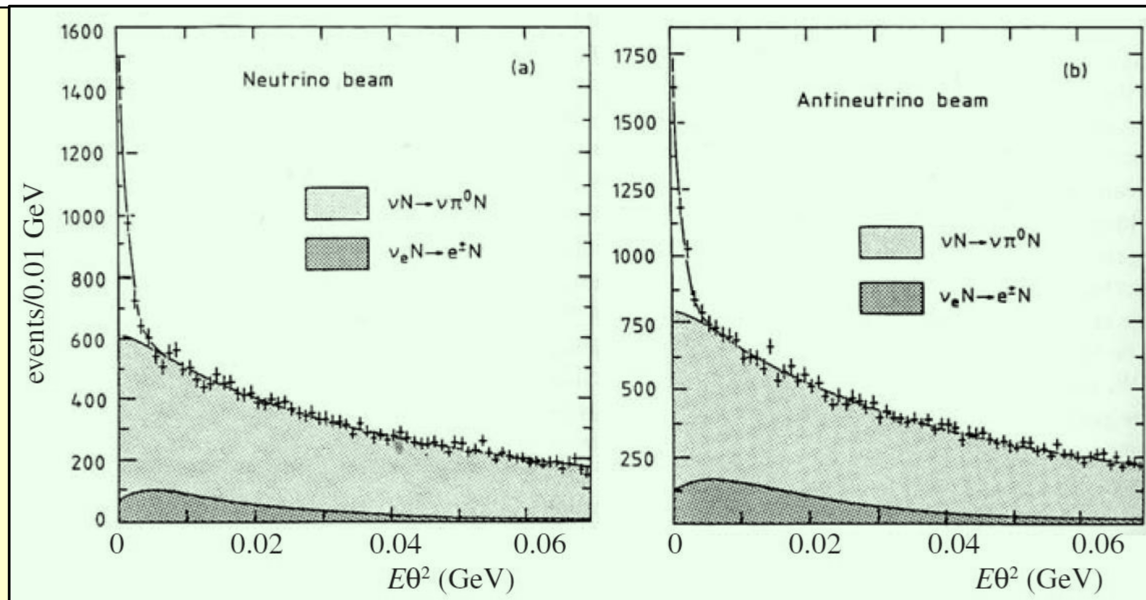
- The extraction of the signal requires the rejection of the background.
- The main one is due to NC hadronic interactions, without $\mu^\pm$ in the final state, with one or more π^0's; the photons due to π^0 decays mimic the electron shower.
- To reject those events, the deposit of energy in the early scintillators is used.
- Since $\pi^0 \to 2\gamma \to 4e^\pm$, a scintillator, if crossed at a very early stage of the shower development, sees 4 minimum ionizing particles, instead of only one.
- In this way, by using only the part of the detector immediately upstream of the scintillator, a much better isolation of the signal is obtained, at the price of a reduced statistics.

Three "populations" :

- the signal;
- hadronic NC;
- CC due to ν_e beam background;

The selection is statistical, NOT on an event-by-event basis.

[NOT because of quantum mechanics, but selection method]



CHARM, Phys. Lett. B 335, 246 (1994)

Pure leptonic ν processes : analysis

- The pure leptonic process is the cleanest and most systematic-free NC process.
- It has been used to measure θ_w.
- The price is a reduction ~2,000 in statistics and a difficult selection procedure.

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[\left(g_L^e\right)^2 + (1-y)^2\left(g_R^e\right)^2\right];$$

$$\frac{d\sigma_{NC}(\bar\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[\left(g_R^e\right)^2 + (1-y)^2\left(g_L^e\right)^2\right];$$

$$\sigma_{NC}(\nu_\mu e^-) = \frac{G_F^2 s}{4\pi}\left[1 - 4\sin^2\theta_w + \frac{16}{3}\sin^4\theta_w\right];$$

$$\sigma_{NC}(\bar\nu_\mu e^-) = \frac{G_F^2 s}{12\pi}\left[1 - 4\sin^2\theta_w + 16\sin^4\theta_w\right];$$

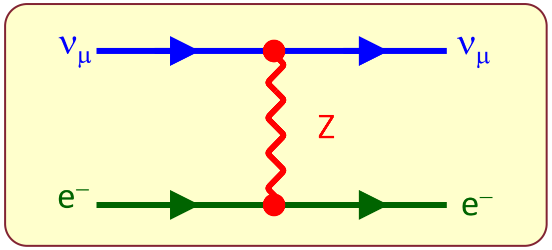
$$R_{NC}^{\nu_\mu e} \equiv \frac{\sigma_{NC}(\nu_\mu e^-)}{\sigma_{NC}(\bar\nu_\mu e^-)} = 3\frac{\left[1 - 4\sin^2\theta_w + \frac{16}{3}\sin^4\theta_w\right]}{\left[1 - 4\sin^2\theta_w + 16\sin^4\theta_w\right]}.$$

- The ratio being really measured is

$$R_{NC}^{\nu_\mu e} \equiv \frac{\sigma(\nu_\mu e^- \to \nu_\mu e^-)}{\sigma(\bar\nu_\mu e^- \to \bar\nu_\mu e^-)} = \frac{\varepsilon_\nu\left[n_\nu^{tot} - n_\nu^{bckg}\right]}{\int \Phi(\nu)dE}\,\frac{\int \Phi(\bar\nu)dE}{\varepsilon_{CC}\left[n_{\bar\nu}^{tot} - n_{\bar\nu}^{bckg}\right]}.$$

- A key point is the ratio of the fluxes, which is computed in many ways (as simulations of the primary interactions + measurements in the decay tunnel, cross-checks with other known processes).
- Final result in the fluxes ratio : ± 2% (syst),

$$\to \Delta \sin^2\theta_w = \pm 0.005.$$

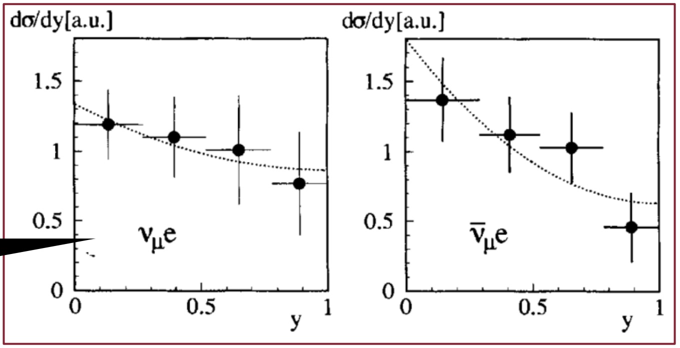


Pure leptonic ν processes : results

Results (from $\nu_\mu e$) :

- $\sin^2\theta_W$ = 0.2324 ± .0058 ± .0059 CHARM
- = 0.2311 ± .0077 (a)
- = 0.2230 ± .0077 (b).

CHARM, Phys. Lett. B 320, 203 (1994).

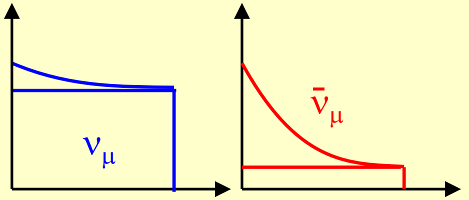


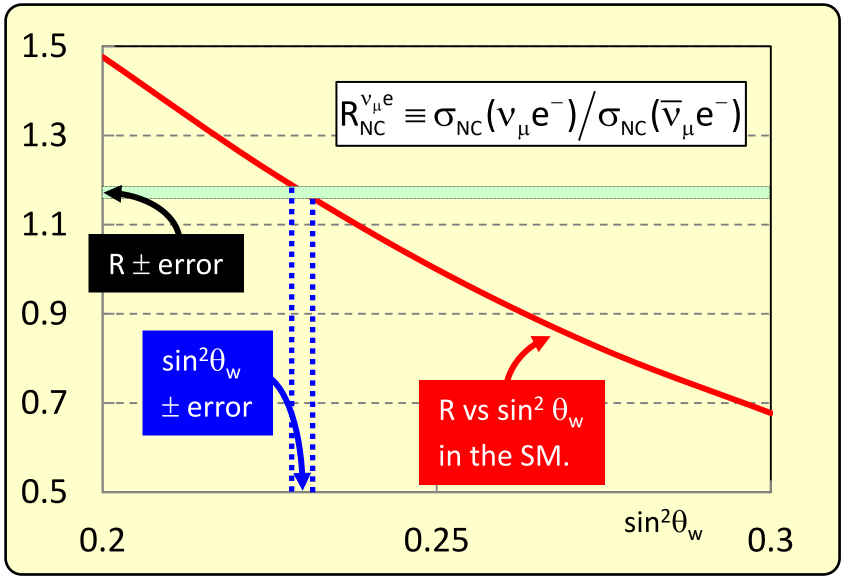
(a) and (b) are from current PDG, for ν's on isoscalar target:

- different because of definition of higher order parameters ("scheme", see the radiative corrections in § LEP).
- the y-distributions contain information on g_L and g_R (i.e. a new determination of the couplings) + a cross-check.

$$\frac{d\sigma_{NC}(\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[g_L^{e2} + (1-y)^2 g_R^{e2}\right];$$

$$\frac{d\sigma_{NC}(\bar\nu_\mu e^-)}{dy} = \frac{G_F^2 s}{\pi}\left[g_R^{e2} + (1-y)^2 g_L^{e2}\right].$$





sketch of the method R → $\sin^2\theta_W$ [see previous page]

sketch of the method dσ (ν,ν̄)/dy → g_L, g_R.

References

1. e.g. [BJ, 14.3], [YN1, 17.7-8], [YN2, 2.1-3];
2. old review : Steinberger, CERN 76-20 (Yellow report);
3. more modern review : Rev.Mod.Phys. 70 (1998) 1341;
4. ν beams : Kopp, Phys.Rep. 439 (2007) 101.

Found on the web – Courtesy of an unknown author.

Gustave Doré (1832–1883) - Pantagruel with his father Gargantua and mother Gargamelle - watercolor

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End of chapter 7