Particle Physics - Chapter 10
LEP — $e^+e^-$ physics
10 – LEP – $e^+e^-$ physics

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**Graph:**
- A graph showing the cross-section $\sigma_{\text{had}}$ for different numbers of vertices ($n_{\nu}$) as a function of $E_{\text{cm}}$.
- The graph includes data from ALEPH, DELPHI, and L3, with OPAL.
- The graph highlights the average measurements, with error bars increased by a factor of 10.

Paolo Bagnaia – PP – 10
1. The LEP Collider
2. Detectors
3. The L3 detector
4. LEP events
5. – 16. [...]
2\pi R \approx 27 \text{ km} \\
\sim 100 \text{ m underground} \\
\text{planar, slightly tilted} \\
\text{wrt surface, because} \\
of \text{geology.}
The LEP collider: $e^\pm$ acceleration

$e^\pm$:
- LIL ($\rightarrow 200/600$ MeV);
- EPA (600 MeV);
- PS ($\rightarrow 3.5$ GeV);
- SPS ($\rightarrow 22$ GeV);
- LEP ($\rightarrow 45\div105$ GeV).
### The LEP collider: parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LEP 1</th>
<th>LEP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference (Km)</td>
<td>26.66</td>
<td>same</td>
</tr>
<tr>
<td>$E_{\text{max}}$ / beam (GeV)</td>
<td>50</td>
<td>105</td>
</tr>
<tr>
<td>max lumi $\mathcal{L}$ ($10^{30}$ cm$^{-2}$ s$^{-1}$)</td>
<td>$\sim$25</td>
<td>$\sim$100</td>
</tr>
<tr>
<td>time between collisions ($\mu$s)</td>
<td>22 (11)</td>
<td>22</td>
</tr>
<tr>
<td>packet length (cm)</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>packet radius (hori.) (µm)</td>
<td>200÷300</td>
<td></td>
</tr>
<tr>
<td>packet radius (vert.) (µm)</td>
<td>2.5÷8</td>
<td></td>
</tr>
<tr>
<td>injection energy (GeV)</td>
<td>22</td>
<td>same</td>
</tr>
<tr>
<td>particles/packet ($10^{11}$)</td>
<td>4.5</td>
<td>same</td>
</tr>
<tr>
<td>packet number</td>
<td>4+4 (8+8)</td>
<td>4+4</td>
</tr>
</tbody>
</table>
The LEP collider: $\sqrt{s}$ vs year

![Graph showing energy (GeV) vs year from 1989 to 2000 for LEP collider experiments.

- 1990-91: 90-91
- 1991-94: 90-91
- 1992-95: 90-91
- 1993-96: 90-91
- 1994-97: 91-136
- 1995-98: 181-172
- 1996-99: 182-184
- 1997-00: 189-189
- 1998-00: 192-192
- 1999-00: 200-209

Energy (GeV)
The LEP collider: $\mathcal{L}_{\text{integrated}}$
The LEP collider: $\mathcal{L}_{\text{int}}$ vs day

1000 pb$^{-1}$ since 1989

Graph showing integrated luminosity over time with data points for different physics years and energy ranges.
The LEP collider: $e^\pm$ brem

- $\Delta E_{\text{orbit}} \propto e^2 E^4 / (M^4 R)$; [§ 8]
  - $\Delta E^\pm_{\text{orbit}} \text{(MeV)} = 8.85 \times 10^{-5} E^4 \text{(GeV)} / R \text{(Km)}$;
- $\langle R_{\text{LEP}} \rangle = 4.25 \times 10^3 \text{ m} \rightarrow \text{see table}$;
- In QED, the bremsstrahlung is not deterministic; the formula gives the average; a further (annoying) effect is the increase of emittance, i.e. the increase of the packets both in space and momentum; this effect is greater in the horizontal plane, as an effect of the magnetic bending:
  - $\sigma_{\text{hori}} = 200 \div 300 \mu\text{m}$;
  - $\sigma_{\text{vert}} = 2.5 \div 8 \mu\text{m}$.

<table>
<thead>
<tr>
<th>$E_{\text{beam}}$ (GeV)</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\Delta E_{\text{orbit}}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>90</td>
<td>$\sim 0.1$</td>
</tr>
<tr>
<td>90</td>
<td>180</td>
<td>$\sim 1.4$</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>$\sim 2.1$</td>
</tr>
</tbody>
</table>

[beam perp. to the page]
Assume $\mathcal{L}_{\text{max}} = 2 \times 10^{31} \text{cm}^{-2}\text{s}^{-1}$:

- $\sigma_{\text{tot}}(e^+e^- \rightarrow Z, \sqrt{s}=m_Z) \approx 40 \text{ nb}$:
  - $R_{\text{max}}(e^+e^- \rightarrow Z, \sqrt{s}=m_Z) = \mathcal{L} \sigma_{\text{tot}} = 0.8 \text{ Hz}$;
  - $6 \times 10^4 \text{ events / day} \rightarrow 10^7 \text{ events / year};$
  - [??? no !!!];

... because ...

- the luminosity normally quoted corresponds to the "peak lumi.", i.e. the first minutes after acceleration and squeezing;
  
  $\mathcal{L}(t) = \mathcal{L}_{\text{max}} \exp(-t/\tau)$ (stochastic effects + optics corrections)

  $\rightarrow \langle \mathcal{L} \rangle \approx \frac{1}{2} \mathcal{L}_{\text{max}}$

  + techn. stops, maintenance, mistakes, ...

  $\Rightarrow$ @ LEP 1:
  
  $4 \times 10^6 \text{ hadronic events} \times 4 \exp =$

  $= 15.5 \times 10^6 \text{ hadronic events} + \text{the corresponding leptons.}$

Problem: use the formulæ of § 8 and the LEP parameters to compute $\mathcal{L}_{bc}$ and $\mu (=\mathcal{P}_{\text{int}})$.

Comment on TDAQ requirements. Is LEP trigger/DAQ "easy" or "difficult"?

[please think before answering]
The LEP collider: the competition - SLC

SLC: Stanford Linear Collider (1989-98):
- the first example of linear $e^+e^-$ collider;
- lower energy (only Z pole) and less intense;
- polarized beams;
- promising new technique ($\sqrt{s} > 500$ GeV $\rightarrow$ a circular $e^+e^-$ requires a huge ring).
A typical detector of LEP / TeVatron / LHC (ATLAS is the only remarkable exception). Notice both the possible measurement of E, $\bar{p}$ and the particle id. capability.
A detector fully operational allows for both the measurement of the 4-momenta of all the particles and their identification ("part.id"). The charge is measured by the sign of the bending.

The ν's are "detectable" from the conservation of the 4-momentum, i.e.:

$$\begin{align*}
\bar{p}_\nu &= -\sum_{\text{all } j} \bar{p}_j; \\
E_\nu &= \sqrt{s} - \sum_{\text{all } j} E_j; \\
\bigoplus m_\nu^2 &= E_\nu^2 - |\mathbf{p}|_\nu^2 = 0
\end{align*}$$

**Problem:** what happens if there are two ν's in the final state?  
An interesting question ... and not uncommon \([Z \rightarrow \tau\tau, ZH \rightarrow \nu\bar{\nu}bb]\).
Detectors:

ALEPH

1. Beam Pipe
2. Silicon Vertex Detector
3. Inner Tracking Chamber
4. Luminosity Monitor
5. TPC Endplate
6. Electromagnetic Calorimeter
   6a. Barrel
   6b. Endcap
7. Superconducting Coil
8. Hadron Calorimeter
   8a. Barrel
   8b. Endcap
9. Muon Chambers
Detectors : DELPHI
Detectors: OPAL

- Muon Chambers
- Time of Flight and Presampler
- Z-Chambers
- Vertex Chambers
- Jet Chamber
- Hadron Calorimeters
- Forward Detector
- Electromagnetic Calorimeters
The L3 detector: SMD

- 96 silicon wafers
- 70 mm × 40 mm × 300 µm
- two layers:
  - Ø inner layer: 120 mm
  - Ø outer layer: 150 mm
  - zenith coverage: |cosθ| < 0.93.

2 read outs:
- 50 µm in rφ;
- 150÷200 µm in z
The L3 detector: TEC

- ext. – int. radius = 317 mm;
- two separate concentrical regions: inner 8 wires + outer 54 wires;
- 80% CO₂, 20% iC₄H₁₀, 1.2 bar (abs);
- \( v_{\text{drift}} = 6\mu m / \text{ns} \) ("TEC" = Time Expansion Chamber);
- \( \alpha_{\text{Lorentz}} = 2.3^\circ \);
- z-detector (\( \sigma = 320\mu m \)).
The residuals are the distances (with sign) between the measurements and the fitted trajectory. Assuming "many" measurements with the same resolution, their distribution is expected to be gaussian with mean=0 and RMS=resolution.
The L3 detector: SMD + TEC

Why plot \( \frac{1}{E} - \frac{1}{p} \), instead of \( E - p \)?
Answer in few slides, but you should be able to understand yourself.

\[ 1/E_T - 1/p_T \]
\[ \sigma_{\text{fit}} = 0.01 \text{ GeV}^{-1} \]

Distance line-vertex
\[ \sigma_{\text{fit}} = 30 \mu m \]

Tracks, which miss the interaction point, are a signal of secondary vertices (\( \tau \)'s, heavy flavors...)
\[ \rightarrow \text{the resolution on the "impact parameter" is important.} \]
• 11,000 BGO (Bismuth germanium oxide $\text{Bi}_4\text{Ge}_3\text{O}_{12}$) scintillating crystals;
• pyramids 20×20 → 30×30 mm$^2$, length 240 mm;
• $X_0 = 11.3$ mm → 21 $X_0$. 
the mass resolution for particles decaying into $\gamma$'s is the traditional figure of merit of the e.m. calo (true also for $H \rightarrow \gamma\gamma$ at LHC !!!).
The L3 detector: HadCal

- plates of depleted U ($U_{238}$) + proportional wire chambers (370,000 wires);
- brass $\mu$-filter (65%Cu, 35% Zn) + prop. tubes;
- BGO + hadcal in calo trigger (few algorithms in .OR., e.g. $E_{\text{tot}}$, $E_{\text{tot}}^{\text{BGO}}$, cluster, single $\gamma$, ....
The L3 detector: HadCal results

• \( Z \rightarrow q\bar{q} \) at \( \sqrt{s} = m_Z \);
• \( E_{\text{tot}} \) is known and used to calibrate the detector;
• \( E_{\text{vis}} / \sqrt{s} = \sum_i E_i / \sqrt{s} \) in two cases:
  - calo e.m. + had;
  - calo e.m. + had + TEC (− double-counting);
• resolution = 10.2% with calos only;
• resolution = 8.4%, when TEC is also used (avoiding double counting).
The L3 detector: $\mu$ chambers

- octants, each with three chamber types: MO + MN + MI (16 + 24 + 16 wires);
- effective length of measurement: 2.9 m
- mechanical accuracy: $\sim$10$\mu$m;
- alignment with optical sensors.
Why plot $E_{\text{beam}} / E_{\text{measured}}$?

- the sagitta ($\propto 1/p$) is the measured parameter;
- therefore $1/p$ expected gaussian, while $p$ is strongly asymmetric in the tails;
- $E_{\text{beam}} / E_\mu = \sqrt{s} / (2 p_\mu)$;
- $\sigma(m_Z)/m_Z = \sigma[E_{\text{beam}} / E_\mu] / \sqrt{2}$.

For $Z$ events, error from the machine, i.e. $\sigma(m_Z) = \sigma(\sqrt{s}) = \text{few MeV}$.

This method is used to check $p_\mu$, which is used in other channels (e.g. Higgs search).

And why $(1/E - 1/p)$, or $(1/E_T - 1/p_T)$?

Similar, but more elaborated.

$E$ (and $E_T$) comes from a calo, so it is normal, while $p$ (and $p_T$) comes from a spectrometer, so it is normal in $1/p$.

Plot $(E - p)$ if $\sigma(E) \gg \sigma(p)$, but $(1/E - 1/p)$ if $\sigma(p) \gg \sigma(E)$. 
The L3 detector: trigger / DAQ

- chmb data
- cal. data
- μ data
- lum data
- ...

- ℓ1 drift
- ℓ1 cal
- ℓ1 μ
- ℓ1 lum
- ℓ1 ...

- ℓ1 OR

- ℓ1 must finish before the next b.c., ℓ2 + ℓ3 produce dead time.

- ℓ1 - ℓ2 work on “semplified” (fast) data

- reset + next bunch crossing

- ℓ2

- all ℓ1 data

- acquisition

- complete data
The L3 detector: trigger requirements

- crossing @ 44/88 KHz ↔ physics ≤ 1 Hz, i.e. "μ" ≈ 10^{-4} ÷ 10^{-5};
- event trigger (no selection on process type, unlike LHC);
- 3 levels of trigger;
  - 1st level: simplified readout (e.g. faster ADC less precise), logical OR among:
    - TEC (e.g. 2 opposite tracks);
    - μ (at least one candidate);
    - ...
    - energy (see next slides);
  - 2nd level: same data as 1st lvl, but combine different detectors (e.g. a track + corresponding calo deposit);
  - 3rd level: final data.
- fake triggers sources (~10÷20 Hz at 1st level):
  - electronic noise;
  - beam halo + "beam-gas" interactions, brem photons, ...;
  - cosmics, ...
- 1st level is cabled + home-made processors [home: THIS building];
- 2nd level: (quasi-)commercial processor;
- 3rd level: standard computer (vax-station at the time, today would use pc server + LINUX).
  → inefficiency ≤ 10^{-3} for Z → e^+e^−, μ^+μ^−, hadrons;
  → dead time ≈ 5%.
The L3 detector: energy trigger

- CAMAC(*) processor, built by "Sezione INFN" (this building, ground floor);
- Fast digitization of calo signals;
- Decision algorithm based on a digital programmable processor, realized with logic and arithmetic units;
- ~200 CAMAC modules;
- Decision in ~22 μs →

(*) CAMAC was an electronic standard, widely used in the '70s – 90's, now almost completely replaced by VME and other systems.
The L3 detector: energy trigger scheme
The $e^+e^-$ initial state produces very clean events (parton system = CM system = laboratory, no spectators).

In these four LEP events the beams are perpendicular to the page.

The recognition of the events is really simple, also for non-experts.

Great machines for high precision physics ...
$e^+ e^- \rightarrow \mu^+ \mu^-$

+ signals in SMD
+ track in TEC (→ momentum and charge)
+ mip in calos
+ signals in $\mu$ chambers (→ momentum and charge)
= identified and measured $\mu^\pm$. 

LEP events: $\mu^+ \mu^-$
LEP events : $e^+e^-\gamma$

$e^+e^-\rightarrow e^+e^-\gamma$

+ signals in SMD
+ track in TEC (→ momentum and charge)
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in $\mu$ chambers

= identified and measured $e^\pm$.

+ no signal in SMD
+ no signal in TEC
+ e.m. shower in e.m. calo
+ (almost) nothing in had calo
+ absolutely nothing in $\mu$ chambers

= identified and measured $\gamma$. 
**LEP events : $\tau^+\tau^-$**

$$e^+ e^- \rightarrow \tau^+ \tau^-$$

$\tau^\pm$ id. does depend on decay:
- 1/3/5 had tracks;
- [ or identified single $\ell^\pm$;]
- $\ell^\pm + E$ (i.e. a $\nu_{\tau}/\bar{\nu}_{\tau}$)

(the evidence comes from the combination of the two decays in the opposite emispheres).
LEP events: 3 jets

\[ e^+ e^- \rightarrow q \bar{q} g \]

A (anti-)quark or a gluon gives a hadronic jet:
- Many collimated tracks
- Large splashes in e.m. and had calos
- (Possibly) low momentum associated e^±/\mu^±
a heavy flavor quark is a quark (i.e. a jet) with:
+ displaced secondary vertices (SMD)
+ high momentum leptons from quark semileptonic decays
[not all h.f. have one or both characteristics → h.f. id. efficiency not complete (see next)]
LEP events: $b\bar{b}$, $b \rightarrow \mu^-$

$e^+ e^- \rightarrow b \bar{b}$

identified $\mu^+$

Muon Detector
ii. Exp. methods

1. – 4. [...]  
5. Data analysis  
6. Secondary vertices  
7. Efficiency and purity  
8. The luminosity  
9. – 16. [...]
data analysis

data samples (3rd level+pre-an.)

- $e^+e^- \rightarrow e^+e^-$
- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^+e^- \rightarrow \tau^+\tau^-$
- $e^+e^- \rightarrow \gamma\gamma$
- $e^+e^- \rightarrow \ell\chi$

analysis

- $d\sigma/d\cos\theta$
- $d\sigma/d\cos\theta$
- $d\sigma/d\cos\theta$
- $d\sigma/d\cos\theta$

physics

- $M_Z, \Gamma_Z, BR_{e,\mu,\tau}$
- $A_{FB}^{e,\mu,\tau}$
- $g_A^f, g_V^f$
- lifetimes
- polarization
- QCD
- resonances
- SUSY
- other exotica

these slides

very simplified – just for the lectures

(*) "lineshape" : $\sigma = \sigma(\sqrt{s})$
At LEP, as in any other experiment, a number of events \( N_{\text{exp}} \) has to be translated to a cross section \( \sigma_s \) ("signal");

- \( dN_{\text{exp}}/d\Omega \to d\sigma_s/d\Omega; \)
- straightforward: \( \sigma_s = N_{\text{exp}} / \mathcal{L}_{\text{int}}; \)
- but (at least) two problems:
  - the selection algorithm loses true- and gains spurious-events:
    \( N_{\text{exp}} = N_{\text{true}} - N_{\text{lost}} + N_{\text{sp}}; \)
  - the determination of \( \mathcal{L}_{\text{int}}, \) the luminosity.
- the experiment must measure/compute:
  - \( N_{\text{exp}} \) : number of selected events;
  - \( \sigma_b \) : cross-section of bckgd;
  - \( \varepsilon_{s,b} \) : efficiency (signal and bckgd);
  - \( \Delta N_{\text{exp}} = \sqrt{N_{\text{exp}}} \) (statistical error);
  - \( \Delta \varepsilon_{s,b} \) = "systematics";
  - \( \mathcal{L}_{\text{int}} \) = int. luminosity.

then (next slides):

- \( N_{\text{exp}} = \mathcal{L}_{\text{int}} (\varepsilon_s \sigma_s + \varepsilon_b \sigma_b) \to \sigma_s = (N_{\text{exp}}/\mathcal{L}_{\text{int}} - \varepsilon_b \sigma_b) / \varepsilon_s; \)
- \( d\sigma_s/d... = [...]; \)
- the luminosity \( \mathcal{L}_{\text{int}} \) is equal for signal and bckgd and must be measured;
- LEP measures \( \mathcal{L}_{\text{int}} \) from a process ("lumi process"), with a calculable cross section, triggered and acquired at the same time as other data (\( \to \) so DAQ inefficiencies cancel out):
  \( \mathcal{L}_{\text{int}} = N_{\text{lumi}} / (\varepsilon_{\text{lumi}} \sigma_{\text{lumi}} + \varepsilon_{\text{b-lumi}} \sigma_{\text{b-lumi}}) \)
- therefore three new errors:
  (statistics) \( \Delta N_{\text{lumi}} = \sqrt{N_{\text{lumi}}}, \)
  (sistematics) \( \Delta \varepsilon_{\text{lumi,b-lumi}}, \Delta \sigma_{\text{b-lumi}} \)
  ("theory") \( \Delta \sigma_{\text{lumi}}^{\text{theory}}. \)

NB. In an ideal experiment, \( N_{\text{lost}} = N_{\text{sp}} = 0 \to \varepsilon_s = 1, \varepsilon_b = 0. \)
data analysis: theory ↔ exp. data

An example: $e^+e^- \rightarrow \mu^+\mu^-$:

- studies for efficiency and purity with MC simulation [see later].
- **signal**: true events $e^+e^- \rightarrow \mu^+\mu^-$; the yield depends on $m_Z$, $\Gamma_Z$, $\Gamma_\mu$ (unknown);
- **bckgd**: events from other sources, with similar final state (because really the same or similar in the detector), e.g.:
  - $e^+e^- \rightarrow Z \rightarrow \tau^+\tau^-$
    $$\rightarrow (\mu^+\bar{\nu}_\tau \nu_\mu, \mu^-\bar{\nu}_\tau \bar{\nu}_\mu)$$
    $$\rightarrow (\mu^+\mu^-) (+ \text{not-visible});$$
  - $e^+e^- \rightarrow e^+e^- \mu^+\mu^-$
    $$\rightarrow (e^+e^-)_{\text{beam chamber}} (\mu^+\mu^-)^{\text{detected}};$$
    $$\rightarrow (\mu^+\mu^-) (+ \text{not-detected});$$

\[ N_{\exp} = \mathcal{L}_{\text{int}} [\varepsilon_s \sigma_s + \varepsilon_b \sigma_b]. \]
In 1989, when LEP started, the SM was completely formulated and computed;
• the only missing pieces (at that time) were the top quark and the Higgs boson (both now discovered);
• the values of $m_{\text{top}}$ and $m_{\text{Higgs}}$ are such that they (in lowest order) have no role at LEP $\sqrt{s}$ [but for $H$ we did NOT know];
• twelve years of LEP physics gave NO major surprise, but general agreement with SM predictions;
• tons of measurements, a superb unprecedented work of precision physics: the number of light $\nu$'s and the predictions of $m_{\text{top}}$ and $m_{\text{Higgs}}$ via higher orders are [imho] the LEP masterpieces.
data analysis: comparison theory ↔ data

Therefore, a **measurement** means:

- select a pure (as much as possible) sample of events \( N_i \);
- measure the statistical significance of the experiment (\( \to \mathcal{L}_{\text{int}} \));
- measure/compute the associated efficiency and purity (\( \to \varepsilon, p \));
- compute \( \sigma_i \equiv \sigma_i^{\text{exp}} = [\text{previous slide}] \)
  \[ \text{or } d\sigma_i^{\text{exp}}/dk = (...) \];
  \( \to \) finally **theory ↔ experiment**:
  - compute \( \sigma_i^{\text{theo}} \) from theory;
  - **compare** \( \sigma_i^{\text{theo}} \leftrightarrow \sigma_i^{\text{exp}} \).

["limits" require a different method, see § limits].
data analysis: results

SM predictions:
- $\sigma(f\bar{f})$, $\sigma(e^+e^-)$, $d\sigma/d\cos\theta$ ... ("Born");
- radiative corrections;
- approximations;

experiment(s) (LEP, L3 as an example):
- cross sections $\sigma(e^+e^-\rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \text{hadrons, } \nu\bar{\nu})$;
- differential cross sections $d\sigma(e^+e^- \rightarrow ...) / d\cos\theta$;
- "lineshape" (i.e. $\sigma(e^+e^- \rightarrow ...)$ as a function of $\sqrt{s}$ [also $d\sigma(e^+e^- \rightarrow ...) / d\cos\theta$ vs $\sqrt{s}$].

data analysis and interpretations: global fit (4 exp. data) $\leftrightarrow$ (SM):
- $Z$ mass, full and partial width ($m_Z$, $\Gamma_Z$, $\Gamma_f$);
- number of $\nu$'s from $\Gamma_{\text{invisible}}$ and from $\gamma_{\text{single}}$;
- asymmetries $A_{\text{forward-backward}}$ for $e^+e^-\rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \text{hadrons}$;
- global fit data $\leftrightarrow$ SM ($\rightarrow$ consistency);
- global fit data $\leftrightarrow$ SM ($\rightarrow$ predictions of $m_{\text{top}}$, $m_{\text{Higgs}}$ from radiative corrections).
how to detect and identify c / b / τ's with a μ-vertex

the detector

typical event: case 1

doi you see the difference?

typical event: case 2

heavy quark (e.g. b) decay

μ-vertex

beam pipe

few cm

it needs a great accuracy in the "impact parameter" measurement.
Analysis method (B as an example, similar for c-mesons/baryons, $\tau^\pm$):

- [B conservation $\rightarrow$ 2 B in the event $\rightarrow$ 2 sec. vtxs];
- B ref. sys: $\tau(B^{\pm,0}) \approx 1.5 \times 10^{-12}$ s $\rightarrow \ell^* = c \tau_B \approx 500 \mu$m;
- $\beta_B \approx 1 \rightarrow \ell = \ell_B, \beta_B \gamma_B \approx c \tau_B \gamma_B \approx$ few mm [see];
- $\ell_T = (\ell \tan \theta)$ is invariant wrt a $\mathbb{L}$-transform along $\beta_B$
  $\rightarrow \ell_T = \ell_T^* = \ell^* \sin \theta^* \approx 100 \div 500 \mu$m
  ($\theta^*$ is the angle B/\pi in the B ref. sys., NOT small);
- $\ell_T$ has large errors, but $\ell'_T$, the transverse distance (extrapolation of a track) $\leftrightarrow$ (primary vtx) can be meas.;
- $\theta \sim m_B/E_B \approx 1/\gamma_B$ = small $\rightarrow \sin \theta \approx \tan \theta \rightarrow \ell'_T \approx \ell_T$;
- [call both $\ell'_T$ and $\ell_T$ "impact parameter $\ell_T"$];
- need a detector with an accuracy $<< 100 \mu$m in $\ell_T$ (i.e. in the extrapolation of the line of flight of a charged particle after 20÷30 mm from the last meas;
- i.e. a very precise microvertex detector may identify and reconstruct b, c, $\tau$ decays.
efficiency and purity

- No selection method is fully "pure" and "efficient", i.e. in a selected sample of events of type "i", there are some events "j" (j≠i), while some events "i" have been rejected;
- if \( N_{i}^{sel} \) is the number of events of the sample, define:
  - **efficiency**: \( \varepsilon_i = N_{i}^{sel,true} / N_{i}^{true,all} < 1 \) [ideally = 1];
  - **purity**: \( p_i = N_{i}^{sel,true} / N_{i}^{sel,all} < 1 \) [ideally = 1];
  - [contamination]: \( k_i = N_{i}^{sel,false} / N_{i}^{sel,all} = 1 - p_i \);
- in general, \( \varepsilon_i \) and \( p_i \) are anti-correlated (see below);
- an algorithm (e.g. a cut in a kin. variable) produces \( \varepsilon_i + p_i \);
- the "optimal" **choice** depends on the analysis and on \( L_{int} \).

Example [no "i" in the plots]:
- two cases of \( p_i \) vs \( \varepsilon_i \), when the cut varies.
- exp. A "is better" than B.
- "★" shows a possible choice for \( (p_i, \varepsilon_i) \) in A.
N_{i}^{\text{sel, true}} and N_{i}^{\text{true, all}} are NOT directly measurable. Few methods to determine the relation \( \varepsilon / p \), e.g.:

- **Monte Carlo (commonly used):**
  - 3 steps: "physics" \([\rightarrow 4\text{-mom.}] + \text{detector} \[\rightarrow \text{pseudo-meas.}] + \text{analysis} \[\text{exactly the same as in real data}];
  - pros: large statistics, flexible, easy;
  - cons: (some) systematics cannot be studied;

- **Test beam:**
  - intrinsic purity + large statistics;
  - pros: systematics;
  - cons: not flexible, difficult, expensive;

- "data themselves"
  - e.g. \( \mu \) from Z\(\rightarrow\mu\mu \) to study \( b\rightarrow\mu X \):
    - "tag and probe" \( [p \approx 1 \text{ even if } \varepsilon \text{ small}] \) to force purity;
    - ok for systematics;
    - difficult reproduction of the required case [in the example isolated \( \mu \)'s 45 GeV instead of low-\(p_T\) \( \mu \) in a jet].

∴ Combination of the above, iterations, new ideas (i.e. \textit{you 😊})...
efficiency and purity: example

An example of the computation of $\varepsilon$ vs $p$ (secondary vtxs with impact parameter):

- use a mc (not shown) to define the distribution of impact parameter $b$ in events with sec. vtxs;
  - a cut on $b \rightarrow \varepsilon = \varepsilon(b_{\text{cut}})$;
- use a process without secondaries ($Z \rightarrow \mu^+\mu^-$) to define the distribution of the variable $b$;
  - a cut on $b \rightarrow p = p(b_{\text{cut}})$;
- $\varepsilon = \varepsilon(b_{\text{cut}}) \oplus p = p(b_{\text{cut}})$ are parametric equations;
- repeat with more info $\rightarrow$ "3D" $\rightarrow$ better curve.
efficiency and purity: the bckgd

• The background ["bckgd"] may be conceptually divided into two categories:
  ➢ **irreducible bckgd\(^(*)\)**: other processes with the same final state [e.g. \(e^+e^-\rightarrow ZH, Z\rightarrow\mu^+\mu^-, H\rightarrow b\bar{b}\) (signal) \(\leftrightarrow\) \(e^+e^-\rightarrow Z_1\ Z_2, Z_1\rightarrow\mu^+\mu^-, Z_2\rightarrow b\bar{b}\) (bckgd)];
  ➢ **reducible bckgd**:
    ▪ badly-measured events,
    ▪ detector mistakes,
    ▪ physics processes which appear identical (with given selection criteria) to the process under study [e.g. because part of the final state is undetected, \(e^+e^-\gamma_{\text{unseen}}\leftrightarrow e^+e^-\nu\)];

• the meaning of the distinction is that r.b. can be disposed with a better detector, or a more accurate selection (maybe with a loss in \(\varepsilon_s\)), while i.b. is intrinsic, and can only be subtracted statistically, by comparing \(N^{\text{exp}}\leftrightarrow(\text{expected bckgd})\) and \(N^{\text{exp}}\leftrightarrow(\text{expected signal+bckgd})\);

\(\text{(*) Similar to the "resonances" of the strong interactions, where a mass distribution exhibits peaks, interpreted as short-lived particles. However, it is impossible to assign single events to the resonating peak or to the non-resonant bckgd.}\)
[few slides ago:

*LEP measures $\mathcal{L}_{\text{int}}$ from a process (...):

$$\mathcal{L}_{\text{int}} = \frac{N_{\text{lumi}}}{(\varepsilon_{\text{lumi}} \sigma_{\text{lumi}} + \varepsilon_{\text{b-lumi}} \sigma_{\text{b-lumi}})}$$

* the "lumi" process ($\sigma_{\text{lumi}}$) is $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering) at small $\theta$;

* we **assume** that, when $\theta \rightarrow 0^\circ$, the Bhabha scattering is dominated by the $\gamma^*$ exchange in the t-channel, while both
  (a) the $\gamma^*/Z$ exchange in the s-channel;
  (b) the $Z(*)$ exchange in the t-channel are negligible;

* therefore, the LEP experiments have e.m. calorimeters at small $\theta$, to both

identify and measure $e^\pm$ ("luminometers", ring-shaped ♦);

* it is essential that the "ring" reaches very small $\theta$, to minimize $\Delta \sigma_{\text{stat}}$

  $$(d\sigma_{\text{Rutherford}} / d\cos \theta \propto \theta^{-4});$$

* their position and efficiency must be known (= measured) very reliably, in order to minimize systematics;

* typically at LEP, $25 \leq \theta_{\text{lumi}} \leq 60$ mrad:

  $$\sigma_{\text{lumi}} = \frac{16\pi\alpha_{\text{em}}^2}{s} \left(1/\theta_{\text{min}}^2 - 1/\theta_{\text{max}}^2\right);$$

  $$\Delta \mathcal{L} / \mathcal{L} = \Delta \sigma_{\text{lumi}} / \sigma_{\text{lumi}} \approx 2\Delta \theta_{\text{min}} / \theta_{\text{min}}.$$
• at the end of LEP, using sophisticated silicon calos, the final results on luminosity was:

\[ \frac{\Delta \mathcal{L}_{\text{int}}}{\mathcal{L}_{\text{int}}} = \left[ \text{see box} \right] \quad (\text{statistical}); \]

\[ \oplus [0.03 \div 0.1 \%] \quad (\text{syst. exp : } \Delta \theta, \text{ alignment, ...}); \]

\[ \oplus [0.11 \%] \quad (\text{theory, higher orders like } e^+e^- \rightarrow e^+e^- \gamma \text{ unseen}); \]

• some of the LEP measurements, as number of \( \nu \)'s, asymmetries, do NOT depend on \( \Delta \mathcal{L}_{\text{int}} \) : because can be expressed as ratios "\( \sigma_1/\sigma_2 \) [\( = N_1/N_2 \)];"

• [the luminosity data are an important fraction of all LEP1 data].

---

An estimate of the importance of the statistical error comes from the comparison:

• \( \sigma(e^+e^- \rightarrow \text{hadrons}, \sqrt{s} = m_Z) \approx 30 \text{ nb, the largest cross-section among all LEP processes}; \)

• \( \sigma(e^+e^- \rightarrow e^+e^-, 25 \leq \theta \leq 60 \text{ mrad}) \approx 100 \text{ nb.} \)

Therefore the statistical error on the luminosity is negligible, but for the hadronic cross section at \( \sqrt{s} = m_Z \), where it is \(~ \sqrt{3/10} \) of the statistical error on the hadron data [but for this process the stat. error is irrelevant wrt systematics].
1. – 8. [...]  
9. $e^+e^- \rightarrow Z \rightarrow ff$  
10. $d\sigma(e^+e^- \rightarrow ff) / d\Omega$  
11. $e^+e^- \rightarrow Z \rightarrow e^+e^-$  
12. Radiative corrections  
13. LEP1 SM fit  
14. $e^+e^- \rightarrow W^+W^- @ LEP2$  
15. Global LEP(1+2) fit  
16. [...]
Many possibility from $e^+e^-$ initial state;
- similar couplings wrt already considered processes [$\S 3$, $\S 4$, $\S 6$, $\S 7$];
- at low energy, QED only (exchange of $\gamma^*$ in the s-channel);

at $\sqrt{s} \approx m_Z$:
- $\sigma_{\text{res}}(e^+e^- \rightarrow \bar{f}f) \propto \Gamma_f / [ (s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2 ]$;
- for each fermion pair, two (four for $e^+e^-$) diagrams + interferences);
- at higher energy, new phenomena ($W^\pm$, exchange, IVB pairs in the final state, ...).
In the SM, at lowest order, for $f \neq e^\pm$, $m_f \ll m_Z$:

- $\sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f}) = \sigma_{Zs} + \sigma_{\gamma s} + J_f$;

$$\sigma_{Zs} = \frac{s \Gamma^2 Z}{(s - m_Z^2)^2 + s^2 \Gamma^2 Z / m_Z^2} \frac{12 \pi \Gamma e \Gamma_f}{m_Z^2 \Gamma^2_Z},$$

- $\sigma_{\gamma s} = \frac{4 \pi \alpha^2}{3s} c_f Q_f^2$; \quad \left[ c_f = 1 \text{ (leptons)}, 3 \text{ (quark)} \right]$;

$$J_f = -\frac{(s - m_Z^2)m_Z^2}{(s - m_Z^2)^2 + s^2 \Gamma^2 Z / m_Z^2} \frac{2 \sqrt{2} \alpha}{3} c_f Q_f G_f g_V^e g_V^f;$$

- $\Gamma_Z = \Gamma_{\text{tot}} = \sum_f \Gamma(Z \rightarrow f\bar{f})$;

- $\Gamma_f \equiv \Gamma(Z \rightarrow f\bar{f}) = \frac{G_f m_Z^3 c_f}{6 \sqrt{2} \pi} \left[ g_V^{f^2} + g_A^{f^2} \right]$;

- for $\sqrt{s} \approx m_Z$ → interference and $\gamma^*$ negligible;

- $\sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f}, \sqrt{s} = m_Z) = \frac{12 \pi \Gamma e \Gamma_f}{m_Z^2 \Gamma^2_Z}$. 

\[
\begin{cases}
\text{i.e. neglect t-channel, both } Z^* \text{ and } \gamma^* \\
\text{= bell-normalized-to-1} \\
\times \sigma(\sqrt{s} = m_Z) \\
\text{[well known, see §3]}
\end{cases}
\]

[well known, see §3]

\[
\text{new entry, possibly important for } \mathbb{P} \text{-violation}
\]

\[
\Gamma_{\text{tot}} = \sum_f \Gamma(Z \rightarrow f\bar{f})
\]

\[
\Gamma_f = \Gamma(Z \rightarrow f\bar{f}) = \frac{G_f m_Z^3 c_f}{6 \sqrt{2} \pi} \left[ g_V^{f^2} + g_A^{f^2} \right]
\]

\[
\sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f}, \sqrt{s} = m_Z) = \frac{12 \pi \Gamma e \Gamma_f}{m_Z^2 \Gamma^2_Z}
\]
• the partial widths $\Gamma_f$ (e.g. $\Gamma_\mu$) are also easily computed in lowest order:

$$\Gamma_f = \frac{G_F m_Z^3 c_f}{6\sqrt{2}\pi} \left[ g_{V_f}^2 + g_A^2 \right] \rightarrow \left( f = \mu^\pm \right) \rightarrow \Gamma_\mu \approx \frac{1}{4} \frac{G_F m_Z^3}{6\sqrt{2}\pi} \approx 83\text{MeV};$$

• for the other $\Gamma$'s it is found [lowest order values, NOT "the best"]:

<table>
<thead>
<tr>
<th>$f$</th>
<th>$Q_f$</th>
<th>$g_A^f$</th>
<th>$g_V^f$</th>
<th>$\Gamma_f$ (MeV)</th>
<th>$\Gamma_f / \Gamma_\mu$</th>
<th>$R_f$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$</td>
<td>0</td>
<td>$+\frac{1}{2}$</td>
<td>$+\frac{1}{2}$</td>
<td>166</td>
<td>1.99</td>
<td>6.8</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>-1</td>
<td>$-\frac{1}{2}$</td>
<td>-.038</td>
<td>83</td>
<td>[1]</td>
<td>3.4</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>$\frac{2}{3}$</td>
<td>$+\frac{1}{2}$</td>
<td>+.192</td>
<td>286</td>
<td>3.42</td>
<td>11.8</td>
</tr>
<tr>
<td>$u$</td>
<td>$\frac{2}{3}$</td>
<td>$-\frac{1}{2}$</td>
<td>-.346</td>
<td>368</td>
<td>4.41</td>
<td>15.2</td>
</tr>
<tr>
<td>$c$</td>
<td>$\frac{1}{3}$</td>
<td>$+\frac{1}{2}$</td>
<td>+.192</td>
<td>286</td>
<td>3.42</td>
<td>11.8</td>
</tr>
<tr>
<td>$s$</td>
<td>$\frac{1}{3}$</td>
<td>$-\frac{1}{2}$</td>
<td>-.346</td>
<td>368</td>
<td>4.41</td>
<td>15.2</td>
</tr>
<tr>
<td>$b$</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{2}$</td>
<td>-.346</td>
<td>368</td>
<td>4.41</td>
<td>15.2</td>
</tr>
</tbody>
</table>

In Born approx. [B = "Born"]:

- $\Gamma^B_Z = 2423 \text{ MeV}$, $\Gamma^B_{\text{hadr}} = 1675 \text{ MeV}$, $\Gamma^B_{\text{invis}} = \Gamma^B_\nu = 498 \text{ MeV}$;
- $R^B_{\text{hadr}} = 69.1 \%$, $R^B_{\text{lept}^\pm} = 10.2 \%$, $R^B_{\text{invis}} = 20.5 \%$,
- $R^B_{\text{hadr}} / R^B_{\text{vis}} = 87.0 \%$.

- $\Gamma_Z \approx 2.4 \text{ GeV}$, $\Gamma_\nu \approx 0.5 \text{ GeV}$, remember!
- $\nu : \ell^\pm : u : d \approx 2 : 1 : 3.4 : 4.4$, hadr : $\ell^\pm : \nu \approx 70 : 10 : 20.$
$e^+e^- \rightarrow Z \rightarrow f\bar{f}$: predictions

$Z/Z$ and $\gamma^*/\gamma^*$ are +ve by definition,

$|\gamma^*/Z|$ is plotted ($<0$ @ $\sqrt{s}<m_Z$, $>0$ @ $\sqrt{s}>m_Z$).
$e^+e^- \rightarrow Z \rightarrow \bar{f}f$: home-made predictions

$\sqrt{s}$ (GeV) | $\sigma$ (nb)
---|---
$< 0$ | $> 0$

$m_Z = 91.1876$ GeV
$\Gamma_Z = 2.4952$ GeV
$\Gamma_e = 0.083984$ GeV
$\Gamma_\mu = 0.083984$ GeV
$1/\alpha_{em} = 128.877$
$q_\mu = -1$
$c_\mu = 1$
$g^e_v = -0.03783$
$g^\mu_v = -0.03783$
$G_F = 1.1664 \times 10^{-5}$ GeV$^{-2}$
$(\hbar c)^2 = 3.8938 \times 10^5$ GeV$^2$ nb

$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$
$ZZ, \gamma^*\gamma^*, |\gamma^*Z|$.
Introduce a different process: "2 γ physics":

- it is so called because the initial state of the hard collision is given by two γ's;
- the two e± of the initial state retain much of the energy, and in most cases escape undetected in the beam chamber;
- classify events in "untagged", "single tag" and "double tag", depending on whether 0, 1, 2 and e± are detected;
- lot of nice kinematics [try it];

- events studied using two variables:
  - √s = m_{ini}(e^+e^-);
  - W = m(γγ) = m(hadrons);
- the study of σ_{γγ} requires a cut on W, i.e. σ_{γγ} = σ_{γγ}(W > W_{cut}), both for theory and detection:
  - σ_{γγ} weakly dependent on √s;
  - σ_{γγ} strongly dependent on W, σ_{γγ} ∼ e^{-W}.

Why study "2 γ physics"? Two main goals:

1. intrinsic interest:
   - any process deserves a study;
   - rich "factory" of hadron resonances;
   - other low-energy processes;
2. σ_{γγ} is large:
   - LEP1: subtract from high precision meas.;
   - LEP2: typically tiny cross sections → an important background, especially if large E required.
Example: $e^+e^- \rightarrow \text{hadrons}$ (i.e. $e^+e^- \rightarrow q\bar{q}$) in L3 1994 (an old paper, chosen because well written). Selection:

- $0.5 < E_{\text{vis}} / \sqrt{s} < 2.0$;
- $|E_{||}| / E_{\text{vis}} < 0.6$;
- $|E_{\perp}| / E_{\text{vis}} < 0.6$;
- $N_{\text{clusters}} > 13$ (barrel), > 17 (endcap) [next]
Example: \( e^+e^- \rightarrow \text{hadrons} \) (i.e. \( e^+e^- \rightarrow q\bar{q} \)) in L3 1994 – pag. 2

\[ [N_{\text{clusters}} > 13 \text{ (barrel), } > 17 \text{ (endcap)}] \]
\( \text{e}^+\text{e}^- \rightarrow \text{Z} \rightarrow \bar{f}f: \mu^+\mu^- \)

Other example (same paper) : \( \text{e}^+\text{e}^- \rightarrow \mu^+\mu^- \)

Selection :
- \( \geq 1 \mu \) identified;
- \( |p_\mu| > 0.6 (\sqrt{s}/2) \);
- \( \alpha(\mu\mu) \) “small”;
- \( N_{\text{clusters}} < 15 \);
- \( \text{time}_{\text{scintillators}} \).

Q. : why \( \mu \)'s have smaller acollinearity than \( \tau \)'s?
Problem. Two variables \((x, y)\) are normally\(^{1}\) (=Gauss) distributed with mean \((m_x, m_y)\) and standard deviation \(\sigma_x = \sigma_y = \sigma\). Find the distribution of the distance from the center

\[
r = \sqrt{(x - m_x)^2 + (y - m_y)^2}.
\]

Solution:

\[
\begin{align*}
(x - m_x) = r \cos \theta; \\
(y - m_y) = r \sin \theta;
\end{align*}
\]

\[
f_{\text{Gauss}}(t | \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{t^2}{2\sigma^2} \right];
\]

\[
f(x, y) = f(x | \sigma) \times f(y | \sigma) = \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{x^2 + y^2}{2\sigma^2} \right];
\]

\[
J \left( \begin{array}{c} x, y \cr r, \theta \end{array} \right) = \begin{vmatrix}
\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\
\frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta}
\end{vmatrix} = \begin{vmatrix}
\cos \theta & \sin \theta \\
-r\sin \theta & r\cos \theta
\end{vmatrix} = r;
\]

\[
f(r, \theta) = \frac{r}{2\pi\sigma^2} \exp \left[ -\frac{r^2}{2\sigma^2} \right];
\]

\[
f(r) = \int_0^{2\pi} d\theta [f(r, \theta) = 2\pi f(r, \theta) = \frac{r}{\sigma^2} \exp \left[ -\frac{r^2}{2\sigma^2} \right]].
\]

\(^{1}\)W. Tell's crossbow; \(^{2}\)the event \(\mathcal{E}_T\) at LEP/LHC; \(^{3}\)the sum of momenta of the charged particles wrt the jet axis, …

\(m_x\) and \(m_y\) are translations wrt centre; they do NOT influence the result.
$e^+e^- \rightarrow Z \rightarrow \bar{f}f$: a W. Tell tale

$$f(r) = r e^{-r^2/(2\sigma^2)} / \sigma^2$$

**Graph:**
- $f_{\text{Gauss}}(x, \sigma=1)$
- $f(r, \sigma=1)$
- $f_{\text{Gauss}}(x, \sigma=2)$
- $f(r, \sigma=2)$

- max at $r = \sigma$
- $f = 0.607 / \sigma$

**Equation:**
$$J = r \rightarrow$$
at small $r$, no space left
surface $= 2\pi r dr$

**Next question:**
the case $\sigma_x \neq \sigma_y$ [easy, needs only one smart trick]
\[ e^+ e^- \rightarrow Z \rightarrow f \bar{f} \text{ lineshape} \]

**Notice:**
- \( \sigma(\text{had}) \gg \sigma(\mu\mu); \)
- fit quality;
- strategy change in 1993;
- the line is the SM fit (see later).

For \( e^+ e^- \rightarrow \tau^+ \tau^- \) see later.
Differential cross-section in lowest (Born) order:

\[
\frac{d\sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f})}{d\cos\theta} = \frac{\pi\alpha^2(s)C_f}{2s} \left[ 1 + \cos^2\theta \right] \times \left[ Q^2_{e}Q^2_{f} - 2\left[\chi\right]Q_eQ_\gamma e\gamma f\cos\delta_R + \left[\left(\chi\right)^2\left(\frac{g^e_A}{g^e_V}\right)^2 + \left(\frac{g^f_A}{g^f_V}\right)^2\right] \left[\left(\frac{g^f_A}{g^f_V}\right)^2 + \left(\frac{g^f_A}{g^f_V}\right)^2\right] \right] + \\
2\cos\theta \times \left[-2\left[\chi\right]Q_eQ_\gamma e\gamma f\cos\delta_R + 4\left[\chi\right]^2\left(\frac{g^e_A}{g^e_V}\right)^2 + \left(\frac{g^f_A}{g^f_V}\right)^2\right]\right];
\]

\[\chi = \frac{G_F}{2\sqrt{2}\pi\alpha(s)} \times \frac{sm^2}{(m_z^2 - s)^2 + m_z^2\Gamma^2_z};\]
\[\tan\delta_R = \frac{m_z\Gamma_z}{m_z^2 - s};\]

\[A_{f}^{\text{FB}}(\sqrt{s}) \equiv \frac{\sigma(\cos\theta > 0,\sqrt{s}) - \sigma(\cos\theta < 0,\sqrt{s})}{\sigma(\cos\theta > 0,\sqrt{s}) + \sigma(\cos\theta < 0)};\]

\[A_{f}^{\text{FB}}(\sqrt{s} = m_z, Z_{s\text{-channel only}}) = 3 \times \frac{g^e_Vg^e_A}{\left(g^e_V\right)^2 + \left(g^e_A\right)^2} \times \frac{g^f_Vg^f_A}{\left(g^f_V\right)^2 + \left(g^f_A\right)^2};\]

\[A_{f}^{\text{FB}}\] is the "forward-backward asymmetry" for \(e^+e^- \rightarrow f\bar{f}.\)
\[ \frac{d\sigma_{\text{Born}}(e^+e^- \to \bar{f}f)}{d\cos\theta} = \frac{\pi\alpha^2(s)c_f}{2s} \left[ (1+\cos^2\theta) \times \left[ \frac{Q_e^2Q_f^2 - 2\chi Q_e g_V g_f^f \cos\delta_R}{\chi} + \frac{\chi^2 (g_A^e)^2 + (g_V^e)^2}{(g_A^f)^2 + (g_V^f)^2} \right] + 2\cos\theta \times -2\chi Q_e Q_f g_A^e g_A^f \cos\delta_R + 4\chi^2 g_A^e g_A^f g_V^e g_V^f \right] \right]; \]

\[ A_{\text{FB}}^f(\sqrt{s}) = \frac{\sigma(\cos\theta > 0, \sqrt{s}) - \sigma(\cos\theta < 0, \sqrt{s})}{\sqrt{s-m_Z^2}} \cdot \frac{3}{\sigma(\cos\theta > 0, \sqrt{s}) + \sigma(\cos\theta < 0, \sqrt{s})} \times \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \times \frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}. \]

- Standard SM computation for \( Z_s \oplus \gamma_s \) only (average on initial and sum on final polarization), then sum on \( \phi \):
  - Notice: the term \( \propto \cos\theta \) is anti-symmetric; it does NOT contribute to \( \sigma_{\text{tot}} \) (\( \int \cos\theta \, d\cos\theta = 0 \)), but only to the (\( \mathcal{P} \)-violating) forward-backward asymmetry;
  - The \( \mathcal{P} \)-violation clearly comes from the interference between the vector (\( \gamma + Z_V \)) and axial (\( Z_A \)) terms.

- At the pole (\( \sqrt{s}=m_Z \)):
  - \( \cos\delta_R = 0 \);
  - The asymmetry, i.e. the term \( \propto \cos\theta \), is \( \propto g_V^e \) (very small) for all fermions;
  - For the \( \mu^+\mu^- \) case [easily measurable], it is even smaller (\( \propto g_V^e g_V^\mu \)).
• Experimentally, the main problem is the selection $f \leftrightarrow \bar{f}$ (i.e. $\theta \leftrightarrow \pi - \theta$). This is
  - essentially impossible for light quarks $u \leftrightarrow \bar{u}$, $d \leftrightarrow \bar{d}$ (despite heroic efforts based on charge counting);
  - difficult for heavy quarks $c, b$ (based on lepton charge in semileptonic quark decays, e.g. $c \rightarrow s\ell^+\nu$, $\bar{c} \rightarrow \bar{s}\ell^-\bar{\nu}$);
  - "simple" for $\mu^\pm$ (only problem: wrong sagitta sign because of high momentum);
  - best channel for $d\sigma/d\cos\theta$ and $A_{FB}$: $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$;
• unfortunately, $A_{FB}(\sqrt{s}=m_Z)$ is very small in the $\ell^+\ell^-$ channels, due to the extra small factor $g_V^\mu$;
• notice the asymmetry change for peak $\pm 2$ GeV.
$$d\sigma(e^+e^- \rightarrow f\bar{f}) / d\Omega: A_{\text{fb}}(\mu^+\mu^-)$$

- **SPEAR**
- **PEP**
- **PETRA**
- **TRISTAN**
- **LEP**

- **MARK I**
- **HRS**
- ** CELLO**
- **AMY**
- **L3 (1993)**
- **MAC**
- **JADE**
- **TOPAZ**
- **MARK II**
- **MARK J**
- **VENUS**
- **PLUTO**
- **TASSO**

- $$A_{\text{FB}}^\gamma(e^+e^- \rightarrow \mu^+\mu^-)$$

- **L3**

- e^+e^- \rightarrow \mu^+\mu^- (\gamma)

- **1990-92**
- **1993**
- **1994**
- **1995**

- **\gamma only \rightarrow V only**
  \rightarrow $$A_{\text{FB}}^\gamma = 0$$

- $$\sqrt{s} \approx m_Z \rightarrow A \text{ dominates}$$
  \rightarrow $$A_{\text{FB}}^\gamma \approx 0$$

- **Z \approx A, \gamma = V \rightarrow A_{\text{FB}}^\gamma \text{ max}$$
  @ max interference
  [no exp ever]

- **full \sqrt{s} range + SM prediction**
Problem. Compute $d\sigma/d\cos\theta$ and $A_{FB}^\text{peak}$ at lowest order from the formulæ. This is a case where the "tree approx." fails. Explain where and why.

If no success, look to Grünewald, op. cit., pag. 230-232 [simplified explanation: higher orders and selection criteria are important, especially for peak+2 ($\rightarrow$ init. state brem). The correct approach is to use higher orders also in the prediction].
• Bhabha scattering is more difficult, due to the presence of another Feynman diagram: the $\gamma^* / Z$ exchange in the t-channel;

• 4 Feynman diagrams $\rightarrow$ 10 terms:
  - $Z$ s-channel ($Z_s$);
  - $\gamma^*$ s-channel ($\gamma_s$);
  - $Z$ t-channel ($Z_t$);
  - $\gamma^*$ t-channel ($\gamma_t$);
  - 6 interferences;

• qualitatively:
  - $\sqrt{s} \approx m_Z$ and $\theta >> 0^\circ$, $Z_s$ dominates.
  - $\theta \approx 0^\circ$, $\gamma_t$ dominates for all $\sqrt{s}$;
  - $\sqrt{s} << m_Z$ and $\theta >> 0^\circ$, $\gamma_s$ and $\gamma_t$ are both important, while $Z_s$ is negligible.
$e^+e^- \rightarrow Z \rightarrow e^+e^-$: $\sigma_{SM}$

- $s$, $t$, interference vs $\sqrt{s}$, with a $\theta$ cut ($|\cos\theta| < 0.72$, i.e. $44^\circ < \theta < 136^\circ$);
- data @ $|\cos\theta| \approx 1$ taken, but not used here [used for lumi];

- notice: the cut on $\cos\theta$ is NOT instrumental, but used OFFLINE to enhance $Z_s$ over $\gamma_t$, considered as bkkgd.
$e^+e^- \rightarrow Z \rightarrow e^+e^-$: results

L3

$e^+e^- \rightarrow \tau^+\tau^- (\gamma)$

$|\cos\theta| < 0.72$

$\sigma$ [nb]

$\sqrt{s}$ [GeV]

ratio
radiative corrections

ISR
FSR
"box"
loop

init. state
top quark
higher orders

final state

+ many others ...
radiative corrections: what? why?

**what?**

- higher orders (both SM and bSM);
- dependent on full SM, QCD included;
- conventionally, classified into QED, weak, QCD, bSM (if any);
- ... or initial and final state;
- also particles not kinematically allowed at lower $\sqrt{s}$ (e.g. top, Higgs);

**computable?**

- in principle yes, if all parameters known;
- in practice, successive approximations ("order n");

**necessary?**

- yes, because required by the measurement accuracy;

**useful?**

- yes, because they give an indirect access to higher energy, by making lower energy observables (like $m_z$) dependent on higher energy parameters (like $m_{top}$ or $m_H$);
- i.e., they "raise" the accessible $\sqrt{s}$;
- + more accurate and powerful test of the theory;
- [much work, theses, papers, ...];

**how to use the bSM part** (e.g. SUSY), both tree-level and higher orders?

- first, do not include it, and look for discrepancies;
- if disagreement (ευρήκα !!!), include physics bSM and look for agreement;
- if not $\rightarrow$ put a limit on physics bSM.
radiative corrections: ISR kinematics

One of the simplest r.c. is the QED brem of a (real) $\gamma$ from one of the initial state $e^{\pm}$ : **ISR** (Initial State Rad.);

- the kinematics is :

$$e^+ e^- ( \sqrt{s}, \ 0, \ 0 );$$

$$\gamma ( \ E_\gamma, \ E_\gamma \cos \alpha_\gamma, \ E_\gamma \sin \alpha_\gamma );$$

$$\bar{f} f ( \sqrt{s} - E_\gamma, \ -E_\gamma \cos \alpha_\gamma, \ -E_\gamma \sin \alpha_\gamma );$$

$$s'/m_{\bar{f}f}^2 = \left( \sqrt{s} - E_\gamma \right)^2 - E_\gamma^2 = s \left( 1 - 2E_\gamma / \sqrt{s} \right);$$

$$z \equiv s'/s = 1 - 2E_\gamma / \sqrt{s}; \quad [s' < s \rightarrow z < 1]$$

$E_\gamma$ is fixed and $\alpha$-independent:

$$E_\gamma = \frac{\sqrt{s} \ s - s'}{2} = \frac{s - s'}{2\sqrt{s}} = \frac{s - m_{\bar{f}f}^2}{2\sqrt{s}}.$$ 

**LEP 1** ($\sqrt{s} < m_z + \text{few GeV}$):

- $\sqrt{s'} \approx m_z$, (but $\Gamma_z$) $\rightarrow$ large $\Delta E_\gamma / E_\gamma$;
- $\alpha_\gamma$ small (brem. dynamics), $\gamma$'s mostly in the beam pipe;
- condition : $2m_f \leq \sqrt{s'} \leq \sqrt{s}$;

**LEP 2** ($\sqrt{s} >> m_z$):

- $\sqrt{s'} \approx m_z$ (because of resonance), known as "return to the Z";
- photon is really monochromatic ($\Gamma_z << E_\gamma$) and very energetic;
- $\alpha_\gamma$ small (brem. dynamics), $\gamma$'s mostly in the beam pipe, Z's with high longitudinal momentum, event very unbalanced;
- events easily removed in the analysis, but it decreases the effective event yield.
Theoretical treatment:

- Assume factorization (ISR) ↔ (Z formation);
- The Z formation at $\sqrt{s}'$ is equivalent to the standard process at $\sqrt{s}$, without ISR:

$$\sigma_{\text{ISR}}(e^+e^- \rightarrow f\bar{f}\gamma; \sqrt{s}) =$$

$$= \int_{4m_f^2/s}^{1} dz \left( R(z,s,\alpha_\gamma) \times \sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f}; \sqrt{zs}) \right);$$

- $R(z,s,\alpha_\gamma) = \text{radiator}$, i.e. probability (function of $\sqrt{s}$, $z$, $\alpha_\gamma$) for $\gamma$ brem;
- $R$ calculable in QED at a given order.

At LEP 2, cut on $z (\approx E_{\text{vis}}/\sqrt{s})$, typically $z<0.85$.

$$\gamma + \rightarrow \gamma + \left[ \begin{array}{c} \sigma = \int_{2}^{4m_f^2/s} dz \end{array} \right]$$
A precise computation requires much tedious work: these values are just for understanding [see fig.]:

- \( \sqrt{s} \big|_{\text{Born}}^{\text{max}} \approx m_Z (1 + \gamma^2)^{1/4} \approx m_Z (1 + \frac{1}{4} \gamma^2) \approx m_Z + 17 \text{ MeV} \); [slightly larger]

- \( \sqrt{s} \big|_{\text{ISR}}^{\text{max}} \approx m_Z (1 - \frac{1}{4} \gamma^2) + \pi \beta \Gamma_Z / 8 \approx m_Z + 89 \text{ MeV} \); [slightly larger];

- \( \sigma^f_0 \equiv \sigma_{\text{Born}}(e^+ e^- \rightarrow f \bar{f}; \sqrt{s}=m_Z) = 12 \pi \Gamma_e \Gamma_f / (m_Z^2 \Gamma_Z^2) \);

- \( \sigma(e^+ e^- \rightarrow f \bar{f}) \big|_{\text{Born}}^{\text{max}} \approx \sigma^f_0 (1 + \frac{1}{4} \gamma^2) \approx \sigma^f_0 (1 + 0.00019) \approx 0.75 \sigma^f_0 \); [much smaller];

- \( \sigma(e^+ e^- \rightarrow f \bar{f}) \big|_{\text{ISR}}^{\text{max}} \approx \sigma^f_0 \gamma \beta (1 + \delta_{\text{sup}}) \approx 0.75 \sigma^f_0 \); [much smaller];

- similar method for \( \Gamma_Z \):
  - \( \Gamma_Z \) s-dependent: \( \Gamma_Z \rightarrow s \Gamma_Z / m_Z^2 \);
  - (references); 

\( \gamma \equiv \Gamma_Z / m_Z \approx 0.027 \);
\( \beta \equiv 2 \alpha [2 \ln (m_Z / m_e) - 1] / \pi \approx 0.108 \);
\( \delta_{\text{sup}} \equiv [\text{soft- and virtual-}\gamma'\text{s, calculable}]. \)

\( \sqrt{s} \) naïve BW
- Born
- Born+ISR

notice also that the lineshape is dependent on the type of the fermion (e.g., for \( e^+ e^- \rightarrow \nu \bar{\nu} \) no \( \gamma \) in final state).
[an example: radiative corrections for $W^\pm$ and $Z$ mass]

- in the SM, $m_W$ and $m_Z$ are related by:
  \[ m_W^2 \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_F} ; \sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} ; \]
- radiative corrections modify the formulæ;
- define the parameters $\Delta r$ (radiative correction parameter), $\Delta \alpha$ (QED rad. corr.), $\Delta r_w$ (weak rad. corr.):
  \[ m_W^2 \sin^2 \theta_W = \frac{\pi \alpha}{\sqrt{2} G_F} \times \frac{1}{1 - \Delta r} \rightarrow \]
  \[ \Delta r = 1 - \frac{\pi \alpha}{\sqrt{2} G_F} \times \frac{m_Z^2}{m_W^2 (m_Z^2 - m_W^2)} ; \]
  \[ \frac{1}{1 - \Delta r} = \frac{1}{1 - \Delta \alpha} \times \frac{1}{1 - \Delta r_w} ; \]
- $\Delta \alpha$ is reabsorbed in $\alpha_{(s)}$, running coupling constant [the (s) means "function of $\sqrt{s}$"]:
  \[ \Delta \alpha_{(s)} = (\alpha_{(s)} - \alpha_{(s=0)}) / \alpha_{(s)} ; \]
- from QED:
  \[ \Delta \alpha_{(m_Z^2)} \approx 0.07 \rightarrow \alpha_{(m_Z^2)} \approx [128.89 \pm 0.09]^{-1} ; \]
  [error from $\int \sigma(e^+e^-\rightarrow \text{hadr.}) @ \sqrt{s} < m_Z$]
- the equation with $m_w + m_z$ becomes:
  \[ m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi \alpha_{(s=m_Z^2)}}{\sqrt{2} G_F} \times \frac{1}{1 - \Delta r_w} ; \]
- [to select top and Higgs terms] expand $\Delta r_w$ into parts, dependent on $m_t (\propto m_t^2)$ and $m_H (\propto \ln m_H)$, and the rest ($\Delta \bar{r}_w$):
  \[ \Delta r_w = \Delta \bar{r}_w \big|^{\text{calc.}}_{m_t = \hat{m}} + \frac{\partial \Delta r_w}{\partial m_t} \big|^{\delta m_t}_{m_t = \hat{m}} + \frac{\partial \Delta r_w}{\partial m_H} \delta m_H ; \]
  [$\hat{m} = 175$ GeV].
• assume we are in the "post-top, pre-Higgs" era [i.e. 1995-2011]:

• numerically, the sensibility is:

\[ \Delta r_W \approx \Delta r_W^{\text{calc.}} + -0.0019 \left( \frac{m_t}{175 \text{GeV}} \right) \left( \frac{\delta m_t}{5 \text{GeV}} \right) + \]

\[ +0.0050 \left( \frac{\delta m_H}{m_H} \right); \]

[the two terms have opposite sign and very different size]

• the meas. of \( m_W, m_Z, m_t \) + the calculation of higher orders of SM allow for a "measurement" of \( m_H \) á la Hollik;

• in reality, many observables \( \rightarrow \) global fit.
LEP1 SM fit

The LEP game

single channel
[e.g. e^+e^- → hadrons
@ √s = 95 GeV]

all e.w.
parameters
[really]

LEP e.w. fit

⊗ many exp. [χ, Delphi, L3, OPAL]

⊗ many distributions [σ, dσ/dΩ, ...]

⊗ many channels [q, μ, e, τ, ...]

⊗ many √s

“lineshape”
in the SM, the observables [e.g. \(\sigma\)'s, \(\sigma\)/dcos\(\theta\)'s, asymmetries, ...] are (functions of few) parameters like \(m_Z\), \(\Gamma_Z\), \(\Gamma_f\), \(\theta_w\) ...;

- in an experiment: \(N\) observables \(t_i\) (\(i = 1, ..., N\)) and \(M\) SM parameters \(\lambda_k\) (\(k=1,...,M\));
- [at LEP 1, \(N = \text{few} \times 100\), \(M \leq 10\), see later);
- [\(M\) is fixed, but the choice is free, e.g. one among \(m_Z\), \(m_W\) and \(\theta_w\) is redundant]
- the dependence of \(t_i\) from \(\lambda_k\) is known: \(t_i = t_i(\lambda_k) \pm \Delta t_i\) (\(\Delta t_i\) = the theoretical error);
- the \(N\) observables are measured: \(m_i \pm \Delta m_i\) (\(\Delta m_i\) = the convolution of stat. and sys.);
- a (difficult) numerical program computes the "best" \(\lambda_k\)'s which fit the observations;
- then the same values of \(\lambda_k\) are used for all the computations (shown as the "SM fits").
- [since \(N \gg M\), the dependence of any \(\lambda_k\) on the single \(i^{th}\) meas. is very small.]
- [also test the agreement SM ↔ data.]

\[
\chi^2 = \sum_i \left[ \frac{t_i(\lambda_k) - m_i}{\Delta t_i^2 + \Delta m_i^2} \right]^2;
\]
\[
\frac{\partial \chi^2}{\partial \lambda_k} = 0 \quad (M \text{ equations})
\]

solve the system \(\text{all} \lambda_k\)’s

\[
\Delta \chi^2 = 1
\]

\[
\lambda_{fit}
\]

\[
M = 1
\]
• in LEP jargon, "lineshape" means \( \sigma(e^+e^- \rightarrow Z \rightarrow ff) \) vs \( \sqrt{s} \) (*) for a given fermion pair of type \( f \);

• the lineshape shows the characteristic "bell shape", due to the resonance;

• both the height and the width of the bell depend on the e.w. parameters;

• the strategy is
  
  a) first, measure mass, full and partial widths of the \( Z \);
  
  b) then, fit:

  - number of light \( \nu \)'s (= fermion families);
  - electro-weak couplings.

\[ \sigma_{\text{Born}}(e^+e^- \rightarrow ff, \sqrt{s} = m_Z) = \frac{12\pi \Gamma_e \Gamma_f}{m_Z^2 \Gamma_Z^2}. \]

(*) warning: NOT "d\sigma/d\sqrt{s}", which is meaningless.
**LEP1 SM fit: \( m_Z, \Gamma_Z \)**

### Without lepton universality

<table>
<thead>
<tr>
<th>( \chi^2/N_{df} = 32.6/27 )</th>
<th>( m_Z )</th>
<th>( \Gamma_Z )</th>
<th>( \sigma^0_h )</th>
<th>( R^0_e )</th>
<th>( R^0_\mu )</th>
<th>( R^0_\tau )</th>
<th>( A^{0,e}_{FB} )</th>
<th>( A^{0,\mu}_{FB} )</th>
<th>( A^{0,\tau}_{FB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_Z ) [GeV]</td>
<td>91.1876( \pm ) 0.0021</td>
<td>1.00</td>
<td>-0.024 1.00</td>
<td>-0.044 -0.297 1.00</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( \Gamma_Z ) [GeV]</td>
<td>2.4952 ( \pm ) 0.0023</td>
<td>-0.041 1.00</td>
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</tr>
<tr>
<td>( \sigma^0_h ) [nb]</td>
<td>41.541 ( \pm ) 0.037</td>
<td>-0.044 -0.297 1.00</td>
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<tr>
<td>( R^0_e )</td>
<td>20.804 ( \pm ) 0.050</td>
<td>0.078 -0.11 0.105 1.00</td>
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<tr>
<td>( R^0_\mu )</td>
<td>20.785 ( \pm ) 0.33</td>
<td>0.000 0.008 0.131 0.069 1.00</td>
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<tr>
<td>( R^0_\tau )</td>
<td>20.764 ( \pm ) 0.45</td>
<td>0.002 0.006 0.092 0.046 0.069 1.00</td>
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<tr>
<td>( A^{0,e}_{FB} )</td>
<td>0.0145 ( \pm ) 0.0025</td>
<td>-0.014 0.007 0.001 -0.371 0.001 0.003 1.00</td>
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<tr>
<td>( A^{0,\mu}_{FB} )</td>
<td>0.0169 ( \pm ) 0.0013</td>
<td>0.046 0.002 0.003 0.020 0.012 0.001 -0.024 1.00</td>
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<tr>
<td>( A^{0,\tau}_{FB} )</td>
<td>0.0188 ( \pm ) 0.0017</td>
<td>0.035 0.001 0.002 0.013 -0.003 0.009 -0.020 0.046 1.00</td>
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</tbody>
</table>

### With lepton universality

<table>
<thead>
<tr>
<th>( \chi^2/N_{df} = 36.5/31 )</th>
<th>( m_Z )</th>
<th>( \Gamma_Z )</th>
<th>( \sigma^0_h )</th>
<th>( R^0_\ell )</th>
<th>( A^{0,\ell}_{FB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_Z ) [GeV]</td>
<td>91.1875( \pm ) 0.0021</td>
<td>1.00</td>
<td>-0.023 1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Gamma_Z ) [GeV]</td>
<td>2.4952 ( \pm ) 0.0023</td>
<td>-0.045 -0.297 1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma^0_h ) [nb]</td>
<td>41.540 ( \pm ) 0.037</td>
<td>-0.045 -0.297 1.00</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( R^0_e )</td>
<td>20.767 ( \pm ) 0.25</td>
<td>0.033 0.004 0.183 1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A^{0,\ell}_{FB} )</td>
<td>0.0171 ( \pm ) 0.0010</td>
<td>0.055 0.003 0.006 -0.056 1.00</td>
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</tbody>
</table>

---

**for updated values, check [pdg]**

**two fits:**

a) "without lepton universality", 9 parameters: larger errors, more general;

b) "with l. u.", 5 parameters, smaller errors, assume lepton universality.

\[
R_x \equiv \frac{\Gamma_{\text{hadr}}}{\Gamma_x} = \frac{\sigma_{\text{hadr}}}{\sigma_x};
\]

all values computed at the pole.
LEP1 SM fit: $m_z$
LEP1 SM fit: $\Delta m_z$, $\Delta \Gamma_z$
• Neutrinos are the lightest component of the fermion families [in SM no theor. explanation, just matter of fact];

• assuming this case also for (hypothetical) further families, i.e. additional $\nu$'s lightest member of a family;

• the decay $Z \rightarrow \nu\bar{\nu}$ is important ($\sim$20%), but not observable (but "single $\gamma$", not treated here);

• but it contributes to $\Gamma_z$ (observable);

• indirect detection: measure $\Gamma_z$, subtract the contribution of observable decays ("$\Gamma_{\text{visible}}$"), get "$\Gamma_{\text{invisible}}$" and compute $n_\nu$ (more precisely the number of light $\nu$, i.e. $m_\nu < m_z/2$):

$$\Gamma_{\text{inv}} \equiv \Gamma_z - \sum_{j=q,\ell} \Gamma_j = \Gamma_z - \Gamma_{\text{hadr}} - 3 \Gamma_{\ell^\pm};$$

$$n_\nu = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu}^{\text{SM}}} \left( \frac{\Gamma_{\text{exp}}^{\text{inv}}}{\Gamma_z^{\text{SM}}} \right) \left( \frac{\Gamma_z^{\text{SM}}}{\Gamma_{\nu}^{\text{SM}}} \right).$$

• [the last step to decrease stat and syst errors]

• it turns out:

$$n_\nu = 2.9840 \pm 0.0082$$

i.e. $n_\nu = 3$, no other families

[probably the best, most known, most quoted LEP result, see fig on pag. 2].

NB strictly speaking, $n_\nu = \text{width of invisible decays normalized to } \Gamma_{\nu}$, i.e. it could get contributions from other invisible decays (physics bSM, e.g. neutralino); in such cases, "$n_\nu$" not an integer.

$$\sigma_{\text{Born}}(e^+e^- \rightarrow f\bar{f}, \sqrt{s} = m_z) = \frac{12\pi \Gamma_e \Gamma_f}{m_z^2 \Gamma_z^2};$$

$$\Gamma_{\nu}^{\text{SM}} = \frac{G_F m_z^3 c_f}{12 \sqrt{2} \pi},$$

$$\Gamma_z = \sum_i \Gamma_i.$$
Example of global fit result: $g_A$ vs $g_V$ for leptons:

- 68% (i.e. 1 $\sigma$) contours;
- computed after top and before Higgs discovery;
- the "→" shows ± 1 $\sigma$ in $\alpha_{em}$, $m_t$...
- ... and 114, 300, 1000 GeV for $m_H$.
- the red dot shows the SM Born point, with the QED corr. only (i.e. $\alpha_{em}(m_Z) \approx 1/128$ → weak rad. corr. are important.

Notice:

- good compatibility among leptons (→ universality);
- preference for light Higgs (...wow)
LEP1 SM fit: $\sin^2 \theta$ vs $\Gamma_\ell$

SM-with-rad-corr:
- $m_H = 114, 300, 1000$ GeV;
- $m_t = 174.3 \pm 5.1$ GeV.

meas. (68%)

SM-no-rad-corr, but $\alpha_{em}(m_Z) \pm \Delta \alpha_{em}(m_Z)$

+ tons of similar plots, with all the e.w. parameters vs all others, compared with SM → agreement.
In 1994-2000 LEP gradually $\sqrt{s} = m_z \rightarrow 200$ GeV

- LEP1 was dominated by the Z pole;
- on the contrary, LEP2 is "democratic";
- many final states:
  - "2 photons", e.g. $e^+e^- \rightarrow e^+e^- q\bar{q}$;
  - "2 fermions", e.g. $e^+e^- \rightarrow Z^* \gamma^* \rightarrow q\bar{q}$;
  - "4 fermions", e.g. $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q} q\bar{q}$;
  - $e^+e^- \rightarrow \gamma\gamma$;
- Higgs searches (special case of 4 fermions).

- only $W^+W^-$ and Higgs in these lectures.
• the process $e^+e^- \rightarrow W^+W^- \rightarrow \bar{f}f\bar{f}f$ dominates the 4 fermions sample;

• in lowest order, there are three Feynman diagrams;

• all the vertices of the e.w. theory: $ffW$, $ffZ$, $ff\gamma$, $ZWW$, $\gamma WW$;

• the overall (finite) cross section results from delicate cancellations among the 6 terms ($3 |\text{module}|^2 + 3$ interferences) [next slide];

• therefore, any possible anomaly (discrepancy wrt SM, e.g. an anomaly in the couplings) would result in evident deviations from the predictions.
$e^+e^- \rightarrow W^+W^- @ LEP2$: cross section

In the plot $\Gamma_W = 0$
\[ e^+e^- \rightarrow W^+W^- @ LEP2: \text{cross section results} \]

**Question:** Are all SM couplings needed?

**Answer:** Yes!!
Technically clever and simple:

- compute \( \sigma(e^+e^- \rightarrow W^+W^-) = \sigma(m_W) \);
- compute the "best" \( \sqrt{s} \), by combining
  - sensitivity \( (\partial \sigma / \partial m_W = \text{max}) \rightarrow \sqrt{s} \approx \) threshold;
  - \( (\Delta \sigma^{\text{stat}} \downarrow) \rightarrow (\sigma \uparrow) \rightarrow (\sqrt{s} \uparrow) \);
  - take into account \( \Delta_{\text{theory}} \) and syst.;
- measure.
e^+e^- → W^+W^- @ LEP2: constraints

- kinematical constraints (e.g. 4-mom conservation) help in the analysis:
  - selection criterion (rejection of bad measurements or event classification in other processes);
  - improve resolution (see next);
- this case as an example: likelihood fit to m_W, Γ_W;
- compare analysis/fit on real data wrt same procedure on "pseudo-events" (physics + detector mc);
- Γ_W strongly (anti-)correlated with experimental resolution ["pessimistic" detector mc → σ_{meas} too large → Γ_W too small !!!];

- systematics from:
  - ISR/FSR parameterization;
  - reconstruction algorithms (especially jets, ex. color reconnection, Bose-Einstein correlations);
  - many other sources...
- consistency checks: in this case m_Z, Γ_Z from e^+e^- → ZZ (with smaller stat).
In the parameter space:

- $n \text{ unkn.} = 4 \times n_{\text{body}} = 16$;
- $N \text{ meas.} \text{[e.g. } E, \vec{p} \text{ for jets / } \ell^\pm \text{s]}$;
- $K \text{ equations } = 4 \text{ mom } + \text{ masses}^{(*)}$;
- $C (=N+K-n) \text{ constraints};$
- E.g. : $e^+e^- \rightarrow W^+W^- \rightarrow f_1f_2f_3f_4, n=16$:
  - 4 jets : $N=16, K=5 \rightarrow C = 5$;
  - $\ell^\pm \nu j j : N=12, K=6 \rightarrow C = 2$;
  - $\ell^+ \nu \ell^- \bar{\nu} : N=8, K=7 \rightarrow C < 0$;
- If $C > 0$, a kinematical fit is possible (a simplified sketch in $x_1, x_2$, i.e. $n=2$)

[the red arrow "$\rightarrow"$ represents a statistical estimate ($\chi^2$, likelihood) and a computation method (e.g. Lagrange multipliers)].

___________________________

(*) $m_{W^+} = m_{W^-}$ and $m_\nu \approx 0$. 

Paolo Bagnaia – PP – 10
\( e^+ e^- \rightarrow W^+ W^- \rightarrow q\bar{q}e\nu \)

\( e^+ e^- \rightarrow W^+ W^- \rightarrow q\bar{q}\mu\nu \)

\( e^+ e^- \rightarrow W^+ W^- \rightarrow q\bar{q}q\bar{q} \)

- the effects of kinematical fits:
- "C" (=constraints) from bubble chamber jargon;
- higher C, more constraints, more improvement from fit.
$e^+e^- \rightarrow W^+W^-$ @ LEP2: $m_w$, $\Gamma_w$ results

- **ALEPH [1996-2000]**: $80.404 \pm 0.074$
- **DELPHI [1996-2000]**: $80.379 \pm 0.058$
- **L3 [1996-2000]**: $80.376 \pm 0.077$
- **OPAL [1996-1999]**: $80.490 \pm 0.065$
- **LEP** (LEP working group): $80.412 \pm 0.042$

$\chi^2$/dof = 29.6 / 37

- **ALEPH [1998-2000]**: $2.13 \pm 0.14$
- **DELPHI [1997-2000]**: $2.11 \pm 0.12$
- **L3 [1996-2000]**: $2.24 \pm 0.19$
- **OPAL [1996-1998]**: $2.04 \pm 0.18$
- **LEP** (LEP working group): $2.150 \pm 0.091$

$\chi^2$/dof = 19.7 / 24

**NB**: 2003 values, WW events only [no LEP global fit]

$m_w = 80.412 \pm 0.029 \pm 0.031$ GeV; $\Gamma_w = 2.150 \pm 0.068 \pm 0.060$ GeV.
• in the SM the $W^{\pm}$ boson decays through CC interactions (V-A);

• therefore the coupling is the same for all $ff'$ pairs, providing:
  - $m(ff') < m_w$ (→ no t decays);
  - $qq\bar{q}$ mixing (à la CKM) must be used;

• ASSUMING (just for the discussion) a diagonal CKM matrix, $W^+$ decays into $e^+\nu, \mu^+\nu, \tau^+\nu, u\bar{d}, c\bar{s}$, (t$b$ forbidden);

• [if $W^-$, then corresponding antiparticles];

• ($m_f \ll m_w$ and CKM ≈ diagonal) → same BR for all channels (but color factor);

• the V-A theory gives in lowest order:
  $$\Gamma(W\rightarrow ff') = G_F m_w^3 / (6\sqrt{2}\pi) \approx 226 \text{ MeV};$$

• (3 leptons + 2 quarks × 3 colors = 9):

$$\Gamma_W = \Sigma \Gamma_i(W\rightarrow ff') \approx 9 \times 226 \text{ MeV} = 2.05 \text{ GeV};$$

$$\text{BR}(W\rightarrow \ell^\pm\nu) \approx 1/9 \approx 0.11;$$

$$\text{BR}(W^+\rightarrow u\bar{d}) \approx \text{BR}(W^+\rightarrow c\bar{s}) \approx 1/3 \approx 0.33;$$

• if the correct quark mixing is used, the CKM matrix element $V_{qq'}$ must be considered:

$$\Gamma(W\rightarrow qq') = |V_{qq'}|^2 G_F m_w^3 / (6\sqrt{2}\pi);$$

$$\Gamma_W = \Sigma \Gamma_i(W\rightarrow ff') = \text{unchanged};$$

$$\text{BR}(W\rightarrow qq') \approx |V_{qq'}|^2 / 3.$$
e^+e^- \rightarrow W^+W^- @ LEP2: W^\pm decay results

**W Leptonic Branching Ratios**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Dilepton</th>
<th>Muon</th>
<th>Tau</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>10.71 ± 0.16</td>
<td>10.63 ± 0.15</td>
<td>11.38 ± 0.21</td>
</tr>
<tr>
<td>DELPHI</td>
<td>10.65 ± 0.27</td>
<td>11.46 ± 0.43</td>
<td>11.14 ± 0.31</td>
</tr>
<tr>
<td>L3</td>
<td>10.03 ± 0.31</td>
<td>10.86 ± 0.45</td>
<td></td>
</tr>
<tr>
<td>OPAL</td>
<td>10.71 ± 0.27</td>
<td>10.86 ± 0.32</td>
<td></td>
</tr>
</tbody>
</table>

**W Hadronic Branching Ratio**

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Hadronic</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>67.13 ± 0.40</td>
</tr>
<tr>
<td>DELPHI</td>
<td>67.45 ± 0.48</td>
</tr>
<tr>
<td>L3</td>
<td>67.50 ± 0.52</td>
</tr>
<tr>
<td>OPAL</td>
<td>67.41 ± 0.44</td>
</tr>
<tr>
<td>LEP</td>
<td>67.41 ± 0.27</td>
</tr>
</tbody>
</table>

\[ \chi^2/\text{ndf} = 6.3 / 9 \]

\[ \chi^2/\text{ndf} = 15.4 / 11 \]
In the SM, $m_W$ and $\Gamma_W$ are correlated:

- are the previous measurements consistent?
  
  - yes, see the plot;

- can do better? i.e. check the SM with all the LEP measurement?
  
  - yes;

- even better? i.e. add also the other SM non-LEP measurement, i.e. $v$'s and low-energy?
  
  - yes, see next slide;

- is the fit producing a value for the (still) unknown parameters, e.g. $m_H$?
  
  - yes.
global LEP(1+2) fit

circa A.D. 2000, at the end of LEP era.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Pull $(O_{\text{meas}} - O_{\text{fit}}) / \sigma_{\text{meas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_{\text{had}}^{(5)}(m_Z)$</td>
<td>0.02761 ± 0.00036 -0.16</td>
</tr>
<tr>
<td>$m_Z$ [GeV]</td>
<td>91.1875 ± 0.0021 0.02</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4952 ± 0.0023 -0.36</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>41.540 ± 0.037 1.67</td>
</tr>
<tr>
<td>$R_i$</td>
<td>20.767 ± 0.025 1.01</td>
</tr>
<tr>
<td>$A_i^{0,b}$</td>
<td>0.01714 ± 0.00095 0.79</td>
</tr>
<tr>
<td>$A_i^{0,c}$</td>
<td>0.1465 ± 0.0032 -0.42</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21644 ± 0.00065 0.99</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.1718 ± 0.0031 -0.15</td>
</tr>
<tr>
<td>$A_i^{0,b}$</td>
<td>0.0995 ± 0.0017 -2.43</td>
</tr>
<tr>
<td>$A_i^{0,c}$</td>
<td>0.0713 ± 0.0036 -0.78</td>
</tr>
<tr>
<td>$A_i$</td>
<td>0.922 ± 0.020 -0.64</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.026 0.07</td>
</tr>
<tr>
<td>$A_i$(SLD)</td>
<td>0.1513 ± 0.0021 1.67</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{M(2)}})$</td>
<td>0.2324 ± 0.0012 0.82</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>80.426 ± 0.034 1.17</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>2.139 ± 0.069 0.67</td>
</tr>
<tr>
<td>$m_t$ [GeV]</td>
<td>174.3 ± 5.1 0.05</td>
</tr>
<tr>
<td>$\sin^2 \theta_W(\nuN)$</td>
<td>0.2277 ± 0.0016 2.94</td>
</tr>
<tr>
<td>$Q_W$(Cs)</td>
<td>-72.83 ± 0.49 0.12</td>
</tr>
</tbody>
</table>

experiment - theory pull $\equiv \frac{O_{\text{meas}} - O_{\text{fit}}}{\sigma_{\text{meas}}}$;

expected gaussian, $\mu=0$, $\sigma=1$;

$\chi^2 = \sum_i (\text{pull}_i)^2$;

$\chi^2 / \text{dof} = 25.5 / 15 \rightarrow P(\chi^2)=4.4\%$.

This nice agreement was mainly used to:
- claim the quality of the SM (and exp.'s);
- predict the (unknown) mass of the Higgs.
global LEP(1+2) fit : $m_H$ prediction

$\Delta\chi^2 = \chi^2(m_H) - \chi^2_{\text{min}}$

$\Delta\alpha^{(5)}_{\text{had}} =
- 0.02761 \pm 0.00036$
- 0.02738 \pm 0.00020

excl. by direct search [see];

$\Delta\alpha^{(5)}_{\text{had}}$ : contribution of light quarks to photon vacuum polarization (two computations).

$\chi^2_{\text{min}} / \text{dof} = 22.9 / 15$

$m_H = 88^{+53}_{-35}$ GeV

$m_H < 196$ GeV @ 95% CL

Just an example, often remade with small variations before LHC. The 2nd most quoted LEP plot (after $n_\nu$) will disappear soon.

"$\mathcal{H}$" (="history") in the right corner means "now obsolete" (here and in the following).
1. – 15. [...] 

16. **Higgs search at LEP1**

17. **Higgs search at LEP2**

- The Higgs boson has been *(very likely)* discovered at LHC, definitely not at LEP.

- Why remember an old and not-so-nice story, like the LEP search of the Higgs?

- Because it is very instructive – almost all searches are unsuccessful → in practice limits and exclusions are much more frequent than discoveries;

- *[also, in the past fluctuations/mistakes have been rare, but not null]*

- go → § 11, then come back;

- Higgs properties are treated in § LHC [+RQM + EWI];

- here only an incomplete discussion for Higgs production in $e^+e^-$ at LEP1 & LEP2 energies.
In the SM the Higgs boson is at the origin of fermion masses;

- at least one H, neutral, spin-0;
- only 1 H → "minimal SM" (MSM, the case discussed in these lectures);
- $m_H$ free parameter of SM (but $m_H < 1$ TeV);
- in the MSM, if $m_H$ is given, the dynamics is completely determined and calculable (couplings, cross sections, BR's, angular distributions, ...);

properties:
- charge : 0; spin : 0; $J^P = 0^+$;
- coupling with fermions $f$:
\[
\Gamma(H \rightarrow ff) = \frac{c_f}{4\pi\sqrt{2}} G_F m_H m_f^2 \beta_f^3 ;
\]
\[
\beta_f = \sqrt{1 - 4m_f^2 / m_H^2} ; \quad c_f = \begin{cases} 1 \text{ [leptons]} \\ 3 \text{ [quarks]} \end{cases}
\]
- [notice: $\Gamma_f \propto m_f^2$);
- therefore, H decays mainly in the fermion pair of highest mass kinematically allowed;
- therefore, if $m_H > 2m_b$ (i.e. > 10 GeV), mainly $H \rightarrow b\bar{b}$.

- $Z \rightarrow HH$ (spin-statistics, like $\rho^0 \rightarrow \pi^0\pi^0$);

- in lowest order only:
  - $Z \rightarrow H \gamma$ (Z, H neutral !!!) [or $H \rightarrow Z\gamma$];
  - $H \rightarrow \gamma\gamma$ (H neutral !!!)
- however, $(H \rightarrow \gamma\gamma)$ essential for the discovery (see § LHC).
- $H \rightarrow gg$ (no strong interactions);
- but $H \rightarrow Z\gamma, \gamma\gamma, gg$ through higher order processes.

more complete discussion in § LHC, e.g. discussion of $H \rightarrow Z, W$ decays.
• LEP 1 ($\sqrt{s} \approx m_Z$): $e^+ e^- \rightarrow Z \rightarrow HZ^* \rightarrow (f \bar{f})(f \bar{f})$; i.e. the Higgs production is one of the possible Z decays:

$$\Gamma(Z \rightarrow Hf\bar{f}) \frac{d\Gamma(Z \rightarrow f\bar{f})}{dx} = \frac{G_F m_Z^2 (12 - 12x + x^2 + 8y^2)\sqrt{x^2 - 4y^2}}{24\sqrt{2\pi^2} (x - y^2)^2};$$

$$x = \frac{2E_H}{m_Z} = \frac{m_Z^2 + m_H^2 - m_{ff}^2}{m_Z^2}; \quad y = \frac{m_H}{m_Z}.$$

• Kinematical constraint:

$$\sqrt{s} \approx m_Z > m_{Z^*} + m_H \rightarrow m_H < m_Z$$

• Best observable when:

$Z^* \rightarrow \ell^+ \ell^-$ (no bckgd),

$H \rightarrow b \bar{b}$ (BR $\geq 80\%$);

• $\text{BR}(Z \rightarrow H\ell^+ \ell^-) \approx 10^{-4}$ @ $m_H = 8$ GeV

$\approx 10^{-7}$ @ $m_H = 70$ GeV;

• Kinematics not difficult, e.g. $Z^* \rightarrow \mu^+ \mu^-$,

$m(Z^*) = m_{\mu\mu}, \quad E(Z^*) = E_{\mu\mu},$

$m_H^2 = s + m_{\mu\mu}^2 - 2\sqrt{s}E_{\mu\mu}$.
The main decay product of H is the $f\bar{f}$ of largest mass compatible with $m_H$: e.g. $s$ means $H \rightarrow s\bar{s}$.

When a new threshold opens up, there is a "step" in $c\tau$ ($\sim 1/\Gamma$), rounded by phase space.
For $\sqrt{s} \approx m_Z$ (real $Z$) and $m_H << m_Z$, the Bjorken process ($e^+e^- \rightarrow Z \rightarrow HZ^*$) has a sizeable cross section, but at larger $m_H$ it essentially disappears → go to larger $\sqrt{s}$.

The predictions at $\sqrt{s} >> m_Z$ come from a similar process ($e^+e^- \rightarrow Z^* \rightarrow HZ$, virtual $Z^*$), known as "higgs-strahlung" [next slides].
Higgs search @ LEP1: results

- This plot summarizes the limits of the four experiments:
  - A: 63.1 GeV
  - D: 55.4 GeV
  - L: 60.2 GeV
  - O: 59.1 GeV

- The candidate at $m_H = 67$ GeV (OPAL) reduces the limit by few $\times$ 100 MeV;

- A test case for the method, discussed in § limits; notice:
  - The combined limit is "better" than any single exp.;
  - The "worst" observed limit does not come necessarily from the "worst" exp.;
  - ... because it is a random variable;

- Conclusion: move to higher $\sqrt{s}$, i.e. Bjorken process $\rightarrow$ higgs-strahlung.

J.F. Grivaz, Bruxelles '95

LEP 1, $\sqrt{s} \approx m_z$:

~3.7 M [Z$\rightarrow$ hadrons] / exp in 1989-94;

$m_H > 65.2$ GeV @ 95% CL
Higgs search @ LEP2

• LEP 2: process of "higgs-strahlung" (= radiative emission of a Higgs boson from a $Z^*$);

• i.e. the higgs production is a 4-fermion final state, mediated by a virtual $Z^*$ [like $e^+e^- \rightarrow W^+W^- \rightarrow 4f$];

• kinematical constraint:
  \[ \sqrt{s} = m_{Z^*} > m_Z + m_H \]

• [no $\mathcal{K}$ here, because of possible future colliders, see later].

\[
\sigma_0(e^+e^- \rightarrow Z^* \rightarrow HZ) = \frac{G_F^2m_Z^4}{24\pi s} \left[ (g_V^f)^2 + (g_A^f)^2 \right] \sqrt{\lambda + \frac{12m_Z^2/s}{(1-m_Z^2/s)^2}}; \\
\lambda = (1 - m_H^2/s - m_Z^2/s)^2 - 4m_H^2m_Z^2/s^2; \\
\frac{1}{\sigma_0} \frac{d\sigma_0}{d\cos\theta} = \frac{\lambda^2 \sin^2\theta + 8m_Z^2/s}{4\lambda^2/3 + 16m_Z^2/s} \text{ for } s \text{ large, } \frac{3}{4} \sin^2\theta.
\]
• $-2\ln Q = -2\ln(\Lambda_s/\Lambda_b)$;

• $-2\ln Q(m_H=115) = -7$;

• if interpreted as a discovery
  - $m_H = 115^{+1.3}_{-0.9}$ GeV;
  - $1-CL_b = 4.2\times10^{-3}$;
  - i.e. "2.9 $\sigma$";

• if interpreted as a limit :
  - $m_H > 113.5$ GeV @ 95%CL.
Given the consistency for the combined results with the hypothesis of the production of a SM Higgs boson with a mass of 115 GeV, and an observed excess in the combined data set of $2.9\sigma$, a further run with $200 \text{ pb}^{-1}$ per experiment at 208 GeV would enable the four experiments to establish a $5\sigma$ discovery.

The four experiments consider the search for the SM Higgs boson to be of the highest importance, and CERN should not miss such a unique opportunity for a discovery.

Therefore, we request to run LEP in 2001 to collect $\mathcal{O}(200 \text{ pb}^{-1})$ at $\sqrt{s} = 208$ GeV.

**ALEPH, DELPHI, L3, OPAL**

The LEP Higgs Working Group

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**8/11/2000**

These are the measurements taken of LEP’s final beam. The accelerator was switched off for the last time at 8:00 am on 2 November. (Click on photo for enlargement)

After extended consultation with the appropriate scientific committees, CERN’s Director-General Luciano Maiani announced today that the LEP accelerator had been switched off for the last time. LEP was scheduled to close at the end of September 2000 but tantalising signs of possible new physics led to LEP’s run being extended until 2 November. At the end of this extra period, the four LEP experiments had produced a number of collisions compatible with the production of Higgs particles with a mass of around 115 GeV. These events were also compatible with other known processes. The new data was not sufficiently conclusive to justify running LEP in 2001, which would have inevitable impact on LHC construction and CERN’s scientific programme. The CERN Management decided that the best
Higgs search @ LEP2: July 2001

-2\ln(Q)

-20 -15 -10 -5 0 5 10 15 20 25

-2 \ln Q|_{\text{actual events}} = -2.9;

1-CL_b = 3.5 \times 10^{-2};

"2.1 \sigma";

if interpreted as a discovery:

- \text{m}_H = 115.6 \text{ GeV;}

- \text{m}_H > 114.1 \text{ GeV @ 95\%CL.}

if interpreted as a limit:

- \text{m}_H > 114.1 \text{ GeV @ 95\%CL.}
Higgs search @ LEP2: the end

- method "gedanken-experiment" [i.e. produce via mc many experiments, with the same quality and $L_{\text{int}}$ of the present one]:
  - $m_H^{\text{test}} = 115.6$ GeV;
  - $\int f_{b,s} \, d(-2\ell nQ) = 1$;
  - $\star \star = 1-\text{CL}_b = 3.5\%$;
  - $\star \star \star = \text{CL}_{s+b} = 43\%$.

Comments/questions (imho):
- (just for history, now irrelevant) why was the first analysis wrong? well, ... ?
- why to show it to students? because it is very instructive, normal classes see only the standard (discovery vs limits).
• the "LEPC result" is difficult to explain (NOT only to students) : stat. fluctuations, mistakes, systematics out-of-control, ...

• the CERN management (L. Maiani) took the right decision at a high risk;

• the real threat was a delay of LHC, a huge human and economic price;

• instead, the final results are relatively simple to explain: a honest fluctuation at 3.5% does not deserve a discussion;

• the Higgs boson search crossed the ocean, but the TeVatron did not really enter in the game;

• and finally LHC ... [you know].

Other more personal comments:
• unlike theoretical physics, statistics (and human behavior) require risk evaluation;
• experimental physics lies in the middle;
• you should understand and judge the decisions of the experiments and the management (often they did NOT agree);
• ... while the landscape was changing (November '00, July '01, post-LEP, now);
• you might conclude that the "right decision" is a function of role and time (???);
• ... and that searches are risky, not for gutless people.
**AFTER the LHC discovery:**

Q: could LEP see a 126 GeV Higgs?

plot the cross section:

- $\sigma = 0.2 \div 1.8 \text{ pb}$;
- strongly $m_H$ dependent;
- $\mathcal{L}_{\text{int}} \approx 200 \text{ pb}^{-1}/\text{year}$;
- i.e. $n = 40 \div 200$ events/yr, shared among many decay channels (some undetectable).

A: the plot is very clear: you should be able to judge yourself!

**warning:**
- tree level,
- $\Gamma_H = \Gamma_Z = 0$; but ok for discussion.
Plot $\sigma(e^+e^- \rightarrow Z^* \rightarrow HZ)$ vs the "kinetic" energy, i.e. $(T = \sqrt{s} - m_H - m_Z)$, in the approx. $\Gamma_Z = \Gamma_H = 0$:

- $T \leq 0 \rightarrow \sigma = 0$ (obvious);
- the $\times$'s show $\sqrt{s} = 209$ GeV;
- $\sigma_{\text{max}}(T)$ at $T \approx 15\div20$ GeV, slightly increasing with $m_H$;
- $\sigma_{\text{max}}(m_H)$ decreases a lot when $m_H$ increases;
- for $\sqrt{s} \gg m_H + m_Z$, $\sigma \propto s^{-1}$ (obvious);
- for $m_H > 110$ GeV, other processes (not shown), other than higgsstrahlung;
- if $m_H = 126$ GeV (LHC), $H$ not produced at LEP 2.
In the post-LEP (and post-H-discovery) era, the interest has shifted to a possible higher energy $e^+e^-$ collider (circular or linear).

In this case:

- consider also other processes (e.g. the so-called "WW-fusion" $e^+e^- \rightarrow H\bar{\nu}_e\nu_e$ [see];
- compute the cross-section for $m_H=126$ GeV, as a function of $\sqrt{s}$ [see];
- study the physics of (say) $\sim 1$ million $H$:
  - measure $\Gamma_H$ à la $J/\psi$;
  - measure all $H$ couplings;
- [obviously no $\mathcal{K}$ here].
1. LEP predictions from SM : Yellow report CERN 89-08.
End of chapter 10