Quantities to measure in EPP

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- ***Physics quantities*** (to be compared with theory expectations)
  - Cross-section
  - Branching ratio
  - Asymmetries
  - Particle Masses, Widths and Lifetimes
- ***Quantities related to the experiment*** (BUT to be measured to get physics quantities)
  - Efficiencies
  - Luminosity
  - Backgrounds

Cross-section - I

- Suppose we have done an experiment and obtained the following quantities for a given final state:
  - N_{cand}, N_b, ε, ϕ
- What is ϕ ? It is the "**flux**", something telling us how many collisions could take place per unit of time and surface.
  - Consider a "**fixed-target**" experiment (transverse size of the target >> beam dimensions):

$$\phi = \dot{N}_{proj} N_{tar} \delta x = \frac{\dot{N}_{proj} \rho \delta x}{A m_N} = \frac{\dot{N}_{proj} \rho (g/cm^3) N_A \delta x(cm)}{A}$$

  - Consider a "**colliding beam**" experiment

$$\phi = f_{coll} \frac{N_1 N_2}{4\pi \Sigma_X \Sigma_Y} = L$$

(head-on beams: N_1 and N_2 number of particles per beam, Σ_X, Σ_Y beam transverse gaussian areas, f_{coll} collision frequency) In this case we normally use the word "**Luminosity**". Flux or luminosity are measured in: $\mathbf{cm^{-2}s^{-1}}$

Cross-section - II

- In any case, the rate of events due to final state X is:

$$\dot{N}_X = \phi \sigma_X$$

- **σ_X is the cross-section**, having the dimension of a surface.
  - it doesn't depend on the experiment but on the process only
  - can be compared to the theory
  - for a given $\boldsymbol{\sigma_X}$, the higher is $\boldsymbol{\phi}$, the larger the event rate
  - given an initial state, for every final state X you have a specific cross-section
  - the "**total cross-section**" is obtained by adding the cross-sections for all possible final states: *the cross-section is an* ***additive*** *quantity*.
  - The unit is the "**barn**". 1 barn = 10^{-24} cm^2.

Cross-section - III

- Suppose we have taken data for a time $\boldsymbol{\Delta t}$: the total number of events collected will be:

$$N_X = \sigma_X \times \int_{\Delta t} \phi \, dt$$

The flux integral over time is the ***Integrated Flux*** or (in case of colliding beams) ***Integrated Luminosity***. Integrated luminosity is measured in: $\mathbf{b^{-1}}$

- How can we measure this cross-section ?

$$\sigma_X = \frac{N_X}{\int \phi dt} = \frac{1}{\int \phi dt} \frac{N_{cand} - N_b}{\varepsilon}$$

- Sources of uncertainty: we apply the uncertainty propagation formula. We assume no correlations btw the quantities in the formula (L_{int} = integral of flux)

$$\left(\frac{\sigma(\sigma_X)}{\sigma_X}\right)^2 = \left(\frac{\sigma(L_{\text{int}})}{L_{\text{int}}}\right)^2 + \left(\frac{\sigma(\varepsilon)}{\varepsilon}\right)^2 + \frac{\sigma^2(N_{cand}) + \sigma^2(N_b)}{(N_{cand} - N_b)^2}$$

Branching ratio measurement

- Given an unstable particle ***a***, it can decay in several (say N) final states, k=1,…,N. If $\mathbf{\Gamma}$ is the ***total width*** of the particle (Γ=1/τ with τ particle lifetime), for each final state we define a "***partial width***" in such a way that

$$\Gamma = \sum_{k=1}^{N} \Gamma_k$$

- The ***branching ratio*** of the particle a to the final state X is

$$B.R.(a \to X) = \frac{\Gamma_X}{\Gamma}$$

- To measure the B.R. the same analysis as for a cross-section is needed. In this case we need the number of decaying particles $\boldsymbol{N_a}$ (not the flux) to normalize:

$$B.R.(a \to X) = \frac{N_{cand} - N_b}{\varepsilon} \frac{1}{N_a}$$

Differential cross-section - I

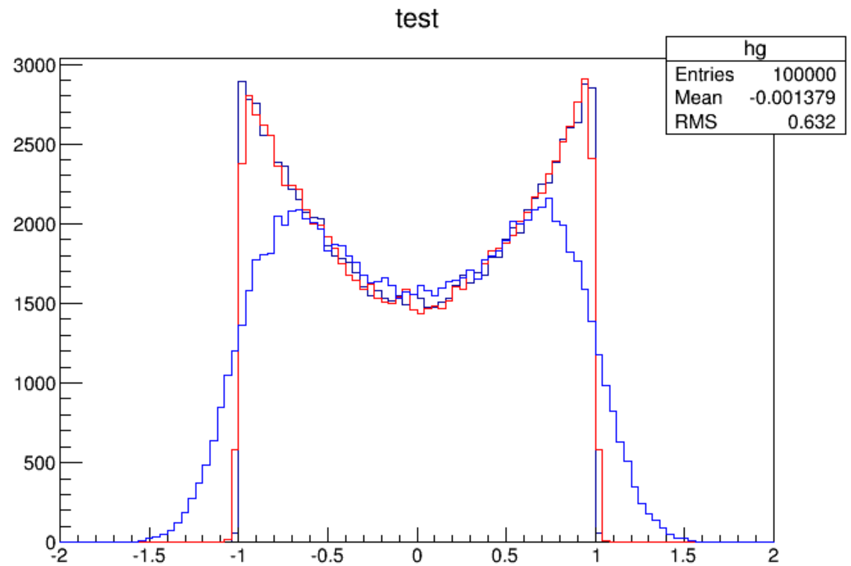
- If we want to consider only final states with a given kinematic configuration (momenta, angles, energies,…) and give the cross-section as a function of these variables
- Experimentally we have to divide in bins and count the number of events per bin.
- Example: diferential cross-section vs. scattering angle

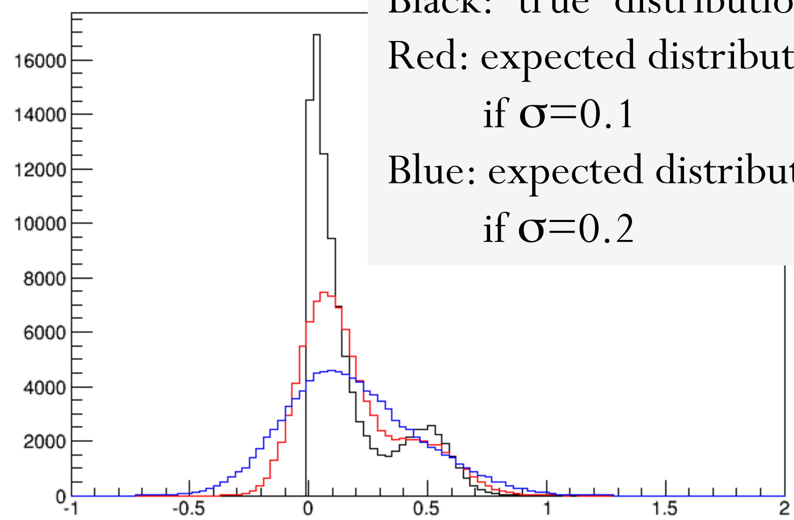
$$\left(\frac{d\sigma}{d\cos\theta}\right)_i = \frac{1}{\int \phi dt}\left(\frac{N_{cand}^i - N_b^i}{\varepsilon_i}\right)\frac{1}{\Delta\cos\theta_i}$$

- NB: N_{cand}, N_b and ε as a function of θ are needed.

Differential cross-section - II

- Additional problems appear.
  - Efficiency is required per bin (can be different for different kinematic configurations).
  - Background is required per bin (as above).
  - The migration of events from one bin to another is possible:





Black: "true" distribution
Red: expected distribution if $\sigma=0.1$
Blue: expected distribution if $\sigma=0.2$

Folding - Unfolding

- In case there is a substancial migration of events among bins (resolution larger than bin size), this affects the comparison btw exp.histo ($\boldsymbol{n_i^{exp}}$) and theory ($\boldsymbol{n_i^{th}}$). This can be solved in two different ways:
  - **Folding** of the theoretical distribution: the theoretical function $f^{th}(x)$ is "smeared" through a smearing matrix $\boldsymbol{M}$ based on our knowledge of the resolution; $\boldsymbol{n_i^{th}} \rightarrow \boldsymbol{n'^{th}_i}$

$$n_i'^{th} = \sum_{j=1}^{N} n_j^{th} M_{i,j}$$

$$n_i^{th} = \int_{x_i}^{x_{i+1}} dx f^{th}(x)$$

  - **Unfolding** of the experimental histogram: $\boldsymbol{n_i^{exp}} \rightarrow \boldsymbol{n'^{exp}_i}$. Very difficult procedure, mostly unstable, inversion of $\boldsymbol{M}$ required

$$n_i'^{\exp} = \sum_{j=1}^{N} n_j^{\exp} M_{i,j}^{-1}$$

Asymmetry measurement

- A very useful and powerful observable:

$$A = \frac{N^+ - N^-}{N^+ + N^-}$$

- It can be "charge asymmetry", Forward-Backward asymmetry",…
  - Independent from the absolute normalization
  - (+) and (-) could have different efficiencies, but most of them could cancel:

$$A = \frac{N^+/\varepsilon^+ - N^-/\varepsilon^-}{N^+/\varepsilon^+ + N^-/\varepsilon^-}$$

- Statistical error (**N**=**N**$^+$+**N**$^-$) *(proof on blackboard)*:

$$\sigma(A) = \frac{1}{\sqrt{N}}\sqrt{1 - A^2}$$

Particle properties

- Once a particle has been identified (either directly or through its decay products), it is interesting to measure its properties:
  - Mass $\boldsymbol{M}$
  - Total Decay Width $\boldsymbol{\Gamma}$
  - LifeTime $\boldsymbol{\tau}$
  - Couplings $\boldsymbol{g}$
- If the particle is identified through its decay, all these parameters can be obtained through a dedicated analysis of the kinematics of its decay products.

Invariant Mass - I

- Suppose that a particle *X* decays to a number of particles (***N***), and assume we can measure the quadri-momenta of all them. We can evaluate the Invariant Mass of X for all the candidate events of our final sample:

$$M_{inv}^2 = \left(\sum_{k=1}^{N} \tilde{p}_k \right)^2$$

- It is a relativistically invariant quantity. In case of N = 2

$$M_{inv}^2 = m_1^2 + m_2^2 + 2\left(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2\right)$$

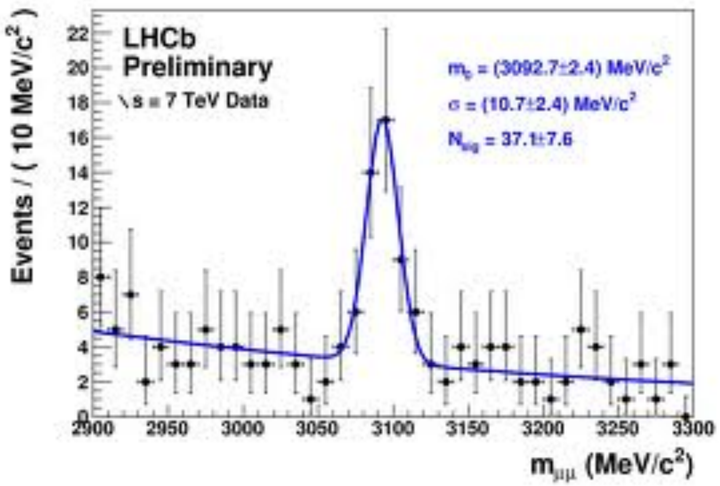
- If N=2 and the masses are 0 or very small compared to p

$$M_{inv}^2 = 2E_1E_2\left(1 - \cos\theta\right) = E_1E_2 \sin^2 \theta/2$$

- Where $\boldsymbol{\theta}$ is the opening angle between the two daughter particles.

Invariant Mass - II

- Given the sample of candidates, we do the invariant mass distribution and we typically get a plot like that:
  - A peak (the signature of the particle)
  - A background (almost flat in this case) ➔ **unreducible** background.



- What information can we get from this plot (by fitting it) ?

(1) Mass of particle;

(2) Width of the particle (BUT not in this case…);

(3) Number of particles produced (related to $\boldsymbol{\sigma}$ or ***BR***)

Parenthesys: 2 kinds of background

- ***Unreducible background***: same final state as the signal, no way to disentangle. The only way to separate signal from unreducible background is to fit the inv.mass spectrum
- ***Reducible background***: a different final state that mimic the signal (e.g. because you are losing one or more particles, or because you are confusing the nature of one or more particles)
- Example:
  - Signal: pp→H→ZZ*→4l
  - Unreducible background: pp→ ZZ*→4l
  - Reducible backgrounds: pp→Zbb with Z→2l and two leptons, one from each b-quark jet; pp→ tt with each t→Wb→lν"l"j

Invariant Mass - III

- Which is the expected invariant mass distribution for an "unstable" particle ? How is the "peak" done ?
- We consider the wave function of a decaying particle and its Fourier transform:

$$\psi(t) = \psi(0)e^{-iWt} = \psi(0)e^{-iMt-\Gamma t/2}$$

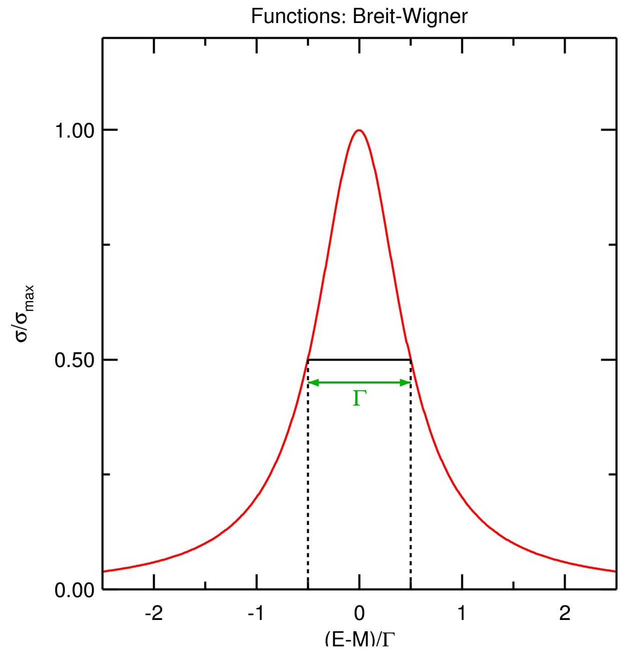
$$\chi(E) = \int \psi(t)e^{-iEt}dt = \psi(0)\int e^{-t(\Gamma/2+i(M-E))}\,dt \propto \frac{1}{(E-M)-i\Gamma/2}$$

- $\Gamma=1/\tau$: the higher is τ the smaller is Γ

$$\sigma(E) = |\chi(E)|^2 = \sigma_{\max}\frac{\Gamma^2/4}{(E-M)^2+\Gamma^2/4}$$

- Relativistic formula (Breit-Wigner):

$$\sigma(E) = \sigma_{\max}\frac{M^2\Gamma^2}{(E^2-M^2)^2+M^2\Gamma^2}$$



NB: the Γ is NOT the σ of an equivalente gaussian

Mass and Width measurement

- Fit of the M_{inv} spectrum with a Breit-Wigner + a continuos background: BUT careful with mass resolution. It can be neglected only if $\sigma(M_{inv}) << \Gamma$
- If $\sigma(M_{inv}) \approx \Gamma$ or $\sigma(M_{inv}) > \Gamma$ there are two approaches (as we already know):
  - Folding: correct the theoretical distribution to be used in the fit:

$$\sigma_{fit}(E) = \int G_{res}(E - E_0)\sigma_{BW}(E_0)dE_0$$

  - Unfolding: correct the experimental data and fit with the theoretical function.
  - Use a gaussian (or a "Crystal Ball" function) neglecting completely the width.
- In many cases only the mass is accessible: the uncertainty on the mass is the one given by the fit (taking into account the statistics) + possible scale systematics.

Gaussian vs. Crystal Ball

- Gaussian: 3-parameters, A, μ, σ. Integral $= A\sigma\sqrt{2\pi}$

$$f(m / A,\mu,\sigma) = A\exp(-\frac{(m-\mu)^2}{2\sigma^2})$$

- Crystal-Ball: 5-parameters, m, σ, α, n, N

$$f_{CB}(m,\bar{m},\sigma,\alpha,n) = N \cdot \begin{cases} e^{\frac{-(m-\bar{m})^2}{2\sigma^2}} & \text{per } \frac{m-\bar{m}}{\sigma} > -\alpha \\ A \cdot (B - \frac{m-\bar{m}}{\sigma})^{-n} & \text{per } \frac{m-\bar{m}}{\sigma} \leq -\alpha \end{cases}$$

$$A = (\frac{n}{|\alpha|})^n e^{-\frac{\alpha^2}{2}},\ B = \frac{n}{|\alpha|} - |\alpha|$$

Essentially takes into account energy losses, useful in many cases.

Template fits: not functions but histograms

In this case the fit is not done with a function with parameters BUT it is a "template" fit:

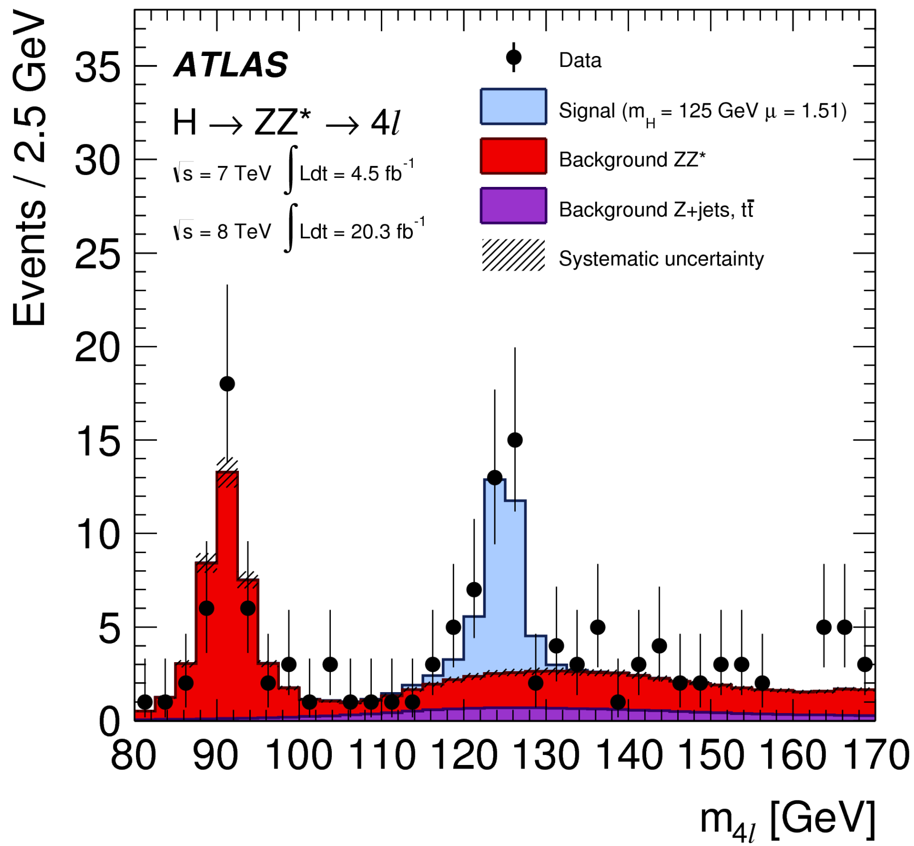
$F = a\text{HIST1}(m_H, \ldots) + b\text{HIST2}$

a, b and m_H are free parameters

The method requires the knowledge (from MC) of the expected distributions. Such a knowledge improves our uncertainties.

NB: HIST1 and HIST2 take into accc experimental resolution: so it is directly the folding method

An example: Higgs mass in the 4l channel.



Effect of the mass resolution on the significativity of a signal

- Let's consider now the case in which we look for a process and we expect a peak in a distribution at a definite mass: when may we say that we have observed that process ?
- Method of assessment: simple fit S+B (e.g. template fit).
  $S \pm \sigma(S)$ away from 0 at least 3 (5) standard deviations.
- Ingredients:
  - Mass resolution;
  - Background

$$\sigma^2(S) = \sigma^2(N) + \sigma^2(B) = N + \sigma^2(B)$$

$$\approx N = S + B = S + 6\sigma_M b$$

- Effect of mass resolution negligible if:

$$\sigma_M << \frac{S}{6b}$$

H→γγ ATLAS: is the resolution negligible ?

Numbers directly from the plot:

$S \approx 1000$

$b \approx 5000/2$ GeV

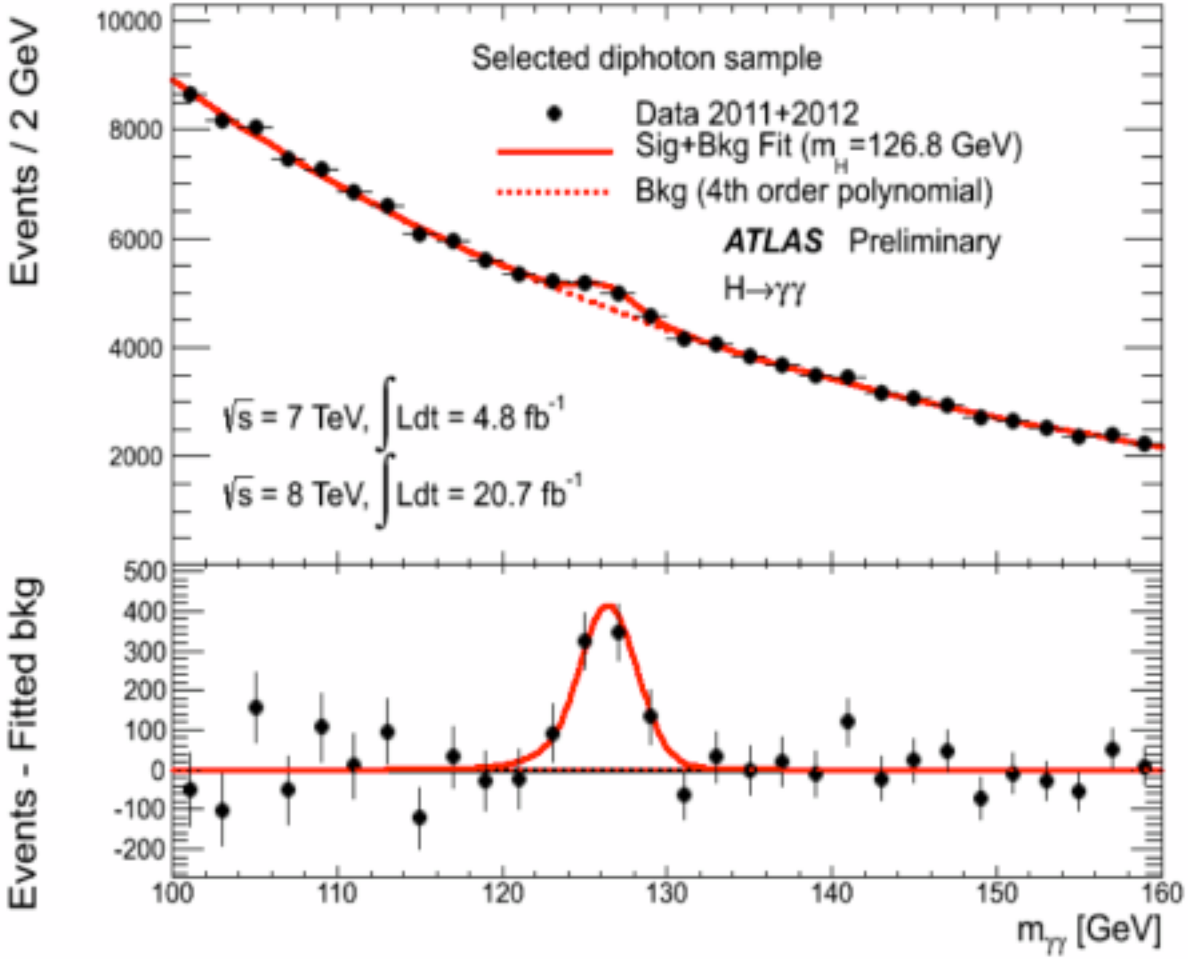
$= 2500/\text{GeV}$

$\sigma_M \approx 10$ GeV/6

$= 1.7$ GeV

➔ $S/6b$

$= 0.07$ GeV $<< \sigma_M$

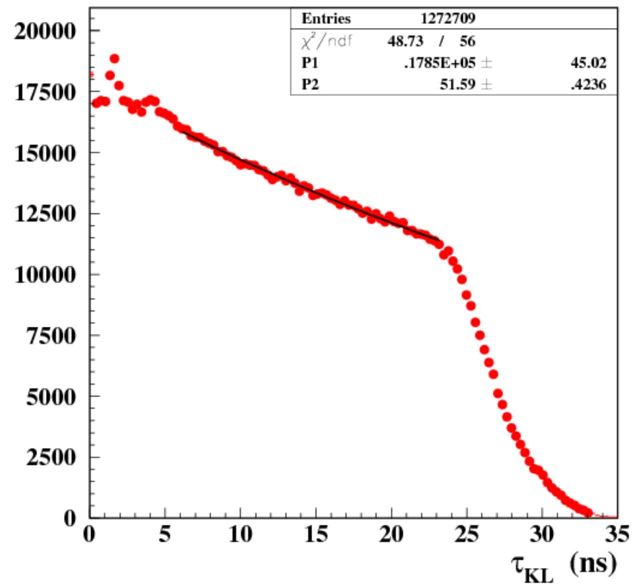


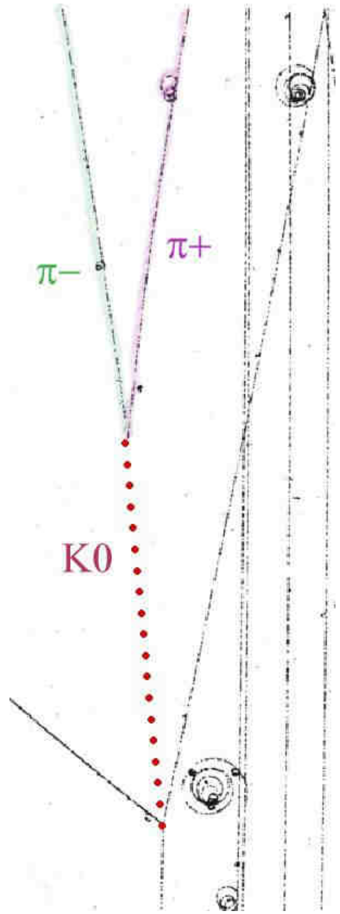
Lifetime measurement - I

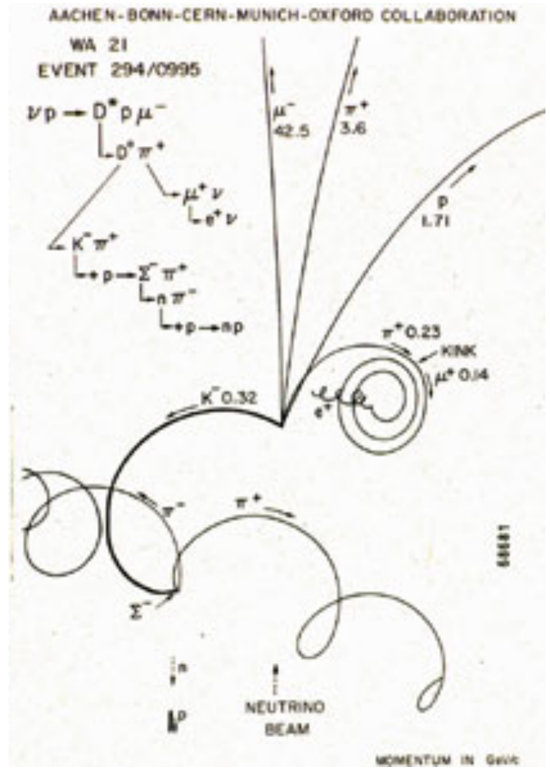
→ In the first decades of EPP, bubble-chambers and emulsions allowed to see directly the decay length of a particle either neutral or charged (see Kaons);

→ The decay length l is related to the lifetime through the $l = \beta\gamma\tau c$ ➔ $\tau = l / \beta\gamma c$

→ For a sample of particles produced we expect an exponential distribution







Lifetime measurement - II

- Example: pions, kaons, c and b-hadrons in the LHC context (momentum range 10 ÷ 100 GeV).

	π	K	D	B
Mass (GeV)	0.140	0.494	1.869	5.279
Life Time (s)	2.6×10^{-8}	1.2×10^{-8}	1.0×10^{-12}	1.6×10^{-12}
Decay length (m) p = 10 GeV	557	72.8	1.6×10^{-3}	9.1×10^{-4}
Decay length (m) p = 100 GeV	5570	728	0.016	0.0091

NB When going to c or b quarks, decay lengths O(<mm) are obtained
➔ Necessity of dedicated "vertex detectors"

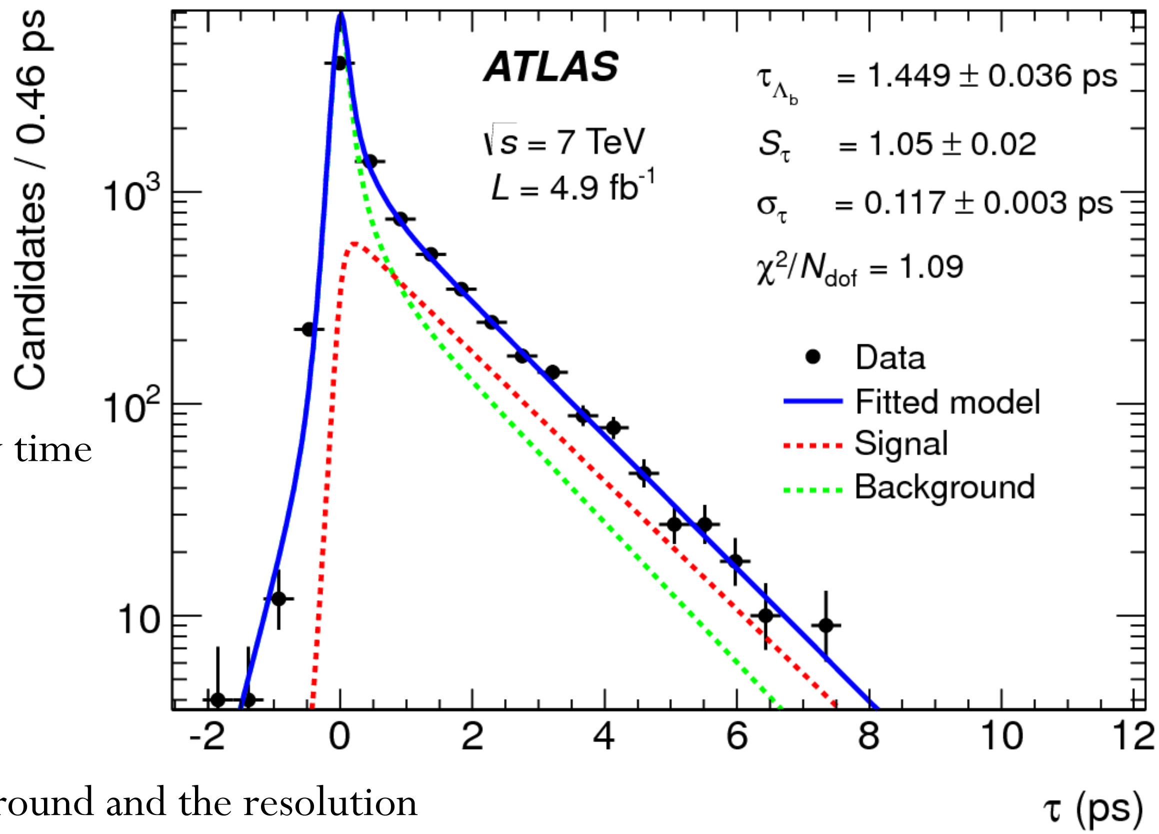
Lifetime measurement - III

For low-τ particles
(e.g. B-hadrons, τ, …):
→ define the proper decay time:

$$\tau = \frac{Lm}{p}$$

At hadron colliders the proper decay time is defined on the transverse plane:

$$\tau = \frac{L_{xy} m}{p_T}$$



The fit takes into account the background and the resolution

Typical resolutions: O(10^{-13} s) ➔ tens of μm

Efficiency measurement - I

- Suppose you want to measure the detection efficiency of a final state X: X contains N particles e.g. $Z \rightarrow \mu\mu$ contains 2 particles and whatever else. How much is the probability to select an event containing a $Z \rightarrow \mu\mu$?
- Let's suppose that:
  - Trigger is: at least 1 muon with $p_T > 10$ GeV and $|\eta| < 2.5$
  - Offline selection is: 2 and only 2 muons with opposite charge and $M_Z - 2\Gamma < M_{inv} < M_Z + 2\Gamma$
- Approach for efficiency
  - Full event method: apply trigger and selection to simulated events and calculate N_{sel}/N_{gen} (validation is required)
  - Single particle method: (see next slides)

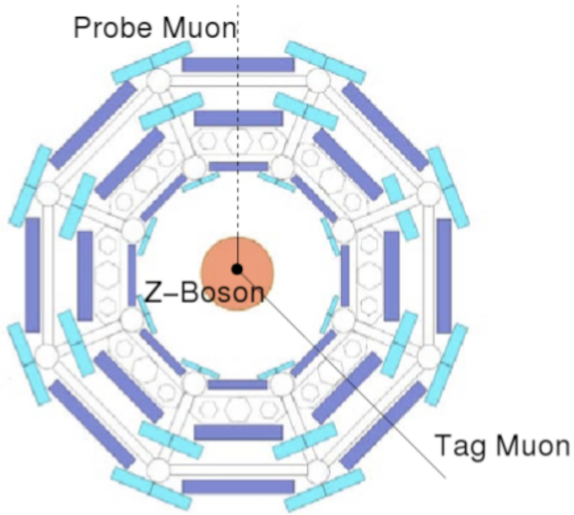
Efficiency measurement - II

- Measure single muon efficiencies as a function of kinematics (p_T, η, …); eventually perform the same "measurement" using simulated data.
  - Tag & Probe method: muon detection efficiency measured using an independent detector and using "correlated" events.
  - Trigger efficiency using "pre-scaled" samples collected with a trigger having a lower threshold.

$$\varepsilon_{trigger} = \frac{\#\,\mu - triggered}{\#\,\mu - total}$$

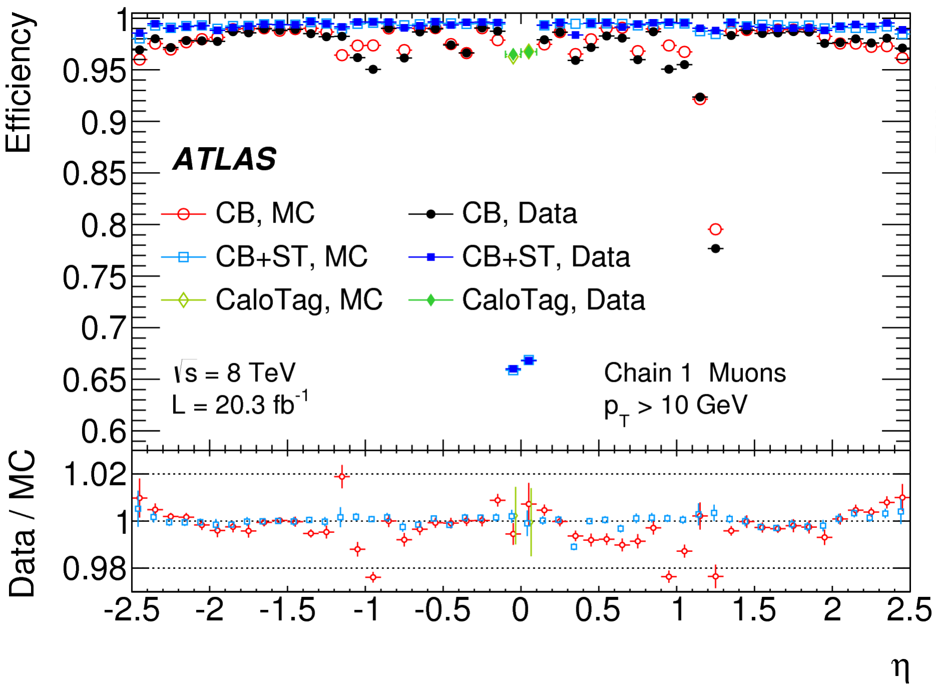
T&P: a "Tag Muon" in the MS and a "Probe" in the ID
Tag+Probe Inv.Mass consistent With a Z boson
➔ There should be a track in the MS

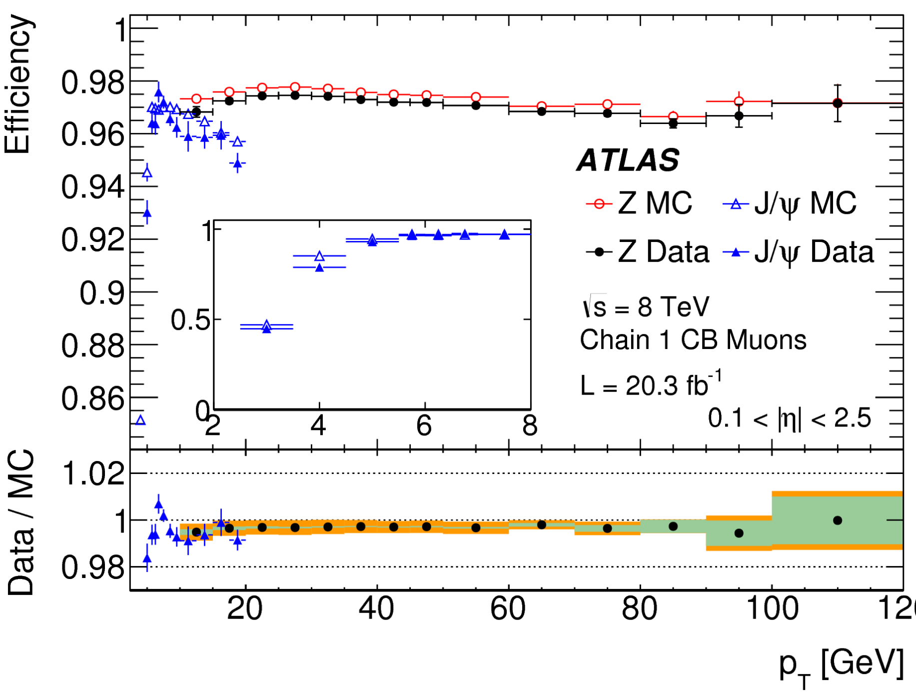
$$\varepsilon_{TP} = \frac{\#\,\mu - reco}{\#\,\mu - \text{expected}}$$



Efficiency measurement - III

- Muon Efficiency – ATLAS experiment.
- As a function of η and p_T – comparison with simulation ➔ Scale Factors





Efficiency measurement - IV

- After that I have: $\varepsilon_T(p_T, \eta, \ldots)$ and $\varepsilon_S(p_T, \eta, \ldots)$
- From MC I get the expected kinematic distributions of the final state muons and I apply for each muon its efficiency depending on its p_T and η. The number of surviving events gives the efficiency for X
- Or I simply apply the scale factors to the MC fully simulated events to take into account data-MC differences.

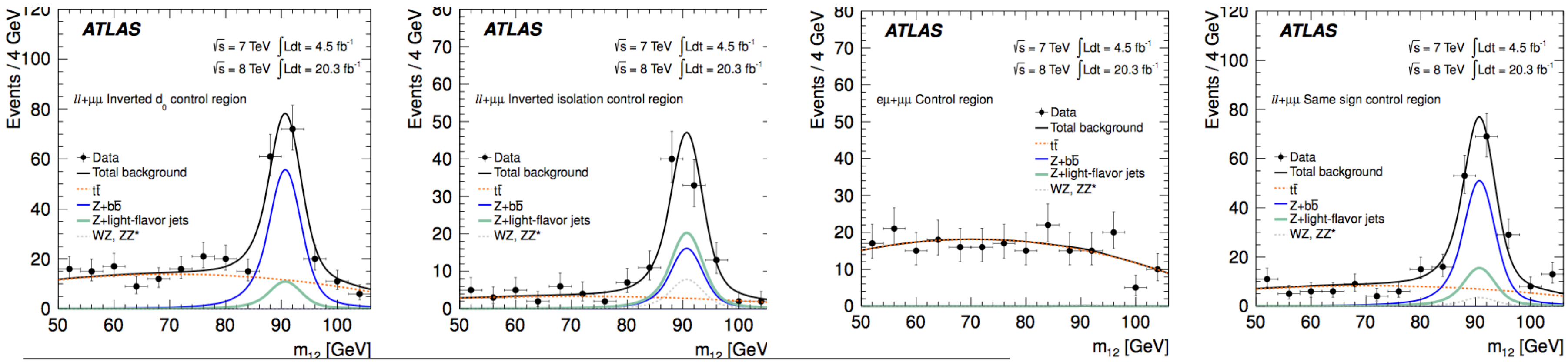
Background measurement - I

- Based on simulations:
  - define all possible background processes (with known cross-sections);
  - apply trigger and selection to each simulated sample;
  - determine the amount of background in the "signal region" after weighting with known cross-sections.
- Data-driven methods:
  - "control regions" based on a different selection (e.g. sidebands);
  - fit control region distributions with simulated distributions and get weigths;
  - then export to "signal region" using "transfer-factors".
- Example: reducible background of H4l ATLAS analysis (next slides)

Background measurement - II

Table 3: Expected contribution of the $\ell\ell + \mu\mu$ background sources in each of the control regions.

	Control region			
Background	Inverted d_0	Inverted isolation	$e\mu + \mu\mu$	Same-sign
$Zb\bar{b}$	$32.8 \pm 0.5\%$	$26.5 \pm 1.2\%$	$0.3 \pm 1.2\%$	$30.6 \pm 0.7\%$
Z + light-flavor jets	$9.2 \pm 1.3\%$	$39.3 \pm 2.6\%$	$0.0 \pm 0.8\%$	$16.9 \pm 1.6\%$
$t\bar{t}$	$58.0 \pm 0.9\%$	$34.2 \pm 1.6\%$	$99.7 \pm 1.0\%$	$52.5 \pm 1.1\%$



(d)

Reducible background yields for 4μ and $2e2\mu$ in reference control region

Control region	$Zb\bar{b}$	Z + light-flavor jets	Total Z + jets	$t\bar{t}$
Combined fit	159 ± 20	49 ± 10	208 ± 22	210 ± 12
Inverted impact parameter			206 ± 18	208 ± 23
Inverted isolation			210 ± 21	201 ± 24
$e\mu + \mu\mu$			–	201 ± 12
Same-sign dilepton			198 ± 20	196 ± 22

Extrapolate to "signal region" using transfer factors
➔ (see next slide)

Background measurement - III

Table 5: Estimates for the $\ell\ell + \mu\mu$ background in the signal region for the full $m_{4\ell}$ mass range for the $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV data. The $Z+$jets and $t\bar{t}$ background estimates are data-driven and the WZ contribution is from simulation. The decomposition of the $Z+$jets background in terms of the $Zb\bar{b}$ and the $Z+$ light-flavor-jets contributions is also provided.

Background	4μ	$2e2\mu$
	$\sqrt{s} = 7$ TeV	
$Z+$jets	$0.42 \pm 0.21(\text{stat}) \pm 0.08(\text{syst})$	$0.29 \pm 0.14(\text{stat}) \pm 0.05(\text{syst})$
$t\bar{t}$	$0.081 \pm 0.016(\text{stat}) \pm 0.021(\text{syst})$	$0.056 \pm 0.011(\text{stat}) \pm 0.015(\text{syst})$
WZ expectation	0.08 ± 0.05	0.19 ± 0.10
	$Z+$jets decomposition	
$Zb\bar{b}$	$0.36 \pm 0.19(\text{stat}) \pm 0.07(\text{syst})$	$0.25 \pm 0.13(\text{stat}) \pm 0.05(\text{syst})$
$Z+$ light-flavor jets	$0.06 \pm 0.08(\text{stat}) \pm 0.04(\text{syst})$	$0.04 \pm 0.06(\text{stat}) \pm 0.02(\text{syst})$
	$\sqrt{s} = 8$ TeV	
$Z+$jets	$3.11 \pm 0.46(\text{stat}) \pm 0.43(\text{syst})$	$2.58 \pm 0.39(\text{stat}) \pm 0.43(\text{syst})$
$t\bar{t}$	$0.51 \pm 0.03(\text{stat}) \pm 0.09(\text{syst})$	$0.48 \pm 0.03(\text{stat}) \pm 0.08(\text{syst})$
WZ expectation	0.42 ± 0.07	0.44 ± 0.06
	$Z+$jets decomposition	
$Zb\bar{b}$	$2.30 \pm 0.26(\text{stat}) \pm 0.14(\text{syst})$	$2.01 \pm 0.23(\text{stat}) \pm 0.13(\text{syst})$
$Z+$ light-flavor jets	$0.81 \pm 0.38(\text{stat}) \pm 0.41(\text{syst})$	$0.57 \pm 0.31(\text{stat}) \pm 0.41(\text{syst})$

The "ABCD" factorization method

- Use two variables (var1 and var2) with these features:
  - For the background they are completely independent
  - The signal is localized in a region of the two variables
- Divide the plane in 4 boxes: the signal is on D only

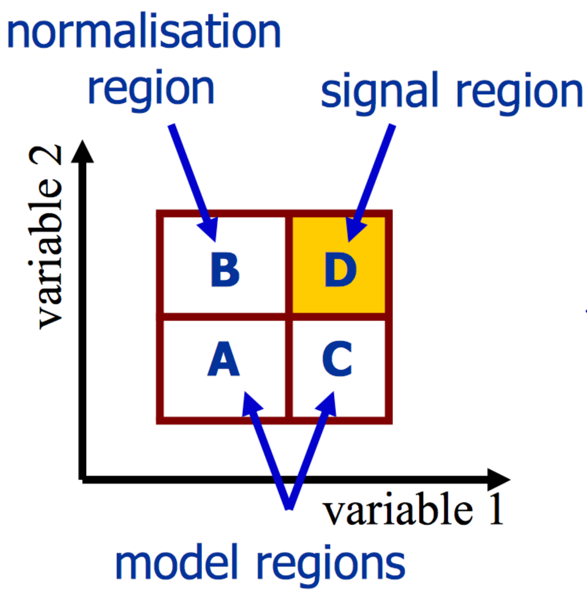
For the background, due to the independence we have few relations:

$$B/D = A/C$$

$$B/A = D/C$$

So: If I count the background (in data) events in regions A,B and C I can extrapolate in the signal region D:

$$D = CB/A$$



Luminosity measurement - I

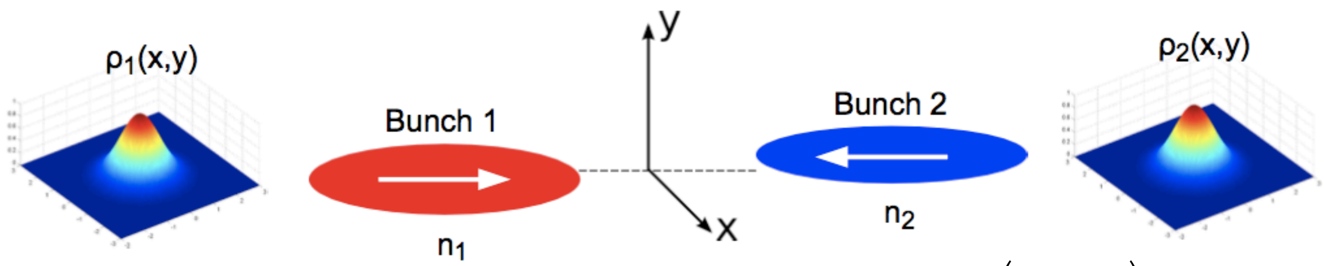
- In order to get the luminosity we need to know the "cross-section" of a candle process:

$$L = \frac{\dot{N}}{\sigma}$$

- In e^+e^- experiments QED helps, since Bhabha scattering can be theoretically evaluated with high precision (< 1%).
- In pp experiment the situation is more difficult.
  - Two-step procedure: continuous "relative luminosity" measurement through several monitors. Count the number of "inelastic interactions";
  - time-to-time using the "Van der Meer" scan the absolute calibration is obtained by measuring the effective σ_{inel}.

Luminosity measurement - II

Van der Meer scan: Measurement of the rate of inelastic interactions as a function of the bunch horizontal and vertical separations:



$$R(\delta x) = \int \rho_1(x,y)\rho_2(x+\delta x,y)dxdy \propto \exp\left(-\frac{x^2}{2\Sigma_x^2}\right)$$

➔Determine the transverse bunch dimensions Σ_x, Σ_y and the inelastic rate at 0 separation.
➔Using the known values of the number of protons per bunch from LHC monitors, one get the ***inelastic cross-section*** that provides the absolute normalization.

$$L = n_b f \frac{N_1 N_2}{4\pi\Sigma_x\Sigma_y} = \frac{\dot{N}_{inel}}{\sigma_{inel}}$$

$$\sigma_{inel} = \left(\frac{\dot{N}_{inel}}{n_b f}\right)\frac{4\pi\Sigma_x\Sigma_y}{N_1 N_2}$$

Recap

- Let's remind at this point that our aim is to learn how to design an experiment.
- We have seen:
  - Definition of the process we want to study
  - Selection of the events correponding to this process
  - Measurement of the quantities related to the process
  - Other measurements related to the physics objects we are studying.
- Now, in order to really design an experiment we need:
  - To see how projectiles and targets can be set-up
  - To see how to put together different detectors to mesure what we need to measure