Particle Physics - Chapter 5

$K^0$ mesons - CKM matrix
5 – $K^0$ mesons – CKM matrix

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*This section belongs to another chapter: It is here because of the similarity between $\nu$ and $K^0$ oscillations.*
• The neutral mesons $K^0$ and $\bar{K}^0$ are special quark systems, in which unusual and surprising phenomena are generated.

• The mathematical interpretation of these phenomena is based almost exclusively on the application of the fundamental principles of q.m., in particular the principle of quantum superposition.

• The experimental observation of the effects of oscillation and regeneration is a further elegant confirmation of the validity of these principles.

• The successes of the experimental physics of the '50s and '60s have been based both on the confirmation of accurate theoretical predictions (like oscillations) and to new and unexpected phenomena (like $CP$ violation).

• They have been possible thanks to new techniques (e.g. regeneration), and to new experimental methods (e.g. the new accelerators, bubble / spark chambers) and by data analysis via computer.

• The study of these particles is possible only by analyzing the symmetry of Nature; $K^0$ physics emerges from the analysis of $CPT$ symmetries, strangeness and isospin.

• In successive years, the $K^0$ meson system has been replicated by the $B^0$ mesons, with further fundamental studies.

• The interpretation in the SM of the flavor and $CP$ violations requires the weak interactions theory and the $CKM$ matrix.

• ... but we hope that experiments show also physics $bSM$ !!!
Quarks and antiquarks of the u and d type can form two **different** neutral mesons: \((u\bar{u})\) (\(d\bar{d}\)), or linear combinations like \(\pi^0\) or \(\eta\) [see § quark model].

The same mechanism holds when heavier families, like \((c\bar{s})\) (\(t\bar{b}\)), are considered. Each heavy flavor has a quantum number which identifies it and its \(\bar{q}\).

These states make sense in a quantum basis of distinct **conserved flavors**, as in strong interactions.

In different quantum bases (e.g. the one where \(\mathbb{C}\mathbb{P}\) is conserved, but not \(\mathbb{C}\) and \(\mathbb{P}\) separately), different states appear, which are linear superposition of the above.

These states may offer a more natural description of the phenomena.

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### Quantum Numbers of Neutral Mesons

<table>
<thead>
<tr>
<th>(q\bar{q})</th>
<th>(K^0)</th>
<th>(\bar{K}^0)</th>
<th>(D^0)</th>
<th>(\bar{D}^0)</th>
<th>(B^0_d)</th>
<th>(\bar{B}^0_d)</th>
<th>(B^0_s)</th>
<th>(\bar{B}^0_s)</th>
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<tr>
<td>(d\bar{s})</td>
<td>(s\bar{d})</td>
<td>(c\bar{u})</td>
<td>(u\bar{c})</td>
<td>(d\bar{b})</td>
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### Quantum Numbers of \(q\bar{q}\) Neutral Mesons

- **\(K^0\) and \(K^+\)** are in the same doublet and contain \(\bar{s}\); **\(B^0/B^+\)** contain \(\bar{b}\), while **\(D^0\)** and **\(D^+\)** contain \(c\) (not \(\bar{c}\)).

### Questions (simple):
- other neutral mesons with heavy quarks? [yes, \(D^0_s\) and \(\bar{D}^0_s\) → write their q.n.]
- why states like \(t\bar{u}, t\bar{c}, \ldots\) are not listed?
The $K^0$-mesons are produced by strong interactions with a fixed strangeness $S$:

$|K^0> = |d\bar{s}>$, $S = +1$; $|\bar{K}^0> = |s\bar{d}>$, $S = -1$.

Problem: get a pure sample of $K^0$'s.

A $K^0$ sample is created, e.g. $(\pi^- p \rightarrow \Lambda K^0)$, with a "threshold energy" [next slide]:

$$E_{\pi^-}^{min} = \frac{(m_\Lambda + m_K)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 0.91 \text{ GeV},$$

to be compared with $(\pi^- p \rightarrow K^0 \bar{K}^0 n)$:

$$E_{\pi^-}^{min} = \frac{(2m_K + m_N)^2 - (m_\pi^2 + m_N^2)}{2m_N} = 1.50 \text{ GeV},$$

Since these processes are the simplest for $K^0 / \bar{K}^0$ respectively, with $0.91 < E_{\pi^-} < 1.50$ GeV only $K^0$'s are produced [the observation of the products of the interaction confirms the conservation of $S$].

However, even when selecting pure $K^0$'s, some unexpected $\bar{K}^0$ mesons show up among the final state particles;

- this effect demonstrates that production and "life" (i.e. decay) of $K^0 / \bar{K}^0$ mesons follow different rules.

- [the weak interactions do NOT conserve $S$, therefore they do NOT distinguish $K^0$ from $\bar{K}^0$ → once produced, their $S$ is "forgotten" and they behave as the same particle, a superposition of different states]
production of $K^0$ mesons : kinematics

Study the reaction $a \ b \rightarrow \ c \ d$ (e.g. $\pi^- p \rightarrow \Lambda K^0$).

If $(m_c + m_d) > (m_a + m_b)$, it requires some kinetic energy to happen.

Study the process in the LAB system, i.e. the system where $b$ (the proton) is at rest:

- the projectile $a$ hits the target $b$, producing $c$ and $d$:
- define $E_a^{\text{min}} = \text{the minimum energy of } a \ \text{IN THE LAB, such that the process happens}$
- in this case, $c$ and $d$ are at rest in the CM frame.

\[
\text{LAB } \begin{cases} 
    a \ (E_a, \ p_a, \ 0) \\
    b \ (m_b, \ 0, \ 0) 
\end{cases} \\
\text{CM } \begin{cases} 
    c \ (m_c, \ 0, \ 0) \\
    d \ (m_d, \ 0, \ 0) 
\end{cases} \\

s_{\text{LAB}}^{\text{ini}} = m_a^2 + m_b^2 + 2E_a m_b = s_{\text{CM}}^{\text{fin}} = (m_c + m_d)^2; \\
E_a^{\text{min}} = \frac{(m_c + m_d)^2 - (m_a^2 + m_b^2)}{2m_b}.
\]

- what, if $E_a^{\text{min}} < m_a$ ??? (an easy question);
- the result does NOT depend on the dynamics, but only on general kinematical constraints : it will be used in similar cases.
production of $K^0$ mesons : comments

To be specific, these strong interactions are **allowed**, because they conserve $S$ :

a. $K^+ \, n \rightarrow K^0 \, p$;
b. $K^- \, p \rightarrow \bar{K}^0 \, n$;
c. $K^0 \, p \rightarrow K^+ \, n$;
d. $\bar{K}^0 \, p \rightarrow \pi^0 \, \Sigma^+$;

• instead, the following s.i. are **forbidden** :
  
  e. $K^+ \, n \rightarrow \bar{K}^0 \, p$;
f. $K^- \, p \rightarrow K^0 \, n$;
g. $\bar{K}^0 \, p \rightarrow K^+ \, n$;
h. $K^0 \, p \rightarrow \pi^0 \, \Sigma^+$.

• Reactions (e-h) are only forbidden by $S$ conservation;

• for a particle-antiparticle pair, because of the $\mathbb{C}\mathbb{P}\mathbb{T}$ symmetry, all the intrinsic properties are exactly correlated (equal or opposite mass, spin, charge, baryon-lepton number, decay channels, BR's).

• However, sometimes, the $K^0$ particle, generated via reaction (a), re-interacts as a $\bar{K}^0$ via reaction (d), or (b) $\rightarrow$ (c) :
  
  i. $K^+ \, n \rightarrow "X^0" \, p$, $"X^0" \, p \rightarrow \pi^0 \, \Sigma^+$;
  ii. $K^- \, p \rightarrow "Y^0" \, n$, $"Y^0" \, p \rightarrow K^+ \, n$;

  $[X^0/Y^0 = K^0$ or $X^0/Y^0 = \bar{K}^0 \,?]$

• it seems that there are transitions "in flight" (i.e. oscillations) $K^0 \leftrightarrow \bar{K}^0$.

• Can this effect show up also in their decay ?

NB Transitions (n $\leftrightarrow$ n) are forbidden because of baryon number, (e$^+$ $\leftrightarrow$ e$^-$) because of electric charge and lepton number. All these "charges" are conserved by all known interactions. Instead the oscillations ($K^0 \leftrightarrow \bar{K}^0$) are only forbidden by $S$ conservation.
A nice oscillation $K^0 \rightarrow \bar{K}^0$:

1. beam of $K^+$;
2. $\pi^- p \rightarrow \Lambda \pi^+ \pi^0$;
3. $\Lambda \rightarrow p \pi^-$ (decay);
4. $\bar{K}^0 p \rightarrow \Lambda \pi^+ \pi^0$;
5. main vertex $K^+ p \rightarrow K^0 p \pi^+ \pi^0$;
6. $K^0 \rightarrow \bar{K}^0$ (???);

[end/right → start/left] $K^0$ and $\bar{K}^0$ unambiguously identified, no other explanation.
In addition, the decay of $K^0$ and $\bar{K}^0$ was not understood and created a puzzle.

- Both $K^0$ and $\bar{K}^0$ can decay into $(\pi^+\pi^-)$ and $(\pi^+\pi^-\pi^0)$ [2$\pi$ and 3$\pi$ states have different G-parity, but G is NOT conserved in w.i.].

The explanation was provided by Gell-Mann and Pais [Phys. Rev. 97, 1387 (1955)], before the discovery that w.i. violate parity:

- $K^0$ and $\bar{K}^0$ are eigenstates of the strong interactions;
- each is the antiparticle of the other, the $\mathbb{C}$ operator transforms ($K^0 \leftrightarrow \bar{K}^0$);
- they have opposite strangeness $S$;
- if $S$ were not there, they would mix (like in $\pi^0$ and $\eta$);
- w.i. do not conserve $S$;
- ... and see a mixture of $K^0$ and $\bar{K}^0$.

Consequences:

- the mixture is interpreted as two new states, quantum superpositions of $K^0/\bar{K}^0$;
- if w.i. conserve $\mathbb{C}\mathbb{P}$, the two new states must be $\mathbb{C}\mathbb{P}$ eigenstates(*);
- since the new states are NOT a particle-antiparticle pair, they may have different properties (masses, lifetimes, decays);
- if the mass difference allows for that, the states oscillate between themselves;
- the only known decay was ("$K^0$" $\rightarrow \pi^+\pi^-$); a possible transition, generated via w.i., is then $[K^0 \leftrightarrow (\pi^+\pi^-) \leftrightarrow \bar{K}^0]$;
- another "$K^0$" must exist, "$K^0" \rightarrow \pi\pi\pi$.

(*) Today we know that the w.i. violate also $\mathbb{C}\mathbb{P}$, but this violation is small, so provisionally we do not take it into account.
the $K^0 \leftrightarrow \bar{K}^0$ puzzle: predictions

(more formally ...)

**TWO "K0" STATES:**

- different values of CP $\rightarrow$ CP $= \pm 1$;
- one with CP=+1 and decay $\rightarrow (\pi\pi)$, another with CP=−1 and decay $\rightarrow (\pi\pi\pi)$;
- other decays are allowed for both states, but they have to conserve $\mathbb{C}P$ (e.g. no $\rightarrow \pi\pi$ for the state CP=−1);
- the state $(\pi\pi\pi)$ is near the kinematical threshold ($m_K \approx 3m_\pi + 70$ MeV) $\rightarrow$ the lifetime of the $(\pi\pi\pi)$ state is much longer than the lifetime of the $(\pi\pi)$ one.
- the obvious proposal was to call "short" the CP=+1 state and "long" the CP=−1;
- so, two new particles have born:
  - they have been discovered;
  - their lifetimes and properties have been measured and found in agreement with the predictions:

1) $K_S^0 : CP = +1$, $\tau = 0.90 \times 10^{-10}$ s,
   decay $\rightarrow \pi^+ \pi^-$, $\rightarrow \pi^0 \pi^0$;

2) $K_L^0 : CP = -1$, $\tau = 0.51 \times 10^{-7}$ s,
   decay $\rightarrow \pi^+ \pi^- \pi^0$, $\pi^0 \pi^0 \pi^0$.

J.W. Cronin and M.S. Greenwood, Physics Today (July 1982):

"So these gentlemen, Gell-Mann and Pais, predicted that in addition to the short-lived K mesons, there should be long-lived K mesons. They did it beautifully, elegantly and simply. I think theirs is a paper one should read sometime just for its pure beauty of reasoning. It was published in Physical Review in 1955. A very lovely thing! You get shivers up and down your spine, especially when you find you understand it. At the time many of the most distinguished theoreticians thought this prediction was really baloney."
the $K^0 \leftrightarrow \bar{K}^0$ puzzle: oscillations

In q.m. or quark model language:

- Both the $K^0$ and $\bar{K}^0$ decay via w.i. in the same final states; the $\pi^+\pi^-$ diagram is shown in the figure, while the others ($\pi^0\pi^0; \pi^+\pi^-\pi^0; \pi\ell\nu$) are similar:

- The oscillations can be understood as a continuous transformation between the $K^0$ and $\bar{K}^0$ themselves, via the second order box-diagrams, or as a mixture, with time-dependent coefficients $\alpha(t), \beta(t)$:

$$|K(t)\rangle = \alpha(t)\ |K^0\rangle + \beta(t)\ |\bar{K}^0\rangle;$$

$$\alpha(t)^2 + \beta(t)^2 = 1 \quad \text{[× a decreasing function of } t, \text{ to account for their decay]}$$
The $K^0_L$ was first observed in 1956 by Lande and coll. with a cloud chamber.

- Brookhaven Cosmotron (3 GeV protons).
- Path between the beam and the cloud chamber (6 meters) is $\sim 100$ $K_S^0 / \Lambda$ lifetimes.
- This path is therefore sufficient for the decay of all strange particles known at the time.

A few months later the same authors confirmed the result. They also observed in the cloud chamber interactions of these particles with the nuclei of He, producing final states with total $S \neq 0$, like $(K^0, 4He \rightarrow \Sigma^- p p n \pi^+)$. These states cannot be generated by a $K^0$, because of the value of $S$.

However, no $K^0$ should be present, because the primary proton energy was chosen to be below the energy threshold for $K^0$ production, which is higher than for $K^0$ [same argument as before].

For some reason, $\bar{K}^0$ mesons have "appeared" $\rightarrow$ oscillation.
The \( K^0_L \) was first observed in 1956 by Lande and coll. with a cloud chamber.

They found 26 events with a "V-zero", incompatible to be \((\pi^+\pi^-)\) because of their \( Q^2 \) (one shown on the right).

[Today we interpret these events as decays \((\pi^\pm e^\mp \nu_e), (\pi^\pm \mu^\mp \nu_\mu), (\pi^\pm \pi^\mp \pi^0)\).]

Events consistent with 3 body decays of neutral mesons of mass \( \sim 500 \text{ MeV} \).

First estimate of the lifetime: \( 10^{-9} \text{ s} < \tau < 10^{-6} \text{ s} \), now \( \tau = 0.53 \times 10^{-7} \text{ s} \).

Another beautiful and "impossible" event (no \( \bar{K}^0 \) in the beam, see previous pages).
In the following slides we assume that the $K^0$ decay conserve $\mathbb{C}\mathbb{P}$, i.e. that both $K^0_S$ and $K^0_L$ are $\mathbb{C}\mathbb{P}$ eigenstates with eigenvalues $= \pm 1$.

Although this is not true (see later), the violation is small and therefore the results obtained with this approximation are in fair agreement with (almost) all observations.

To remember that, the next pages are marked by a little sign "$\mathbb{C}\mathbb{P}$" in the upper right corner.

warning: the sign of $\zeta$ in
$\mathbb{C} |K^0> = \zeta |\bar{K}^0>;$ $\mathbb{C} |\bar{K}^0> = \zeta |K^0>$;
is non-physical; in literature both $\zeta=\pm 1$;
here we (try to) stick to $\zeta = -1$. 

**K^0 decays in CP eigenstates: K_S^0 and K_L^0**

- The states \(|K^0\rangle\) and \(|\bar{K}^0\rangle\) are strong interactions (s.i.) eigenstates:
  \[
  \mathbb{C} |K^0\rangle = - |\bar{K}^0\rangle; \quad \mathbb{C} |\bar{K}^0\rangle = - |K^0\rangle; \\
  \mathbb{P} |K^0\rangle = - |K^0\rangle; \quad \mathbb{P} |\bar{K}^0\rangle = - |\bar{K}^0\rangle; \\
  \mathbb{CP} |K^0\rangle = + |\bar{K}^0\rangle; \quad \mathbb{CP} |\bar{K}^0\rangle = + |K^0\rangle;
  \]

- These equations show that the s.i. states \(K^0 / \bar{K}^0\) are NOT \(\mathbb{CP}\) eigenstates;

- \(|K_1^0\rangle\) and \(|K_2^0\rangle\) are linear combinations of \(|K^0\rangle\) and \(|\bar{K}^0\rangle\), which are \(\mathbb{CP}\) eigenstates:
  \[
  |K_1^0\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle + |\bar{K}^0\rangle \right]; \\
  |K_2^0\rangle = \frac{1}{\sqrt{2}} \left[ |K^0\rangle - |\bar{K}^0\rangle \right]; \\
  |K^0\rangle = \frac{1}{\sqrt{2}} \left[ |K_1^0\rangle + |K_2^0\rangle \right]; \\
  |\bar{K}^0\rangle = \frac{1}{\sqrt{2}} \left[ |K_1^0\rangle - |K_2^0\rangle \right].
  \]

- The \((\pi\pi)\) and \((\pi\pi\pi)\) give (next slide): 
  \[
  \mathbb{CP} |2\pi\rangle = + |2\pi\rangle; \\
  \mathbb{CP} |3\pi\rangle = - |3\pi\rangle;
  \]

- Therefore:
  \(K_S^0 \equiv K_1^0;\quad K_L^0 \equiv K_2^0.\)

- \(K^0 \) and \(\bar{K}^0\) are eigenstates of the strong interactions;
- therefore, the creation process generates one of them [NOT the other];
- but, as soon as they are created, they behave as a linear combination of \(K_S^0\) and \(K_L^0\);
- therefore they "live" (i.e. decay) as them;
- then \(K_S^0 \rightarrow 2\pi\) (lot of phase space, small \(\tau\));
- and \(K_L^0 \rightarrow 3\pi\) (small phase space, long \(\tau\));
- if \(K_{S,L}^0\) interact via strong interactions, they come back to the s.i. eigenstates, as \(K^0\) or \(\bar{K}^0\) with a given probability each.

If \(\mathbb{CP}\) not conserved, NOT true !!!

### Notes

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- if \(K_{S,L}^0\) interact via strong interactions, they come back to the s.i. eigenstates, as \(K^0\) or \(\bar{K}^0\) with a given probability each.
K⁰ decays in $\mathbb{C}\mathbb{P}$ eigenstates : eigenvalues

Compute the eigenvalues of $\mathbb{C}\mathbb{P}$.

For 2$\pi$ systems :

• Since $J^{PC}(\pi^0) = 0^{-+}$ :

  $\mathbb{P} \quad |\pi^0\pi^0> = (-)^2 (-)^L |\pi^0\pi^0> = + |\pi^0\pi^0> ;$
  $\mathbb{C} \quad |\pi^0\pi^0> = (+)^2 |\pi^0\pi^0> = + |\pi^0\pi^0> ;$
  $\mathbb{C}\mathbb{P} \quad |\pi^0\pi^0> = + |\pi^0\pi^0> ;$

• if $L = S_1 = S_2 = 0$ :

  $\mathbb{P}\mathbb{C} \quad |\pi^+\pi^-> = \mathbb{P} \quad |\pi^-\pi^+> = + |\pi^+\pi^-> ;$

• i.e. CP(2$\pi$) = +1, both for the ($\pi^0\pi^0$) and ($\pi^+\pi^-$) systems.

For 3$\pi$ systems :

• $P(\pi^0 \pi^0 \pi^0) = (-)^3 (-)^{L_1} (-)^{L_2} = -1;$
  $C(\pi^0 \pi^0 \pi^0) = (+)^3 \quad = +1;$
  $CP(\pi^0 \pi^0 \pi^0) = -1;$

• $P(\pi^+ \pi^- \pi^0) = (-)^3 (-)^{L_1} (-)^{L_2} = -1;$
  $C(\pi^+ \pi^- \pi^0) = (+) (-)^{L_1} \quad = +1;$
  $CP(\pi^+ \pi^- \pi^0) = -1;$

• i.e. CP(3$\pi$) = −1, both for the ($\pi^0\pi^0\pi^0$) and ($\pi^+\pi^-\pi^0$) systems.

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Conclusion: after strange particle production, expect two neutral particles of (not exactly, but almost) equal mass [actually 498 MeV]:

- the shorter ($K_S^0$) with
  - CP = +1;
  - decay into $2\pi$;
  - "short" lifetime;
  - $[\tau_S = 0.90 \times 10^{-10} \text{ s} = 7.4 \text{ } \mu\text{eV}^{-1}, \ell_S = c\tau_S = 2.68 \text{ cm}]$;

- the longer ($K_L^0$) with
  - CP = −1;
  - decay into $3\pi$;
  - "long" lifetime [$580 \times \tau_S$];
  - $[\tau_L = 0.51 \times 10^{-7} \text{ s} = 0.013 \text{ } \mu\text{eV}^{-1}, \ell_L = 15.5 \text{ m}]$

- therefore:
  - $\Delta\Gamma_K \equiv \Gamma_L - \Gamma_S \approx -\Gamma_S = -7.4 \text{ } \mu\text{eV} = -11.2 \text{ ns}^{-1}$.

$1 \text{ } \mu\text{eV} = 1.52 \text{ ns}^{-1}$; $1 \text{ ns}^{-1} = 0.66 \text{ } \mu\text{eV}$.
• While the $K^0$ and $\bar{K}^0$ masses are equal because of $\mathbb{CPT}$, no symmetry equalizes the masses and lifetimes of $K_S^0$ and $K_L^0$;
• the measurement gives [see later]:
  $\Delta m_K = m(K_L^0) - m(K_S^0) = 3.51 \pm 0.018 \text{\,µeV}$
  $= 5.303 \pm 0.009 \text{\,ns}^{-1}$;
• $\Delta m_K \approx -\frac{1}{2} \Delta \Gamma_K$ [no explanation, but deep phenomenological consequences];
• the mass difference means that the two states [$K_L^0$ and $K_S^0$] evolve with different time constants;
• following the evolution on the basis $(K^0, \bar{K}^0)$, a "desynchronization" is observed between the $K_S^0$ and $K_L^0$ components, interpreted as oscillations ($K^0 \leftrightarrow \bar{K}^0$);
• a little algebra shows that, instead of a pure evolution of a particle of width $\Gamma$, which would give rise to an intensity $N(t) \propto \exp(-\Gamma t)$, we have a different phenomenon:
  $\psi_s(t) = \psi_s^0 \exp\left[-\left(\frac{\Gamma_s}{2} + \text{im}_s\right)t\right]$;
  $\psi_l(t) = \psi_l^0 \exp\left[-\left(\frac{\Gamma_l}{2} + \text{im}_l\right)t\right]$;
• take a pure $K^0$ beam at $t=0$: then, in case of no decay ($\Gamma = 0$, $\tau = \infty$), the probability $P$ to find a $K^0$ or a $\bar{K}^0$, function of $t$, is:
  $P_{K^0}(t) = \frac{1}{4} \left|e^{-\text{im}_s t} + e^{-\text{im}_l t}\right|^2 = \cos^2\left(\frac{\Delta m_K}{2} t\right)$;
  $P_{\bar{K}^0}(t) = \frac{1}{4} \left|e^{-\text{im}_s t} - e^{-\text{im}_l t}\right|^2 = \sin^2\left(\frac{\Delta m_K}{2} t\right)$.
• In addition, the oscillations are damped by the occurrence of the decays ($\tau_L = 1/\Gamma_L >> \tau_S = 1/\Gamma_S$); $\Gamma_S$ dominates, because of the shorter lifetime [next slide].
The amount of $K^0$ and $\bar{K}^0$ can be computed as a function of (proper) time, by simple considerations of quantum mechanics.

E.g. starting with pure $K^0$ (fig.), there is an "oscillation" between the two states, according to $\tau_S$, $\tau_L$, $\Delta m (=|m_S-m_L|)$.

The figure is made with $\tau_S << \tau_L$ and $\Delta m = 1/(2\tau_S)$ (not exact, but realistic and simple).

For the computations, see next page.

\[
R(K^0)(t) = \left|\langle K^0 | \psi(t) \rangle \right|^2 = \frac{1}{4} \left[ \exp \left( -\frac{t}{\tau_S} \right) + \exp \left( -\frac{t}{\tau_L} \right) + 2\exp \left( -\frac{\tau_L + \tau_S}{2\tau_L \tau_S} t \right) \cos(\Delta m_K t) \right];
\]

\[
R(\bar{K}^0)(t) = \left|\langle \bar{K}^0 | \psi(t) \rangle \right|^2 = \frac{1}{4} \left[ \exp \left( -\frac{t}{\tau_S} \right) + \exp \left( -\frac{t}{\tau_L} \right) - 2\exp \left( -\frac{\tau_L + \tau_S}{2\tau_L \tau_S} t \right) \cos(\Delta m_K t) \right].
\]
Some (simple and tedious) algebra. Start with $f K^0$ and $(1-f) \bar{K}^0$. Then put $f=1$:

$$|\psi(t = 0)\rangle = f |K^0\rangle + (1-f) |\bar{K}^0\rangle = \frac{f}{\sqrt{2}} (|K^0_S\rangle + |K^0_L\rangle) + \frac{1-f}{\sqrt{2}} (|K^0_L\rangle - |K^0_S\rangle) = \frac{2f-1}{\sqrt{2}} |K^0_S\rangle + \frac{1}{\sqrt{2}} |K^0_L\rangle;$$

$$|\psi(t)\rangle = \frac{2f-1}{\sqrt{2}} e^{\frac{(\Gamma_S + im_S)t}{2}} |K^0_S\rangle + \frac{1}{\sqrt{2}} e^{\frac{(\Gamma_L + im_L)t}{2}} |K^0_L\rangle \rightarrow \frac{1}{\sqrt{2}} e^{\frac{(\Gamma_S + im_S)t}{2}} |K^0_S\rangle + \frac{1}{\sqrt{2}} e^{\frac{(\Gamma_L + im_L)t}{2}} |K^0_L\rangle;$$

$$R(K^0)(t) = \left| \langle K^0 | \psi(t) \rangle \right|^2 = \frac{1}{4} \left[ \left| \langle K^0 | K^0_S \rangle + \langle K^0 | K^0_L \rangle \right| \right]^2 = \left| \frac{1}{2} e^{-i\gamma} + \frac{1}{2} e^{i\gamma} \right|^2 = \left| \cos \gamma \right|^2 = \cos^2 \left( \frac{\Delta m_K t}{2} \right).$$

**Damped oscillation (previous slide).** If both $\tau_L$ and $\tau_S \gg 1/\Delta m_K$ (not true) $\rightarrow$ simple oscillation.

The computations for $R(\bar{K}^0)(t)$ and for $f \neq 1$ are left to the (patient) reader.
K⁰ oscillations: semileptonic decays

- To test this prediction, the experimental problem [Bettini] is to distinguish K⁰ ↔ K⁰ when they decay. It is not possible from the 2π or 3π states, because these channels have definite CP, not definite strangeness.

- To select definite strangeness states, select semileptonic decays of K⁰. These decays obey the "ΔS = ΔQ rule": the difference between the strangeness of the hadrons in the final and initial states is equal to the difference of their electric charges. The rule is a consequence of the quark contents of the states [K⁰ = ̅s d]:

\[ \bar{s} \rightarrow \bar{u} \ell^+ \nu_\ell \Rightarrow K^0 \rightarrow \pi^- \ell^+ \nu_\ell; \ K^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell; \]
\[ s \rightarrow u \ell^- \bar{\nu}_\ell \Rightarrow \bar{K}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell; \ \bar{K}^0 \rightarrow \pi^- \ell^+ \nu_\ell. \]

- The sign of the charged lepton flags the strangeness of the K⁰/\bar{K}⁰. The semileptonic decays are called K⁰_{e3} and K⁰_{μ3} depending on the lepton. Their branching ratios are large:

BR(K⁰_{e3}) = 41%, BR(K⁰_{μ3}) = 27%.

- The experimental measure regards the charge asymmetry δ, i.e. the difference between +ve and −ve leptons, which is directly related to the oscillations. The results agree very well with the expectations, but the tail.
The regeneration (Pais and Piccioni, 1956) consisted in a clever use of an absorber (the "regenerator"), positioned at a distance determined by $\tau_S$ and $\tau_L$, to demonstrate the superposition of $K^0$ and $\bar{K}^0$.

[explanation on the next slide]
K° regeneration: the idea

- Start with a pure K° beam in vacuum (equal amounts of K° and K°).
- After \( t \approx 10 \tau_S \) the K° intensity down by factor \( e^{(-t/\tau_S)} = e^{-10} \approx 45 \times 10^{-6} \) (none left).
- \( \text{[For K° with 1 GeV momentum this corresponds to \( \sim 0.5 \) m.]} \)
- The K° intensity is down by \( e^{(-t/\tau_L)} \approx 0.98 \), i.e. all left.
- After 0.5 m, 100% K° (50% K° + 50% K°).
- If we put another target at [say] \( t = 20 \tau_S \) [1 m downstream], we will get K° interactions as well as K°.
- K° and K° interact (strongly) differently in the target:
  
- K° p \( \rightarrow \) K° p, K° n;
- K° n \( \rightarrow \) K° n;
- K° p \( \rightarrow \) K° p, \( \Lambda \pi^+ ; \rightarrow \Sigma^0 \pi^+, \Sigma^+ \pi^0 \);
- K° n \( \rightarrow \) K° n, \( \Lambda \pi^0 ; \rightarrow \Sigma^+ \pi^-, \Sigma^0 \pi^0, \Sigma^- \pi^+ \);
- The s quark from the K° can swap with one of the quarks in the proton or neutron, but the \( \bar{s} \) from the K° cannot [e.g. K° p \( \rightarrow \Lambda \pi^+ \), but \( K^0_p \rightarrow \Lambda X \)] .
- Hence there are more K° processes, so the K° are more strongly absorbed.
- Then, no longer 50% K° + 50% K° (as in K°), but an amount of K° has "born".
- So will have some K° decays again.
The experiment used a beam of 1.1 GeV $\pi^-$ from the "Bevatron", the 6.2 GeV ("BeV", old American) proton synchrotron at LNL, Berkeley.

The propane bubble chamber was able to measure the $\pi^\pm$ momenta by their curvature in magnetic field.

Therefore the angle $\theta$ (shown in the fig) is measured.
K⁰ regeneration: results

A study of the phenomenon by M. Good (1957) considered three types of regeneration, with different distributions of the angle θ between the incoming and the regenerated particle:

1. Regeneration for transmission ("forward") : θ = 0. No momentum transfer to the nucleus: coherent.
2. Regeneration for diffraction: elastic scattering, θ distribution as in diffraction.
3. Inelastic regeneration: interaction with individual nucleons, θ distribution as in scattering.

• The relative amount of the three depends on the small mass difference ∆m⁰ = m(K_L⁰) – m(K_S⁰);
• 200 observed 2π decays;
• they were able to confirm oscillations and regeneration;
• ... and to measure the mass difference (units ℏ/τₜ) :

  ∆m⁰ = 0.84^{+0.89}_{-0.22};

[very clever result, despite present best value is 2 σ smaller]
Redefine the $K^0$ mesons system:

- $K^0$ and $\bar{K}^0$ as the particle produced in strong interactions (i.e. s.i. eigenstates):
  - $|K^0> = |d\bar{s}>, S = +1$; $|\bar{K}^0> = |s\bar{d}>, S = -1$;
  - $\mathbb{C} |K^0> = -|\bar{K}^0>$; $\mathbb{C} |\bar{K}^0> = -|K^0>$;

- $K_1^0$ and $K_2^0$ as the $\mathbb{CP}$ eigenstates:
  - $|K_1^0> = 1/\sqrt{2} [ |K^0> + |\bar{K}^0> ]$;
  - $|K_2^0> = 1/\sqrt{2} [ |K^0> - |\bar{K}^0> ]$;
  - $\mathbb{CP} |K_1^0> = + |K_1^0>$;
  - $\mathbb{CP} |K_2^0> = - |K_2^0>$;

- $K_S^0$ and $K_L^0$ as the states with lifetimes $\tau_S$, $\tau_L$ [NOT necessarily $\mathbb{CP}$ eigenstates]:
  - $\tau_S = 0.90 \times 10^{-10}$ s; $\tau_L = 0.51 \times 10^{-7}$ s;

- the $(\pi^+\pi^-)$, $(\pi^0\pi^0)$, $(\pi^+\pi^-\pi^0)$ systems are $\mathbb{CP}$ eigenstates:
  - $\mathbb{CP} |2\pi> = + |2\pi> ; \mathbb{CP} |3\pi> = - |3\pi>$;

- Clearly, if $K_1^0 = K_S^0$, $K_2^0 = K_L^0$, then $\mathbb{CP}$ is conserved in the $K^0$ decays; i.e. $\mathbb{CP}$ conservation implies $K_S^0 \rightarrow 2\pi$, $K_L^0 \rightarrow 3\pi$;

- On the contrary, decays $K_L^0 \rightarrow 2\pi$, $K_S^0 \rightarrow 3\pi$ with small, but non-0 BR, would be an experimental evidence of the NON-CONSERVATION of $\mathbb{CP}$.

In the following slides we do NOT assume $\mathbb{CP}$ conservation in $K^0$ decays. The little "$\mathbb{CP}$" in the upper right corner has disappeared.
Consider three possible interactions:

a. **C and P conserved** ["strong i."]:
   - C, P conserved separately,
   - strangeness conserved;
   - eigenstates $K^0, \bar{K}^0$;

b. **CP conserved**:
   - C, P not conserved separately, but CP conserved;
   - strangeness NOT conserved;
   - eigenstates $K_1^0 \to 2\pi, K_2^0 \to 3\pi$ [because $2\pi$ and $3\pi$ states are CP eigenstates];

c. **CP non conserved** ["weak i."]:
   - $K_S^0, K_L^0$ decay with lifetimes $\tau_S, \tau_L$;
   - strangeness NOT conserved;
   - eigenstates $K_S^0, K_L^0$ [$K_S^0$ and $K_L^0$ NOT CP eigenstates].

Strong interactions follow [a].

If weak interactions conserve CP, then they follow [b]:

$$|K_1^0> = |K_S^0>, |K_2^0> = |K_L^0>,$$

$K_S^0 \to 2\pi, K_L^0 \to 3\pi$.

Instead, if CP is violated in w.i., then [b] is only a first approx. of [c].

The discriminant is the existence (at least with a small BR) of the decays:

$K_S^0 \to 3\pi, K_L^0 \to 2\pi$.

Conclusion:

since a small amount of ($K_S^0 \to 3\pi$) is not observable, due to the background ($K_L^0 \to 3\pi$), the key observation is ($K_L^0 \to 2\pi$).
\(d)\) Mass eigenstates in matter:

\[
|K_{S,M}^0\rangle = \frac{|K_1^0\rangle + \varepsilon^M|K_2^0\rangle}{\sqrt{1+|\varepsilon^M|^2}};
\]

\[
|K_{L,M}^0\rangle = \frac{\varepsilon^M|K_1^0\rangle + |K_2^0\rangle}{\sqrt{1+|\varepsilon^M|^2}}.
\]

\(\text{(CP violation in matter)}\)

\(c)\) Mass eigenstates in vacuum:

\[
|K_S^0\rangle = \frac{|K_1^0\rangle + \varepsilon|K_2^0\rangle}{\sqrt{1+|\varepsilon|^2}};
\]

\[
|K_L^0\rangle = \frac{\varepsilon|K_1^0\rangle + |K_2^0\rangle}{\sqrt{1+|\varepsilon|^2}}.
\]

\(\text{(CP violation in vacuum)}\)

\(b)\) CP eigenstates:

\[
|K_1^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle); \quad \text{CP} = +1;
\]

\[
|K_2^0\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle); \quad \text{CP} = -1;
\]

\[
|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle + |K_2^0\rangle);
\]

\[
|\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(|K_1^0\rangle - |K_2^0\rangle).
\]

\(\text{(K^0 oscillations+decay, regeneration)}\)

\(a)\) Flavor eigenstates:

\[
|K^0\rangle = d\bar{s}; \quad S = +1; \quad \text{CP} \quad |K^0\rangle = +|\bar{K}^0\rangle;
\]

\[
|\bar{K}^0\rangle = s\bar{d}; \quad S = -1; \quad \text{CP} \quad |\bar{K}^0\rangle = +|K^0\rangle.
\]

\(\text{(strong interactions)}\)
In 1964 an experiment was built to search for CP violation at the Brookhaven AGS (Alternating Gradient Synchrotron).

The schematic layout is shown in the fig.:

- the primary proton beam (30 GeV) hits a beryllium target;
- secondaries at $\theta = 30^\circ$ are selected;
- if charged, collimated and bent away;
- if neutral, collimated and let decay;
- the resultant $K_L^0$ (long lifetime) hit a second lead target, regenerate and are let decay again in a long decay tube;
- no $K_S^0$ left $\rightarrow$ if CP is conserved, only long lifetime $K_L^0$ [$= K_2^0$] should remain and decay $\rightarrow 3\pi$;
- if (2$\pi$) observed $\rightarrow$ CP is violated !!!

16 years after, in Stockholm

James Cronin Val Fitch
Helium bag for $K_L^0$ decays + two-arm-spectrometer.

Each of the two arms:
- spark chambers (→ position);
- magnetic field (→ momentum measurement);
- scintillators (→ trigger + tof);
- water Cerenkov (→ particle id);

main background: $n$ (→ tof rejects).

Other selection criteria:
- two opposite charged particles, one for each arm;
- measure $\vec{p}_+$ and $\vec{p}_-$ (direction and module);
- assume $m_+ = m_- = m_\pi \rightarrow m_+ \approx m_K \rightarrow \text{test}$;
- angle $\theta$ between $\vec{p}_{\text{sum}} (= \vec{p}_+ + \vec{p}_-)$ and $\vec{\text{dir}}_{\text{collimator}} \approx 0 \rightarrow \text{test}$.

The three-body decays (e.g. $K_L^0 \rightarrow \pi^+\pi^-\pi^0$) do NOT satisfy those conditions:
- $(\vec{p}_+ + \vec{p}_- = \vec{p}_K - \vec{p}_0)$ not collinear with $\vec{\text{dir}}_{\text{collimator}}$;
- $m_+ \leq (m_K - m_\pi) < m_K$. 
b. distribution of $m^*$ [$=\text{mass}(\pi^+\pi^-)$] for real events and MC simulation [OK!];

c. distribution of $\cos \theta$ for 3 mass bins, with improved resolution:

- $484 < m^* < 494$ and $504 < m^* < 514$ MeV: no $K^0$ should be there: therefore few events, no excess at $\cos \theta \approx 1$;
- $494 < m^* < 504$ MeV: the signal region, lot of events, clear peak at $\cos \theta \approx 1$: THE SIGNAL !!!

d. final result (similar result for the neutral decay $\rightarrow \pi^0\pi^0$):

$$R = BR(K_L^0 \rightarrow \pi^+\pi^-) / BR (K^0_L \rightarrow \text{charged}) = (2.0 \pm 0.4) \times 10^{-3}$$

$\Rightarrow \mathbb{CP}$ is violated !!!
Q.: study the mass $m^*$

[a typical kin. problem with ambiguities + mass hypotheses]

- work in the $K_L^0$ ref. system;
- define $m^* = \text{mass}(+ve, -ve)$;
- approx. : $m_\nu \approx 0, m_e^2 \ll m_{\pi}^2$;
- look at the box

\begin{align*}
\text{a)} & \quad K_L^0 \rightarrow \pi^+\pi^- \\
& \quad m^* = m_K \text{ [easy, no problem]}; \\
\text{b)} & \quad K_L^0 \rightarrow \pi^+\pi^-\pi^0 \\
& \quad m^* |_{\text{min}} = 2m_\pi \approx 270 \text{ MeV}; \\
& \quad m^* |_{\text{max}} = m_K - m_\pi \approx 360 \text{ MeV}; \\
\text{c)} & \quad K_L^0 \rightarrow \pi^\pm e^\mp\nu/\bar{\nu} \\
& \quad m^* |_{\text{min}} = m_\pi + m_e \approx m_\pi; \\
& \quad m^* |_{\text{max}} = m_K - m_\nu \approx m_K; \\
& \quad \text{[apparently easy, but ...]} \\
\text{d)} & \quad K_L^0 \rightarrow \pi^\pm e^\mp\nu/\bar{\nu}, \text{ "e$^\mp$" interpreted as } \pi^\mp: \\
& \quad "m^*" |_{\text{min}} = m_\pi + "m_e" = 2m_\pi \approx 270 \text{ MeV}; \\
& \quad \text{for } "m^*" |_{\text{max}} \text{ compute } |\vec{p}_{\pi/e}| \text{ and } E_{\pi/e} \text{ when } |\vec{p}_\nu| \approx 0: \\
& \quad \begin{aligned}
& \quad p_\pi = p_e = \frac{m_K^2 - m_\pi^2}{2m_K} \quad [\text{see e.g. § 4}]; \\
& \quad E_\pi = "E_e" = \sqrt{m_\pi^2 + p_p^2} = \sqrt{m_\pi^2 + \frac{m_K^4 + m_\pi^4 - 2m_K^2m_\pi^2}{4m_K^2}} = \\
& \quad \quad = \frac{\sqrt{m_K^4 + m_\pi^4 + 2m_K^2m_\pi^2}}{4m_K^2} = \frac{m_K^2 + m_\pi^2}{2m_K}; \\
& \quad "m^*" |_{\text{max}} = E_\pi + "E_e" = 2E_\pi \approx m_K(1 + m_\pi^2/m_K^2). \\
\end{aligned}
\end{align*}
\( \mathbb{C}\mathbb{P} \) violation: semileptonic decays

- The \( (K_L^0 \rightarrow \pi^+\pi^-) \) is NOT the only channel, which shows \( \mathbb{C}\mathbb{P} \) violation;

- another important process is the semileptonic decay \( (K_L^0 \rightarrow \pi^\pm\ell^\mp\nu_\ell) \);

- it is an important channel, since:
  
  \[
  BR(K_L^0 \rightarrow \pi^\pm\ell^\mp\nu_\ell) \approx 40.6 \%
  \]
  
  \[
  BR(K_L^0 \rightarrow \pi^\pm\mu^\mp\nu_\mu) \approx 27.0 \%
  \]

- if \( \mathbb{C}\mathbb{P} \) were conserved, the rate with the +ve and the –ve charge would be the same, since they are connected by a \( \mathbb{C}\mathbb{P} \) transformation;

  instead, they are different; it is customary to express the difference as:

  \[
  \delta_L = \frac{\Gamma(K_L^0 \rightarrow \ell^+\nu_\ell\pi^-) - \Gamma(K_L^0 \rightarrow \ell^-\bar{\nu}_\ell\pi^+)}{\Gamma(K_L^0 \rightarrow \ell^+\nu_\ell\pi^-) + \Gamma(K_L^0 \rightarrow \ell^-\bar{\nu}_\ell\pi^+)};
  \]

  it is measured \( \delta_L = (3.32 \pm 0.06) \times 10^{-3} \).

- NOT "just another boring number".

- First evidence for difference matter-antimatter: "the real matter contains the electron with smaller BR in the \( K_L^0 \rightarrow \pi^+\ell^\mp\nu_\ell \) decay".

- In fact, some mechanism MUST have generated the asymmetry matter-antimatter of the Universe [if primordial universe was symmetric].

- However \( \delta \sim 10^{-3} \) is too small to account for the large asymmetry of our world.

- In addition, if the \( K_L^0 \) decay is the only source, at the big bang time who provided all these \( K_L^0 \)'s?
From [Bettini] :

[... A]t late times, when only $K_L$'s survive, they decay through $K_L \to \pi^- \ell^+\nu_\ell$ a little more frequently than through the $\mathbb{C}\mathbb{P}$ conjugate channel $K_L \to \pi^+ \ell^-\bar{\nu}_\ell$. [...] This shows, again and independently, that matter and antimatter are somewhat different.

Let us suppose that we wish to tell an extraterrestrial being what we mean by matter and by antimatter. We do not know whether his/her world is made of the former or the latter.

We can tell him/her: "prepare a neutral $K$ meson beam and go far enough from the production point to be sure to have been left only with the long-lifetime component." At this point s/he is left with $K_L$ mesons, independently of the matter or antimatter constitution of her/his world.

We continue: "count the decays with a lepton of one or the other charge and call positive the charge of the sample that is about three per thousand larger. Humans call matter the one that has positive nuclei."

If, after a while, our correspondent answers that his nuclei have the opposite charge, and comes to meet you, be careful, apologize, but do not shake his/her hand.

Sandro's version of the famous Feynman joke [see § 4].
The previous examples/experiments show $\mathbb{C}\mathbb{P}$ violations in the decay of neutral flavored mesons ($K^0$, in the following $B^0$).

In fact, three different types of $\mathbb{C}\mathbb{P}$ violation have been identified and measured:

a. in the mixing of neutral mesons ($M \leftrightarrow \bar{M}$) (**indirect violation**);

b. difference in the decay of a particle: $\Gamma(M \rightarrow X) \neq \Gamma(\bar{M} \rightarrow \bar{X})$ (**direct violation**);

c. **interference** between direct and indirect violation: $\Gamma(M \rightarrow X) \neq \Gamma(M \rightarrow \bar{M} \rightarrow X)$.

- in the $K^0$ system (a) is important, while in the $B^0$ system b/c dominate; the relative importance of the effect is determined by the values of the $V_{\text{CKM}}$ matrix [*see later*];
- (a) and (b) are usually parametrized by the coefficients $\varepsilon$ and $\varepsilon'$.

[the indirect violation has been discussed before, e.g. for the 1964 experiment; the couplings $qqW$ are regulated by the $V_{\text{CKM}}$ matrix, see later]
• The complex parameter $\varepsilon$ is associated with the indirect $\mathbb{C}\mathbb{P}$ violation;

• this parameter decouples the states with definite lifetimes from the $\mathbb{C}\mathbb{P}$ eigenstates:

\[
K^0_S = \frac{|K^0_1\rangle + \varepsilon |K^0_2\rangle}{\sqrt{1 + |\varepsilon|^2}} = \frac{(1 + \varepsilon)|K^0\rangle + (1 - \varepsilon)|\overline{K}^0\rangle}{\sqrt{2(1 + |\varepsilon|^2)}};
\]

\[
K^0_L = \frac{|K^0_2\rangle + \varepsilon |K^0_1\rangle}{\sqrt{1 + |\varepsilon|^2}} = \frac{(1 + \varepsilon)|K^0\rangle - (1 - \varepsilon)|\overline{K}^0\rangle}{\sqrt{2(1 + |\varepsilon|^2)}};
\]

• no $\mathbb{C}\mathbb{P}$ violation $\rightarrow \varepsilon = 0 \rightarrow \langle |K^0_S\rangle = |K^0_1\rangle, |K^0_L\rangle = |K^0_2\rangle\rangle$;

• other commonly used parameters are:

\[
\eta_{00} \equiv |\eta_{00}| \exp(i\phi_{00}) \equiv \frac{\langle \pi^0\pi^0 |H|K^0_L\rangle}{\langle \pi^0\pi^0 |H|K^0_S\rangle};
\]

\[
\eta_{+-} \equiv |\eta_{+-}| \exp(i\phi_{+-}) \equiv \frac{\langle \pi^+\pi^- |H|K^0_L\rangle}{\langle \pi^+\pi^- |H|K^0_S\rangle};
\]

• the direct violation is parametrized by a complex parameter $\varepsilon'$:

\[
\eta_{+-} = \varepsilon + \varepsilon'; \quad \eta_{00} = \varepsilon - 2\varepsilon';
\]

• no direct $\mathbb{C}\mathbb{P}$ violation $\rightarrow \varepsilon' = 0$ and $|\eta_{00}| \approx |\eta_{+-}| \approx \varepsilon$;

• $\varepsilon'$ is an important parameter for our understanding of Nature;

• as of today, the best measurement, assuming $\mathbb{C}\mathbb{P}\mathbb{T}$ invariance, are:

\[
|\eta_{+-}| = (2.232 \pm 0.011) \times 10^{-3};
\]

\[
|\eta_{00}| = (2.221 \pm 0.011) \times 10^{-3};
\]

\[
|\phi_{+-}| = (43.51 \pm 0.05)\degree;
\]

\[
|\phi_{00}| = (43.7 \pm 0.8)\degree;
\]

\[
|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3};
\]

\[
\Re(\varepsilon'/\varepsilon) = (1.65 \pm 0.26) \times 10^{-3};
\]

which are obtained in a long series of dedicated experiments on $\mathbb{C}\mathbb{P}$ violation.
D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

• The $\mathbb{C}\mathbb{P}$ transformation combines charge conjugation $\mathbb{C}$ with parity $\mathbb{P}$.

• Under $\mathbb{C}$, particles and antiparticles are interchanged, by conjugating all internal quantum numbers, e.g., $Q \rightarrow -Q$ for electromagnetic charge.

• Under $\mathbb{P}$, the handedness of space is reversed, $\vec{x} \rightarrow -\vec{x}$. [... A] left-handed electron $e_L^-$ is transformed under $\mathbb{C}\mathbb{P}$ into a right-handed positron $e_R^+$.

• If $\mathbb{C}\mathbb{P}$ were an exact symmetry, the laws of Nature would be the same for matter and for antimatter. We observe that most phenomena are $\mathbb{C}$- and $\mathbb{P}$-symmetric, and therefore, also $\mathbb{C}\mathbb{P}$-symmetric.

• In particular, these symmetries are respected by the gravitational, electromagnetic, and strong interactions.

• The weak interactions, on the other hand, violate $\mathbb{C}$ and $\mathbb{P}$ in the strongest possible way. For example, the charged $W$ bosons couple to left-handed electrons, $e_L^-$, and to their $\mathbb{C}\mathbb{P}$-conjugate right-handed positrons, $e_R^+$, but to neither their $\mathbb{C}$-conjugate left-handed positrons, $e_L^+$, nor their $\mathbb{P}$-conjugate right-handed electrons, $e_R^-$.

(... continue ...)
D. Kirkby (UC Irvine), Y. Nir (Weizmann Inst.) [PDG 2012] :

(... continued ...)

• While weak interactions violate C and P separately, CP is still preserved in most weak interaction processes.

• The CP symmetry is, however, violated in certain rare processes, as discovered in neutral K decays in 1964 [...], and observed in recent years in B decays. A K_L meson decays more often to π^-e^+ν_e than to π^+e^-ν_ē, thus allowing electrons and positrons to be unambiguously distinguished, but the decay-rate asymmetry is only at the 0.003 level.

• The CP-violating effects observed in B decays are larger: the CP asymmetry in B^0/Ā^0 meson decays to CP eigenstates like J/ψK_S is about 0.7 [...].

• These effects are related to K^0 – Ā^0 and B^0 – Ā^0 mixing, but CP violation arising solely from decay amplitudes has also been observed, first in K → ππ decays [...], and more recently in various neutral [...] and charged B [...] decays.

• Evidence for CP violation in the decay amplitude at a level higher than 3σ (but still lower than 5σ) has also been achieved in neutral D [...] and B_s [...] decays.

• CP violation has not yet been observed in the lepton sector.

LHCb observed CP violation in D decays in 2019 at 5.3σ.
Reinterpret the $\mathbb{C}\mathbb{P}$ violation using the CKM matrix [§ 4]:

- the weak charged current for quarks
  
  \[ j^\mu_{qq} = -i \frac{g}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t}) \gamma^\mu \frac{1}{2} (1 - \gamma^5) V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \]

- therefore, e.g. [notice the "*"; the definition is "$V_{ij}$ when $bds$ is a spinor and $\bar{u}\bar{c}\bar{t}$ the adjoint spinor" and "$V^*_{ij}$ when $uct$ is a spinor and $b\bar{d}\bar{s}$ the adjoint spinor".]

- the $V_{\text{CKM}}$ matrix represents the rotation, i.e. the amount of mixing among rotated quarks.
CKM matrix: $\alpha_{ij}, \delta$

- in a N-family scheme with $N=3$, $V_{\text{CKM}}$ requires $n_{\text{rot}}=3$ real rotations $\alpha_{ij}$ and $n_{\text{ph}}=1$ imaginary phase $\delta$ (see box);
- the rotations $\alpha_{ij}$ are "Euler angles" in the quark space ("Cabibbo angles in 3-dim");

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \]

\[ = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}c_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \]

- $\delta \neq 0 \rightarrow$ some $V_{ij}$ complex
  $\rightarrow \mathbb{CP}$ violation [next slides];
- many representations, give the most common [PDG] ($c_{ij} \equiv \cos \alpha_{ij}$, $s_{ij} \equiv \sin \alpha_{ij}$):

\[ \begin{align*}
\mathcal{K}\mathcal{M} \text{ approach} & \quad [\text{IE, §9}]: \\
\begin{pmatrix} n_{\text{rot}} \\ n_{\text{ph}} \end{pmatrix} & = \begin{pmatrix} N(N-1)/2 \\ (N-1)(N-2)/2 \end{pmatrix} \quad \rightarrow \begin{pmatrix} n_{\text{ph}} \geq 1 \\ (N \geq 3). \end{pmatrix}
\end{align*} \]
The representation is chosen to highlight the agreement with experimental data:

- $\alpha_{ij}$ small $\rightarrow \cos \alpha_{ij} \gg \sin \alpha_{ij}$
  $\rightarrow V_{\text{CKM}} = 1 + "\text{small rotations}\"
  $\rightarrow q'\text{-dynamics} = q\text{-dynamics}$
  + small effects;

- $\alpha_{13}$ small $\rightarrow \alpha_{12} \cong \theta_c$;

- Cabibbo theory works well, when considering $\text{N}=2$ (udsc only);
- $s_{12}$ and $s_{13}$ small $\rightarrow$ matrix almost real
  $\rightarrow \mathbb{C}\mathbb{P}$ violation small.

$$V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} \\
s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}
\end{pmatrix}.$$
The violations associated with $V_{\text{CKM}}$ are usually studied with the Wolfenstein parameterization $V_{\text{CKM}}^W$, which singles out the "small" terms and their physical meaning:

As the "Euler" parameterization, $V_{\text{CKM}}^W$ has 4 independent real parameters ($\lambda, A, \rho, \eta$):

- $\lambda \cong s_{12} \ (\rightarrow \sin \theta_c, \text{mixing } 1^{\text{st}}/2^{\text{nd}});$
- $A\lambda^2 \cong s_{23} \ (\rightarrow \text{mixing } 2^{\text{nd}}/3^{\text{rd}});$
- $A\lambda^3(\rho + i\eta) \cong s_{13}e^{i\delta} \ (\rightarrow \delta \cong \tan^{-1} \eta/\rho);$  
- i.e. $\eta=0 \rightarrow \delta=0 \rightarrow V_{\text{CKM}} \text{ real} \rightarrow \text{no } \mathbb{CP} \text{ violation.}$
The indirect $\mathbb{CP}$ violation in the $K^0$ system can be explained with the CKM formalism [Thoms, 393]:

- for each of the $K^0 \leftrightarrow \bar{K}^0$ diagrams
  - *look the t-channel exchange: 9 couples of diagrams (uu, uc, ut, cu, cc, ct, ...);*
  - *here discuss only (ct) case, others similar;*

  - $M(K^0 \rightarrow \bar{K}^0) \propto V_{cd}^* V_{ts}^* V_{cs}^* V_{td};$
  - $M(\bar{K}^0 \rightarrow K^0) \propto V_{cd}^* V_{ts} V_{cs} V_{td};$

- $V_{ij}$ real $\rightarrow M(K^0 \rightarrow \bar{K}^0) = M(\bar{K}^0 \rightarrow K^0)$
  $\rightarrow$ no $\mathbb{CP}$ violation;

- $V_{ij}$ complex $\rightarrow M(K^0 \rightarrow \bar{K}^0) \neq M(\bar{K}^0 \rightarrow K^0)$
  $\rightarrow$ $\mathbb{CP}$ violation.

- in this case $M(K^0 \rightarrow \bar{K}^0) \neq M(\bar{K}^0 \rightarrow K^0)$:
  $M(K^0 \rightarrow \bar{K}^0) - M(\bar{K}^0 \rightarrow K^0) \propto i\Im(V_{td}) = i\eta A\lambda^3;$
  $[\Delta M$ imaginary, small, $\propto \eta]$
  $\rightarrow$ CP violation $\propto \eta A^2\lambda^6$ [Jarlskog invariant]

It can be shown [Thoms 403] that the $\varepsilon$ parameter of the $\mathbb{CP}$ violation can be written as:

$$|\varepsilon| \propto \eta (1 - \rho + \text{const.})$$
The $\mathbb{CP}$ violation is expected to occur in the SM also in the $D^0$–$\bar{D}^0$ and $B^0$–$\bar{B}^0$ systems through the same dynamical mechanism [see box].

However the importance of the phenomenon depends on the value of the CKM matrix elements, i.e. by the quark mixing.

In the $D^0$–$\bar{D}^0$ case:
- main contribution from $b$ quark exchange;
- but product $V_{cb} V_{ub}$ very small;
- therefore predicted $D^0$–$\bar{D}^0$ mixing minute;
- only been observed in 2019 by LHCb.

Instead $B^0$–$\bar{B}^0$ mixing:
- dominated by $t$ quark exchange;
- expected substantial level of mixing;
- [see next slides for some results].

It could be a golden opportunity: since the SM prediction is small (and computable), a $b$SM effect would not be obscured.
How to measure (the real part of) $V_{ij}$?

- from decays ([YN2, §6], [PDG]):
  - $|V_{ud}| : p \rightarrow n \nu \bar{\nu}$ and other $\beta$ decays;
  - $|V_{cs}| : c$-mesons $C$(abibbo)-allowed;
  - $|V_{us}| : s$-mesons (e.g. $K^\pm$);
  - $|V_{cd}| : c$-mesons $C$-suppressed, : dileptons in $\nu$ scattering;
  - $|V_{ub}| : b$-mesons $\rightarrow$ non-$c$-mesons;
  - $|V_{cb}| : b$-mesons $\rightarrow c$-mesons;
  - $|V_{td}|, |V_{ts}| : (B^0 \leftrightarrow \bar{B}^0)$ oscillations;
  - $|V_{tb}| : t \rightarrow W^\pm b$ [not accurate];
- conceptually simple, the problem is to disentangle the clean weak decay from the dirty hadron corrections;
- semi-leptonic decays cleaner;
- a technically difficult job (hundreds of papers, theses, conferences...);

- nice final result [PDG 2016]:
  - $V_{\text{CKM}}$ quasi-diagonal, as expected;
  - well consistent with SM (unitary, 3 families).

$$|V_{\text{CKM}}| = \begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}|
|V_{cd}| & |V_{cs}| & |V_{cb}|
|V_{td}| & |V_{ts}| & |V_{tb}|
\end{pmatrix} =
\begin{pmatrix}
.97417 & .2248 & .0409
.220 & .995 & .0405
.0082 & .0400 & 1.009
\end{pmatrix} \pm
\begin{pmatrix}
.00021 & .00006 & .0039
.005 & .016 & .0015
.0006 & .0027 & .0031
\end{pmatrix}$$
How to interpret $V_{\text{CKM}}$?

$$|V_{\text{CKM}}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} .97417 & .2248 & .0409 \\ .220 & .995 & .0405 \\ .0082 & .0400 & 1.009 \end{pmatrix} \pm \begin{pmatrix} .00021 & .0006 & .0039 \\ .005 & .016 & .0015 \\ .0006 & .0027 & .0031 \end{pmatrix}$$

- tests of SM from $V^\dagger V = 1$:
  $$\sum_i V_{ij} V_{ik}^* = \delta_{jk}; \quad \sum_j V_{ij} V_{kj}^* = \delta_{ik}.$$  
  (e.g. $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$;)

- if (a) test(s) fail(s)
  - more generations (missing pieces)?
  - general breakdown of the model?

- if all tests succeed
  - general fit imposing unitarity;
  - improved accuracy;
  - stricter tests;
  - more accuracy;
  - and so on, forever [see §LEP].
from one of the unitarity relations:

$$\sum_i V_{i1} V_{i3}^* = V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = \delta_{13} = 0;$$

• add some simple math:

$$V_{ud}, V_{cb}, V_{tb} \text{ real } > 0;$$

$$V_{cd} \text{ real } < 0 \text{ (see } V_{cd}^W);$$

$$\rightarrow |V_{ud}| V_{ub}^* - |V_{cd}| |V_{cb}| + V_{td} |V_{tb}| = 0;$$

$$\rightarrow 1 \frac{|V_{tb}|}{|V_{cd}| |V_{cb}|} V_{td} - \frac{|V_{ud}|}{|V_{cd}| |V_{cb}|} V_{ub}^* = 0;$$

• put the relation in complex plane $\mathbb{R} \mathbb{I}$;

• interpreted it as a triangle (unitarity triangle, u.t.);

• define angles ($\alpha$, $\beta$, $\gamma$) (see fig.);

• relate $V_{ij} \rightarrow$ Wolfenstein param. $\rho^W$, $\eta^W$;

• the vertex is at ($\bar{\rho} \equiv \rho^W$, $\bar{\eta} \equiv \eta^W$)

The exact relation is [check it !]:

$$\bar{\rho} + i \bar{\eta} = (\rho + i \eta) \left(1 - \frac{\lambda^2}{2}\right) + \Theta(\lambda^4).$$

Note:

• u.t. defined by using $V_{ij}$ only;

• nice adimensional parameters (ratios);

• experiments measure triangle "geometry" (sides, angles);

• lot of relations (e.g. $\alpha + \beta + \gamma = 180^\circ$):
  - consistency tests of SM,
  - global fits to parameters assuming SM.
A typical event used for $\mathbb{CP}$ violation in asymmetric $e^+e^-$ at $\sqrt{s} \approx 10.579$ GeV:

$e^+e^- \rightarrow \Upsilon(4S) \rightarrow \bar{B}^0 B^0$;

$\bar{B}^0 \rightarrow \ell^- D^0 X^+$;  \hspace{1cm} $D^0 \rightarrow K^- X^+$;

$B^0 \rightarrow J/\psi K^0_S$;  \hspace{1cm} $J/\psi \rightarrow \mu^+\mu^-$;  \hspace{1cm} $K^0_S \rightarrow \pi^+\pi^-$. 

Unitarity triangle: meas $\beta$ at BaBar, Belle
Unitarity triangle: results for $\beta$ at BaBar

\[ A_{\text{raw}} = \frac{n[\bar{B}(\Delta t) \to J/\psi K^0_s] - n[B^0(\Delta t) \to J/\psi K^0_s]}{n[\bar{B}(\Delta t) \to J/\psi K^0_s] + n[B^0(\Delta t) \to J/\psi K^0_s]} \propto \sin(2\beta) \sin(\Delta m \Delta t). \]

\[
\sin 2\beta = 0.722 \pm 0.040 \\
\pm 0.023
\]

[now improved]

NB: $\sin \beta > 0$ \\
$\rightarrow \eta > 0$ \\
$\rightarrow \mathbb{C}\mathbb{P}$ violated !!!
As of today [PDG 2016]:

- converging measurements (mainly asymmetric $e^+e^-$ factories BaBar, Belle);
- no deviation from $3_f$-SM, e.g. $[\alpha + \beta + \gamma]_{fit} = (183 \pm 8)^\circ$;
- try harder, one of the most promising frontiers !!!
Quarks of same charge and different flavor mix together → composite hadrons "oscillate" (e.g. $K^0 \leftrightarrow \bar{K}^0$).

The CKM matrix parameterizes the process in the context of the SM.

And the lepton sector? Do the $\nu$'s oscillate?

The answer to the previous question is **YES**.

The results are important (Nobel Prize 2015):

- $m_\nu > 0$ (at least for two of them);
- there is mixing in the lepton sector;
- and possibly $\mathbb{C}\mathbb{P}$ violation (not easy to see);
- the first discovery bSM (even though, if $\nu$'s are Dirac fermions, they can be easily incorporated in the SM).

In the following the $\nu$'s will be considered as massive neutral Dirac fermions (sort of neutral electrons), sometimes called "Weyl $\nu$'s":

- this hypothesis is simple, but not the favorite of most physicists;
- (as of today) it is NOT falsified by the exp.;
- other comments on § Standard Model.

**The $\nu$'s are very complicated objects!** many (most ?) of the important discoveries in particle physics of the last 80 years came from them !!!
ν oscillations: toy model

Assume mixing in the ν sector and look for possible observables.

Simple toy model, inspired to Cabibbo angle:

- 2 families ($\nu_1, \nu_2 \rightarrow \nu_e, \nu_\mu$);
  \[
  \begin{pmatrix}
  |\nu_e\rangle \\
  |\nu_\mu\rangle
  \end{pmatrix} = \begin{pmatrix}
  \cos \theta_\nu & \sin \theta_\nu \\
  -\sin \theta_\nu & \cos \theta_\nu
  \end{pmatrix} \begin{pmatrix}
  |\nu_1\rangle \\
  |\nu_2\rangle
  \end{pmatrix};
  \]
- free parameters: masses, mixing angle $\theta_\nu$;
- same formalism as in the ($K_1^0 \leftrightarrow K_2^0$) case;
- time evolution of a pure $\nu_{e,\mu}$ at $t=0$:
  \[
  \begin{align*}
  |\nu_e(t)\rangle &= \cos \theta_\nu e^{-iE_1 t} |\nu_1\rangle + \sin \theta_\nu e^{-iE_2 t} |\nu_2\rangle \\
  |\nu_\mu(t)\rangle &= -\sin \theta_\nu e^{-iE_1 t} |\nu_1\rangle + \cos \theta_\nu e^{-iE_2 t} |\nu_2\rangle
  \end{align*}
  \]
- the oscillation probability $P$ is next slide:

\[
P (\nu_e \rightarrow \nu_\mu) = \sin^2 [2\theta_\nu] \sin^2 \left[ \frac{\Delta m^2 L}{4E} \right];
\]
\[
\Delta m^2 \approx \frac{1.27 \times (m_2^2 - m_1^2) [eV^2] \times L [km]}{E [GeV]}.
\]

Required:
- large $\theta_\nu$;
- $m_\nu > 0$;
- large $L/E$;

→ since $\theta_\nu$ and $m_{1,2}$ are not up to us, the relevant exper. parameter is $L/E$; with present technologies, the observation is:

- difficult (= impossible) with accelerators;
- needs astrophysical exp.

[actual experiments are NOT discussed here: they belong to the astroparticle course]
\[ |\langle \nu_e(t) | \nu_e(0) \rangle|^2 = \left| \left( \cos \theta \nu e^{-iE_1t} \langle \nu_1 \rangle + \sin \theta \nu e^{-iE_2t} \langle \nu_2 \rangle \right) \left( \cos \theta \nu | \nu_1 \rangle + \sin \theta \nu | \nu_2 \rangle \right) \right|^2 = \]
\[ = \left| \cos^2 \theta \nu e^{-iE_1t} + \sin^2 \theta \nu e^{-iE_2t} \right|^2 = \]
\[ = \left| \cos^2 \theta \nu \cos(E_1t) - \sin^2 \theta \nu \sin(E_1t) + \sin^2 \theta \nu \cos(E_2t) - \sin^2 \theta \nu \sin(E_1t) \right|^2 = \]
\[ = \cos^4 \theta \nu \cos^2(E_1t) + \sin^4 \theta \nu \cos^2(E_2t) + 2 \sin^2 \theta \nu \cos^2 \theta \nu \cos(E_1t) \cos(E_2t) + \]
\[ + \cos^4 \theta \nu \sin^2(E_1t) + \sin^4 \theta \nu \sin^2(E_2t) + 2 \sin^2 \theta \nu \cos^2 \theta \nu \sin(E_1t) \sin(E_2t) = \]
\[ = \cos^4 \theta \nu + \sin^4 \theta \nu + 2 \sin^2 \theta \nu \cos^2 \theta \nu \cos[(E_2 - E_1)t] + 1 - (\cos^2 \theta \nu + \sin^2 \theta \nu)^2 = \]
\[ = 1 - 2 \sin^2 \theta \nu \cos^2 \theta \nu \{1 - \cos[(E_2 - E_1)t]\} = 1 - 4 \sin^2 \theta \nu \cos^2 \theta \nu \sin^2 [(E_2 - E_1)t/2] = \]
\[ = 1 - \sin^2(2 \theta \nu) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right). \]

\[ \mathcal{P}_L (\nu_e \rightarrow \nu_\mu) = 1 - |\langle \nu_e(t) | \nu_e(0) \rangle|^2 = \]
\[ = \sin^2(2 \theta \nu) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right). \]

\[ \mathcal{P}_L \] is the oscillation probability after a distance \( L \).

\[ (E_2 - E_1)t = \left( \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_1^2} \right) \approx \]
\[ \approx p \left[ 1 + \frac{m_2^2}{2p^2} - \left( 1 + \frac{m_1^2}{2p^2} \right) \right] \frac{L}{c} \approx \]
\[ \approx \frac{m_2^2 - m_1^2}{2p} \approx \frac{\Delta m^2 L}{2E}. \]
Current $\nu$ oscillation experiments measure:

$$\Delta m_{12}^2 = m_2^2 - m_1^2 \approx 7.37 \times 10^{-5} \text{ eV}^2;$$

$$|\Delta m_{32}|^2 = |m_3^2 - m_2^2| \approx 2.56 \times 10^{-3} \text{ eV}^2;$$

compatible with the two "hierarchies" shown in the box (ambiguity still not solved).

In the SM there are three families $\rightarrow$ the $\nu$ mixing matrix is $3 \times 3$, with the same math properties of the CKM one (three angles + a CP-violating phase).

It is called Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix:

$\nu = V_{PKMS} \nu$;

the present best measurements are [PDG]:

$$V_{PKMS} = \begin{pmatrix}
0.826 & 0.544 & 0.151 \\
0.427 & 0.642 & 0.635 \\
0.368 & 0.540 & 0.757
\end{pmatrix}.$$ 

The CP-violating phase ($\delta_\nu$) is $\approx 3\pi/2$.

Q. why $\nu$'s from the sky and not from an accelerator? compute the value of $L/E$ for the oscillation maxima using these values.
**CPT theorem**

**If** (Quantum field theory) and (Special relativity) and (\( \mathbb{H} \) invariant under Lorentz transformation),

then

the physical states are \( \mathbb{C} \mathbb{P} \mathbb{T} \) invariant, i.e. invariant under the consecutive application of the operators Charge-conjugation, Parity and Time-reversal.

**Nota bene:**

- The states may be invariant for the application of any of the three, like in strong interaction processes.
- In this case, *a fortiori*, they will be invariant under the three together.
- But even processes which violate one (left-handed neutrinos, \( K^0 \) oscillations) or even two (\( K^0 \) semileptonic decays), must be invariant under the combined application of the three together.

**Consequences of the \( \mathbb{C} \mathbb{P} \mathbb{T} \) theorem:**

- Mass, charge and lifetime of a particle and its antiparticle are exactly equal:
  - \(|m(K^0) - m(\bar{K}^0)| / \text{aver.} < 6 \times 10^{-19};
  - \(|m(e^+) - m(e^-)| / \text{aver.} < 8 \times 10^{-9};
  - \(|q(p) - q(\bar{p})| / q(e^-) < 2 \times 10^{-9};
  - \([\tau(\mu^+) - \tau(\mu^-)] / \text{aver.} = (2 \pm 8) \times 10^{-5};

- Any violation in an individual or pair of symmetries must be compensated by an asymmetry in the other operation(s), so to save exact symmetry under \( \mathbb{C} \mathbb{P} \mathbb{T} \).

- (e.g.) The weak interactions violate \( \mathbb{C} \) and \( \mathbb{P} \) separately but in general they are invariant under the combined operation of \( \mathbb{C} \) and \( \mathbb{P} \) (and \( \mathbb{T} \) alone).

- (e.g.) The weak decays of the \( K^0 \) mesons violate \( \mathbb{C} \mathbb{P} \), but this is accompanied by a corresponding violation of \( \mathbb{T} \), so that \([\mathbb{C} \mathbb{P} \mathbb{T}]\) is respected.
References

1. [BJ, 11.13], [YN1, 16];
2. the CPT theorem is discussed in [MQR, 12];
3. the C\(\overline{P}\) violation and the FCNC are discussed in [IE, 12-13]
End of chapter 5