Open Issues in Charmonium Physics

Nora Brambilla & Antonio Vairo

University of Milano and INFN
The system is accessible to QCD studies.

(i) hierarchy of scales (⇒ factorization/effective field theories)
(ii) some of the scales are perturbative.

For these same reasons, charmonium (and quarkonium) are systems where low energy QCD may be studied in a systematic way (e.g. non-perturbative matrix elements, QCD vacuum, confinement, exotica, ... )
Summary

1. Effective Field Theories: NRQCD, pNRQCD
2. Annihilations
   2.1 Inclusive decays
   2.2 Electromagnetic decays
   2.3 Exclusive decays (15% rule)
3. Production
   3.1 Polarization
   3.2 Double charmonium production
4. Radiative transitions
   4.1 $J/\psi \rightarrow \gamma \eta_c$
5. New spectroscopy
   5.1 Hybrids
6. Conclusion
1. EFTs
Quarkonium Scales

Apart from $\alpha_s$, another small parameter shows up near threshold:

$$E \approx 2m + \frac{p^2}{m} + \ldots \quad \text{with} \quad v = \frac{p}{m} \ll 1$$

- The perturbative expansion breaks down when $\alpha_s \sim v$:

$$\alpha_s \left(1 + \frac{\alpha_s}{v} + \ldots\right)$$

- The system is non-relativistic: $p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$. 
Scales get entangled.

\[ E \sim m u^2 \]

\[ p \sim m \]

\[ \sim m u \]
Effective Field Theories for Quarkonium

Whenever a system $H$, described by $\mathcal{L}_{\text{QCD}}$, is characterized by 2 scales $\Lambda \gg \lambda$, observables may be calculated by expanding one scale with respect to the other. An effective field theory makes the expansion in $\lambda/\Lambda$ explicit at the Lagrangian level.

\[ \frac{\lambda}{\Lambda} = \frac{mv}{m} \]

\[ \frac{\lambda}{\Lambda} = \frac{mv^2}{mv} \]
Charmonium Scales

\[ m_c \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}} \]

\[ m_c v \approx 0.8 \text{ GeV} > \Lambda_{\text{QCD}} \quad \text{for } J/\psi, \eta_c \]

\[ m_c v \sim \Lambda_{\text{QCD}} \quad \text{for all higher resonances} \]

As a consequence:

- annihilation, production, hard scale processes happen at a perturbative scale;
- the bound state is perturbative (i.e. Coulombic) perhaps only for the \( J/\psi, \eta_c \);
- for all other charmonium resonances the bound state is non-perturbative. It will be described by matrix elements, (confining) potentials to be determined on the lattice.
NRQCD is the EFT that follows from QCD when $\Lambda = m$.

- The matching is perturbative.
- The Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

  $$\mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times O_n(\mu, \lambda)/m^n$$

Suitable to describe decay and production of quarkonium.
pNRQCD

pNRQCD is the EFT for heavy quarkonium that follows from NRQCD when $\Lambda = \frac{1}{r} \sim m v$

NRQCD + + ... + ... + ...

$E - p^2/m - V(r)$

pNRQCD

- The Lagrangian is organized as an expansion in $1/m$, $r$, and $\alpha_s(m)$:

$$\mathcal{L}_{pNRQCD} = \sum_k \sum_n \frac{1}{m^k} \times c_k(\alpha_s(m/\mu)) \times V(r \mu', r \mu) \times O_n(\mu', \lambda) r^n$$
2. Annihilations
Inclusive Decays in NRQCD

\[ \Gamma(H \rightarrow LH) = -2 \text{Im} \langle H | \mathcal{H} | H \rangle \]

\[ = \sum_n \frac{2 \text{Im} f^{(n)}(n)}{m^{d_n-4}} \langle H | \psi^{\dagger} K^{(n)}(n) \chi | H \rangle \]

\[ \Gamma(H \rightarrow EM) = \sum_n \frac{2 \text{Im} f^{(n)}(n)}{m^{d_n-4}} \langle H | \psi^{\dagger} K^{(n)}(n) \chi | \text{vac} \rangle \langle \text{vac} | \chi^{\dagger} K'^{(n)}(n) \psi | H \rangle \]

Bodwin et al. 95
\[ \langle H | \psi^{\dagger} K^{(n)} \chi \chi^{\dagger} K^{\dagger (n)} \psi | H \rangle = |R(0)|^2 \times \int dt \langle t | G(t) G(0) \rangle \]

Brambilla et al. 02 03
E.m. widths on the lattice

\[
\left( \frac{\Gamma_{ee}(2s)}{\Gamma_{ee}(1s)}\right) \left( \frac{M_{2s}}{M_{1s}} \right)^2
\]

\[\text{Gray et al 05}\]
\[ \frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\gamma)} \]

- Large $\beta_0\alpha_s$ contributions.

\[
\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\gamma)} \approx (1.1 \text{ (LO)} + 1.0 \text{ (NLO)}) \times 10^3 = 2.1 \times 10^3
\]

\[
\frac{\Gamma(\eta_c \to LH)}{\Gamma(\eta_c \to \gamma\gamma)} = (3.3 \pm 1.3) \times 10^3 \text{ (EXP)}
\]
\[
\frac{\Gamma(\eta_c \rightarrow LH)}{\Gamma(\eta_c \rightarrow \gamma \gamma)}
\]

- Large $\beta_0 \alpha_s$ contributions.

\[
\begin{align*}
\frac{\Gamma(\eta_c \rightarrow LH)}{\Gamma(\eta_c \rightarrow \gamma \gamma)} & \approx (1.1 \text{ (LO}) + 1.0 \text{ (NLO)}) \times 10^3 = 2.1 \times 10^3 \\
\frac{\Gamma(\eta_c \rightarrow LH)}{\Gamma(\eta_c \rightarrow \gamma \gamma)} &= (3.3 \pm 1.3) \times 10^3 \text{ (EXP)}
\end{align*}
\]

- Scheme dependence
- Renormalons

\[
\frac{\Gamma(\eta_c \rightarrow LH)}{\Gamma(\eta_c \rightarrow \gamma \gamma)} = (3.01 \pm 0.30 \pm 0.34) \times 10^3
\]

Bodwin Chen 01

* to be done for $P$-wave decays.
\[ \frac{\Gamma(J/\psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} \]

- Large logarithms.

\[ \mathcal{R}_c = \frac{\Gamma(J/\psi \rightarrow e^+e^-)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} \]
P-wave decays at NLO and $mv^5$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>PDG04</th>
<th>PDG00</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(\chi_{c0} \to \gamma\gamma) / \Gamma(\chi_{c2} \to \gamma\gamma)$</td>
<td>$13\pm10$</td>
<td></td>
<td>$3.75$</td>
<td>$\approx5.43$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c2} \to lh_0) - \Gamma(\chi_{c1} \to lh_0)$</td>
<td>$270\pm200$</td>
<td></td>
<td>$347$</td>
<td>$\approx383$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c0} \to \gamma\gamma) / \Gamma(\chi_{c0} \to lh_0)$</td>
<td>$3500\pm2500$</td>
<td></td>
<td>$1300$</td>
<td>$\approx2781$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c1} \to lh_0) - \Gamma(\chi_{c2} \to lh_0)$</td>
<td>$12.1\pm3.2$</td>
<td></td>
<td>$2.75$</td>
<td>$\approx6.63$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c0} \to lh_0) - \Gamma(\chi_{c1} \to lh_0)$</td>
<td>$13.1\pm3.3$</td>
<td></td>
<td>$3.75$</td>
<td>$\approx7.63$</td>
</tr>
</tbody>
</table>

$m_c = 1.5 \text{ GeV} \quad \alpha_s(2m_c) = 0.245$
P-wave decays at NLO and $m\nu^5$

<table>
<thead>
<tr>
<th>Ratio</th>
<th>PDG04</th>
<th>PDG00</th>
<th>LO</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(\chi_{c0} \rightarrow \gamma \gamma) / \Gamma(\chi_{c2} \rightarrow \gamma \gamma)$</td>
<td>$5.1 \pm 1.1$</td>
<td>$13 \pm 10$</td>
<td>$= 3.75$</td>
<td>$\approx 5.43$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c2} \rightarrow l\nu) - \Gamma(\chi_{c1} \rightarrow l\nu) / \Gamma(\chi_{c0} \rightarrow \gamma \gamma)$</td>
<td>$410 \pm 100$</td>
<td>$270 \pm 200$</td>
<td>$\approx 347$</td>
<td>$\approx 383$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c0} \rightarrow l\nu) - \Gamma(\chi_{c1} \rightarrow l\nu) / \Gamma(\chi_{c0} \rightarrow \gamma \gamma)$</td>
<td>$3600 \pm 700$</td>
<td>$3500 \pm 2500$</td>
<td>$\approx 1300$</td>
<td>$\approx 2781$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c0} \rightarrow l\nu) - \Gamma(\chi_{c2} \rightarrow l\nu)$</td>
<td>$7.9 \pm 1.5$</td>
<td>$12.1 \pm 3.2$</td>
<td>$= 2.75$</td>
<td>$\approx 6.63$</td>
</tr>
<tr>
<td>$\Gamma(\chi_{c0} \rightarrow l\nu) - \Gamma(\chi_{c2} \rightarrow l\nu)$</td>
<td>$8.9 \pm 1.1$</td>
<td>$13.1 \pm 3.3$</td>
<td>$= 3.75$</td>
<td>$\approx 7.63$</td>
</tr>
</tbody>
</table>

$m_c = 1.5 \text{ GeV}$ \hspace{1cm} $\alpha_s(2m_c) = 0.245$

mainly from E835 ($\chi_{c0}$, total width and $\gamma \gamma$)
also from Belle ($\chi_{c0} \rightarrow \gamma \gamma$) and CLEO, BES
One should notice that $P$-wave analyses do not include so far either
(i) subtraction of renormalons
(ii) resummation of large logs
(iii) $\mathcal{O}(v^2)$ relativistic effects (numerically of the same magnitude as NLO corrections).

Potentially $P$-wave decays may provide a competitive source of $m_c$ and $\alpha_s(m_c)$.

Mangano Petrelli 95, Maltoni 00, Mussa 03
Exclusive Decay Modes

\[ \kappa_{[h_1 h_2]} = \frac{\mathcal{B}(\psi(2S) \rightarrow h_1 h_2)}{\mathcal{B}(J/\psi \rightarrow h_1 h_2)} \frac{\mathcal{B}(J/\psi \rightarrow e^+ e^-)}{\mathcal{B}(\psi(2S) \rightarrow e^+ e^-)} \frac{\rho_{[J/\psi h_1 h_2]}}{\rho_{[\psi(2S) h_1 h_2]}}. \]

- The phase-space factor \( \rho \approx 1. \)
- If \( \Gamma(J/\psi \rightarrow h_1 h_2) \approx |\psi_{J/\psi}(r = 0)|^2 |A(0)\bar{c}(0) \rightarrow h_1 h_2|^2 \frac{\rho_{[J/\psi h_1 h_2]}}{16\pi M_{J/\psi}} \)

and analogously for the \( \psi(2S) \).

Then

\[ \kappa_{[h_1 h_2]} \approx 1 \]

Also known as 15% rule.
Exclusive Decay Modes

<table>
<thead>
<tr>
<th>Decay mode $h_1 h_2$</th>
<th>$\mathcal{B}[J/\psi \rightarrow h_1 h_2]$ ($\times 10^3$)</th>
<th>$\mathcal{B}[\psi' \rightarrow h_1 h_2]$ ($\times 10^3$)</th>
<th>$\kappa[h_1 h_2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi \pi$</td>
<td>$127 \pm 9$</td>
<td>$&lt; 0.83 (&lt; 0.28)$</td>
<td>$&lt; 0.054 (&lt; 0.18)$</td>
</tr>
<tr>
<td>$\omega \pi^0$</td>
<td>$4.2 \pm 0.6$</td>
<td>$0.38 \pm 0.17 \pm 0.11$</td>
<td>$0.7 \pm 0.4$</td>
</tr>
<tr>
<td>$\phi \eta$</td>
<td>$1.93 \pm 0.23$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega \eta$</td>
<td>$15.8 \pm 1.6$</td>
<td>$&lt; 0.33$</td>
<td>$&lt; 0.17$</td>
</tr>
<tr>
<td>$\phi \eta'$ (958)</td>
<td>$6.5 \pm 0.7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega \eta'$ (958)</td>
<td>$1.05 \pm 0.18$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi \eta''$ (958)</td>
<td>$1.67 \pm 0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^* (892)^+ K^\pm$</td>
<td>$50 \pm 4$</td>
<td>$&lt; 0.54(&lt; 0.30)$</td>
<td>$&lt; 0.089 (&lt; 0.049)$</td>
</tr>
<tr>
<td>$K^* (892)^0 K^0 + c.c.$</td>
<td>$42 \pm 4$</td>
<td>$0.81 \pm 0.24 \pm 0.16$</td>
<td>$0.15 \pm 0.05$</td>
</tr>
<tr>
<td>$\pi^\pm b_1 (1235)^\mp$</td>
<td>$30 \pm 5$</td>
<td>$3.2 \pm 0.8$</td>
<td>$0.79 \pm 0.24$</td>
</tr>
<tr>
<td>$\pi^0 b_1 (1235)^0$</td>
<td>$23 \pm 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K^{\pm} K_1 (1270)^\mp$</td>
<td>$&lt; 30$</td>
<td>$10.0 \pm 2.8$</td>
<td>$&gt; 1.7$</td>
</tr>
<tr>
<td>$K^{\pm} K_1 (1400)^\mp$</td>
<td>$38 \pm 14$</td>
<td>$&lt; 3.1$</td>
<td>$&lt; 0.78$</td>
</tr>
</tbody>
</table>
# Exclusive Decay Modes

<table>
<thead>
<tr>
<th>Decay mode $h_1 h_2$</th>
<th>$\mathcal{B}(J/\psi \to h_1 h_2) \times 10^4$</th>
<th>$\mathcal{B}(\psi' \to h_1 h_2) \times 10^4$</th>
<th>$\kappa[h_1 h_2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDG 04, BES 04, CLEO 04</td>
<td>$\omega \pi^0$</td>
<td>4.2 ± 0.6</td>
<td>0.22 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>$\rho \pi$</td>
<td>127 ± 9</td>
<td>0.46 ± 0.09</td>
</tr>
<tr>
<td></td>
<td>$\omega \eta$</td>
<td>1.93 ± 0.23</td>
<td>0.23 ± 0.12</td>
</tr>
<tr>
<td></td>
<td>$\phi \eta$</td>
<td>15.8 ± 1.6</td>
<td>&lt; 0.11</td>
</tr>
<tr>
<td></td>
<td>$\phi \eta'$ (958)</td>
<td>6.5 ± 0.7</td>
<td>0.35 ± 0.11</td>
</tr>
<tr>
<td></td>
<td>$\omega \eta'$ (958)</td>
<td>1.05 ± 0.18</td>
<td>0.19$^{+0.16}_{-0.11}$ ± 0.03</td>
</tr>
<tr>
<td></td>
<td>$\phi \eta'$ (958)</td>
<td>1.67 ± 0.25</td>
<td>&lt; 0.81</td>
</tr>
<tr>
<td></td>
<td>$K^* (892)^+ K^\pm$</td>
<td>3.3 ± 0.4</td>
<td>0.33 ± 0.13 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>$\bar{K}^* (892)^0 K^0 + c.c.$</td>
<td>50 ± 4</td>
<td>0.26 ± 0.11</td>
</tr>
<tr>
<td></td>
<td>$\pi^\pm b_1 (1235)^\mp$</td>
<td>42 ± 4</td>
<td>1.55 ± 0.25</td>
</tr>
<tr>
<td></td>
<td>$\pi^0 b_1 (1235)^0$</td>
<td>30 ± 5</td>
<td>3.9 ± 1.6</td>
</tr>
<tr>
<td></td>
<td>$K^\pm K_1 (1270)^\mp$</td>
<td>23 ± 6</td>
<td>4.6$^{+0.9}_{-0.8}$ ± 0.6</td>
</tr>
<tr>
<td></td>
<td>$K^\pm K_1 (1400)^\mp$</td>
<td>&lt; 30</td>
<td>10.0 ± 2.8</td>
</tr>
<tr>
<td></td>
<td>$K^\pm K_1 (1270)^\mp$</td>
<td>38 ± 14</td>
<td>&lt; 3.1</td>
</tr>
</tbody>
</table>
Exclusive Decay Modes

Possible explanations include:

- suppression of the $c\bar{c}$ wave function at the origin for a component of $\psi(2S)$ in which the $c\bar{c}$ is in a color-octet $^3S_1$ state.
- suppression of the $\omega\phi$ component of $\psi(2S)$.
- cancellation between $c\bar{c}$ and $D\bar{D}$ components of $\psi(2S)$.
- cancellation between $c\bar{c}$ and glueball components of $\psi(2S)$.
- cancellation between $S$-wave $c\bar{c}$ and $D$-wave $c\bar{c}$ components of $\psi(2S)$.
- cancellation between the amplitudes for the resonant process $e^+e^- \rightarrow \psi(2S) \rightarrow \rho\pi$ and the direct process $e^+e^- \rightarrow \rho\pi$. 
3. Production
There is no formal proof of the NRQCD factorization yet.

The relevant 4-fermion operators are

$$\psi^\dagger K^{(n)} \chi a_H^\dagger a_H \chi^\dagger K'(n) \psi$$

Recently it has been proved that the cancellation of the IR divergences at NNLO requires the modification of the 4 fermion operators into

$$\psi^\dagger K^{(n)} \chi \phi_1(0, \infty) a_H^\dagger a_H \phi(0, \infty) \chi^\dagger K'(n) \psi$$

$$\phi_1(0, \infty) = P \exp \left( -ig \int_0^{\infty} d\lambda \vec{l} \cdot A(\lambda, l) \right), \quad l^2 = 1$$

Nayak Qiu Sterman 05
Charmonium Production at the Tevatron

Octet contributions dominate in production at high $p_T$.

$p p \rightarrow J/\psi + X$

$pp \rightarrow \psi(2S) + X$

Krämer 01, CDF 97
$e^+ e^- \rightarrow e^+ e^- J/\psi X$

$\sqrt{s} = 197$ GeV
$-2 < y_{J/\psi} < 2$

DELPHI prelim.
MRST98 fit
CTEQ5 fit

p _T^2 (GeV ^2)

$d\sigma/dp_T^2$ (pb/GeV ^2)

NRQCD
3P J [8]
3P J [1]
1S 0 [8]
3S 1 [8]

CSM

NRQCD

Klasen Kniehl Mihaila Steinhauser 02
For large $p_T$, quarkonium production via the color-octet mechanism dominates: $O_g^{J/\psi}(3S_1)$.

At large $p_T$ the gluon is nearly on mass shell and so is transversely polarized.

In color octet gluon fragmentation, most of the gluon’s polarization is transferred to the $J/\psi$.

Radiative corrections, color singlet production dilute this.

In the case of the $J/\psi$ feeddown is important:
- Feeddown from $\chi_c$ states is about 30% of the $J/\psi$ sample and dilutes the polarization.
- Feeddown from $\psi(2S)$ is about 10% of the $J/\psi$ sample and is largely transversely polarized.

Spin-flipping terms are assumed suppressed. But this strictly depends on the power counting. If they are not, polarization may dilute at high $p_T$. 
Charmonium Polarization at the Tevatron

\[ \frac{d\sigma}{d\cos \theta} \propto 1 + \alpha \cos^2 \theta \]

\( \alpha = 1 \) is completely transverse \( \alpha = -1 \) is completely longitudinal.

Krämer 01, Braaten et al 01, CDF 97
Charmonium Polarization at the Tevatron

Prompt $J/\psi$ Polarization

CDF 2 Preliminary, $188 \pm 11 \text{ pb}^{-1}$

CDF (preliminary) 05
Double Charmonium Production

\[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) \gtrsim 25.6 \pm 2.8 \pm 3.4 \text{ fb} \quad \text{Belle 04} \]
\[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) \gtrsim 17.6 \pm 2.8^{+1.5}_{-2.1} \text{ fb} \quad \text{BaBar 05} \]
\[ \sigma(e^+e^- \rightarrow J/\psi + \eta_c) = 3.78 \pm 1.26 \text{ fb} \quad \text{NRQCD} \]

- Includes QED interference corrections (-21%)
- Includes uncertainties from h.o. in \( \alpha_s, \nu \) and matrix elements
- In Belle 04 \( \sigma(e^+e^- \rightarrow J/\psi + J/\psi) < 9.1 \text{ fb} \)
- In Brodsky et al 03 \( \sigma(e^+e^- \rightarrow J/\psi + G_J) \approx 1.4 \text{ fb} \)
  where \( G_J \) is a \( J^{++} \) glueball, \( J = 0.2 \)

Braaten Lee 03, Liu et al 03, Bodwin et al 03, Brodski et al 04
Double Charmonium Production

\[
\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} = 0.82 \pm 0.15 \pm 0.14
\]

Belle 03

\[
\frac{\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi + X)} \approx 0.1
\]

Cho Leibovich 96, Baek et al 96, Yuan et al 96
Double Charmonium Production

\[ \sigma(e^+e^- \rightarrow J/\psi + J/\psi) < 9.1 \text{ fb} \]

**Belle 04**

\[ \sigma(e^+e^- \rightarrow J/\psi + J/\psi) = 8.70 \pm 2.94 \text{ fb} \]

**Bodwin et al 03**
4. Radiative Transitions
$J/\psi \rightarrow \gamma \eta_c$

Only one direct experimental measure:

\[
\Gamma(J/\psi \rightarrow \eta_c\gamma) = (1.14 \pm 0.23) \text{ keV} \quad \text{Crystal Ball 86}
\]

Moreover, there are several measurements of the BR $J/\psi \rightarrow \eta_c\gamma \rightarrow \phi\phi\gamma$ and one independent measurement of $\eta_c \rightarrow \phi\phi$ (Belle 03). From them one obtains

\[
\Gamma(J/\psi \rightarrow \eta_c\gamma) = (2.9 \pm 1.5) \text{ keV}
\]

Combining both

\[
\Gamma(J/\psi \rightarrow \eta_c\gamma) = (1.18 \pm 0.36) \text{ keV} \quad \text{PDG 04}
\]

- $\Gamma(J/\psi \rightarrow \eta_c\gamma)$ enters into many charmonium BR.
- Its 30% uncertainty sets typically their experimental errors.
\[ J/\psi \rightarrow \gamma \eta_c \]

\[ \frac{\Gamma(J/\psi \rightarrow \eta_c \gamma)}{\Gamma(J/\psi)} = 0.013 \pm 0.004 \Rightarrow \frac{1 + \kappa_c}{m_c} |M_{i, f}| = 0.40 \pm 0.05 \text{ GeV} \]

If \(|M_{i, f}| = 1\) this implies:

- \(\kappa_c = 0, m_c = 2.3 \pm 0.3 \text{ GeV}\)
- \(\kappa_c = -0.28 \pm 0.09, m_c = 1.8 \text{ GeV}\)
- Large relativistic corrections to the \(S\)-state wave functions

\text{Eichten/QWG 02}
M1 operator at $\mathcal{O}(1)$

$$-c_{\sigma B} \left\{ S^\dagger, \frac{\sigma \cdot eB^m}{2m} \right\} S$$

- $c_{\sigma B}$ behaves like the identity operator.
- To all orders $c_{\sigma B}$ does not get soft contributions.
- Hard contributions are known:

$$c_{\sigma B} \equiv 1 + \kappa_c = 1 + \frac{2\alpha_s(m_d)}{3\pi} + \ldots$$

- No large quarkonium anomalous magnetic moment!

Brambilla Jia Vairo 05
M1 operators at $\mathcal{O}(v^2)$

\[-c_{p^2}\sigma_B \left\{ S^i, \frac{\sigma \cdot e B^m}{4m^3} \right\} \nabla_i^2 S\]

- $c_{p^2}\sigma_B = -1 - \frac{2 \alpha_s(m_c)}{g^2} + \ldots$
M1 operators at $O(v^2)$

$$-c_r \sigma_B \left\{ S^1, \frac{\sigma \cdot e B^{em}}{12m^2} \right\} S$$

\[ \begin{aligned}
&c_F \sigma \cdot B/m \\
&\quad + \\
&\overbrace{A \cdot A^{em}/m}^\text{(hard)} \\
&\quad + \ldots \\
&c_s \sigma \cdot (A^{em} \times E)/m^2 \\
&\quad = (\text{hard}) \times (\text{soft})
\end{aligned} \]

- to all orders (hard) = $2c_F - c_s = 1$; (soft) = $-rV_s'$

- Therefore $c_r \sigma_B = -rV_s'$

- No scalar interaction!

Brambilla Jia Vairo 05
M1 operators at $\mathcal{O}(v^2)$

Coupling of photons with octets: $-c \sigma_B \left\{ O^1, \frac{\sigma \cdot e B^m}{2m} \right\} O$

= 0

- If $mv^2 \sim \Lambda_{\text{QCD}}$, the above graphs are potentially of order $\Lambda_{\text{QCD}}^2/(mv)^2 \sim v^2$.
- The contribution vanishes because $\sigma \cdot e B^m(R)$ behaves like the identity operator.
- There are no non-perturbative contributions at $\mathcal{O}(v^2)$!
$J/\psi \rightarrow \gamma \eta_c$

Up to order $\nu^2$ the transition $J/\psi \rightarrow \gamma \eta_c$ is completely accessible by perturbation theory.

$$
\Gamma(J/\psi \rightarrow \gamma \eta_c) = \frac{16}{3} \alpha_e^2 \frac{k_J^3}{M_{J/\psi}^2} \left[ 1 + C_F \frac{\alpha_s(M_{J/\psi}/2)}{\pi} - \frac{2}{3} \left( C_F \alpha_s(p_{J/\psi}) \right)^2 \right]
$$

The normalization scale for the $\alpha_s$ inherited from $\kappa_c$ is the charm mass $\alpha_s(M_{J/\psi}/2) \approx 0.35 \sim \nu^2$, and for the $\alpha_s$, which comes from the Coulomb potential, is the typical momentum transfer $p_{J/\psi} \approx m_C \alpha_s(p_{J/\psi})/2 \approx 0.8 \text{ GeV} \sim mv$.

$$
\Gamma(J/\psi \rightarrow \eta_c \gamma) = (1.5 \pm 1.0) \text{ keV}.
$$

Brambilla Jia Vairo 05
M1 (hindered and allowed) transitions between higher charmonium states and all E1 transitions involve strongly coupled bound states. The treatment has to be non-perturbative, but may go along the lines outlined before. The matching coefficients will involve Wilson loop amplitudes eventually to be calculated on the lattice.

A first QCD (quenched) lattice study of charmonium radiative transitions has been published 3 days ago. It confirms that there are no large contributions to the charmonium anomalous magnetic moment.
Hadronic Transitions

Hadronic transitions have been very useful to discriminate the nature of the new charmonium resonances.

Relativistic corrections have been studied so far only in phenomenological models:

(i) QCD multipole expansion;

(ii) Chiral Lagrangian for heavy mesons.
5. New Spectroscopy
Many interpretations have been proposed for the X, Y, Z: ordinary quarkonium states, hybrid mesons, $D\bar{D}$ molecules, four-quark states, ...

Even if some of the states are exotic states, like hybrids, due to the heavy mass of the quarks, factorization and analytic approaches may provide insights.
As an example, we consider the $Y(4260)$ case and a possible hybrid interpretation of it.

$Y(4260)$ is generated from initial state radiation in $e^+e^- \rightarrow \gamma J/\psi \pi^+\pi^- \rightarrow J^{PC} = 1^{--}$

$$|Y\rangle = |\Pi_u\rangle \otimes |\phi\rangle$$

- $|\Pi_u\rangle$ is a $1^{+-}$ static hybrid state.
- $|\phi\rangle$ ($P = -1$) is the solution of the Schrödinger equation whose potential is the static energy of $|\Pi_u\rangle$. 

\[ E_r(r) - E_{g(r_0)} \]
<table>
<thead>
<tr>
<th>$J^P_C$</th>
<th>$H$</th>
<th>$\Lambda_H^{RS}$</th>
<th>$\Lambda_H^{RS}/GeV$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^{+-}$</td>
<td>$B_i$</td>
<td>2.25(39)</td>
<td>0.87(15)</td>
</tr>
<tr>
<td>$1^{-+}$</td>
<td>$E_i$</td>
<td>3.18(41)</td>
<td>1.25(16)</td>
</tr>
<tr>
<td>$2^{--}$</td>
<td>$D_i(B_j)$</td>
<td>3.69(42)</td>
<td>1.45(17)</td>
</tr>
<tr>
<td>$2^{+-}$</td>
<td>$D_i(E_j)$</td>
<td>4.72(48)</td>
<td>1.86(19)</td>
</tr>
<tr>
<td>$3^{++}$</td>
<td>$D_i(D_jB_k)$</td>
<td>4.72(45)</td>
<td>1.86(18)</td>
</tr>
<tr>
<td>$0^{++}$</td>
<td>$B^2$</td>
<td>5.02(46)</td>
<td>1.98(18)</td>
</tr>
<tr>
<td>$1^{--}$</td>
<td>$D_i(D_jD_kB_l)$</td>
<td>5.41(46)</td>
<td>2.13(18)</td>
</tr>
<tr>
<td>$1^{-+}$</td>
<td>$(B \wedge E)_i$</td>
<td>5.45(51)</td>
<td>2.15(20)</td>
</tr>
</tbody>
</table>

Foster Michael 99, Bali Pineda 03
Fitting the $II_u$ curve, $E_{II_u} = (0.87 + 0.11/r + 0.24r^2)$ GeV
and solving the Schrödinger equation, one gets

$$M(Y) = 2 \times 1.48 + 0.87 + 0.53 = 4.36 \text{ GeV}$$
Solving the Schrödinger equation provides the wave function, making, in principle, possible the calculation of decay and transition widths.

However, to do a full analysis this is not sufficient. An EFT for states over threshold is needed.
Conclusion

Charmonium physics provides a place where low-energy QCD and its rich structure may be studied in a controlled and systematic way, combining perturbative QCD, lattice calculations, and effective field theory analytical methods.