$$\underbrace{m\frac{d^2x}{dt^2}}\ = -kx \quad \rightarrow \quad \frac{d^2x}{dt^2} = -\underbrace{\frac{k}{m}}_{\omega^2}\, x$$

$$\left[\ \frac{d^2x}{dt^2} = +\xi^2\cdot x \qquad x\nearrow \quad a\nearrow \quad \nearrow\ x'' \quad \searrow a'' \quad ;\ \nearrow x''' \ \right]$$

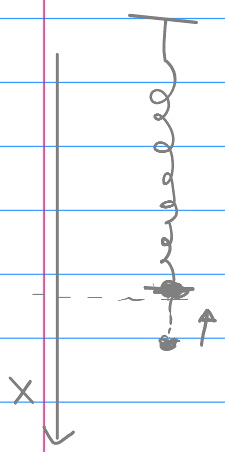
$$\boxed{x = x_0 \cos\omega t}$$

$$\boxed{\begin{cases} x(t=0) = x_0 \\ v(t=0) = 0 \end{cases}}$$

$$x = A\cos(\omega t + \varphi)$$

$$\frac{dx}{dt}$$

$$\frac{d^2x}{dt^2} \rightarrow \frac{d^2x}{dt^2} = -\omega^2 [\bar{x}]$$



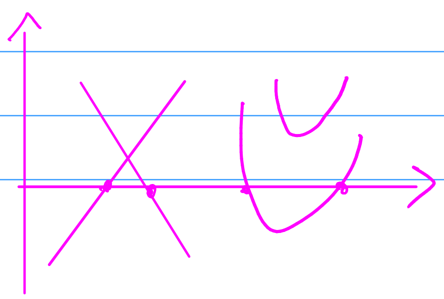
$$F = -kx - \beta v$$

$$a = \frac{F}{m} = -\frac{k}{m}x - \beta/m\, v$$

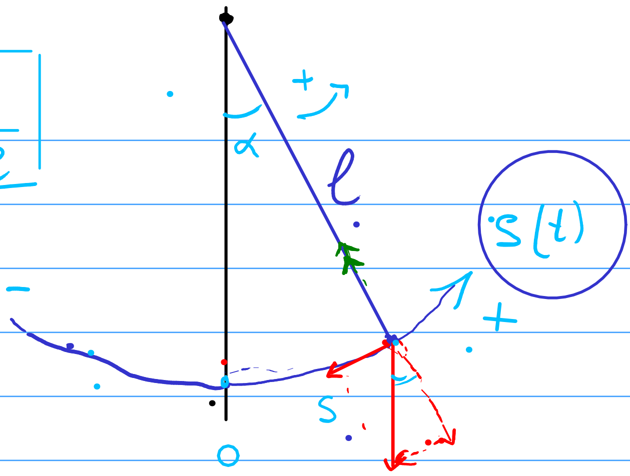
$$\boxed{\frac{d^2x}{dt^2} + \beta/m\,\frac{dx}{dt} + \frac{k}{m}x = 0} + \begin{cases} x_0 \\ v_0 \end{cases}$$

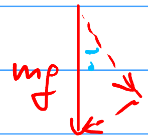
$$x(t)$$

$$x'' + \beta/m\, x' + x = 0$$



$$\boxed{\alpha = \frac{s}{\ell}}$$





$s = \alpha \cdot \ell$

$mg \cdot \sin\alpha$

$$\frac{ds}{dt} = \ell \frac{d\alpha}{dt}$$

$$\frac{d^2 s}{dt^2} = \ell \frac{d^2\alpha}{dt^2}$$

$F = -mg\sin\alpha$

$$a = \frac{\cancel{m} g}{\cancel{m}} = g$$

$$\ell \frac{d^2\alpha}{dt^2} = \frac{d^2 s}{dt^2} = \underbrace{a = \frac{F}{m}} = -\frac{\cancel{m}}{\cancel{m}} g \sin\alpha$$

$$\frac{d^2\alpha}{dt^2} = -\, g/\ell \cdot \sin\alpha \qquad ?!$$

$\alpha << 1$?

$$\boxed{\frac{d^2\alpha}{dt^2} \approx -\, g/\ell \,\alpha}$$

$\omega^2 = g/\ell$

$$t=0 \begin{cases} \alpha = \alpha_M \\ \frac{d\alpha}{dt} = 0 \end{cases} \Rightarrow v\left(= \frac{ds}{dt}\right) = 0; \quad v_\alpha\left(= \frac{d\alpha}{dt}\right) = 0$$

NON VELOCITÀ ANGOLAR

→ PULSAZIONE

$\alpha(t) = \alpha_M \cos(\omega t)$

$s(t) = \ell \cdot \alpha(t)$

$$V_\alpha = \frac{d\alpha}{dt} = -\omega\, \alpha_M \sin(\omega t)$$

x generica

x_g, x_r

$$\frac{dx}{dt} = \alpha (x - x_L)$$

"x" $\rightarrow v \rightarrow v_L$

"x" $\rightarrow T \rightarrow T_L = T_A$

$$\left[\frac{dx}{dt} = \alpha x \Rightarrow x(t) = c\, e^{\alpha t} \right]$$

$$\begin{cases} z(t) = x(t) - x_L \\ \dfrac{dz}{dt} = \dfrac{dx}{dt} \end{cases} \Longrightarrow \frac{dz}{dt} = \alpha \cdot z$$

$$\Rightarrow \underline{z(t) = z_0\, e^{\alpha t}}$$

$$\frac{dz}{dt} = \alpha \left(z_0 e^{\alpha t} \right) = \alpha \cdot z$$

$$x(t) - x_L = (x_0 - x_L)\, e^{\alpha t}$$

$$\boxed{x(t) = x_L + (x_0 - x_L)\, e^{\alpha t}}$$

$\downarrow$ $F = mg - \beta v \Rightarrow m \dfrac{dv}{dt} = mg - \beta v$

$$\frac{dv}{dt} = g - \frac{\beta}{m} v$$

$$= \frac{\beta}{m} \left(\frac{mg}{\beta} - v \right)$$

$$= -\frac{\beta}{m} \left(v - \frac{mg}{\beta} \right)$$

$$= \underbrace{-\frac{1}{\tau}}_{} (v - v_L)$$

$$= \alpha (v - v_L)$$

"X" $\rightarrow V$ $\qquad \alpha = -\beta/m = -\frac{1}{\tau}$ $\rightarrow$ "costante di tempo"

$$v(t) = V_L + (V_0 - V_L)\, e^{-t/\tau}$$

$$\begin{cases} \tau = \dfrac{m}{\beta} \\ V_L = \dfrac{mg}{\beta} \end{cases}$$

$$\rightarrow \begin{cases} t = 0 & \cancel{V_L} + V_0 - \cancel{V_L} = V_0 \\ t \rightarrow \infty & V_L + 0 = V_L \end{cases}$$

"X" $\rightarrow T$ $\qquad \rightarrow \dfrac{dT}{dt} = -\dfrac{1}{\tau}(T - T_L)$

$$v(t) = v_0\, e^{-t/\tau}$$

t	v
0	V_0
τ	$V_0/e = V_0 \cdot 0{,}367 \simeq 37\%\, V_0$
2τ	$V_0\, e^{-2}$

$$\frac{v(t_{1/2})}{V_0} = \frac{1}{2} = e^{-t_{1/2}/\tau}$$

$$\ln\frac{1}{2} = -\frac{t_{1/2}}{\tau} \qquad t_{1/2} = -\tau \cdot \ln\left(\frac{1}{2}\right)$$

$$= \tau \ln 2$$

$$\simeq 0{,}69 \times \tau$$