Introduction to Probabilistic Reasoning

– inference, forecasting, decision –

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– Part 1 –

"It is scientific only to say what is more likely and what is less likely"

(Feynman)

Preamble

- What can we say in just a few hours?
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⇒ Probabilistic approach

An invitation to (re-)think
on foundamental aspects
of data analysis.

Contents

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More 'professional' applications, including computational issues will be presented in the next days

- Allen Caldwell
- Dan
- Roberto Trotta

Software support

- My *extended pocket calulator*:
  - R

  (a 'kind of' Mathlab/Octave; opensource, multiplatform, very easy to start with; numeric computation, probability/statistics functions, graphics; tons of packages. ≈ Nr 1 in statistics, maching learning, etc.)

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⇒ Don't be distracted during the lectures with installations or playing with them (→ better later, or even back home)

Part 1

Claims of discoveries based on 'sigmas'

2011: non only Opera...

- April, CDF: absolutely unexpected excess at about 150 GeV

$$\approx 3.2\,\sigma$$

- September, Opera: neutrinos faster than light

$$\approx 6\,\sigma$$

- December, ATLAS e CMS at LHC: signal compatible with the Higgs at about 125 GeV:

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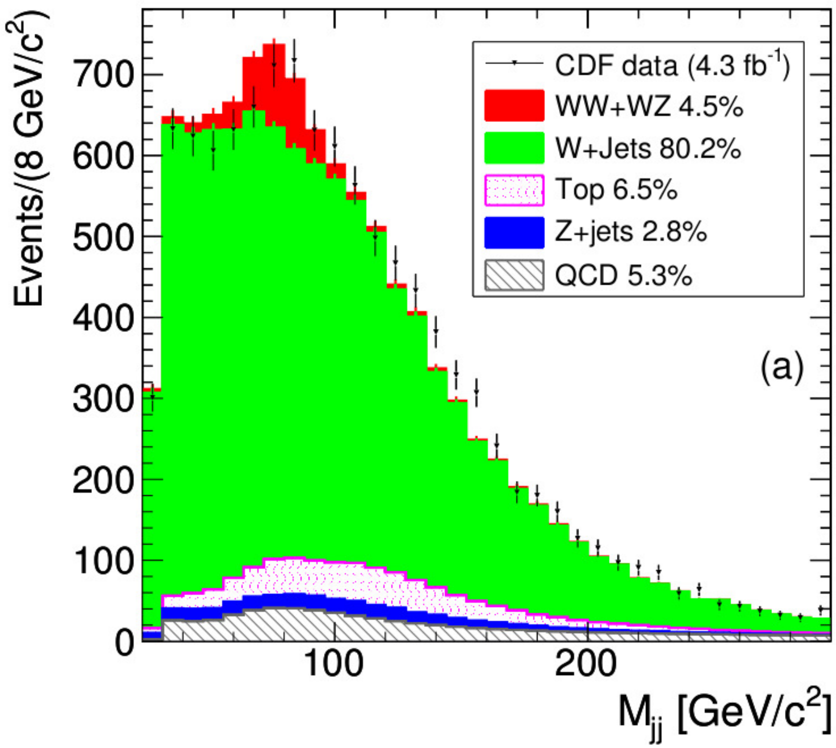
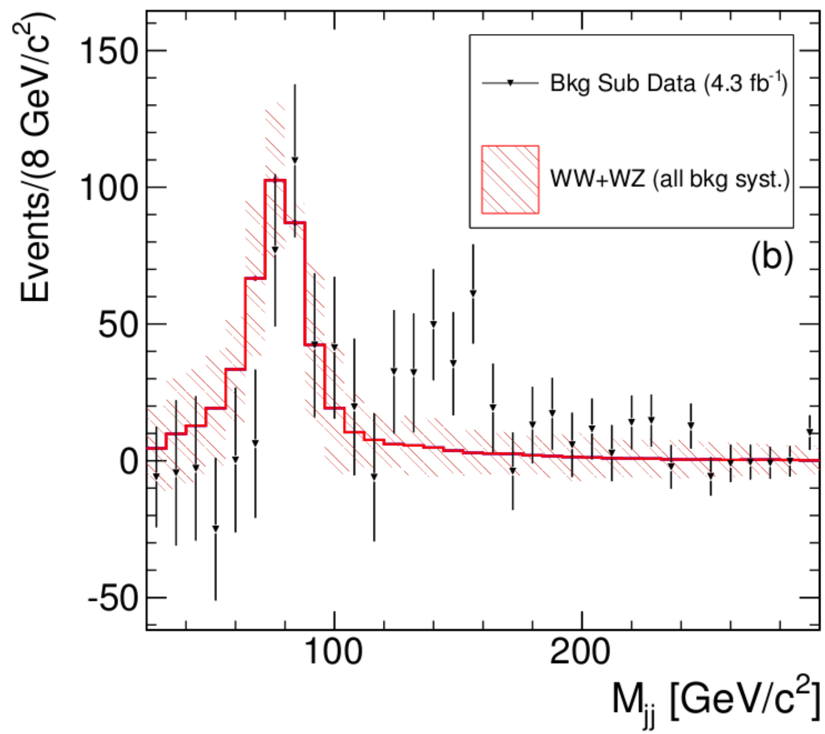
Why there was substancial scepticism towards the first two anouncements, in constrast with a cautious/pronounced optimism towards the third one?

April 2011

CDF Collaboration at the Tevatron





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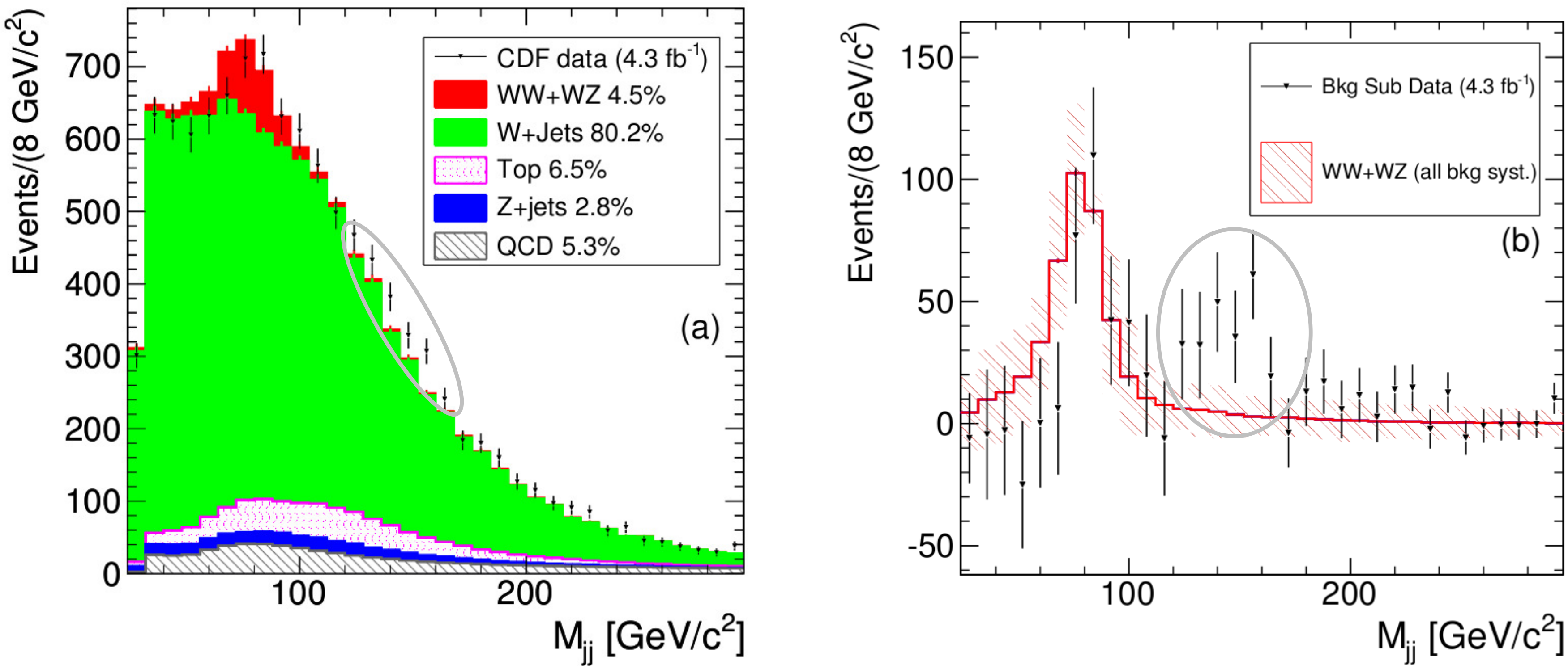

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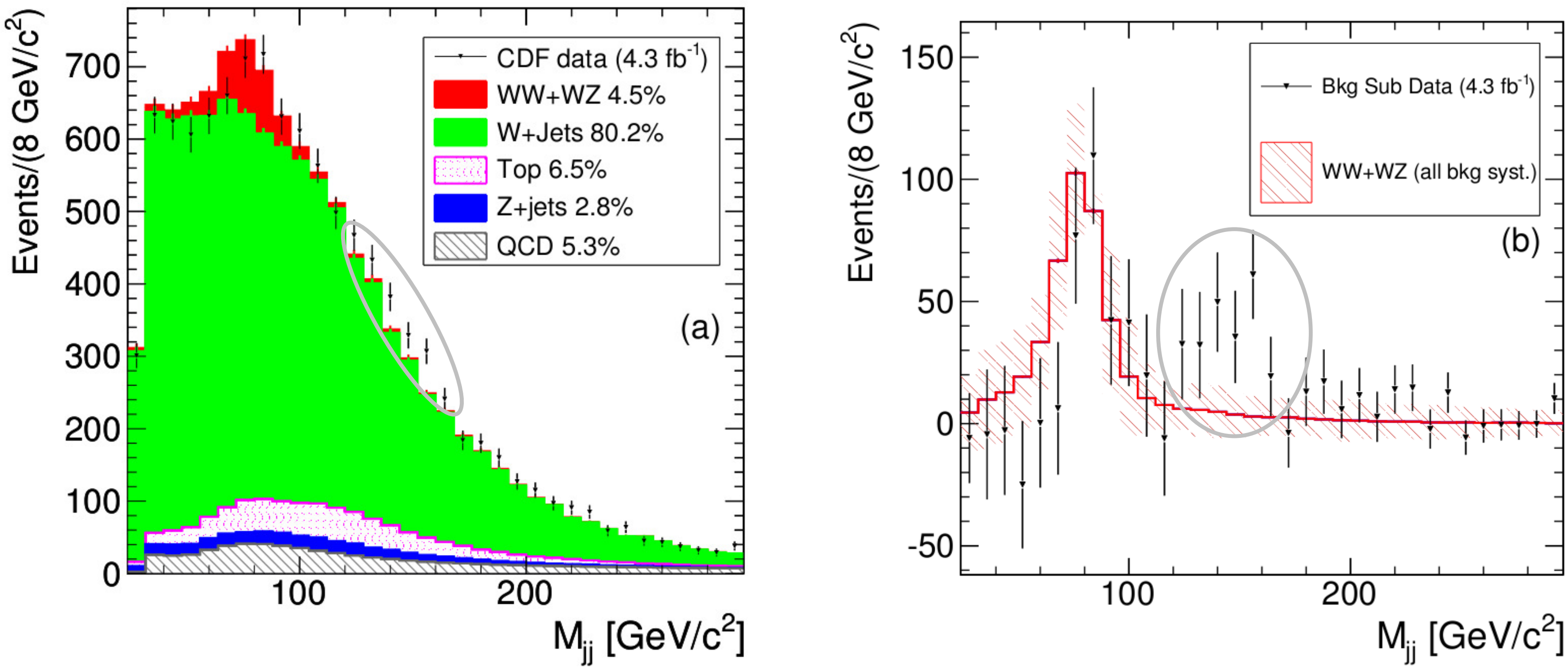

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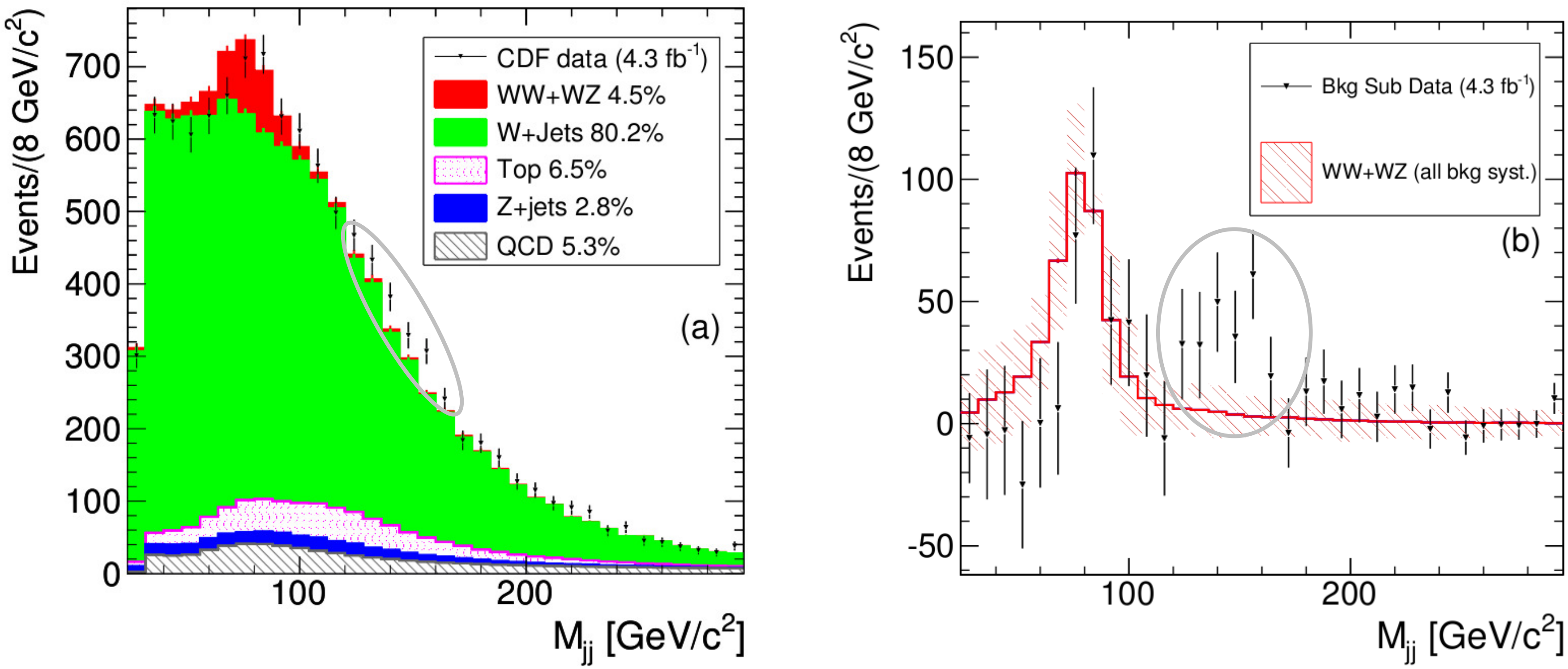

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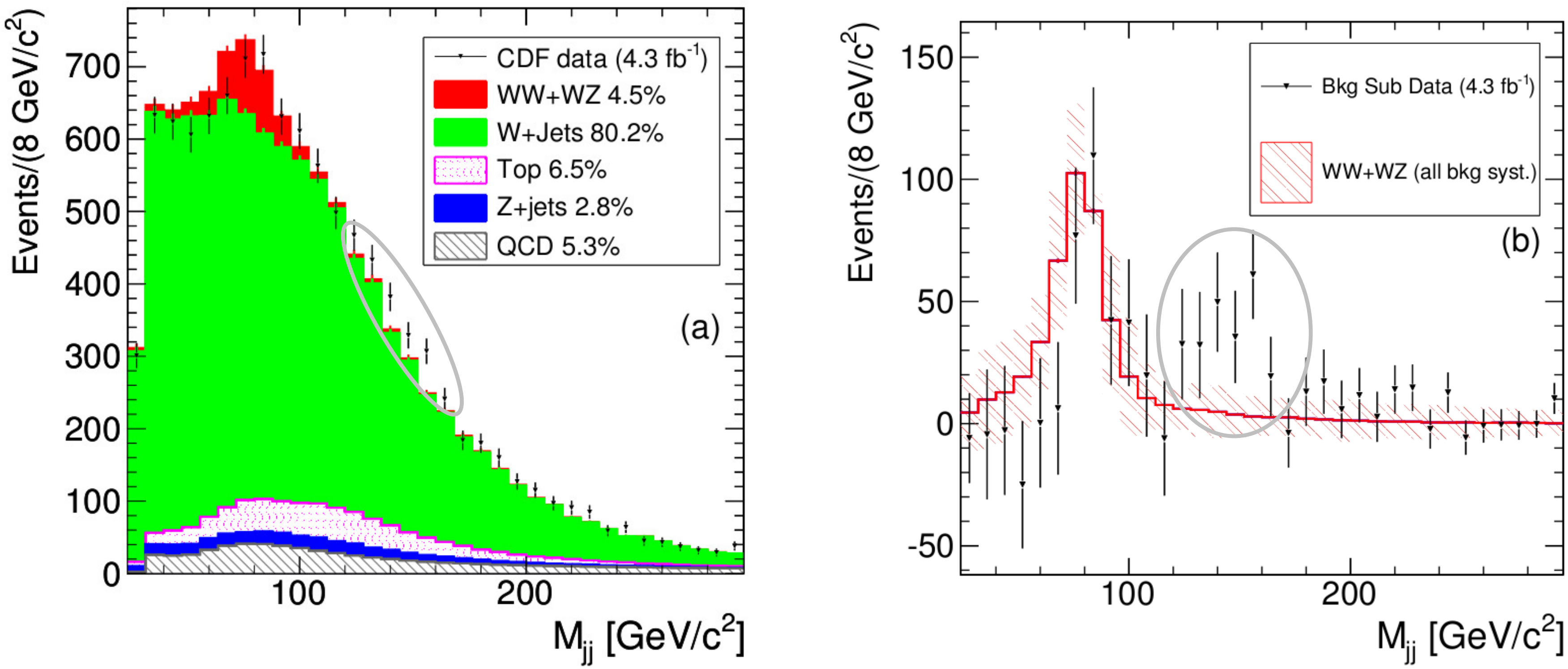
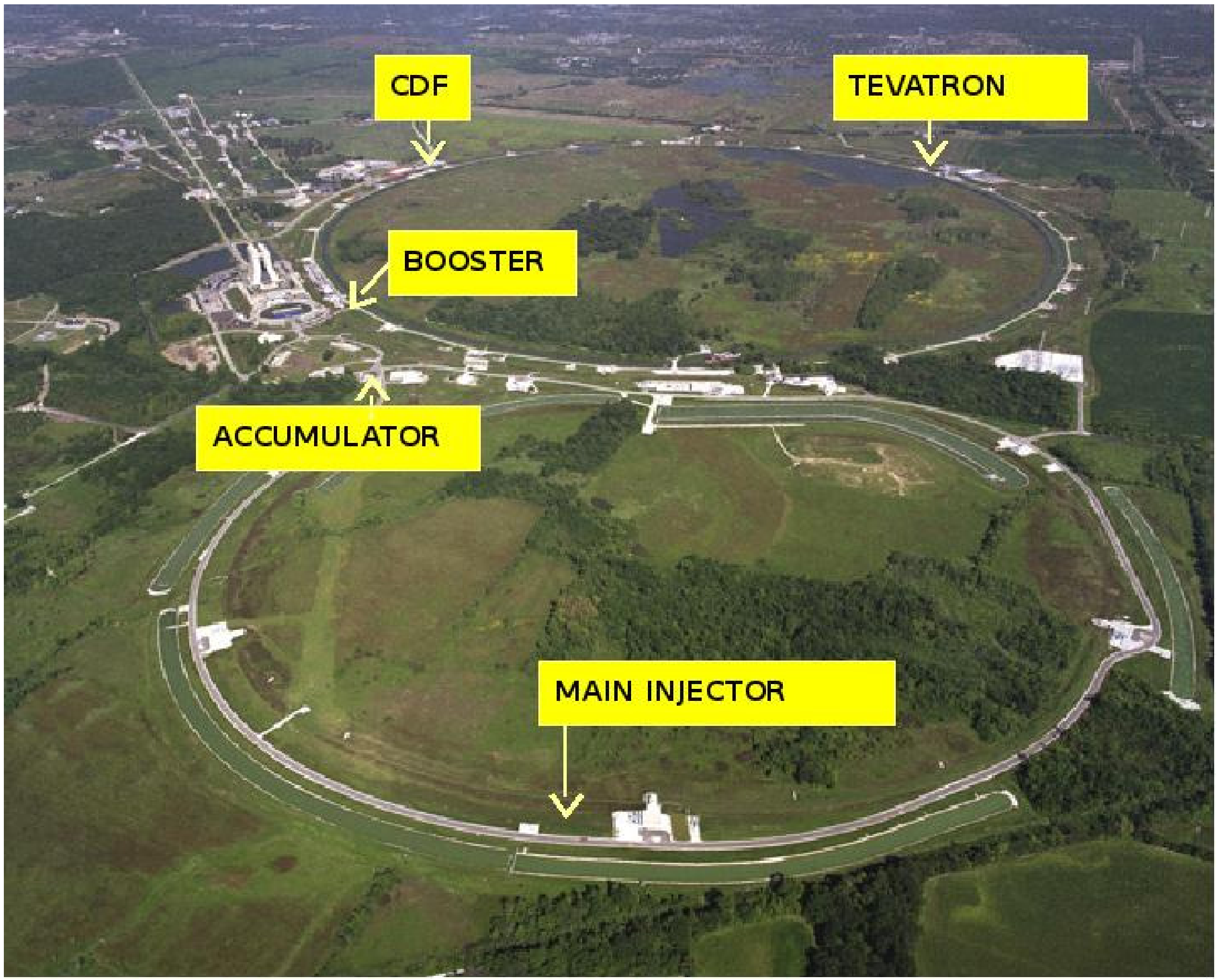


What does it mean?

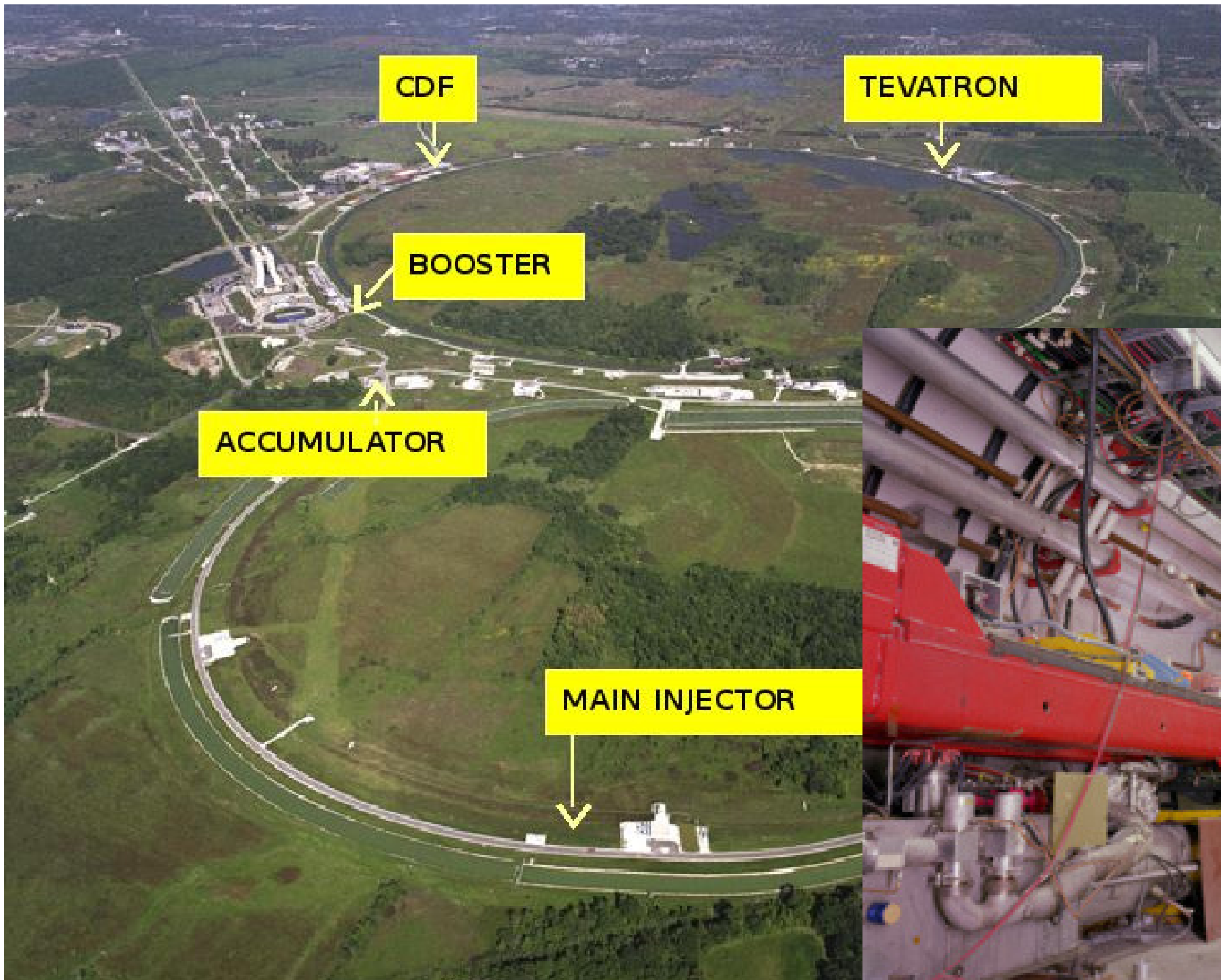
Tevatron and CDF

6.28 km, near Chicago



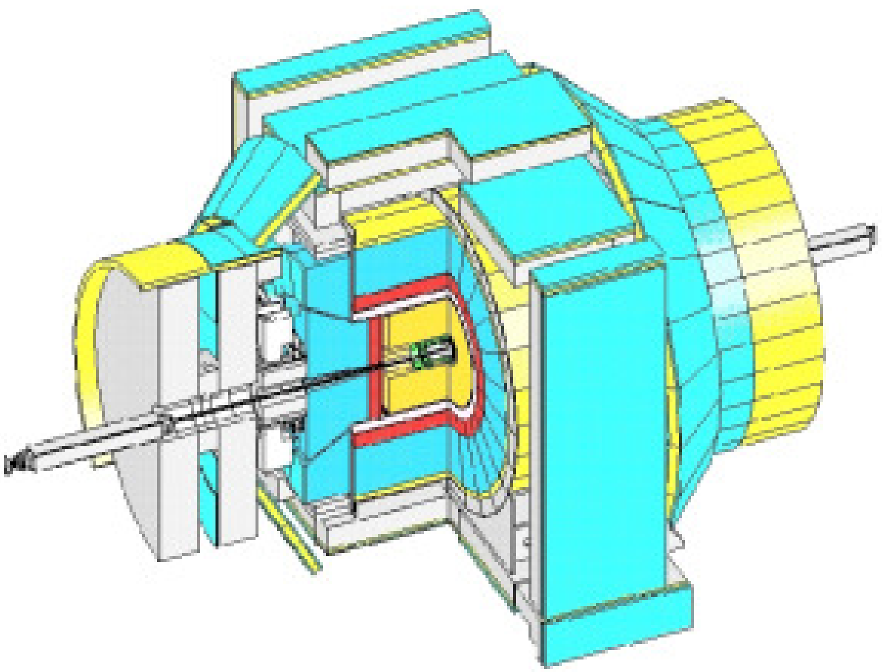
Tevatron and CDF

$p \rightarrow \cdot \leftarrow \bar{p}$ $[\approx 1\,\text{TeV} + 1\,\text{TeV}]$



Tevatron and CDF

CDF: a multipurpose ('hermetic') detector

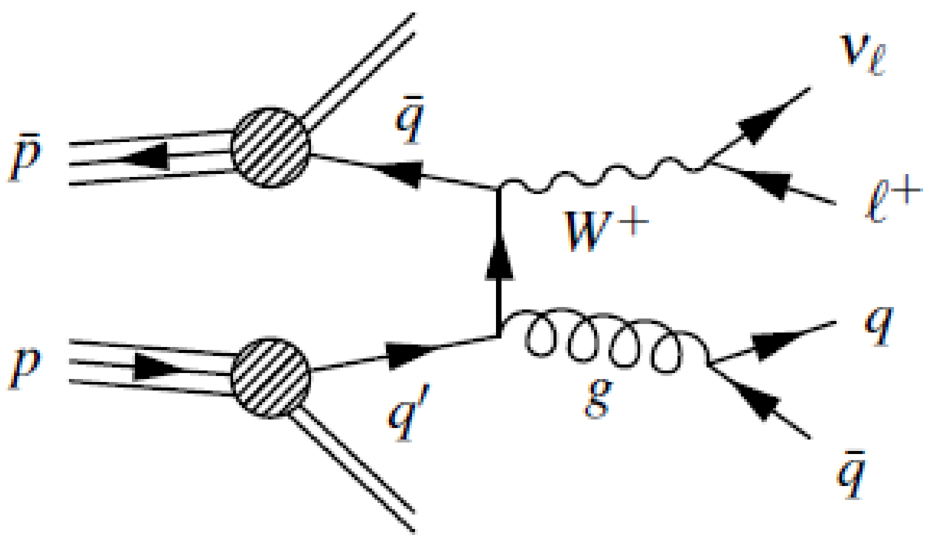


Tevatron and CDF

. . . a large, very sophisticated detector!

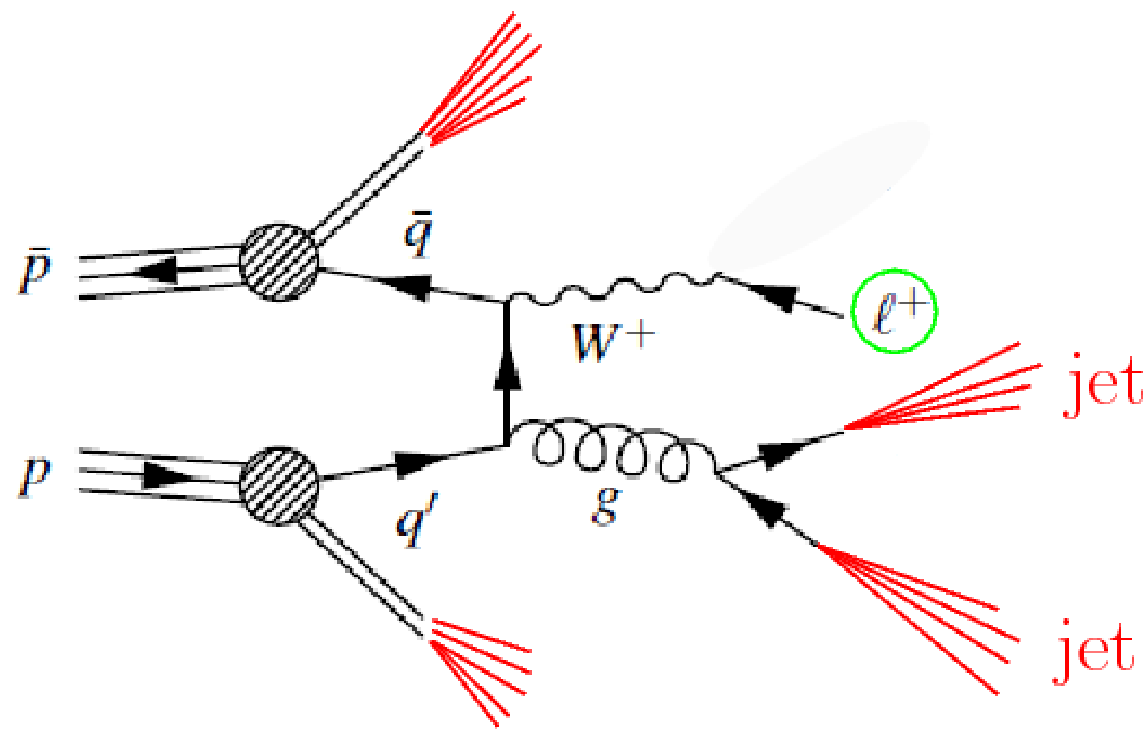
Jet-jet + W

$W + (q\overline{q})$ [+ 'remnants']



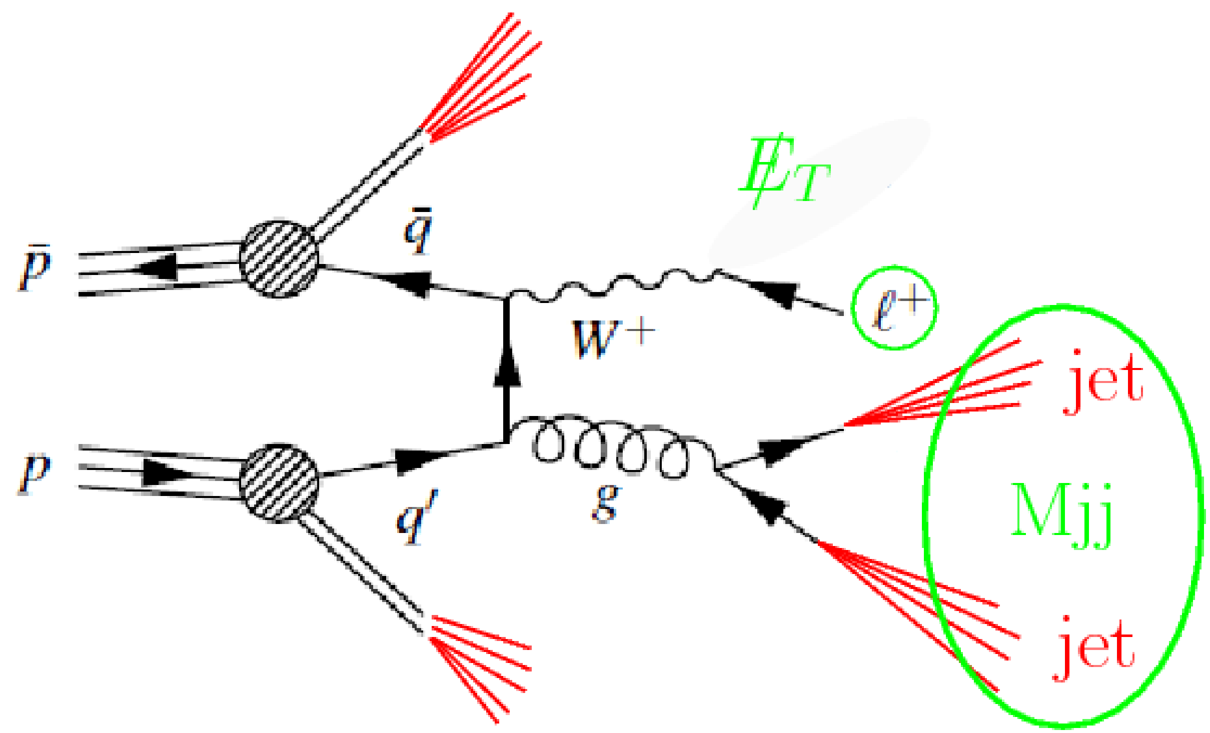
Jet-jet + W

$W + 2$jet [+ much more]



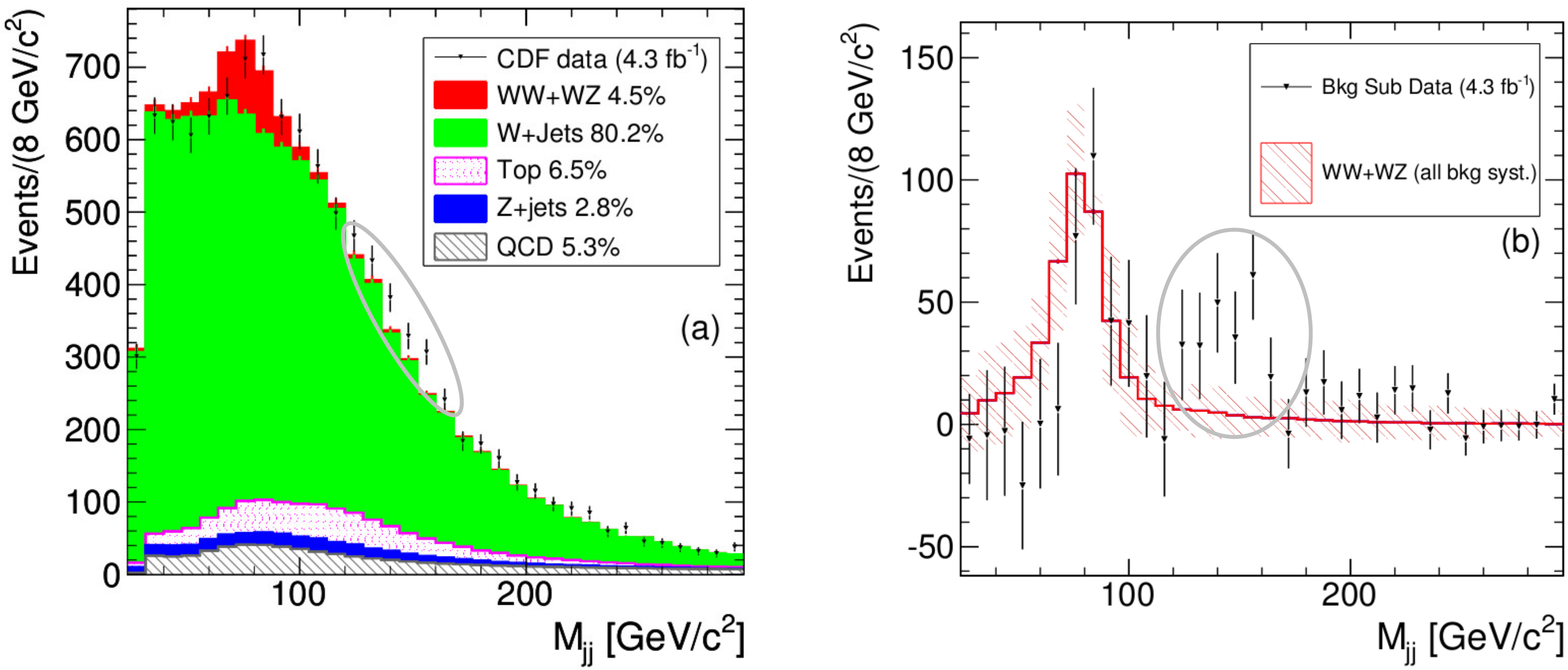
Jet-jet + W

$\Rightarrow M_{jj} + W + \ldots$



The ‘bump’!

Invariant Mass Distribution of Jet Pairs Produced in Association with a W boson in $p\overline{p}$ Collisions at $\sqrt{s} = 1.96$ TeV”, (CDF, 4 aprile 2011)



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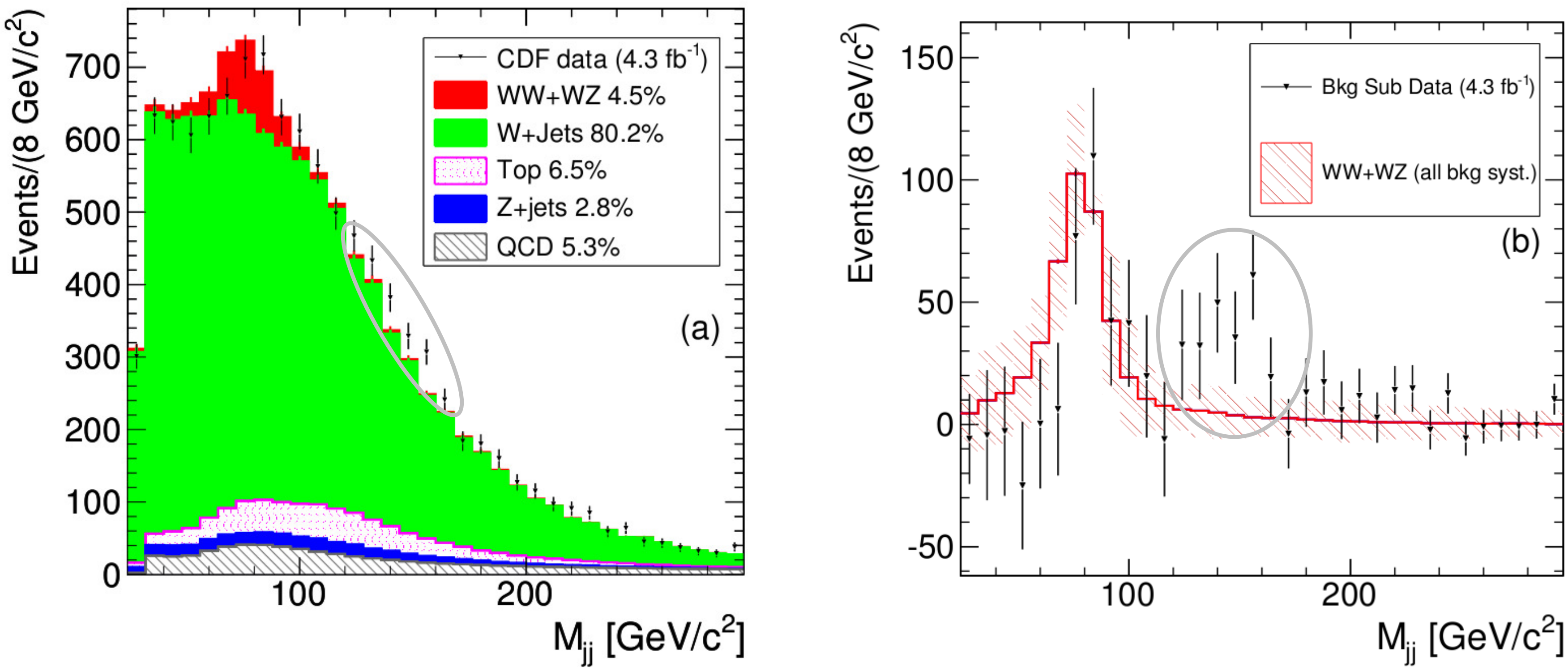

What does it mean?

Sigma and gaussian distribution

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Sigma e probability [gaussian!]

If the random number X is described by a gaussian pdf

$$
\begin{aligned}
P(-\sigma \le X \le +\sigma) &= 68.3\% \\
P(-2\,\sigma \le X \le +2\,\sigma) &= 95.4\% \\
P(-3\,\sigma \le X \le +3\,\sigma) &= 99.73\% \\
1 - P(-3\,\sigma \le X \le +3\,\sigma) &= 0.27\% \\
1 - P(-4\,\sigma \le X \le +4\,\sigma) &= 6.3 \times 10^{-5} \\
\ldots &= \ldots \\
1 - P(-6\,\sigma \le X \le +6\,\sigma) &= 2.0 \times 10^{-9}
\end{aligned}
$$

$$
\begin{aligned}
1 - P(-3.2\,\sigma \le X \le +3.2\,\sigma) &= 1.4 \times 10^{-3} \\
P(X \ge +3.17\,\sigma) &= 7.6 \times 10^{-4} \quad \surd
\end{aligned}
$$

p-value, significance and sigma

"we obtain a p-value of 7.6×10^{-4}, corresponding to a significance of 3.2 standard deviations" ["$3.2\,\sigma$"]

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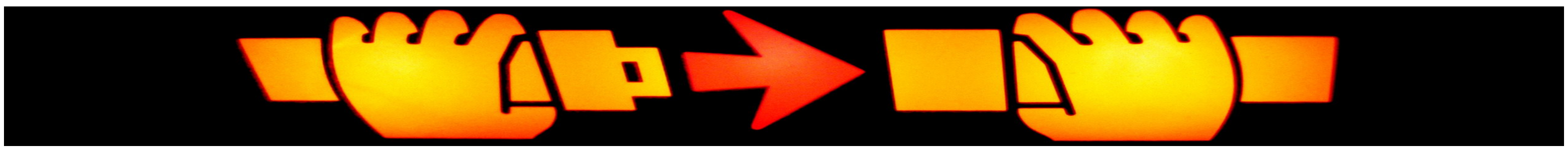

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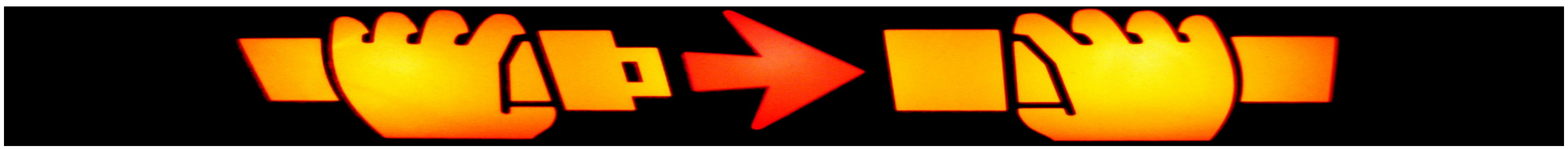

- <u>What is</u> a **p-value**?
- <u>In so far</u> does it provides us a '**significance**'?

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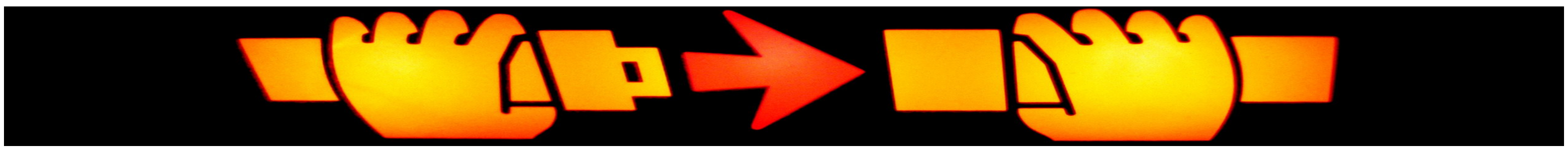

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In short,

- Is 7.6×10^{-4} a **probability**?
- **of what**?

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The New York Times, Tuesday, April 5:

"Physicists at the Fermi National Accelerator Laboratory are planning to announce Wednesday that they have found a suspicious bump in their data that could be evidence of a new elementary particle or even, some say, a new force of nature.

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[Do not ask me how 7.6×10^{-4} becomes $< 2.5 \times 10^{-3}$ (but this can be considere a minor detail. . .)]

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From my experience, journalists might make imprecisions, bad they do not invent pieces of news [. . . at least scientific ones. . . :-)]

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> *"Wednesday afternoon, the CDF collaboration announced that it has evidence of a peak in a specific sample of its data. The peak is an excess of particle collision events that produce a W boson accompanied by two hadronic jets. This peak showed up in a mass region where we did not expect one.*
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$1/1375 = 7.3 \times 10^{-4} \;\Rightarrow\; P(\text{No stat. fluct.}) = 99.93\%$!

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> *This is a big week for particle physicists, and even they will be having many sleepless nights over the coming months trying to grasp what it all means.*
> *That's what happens when physicists come forward, with observational evidence, of what they believe represents something we've never seen before. Even bigger than that: something we never even expected to see.*
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It seems we are understanding well, besides the fact of how 99.9% becomes 99.7%. . .

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Jon Butterworth’s blob on the Guardian, April 9:

> *“The last and greatest breakthrough from a fantastic machine, or a false alarm on the frontiers of physics?*
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But, at the end of the post:

1. "My money is on the false alarm at the moment,. . . "
2. ". . . but I would be very happy to lose it."
3. "And I reserve the right to change my mind rapidly as more data come in!"

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Assolutetly meaningful! (A part from the initial mismatch)

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But how <u>must</u> our convictions <u>rationally</u> change on the light of new experimental data? Is there a logical rule?

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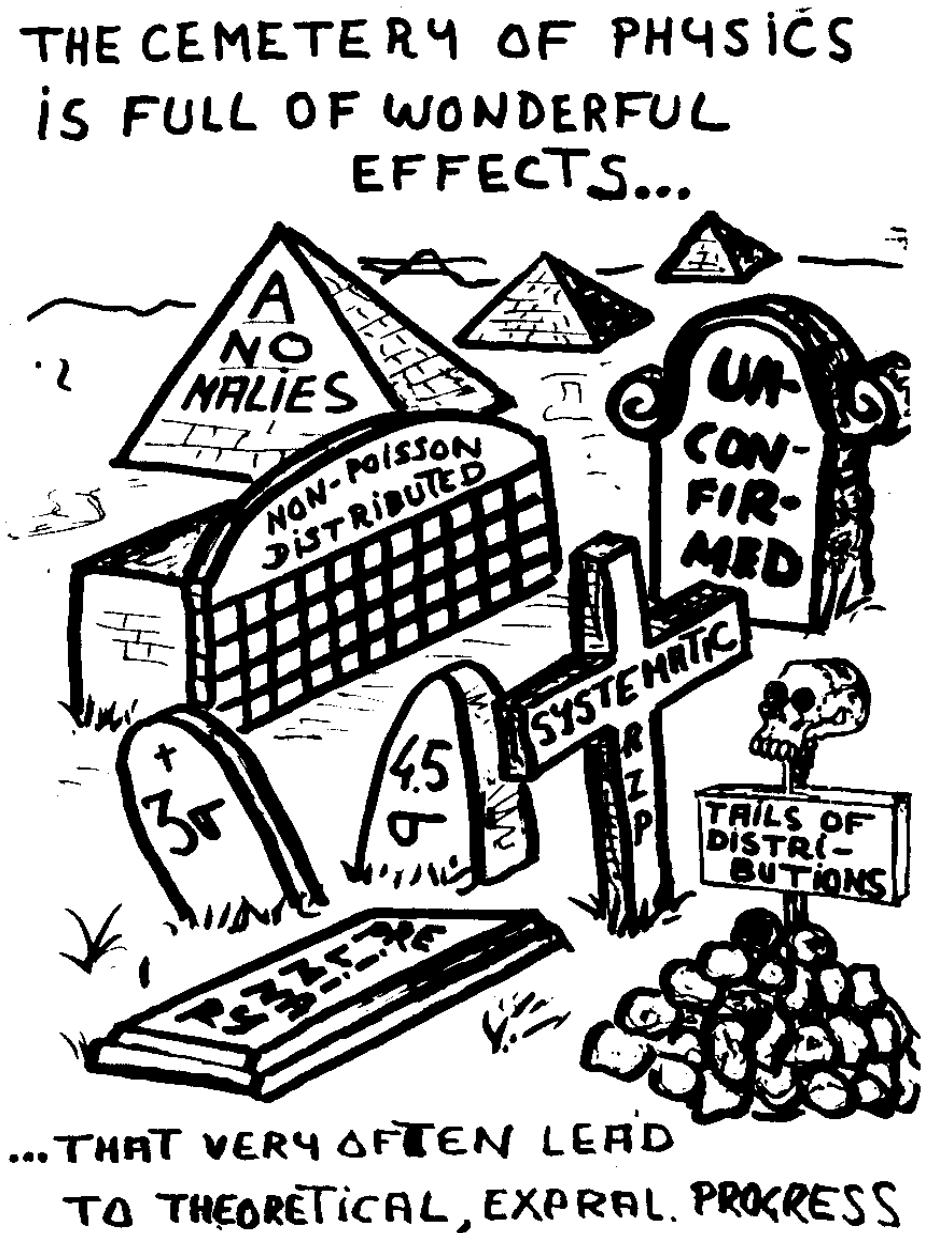
“de Rujula’s paradox”:

> *“If you disbelieve every result presented as having a 3 sigma – or ”equivalently” a 99.7% chance – of being correct. . . You will turn out to be right 99.7% of the times.”*
>
> (Alvaro de Rujula, private communication)

The cemetery of Physics



Alvaro de Rujula

Testing one hypothesis

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Let's review the practice and what is behind it $\Rightarrow$

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It seems OK – *'obvious'*! – but it is indeed naïve for several aspects.

Proof by contradiction . . . ‘extended’. . .

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- Assume that a hypothesis is true;
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is this extension legitimate?

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    E.g. H_i being a Gaussian $f(x \,|\, \mu_i, \sigma_i)$

    $\Rightarrow$ Given any pair or parameters $\{\mu_i, \sigma_i\}$ (i.e. $\forall H_i$), all values of x from $-\infty$ to $+\infty$ are possible.

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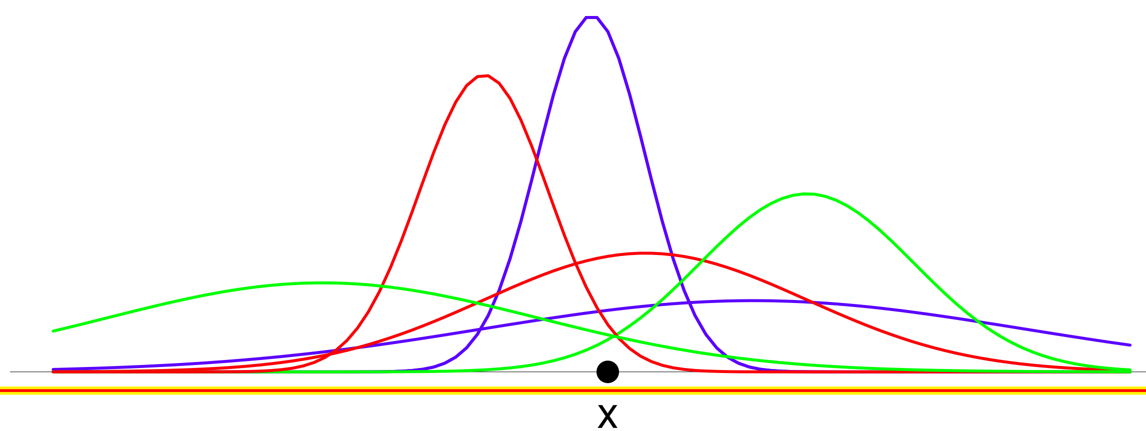
  $\Rightarrow$ Given any pair or parameters $\{\mu_i, \sigma_i\}$ (i.e. $\forall H_i$), all values of x from $-\infty$ to $+\infty$ are possible.

  $\Rightarrow$ Having observed any value of x, none of H_i can be, strictly speaking, falsified.



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⇒ Practically never in the experimental sciences!

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Obviously, this does not means that falsificationism never works, as long as no stochastic processes are involved (randomness inherent to the physical processes, or due to 'errors' in measurement). Certainly it works against itself:

- Science proceeds, in practice, rather differently:

  The natural development of Science shows that researches are carried along the directions that seem more *credibile* (and hopefully fruitful) at a given moment. A behaviour *"179 degrees or so out of phase from Popper's idea that we make progress by falsificating theories"* (Wilczek, `http://arxiv.org/abs/physics/0403115`)

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Obviously, this does not means that falsificationism never works, as long as no stochastic processes are involved (randomness inherent to the physical processes, or due to 'errors' in measurement). Certainly it works against itself:

$\Rightarrow$ logically speaking, falsificationism has to be considered . . . falsified!

Falsificationism and statistics

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But from the impossible to the improbable there is not just a question of quantity, but a question of quality.

This mechanism, logically flawed, is particularly dangerous because is deeply rooted in most scientists, due to education and custom, although not supported by logic.

⇒ Basically responsible of all fake claims of discoveries in the past decades.

[*I am particularly worried about claims concerning our health, or the status of the planet, of which I have no control of the experimental data.*]

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B) ~~**if** $C_i \xrightarrow[\text{small probability}]{} E$, and **we observe** E~~ — NO

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~~"most likely false"~~

But it is behind the rational behind the statistical hypothesis tests!

Example

An Italian citizen is chosen at random and sent to take an AIDS test (test is not perfect, as it is the case in practice). *Simplified model*:

$$
\begin{aligned}
P(\text{Pos}\,|\,\text{HIV}) &= 100\% \\
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H_1='HIV' (Infected) E_1 = Positive

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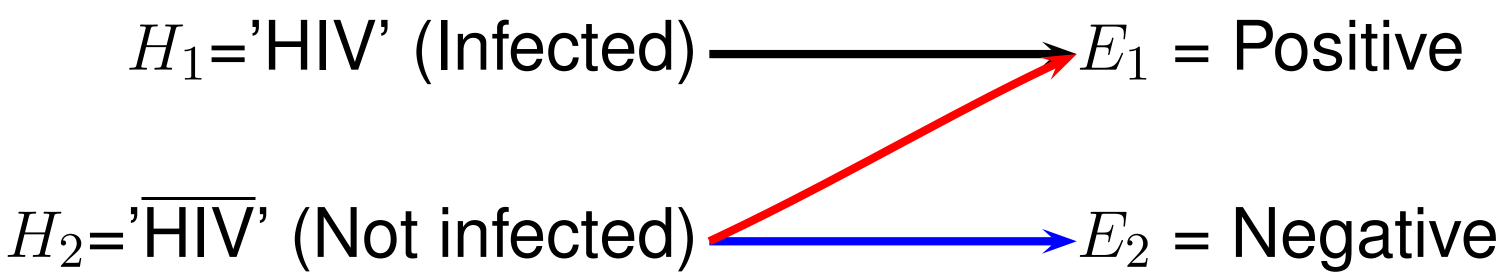

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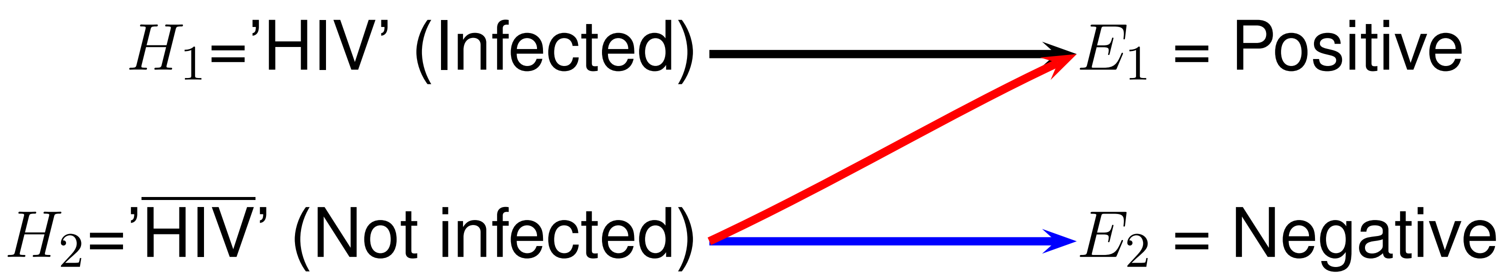

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Result: ⇒ Positive

HIV or not HIV?

What shall we conclude?

Being $P(\text{Pos} \,|\, \overline{\text{HIV}}) = 0.2\%$ and having observed 'Positive', can we say

- "It is practically impossible that the person is healthy, since it was practically impossible that an healthy person would result positive"?

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Instead, $P(\text{HIV}\,|\,\text{Pos, randomly chosen Italian}) \approx 45\%$

Think about it (a crucial information is missing!)

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$\Rightarrow$ Serious mistake! (not just 99.8% instead of 98.3%)

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Pay attention no to arbitrary revert conditional probabilities:

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In particular

- A cause might produce a given effect with very low probability, and nevertheless could be the most probable cause of that effect, often the only one!

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→ What was the probability to give exactly that number?:

$$\begin{aligned} P(X = 3.1416 \,|\, H_0) &= \int_{3.14155}^{3.14165} f_{\mathcal{G}}(x \,|\, \mu, \sigma) dx \\ &\approx f_{\mathcal{G}}(3.1416 \,|\, \mu, \sigma) \times \Delta x \\ &\approx f_{\mathcal{G}}(3.1416 \,|\, \mu, \sigma) \times 0.0001 \\ &\approx 39 \times 10^{-6} \end{aligned}$$

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→ What is the probability that X comes from H_0?

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- Certainly NOT $\approx 39 \times 10^{-6}$;
- Indeed, it is exactly 1, since H_0 is the only cause which can produce that effect:

$$\begin{aligned} P(X = 3.1416 \,|\, H_0) &\approx 39 \times 10^{-6} \\ P(H_0 \,|\, X = 3.1416) &= 1\,. \end{aligned}$$

Probability of something else. . .

Besides the fact that the reasoning based only on the probability of the event given the cause is logically flawed, the ‘techical issue’ of low probability events which would lead to reject any hypothesis forces the statistician to rethink the question. . .

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$\rightarrow$ what matter is not the probability of the X, but rather the probability of X or of any other less probable number (or a number farther than X from the expected value – the story is a bit longer…):

$$P(X \geq 3.1416) \;=\; \int_{3.14155}^{+\infty} f_{\mathcal{G}}(x\,|\,\mu,\sigma)dx \approx 44\%$$

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$$P(X \geq 3.1416) \; [= P(X \geq x_{obs})] \;\; \Rightarrow \;\; \text{`p-value'}$$

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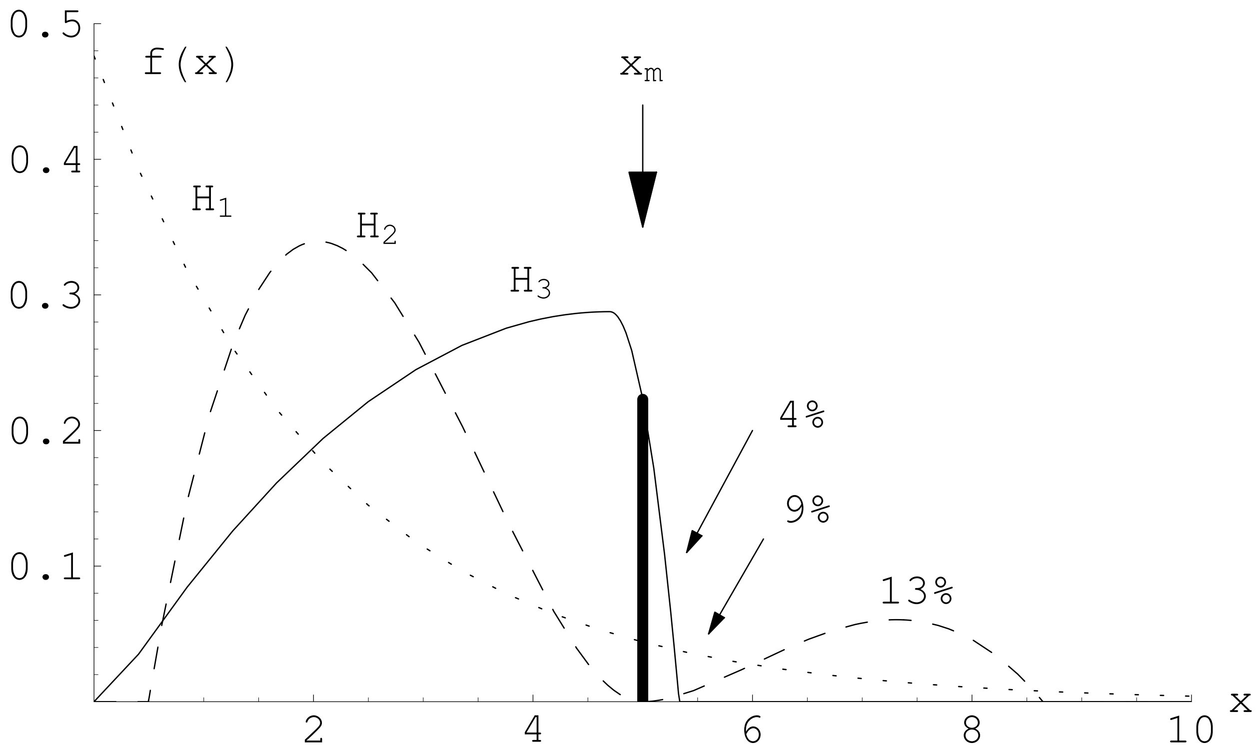
- ⇒ The statisticians are happy... scientists and general public cheated...

Comparing three hypotheses

Which hypothesis is favored by the experimental observation x_m?



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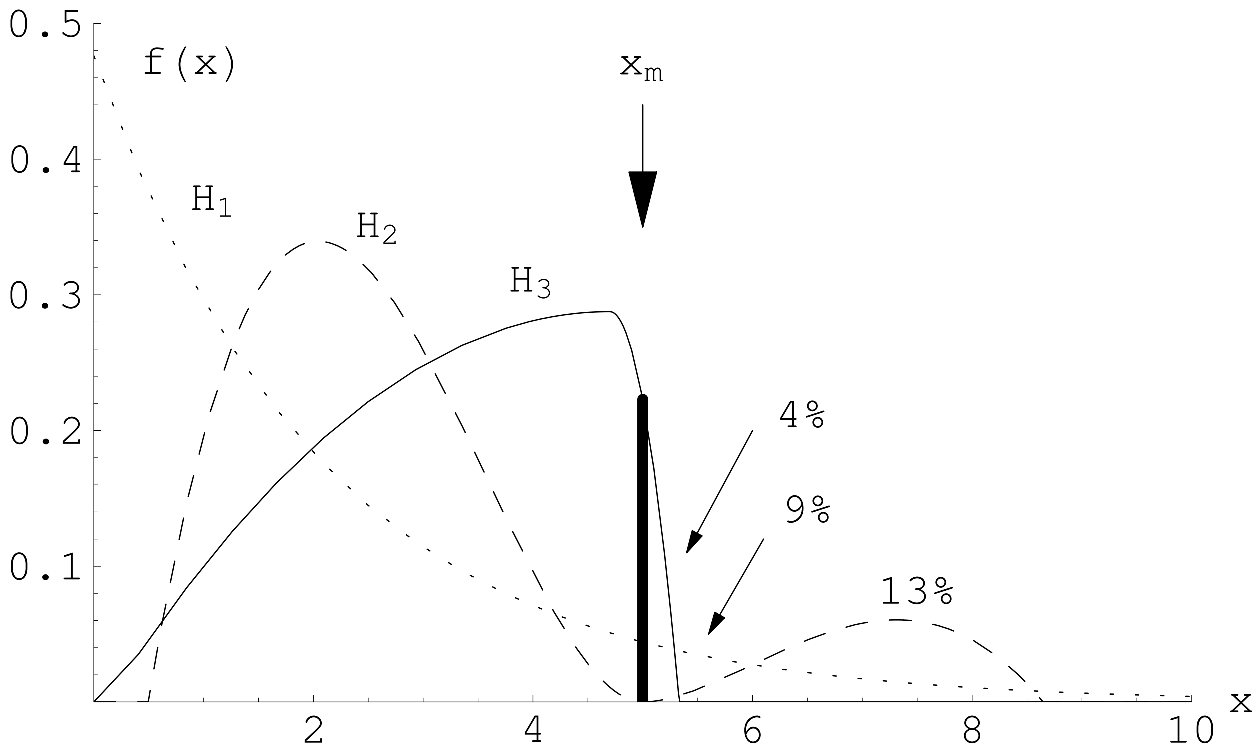

$$P(x_m \,|\, H_3) > P(x_m \,|\, H_1) > P(x_m \,|\, H_2) = 0 \quad (\,!\,)$$

Even if $P(x_m \,|\, H_i) \rightarrow 0$ (it depends on resolution)

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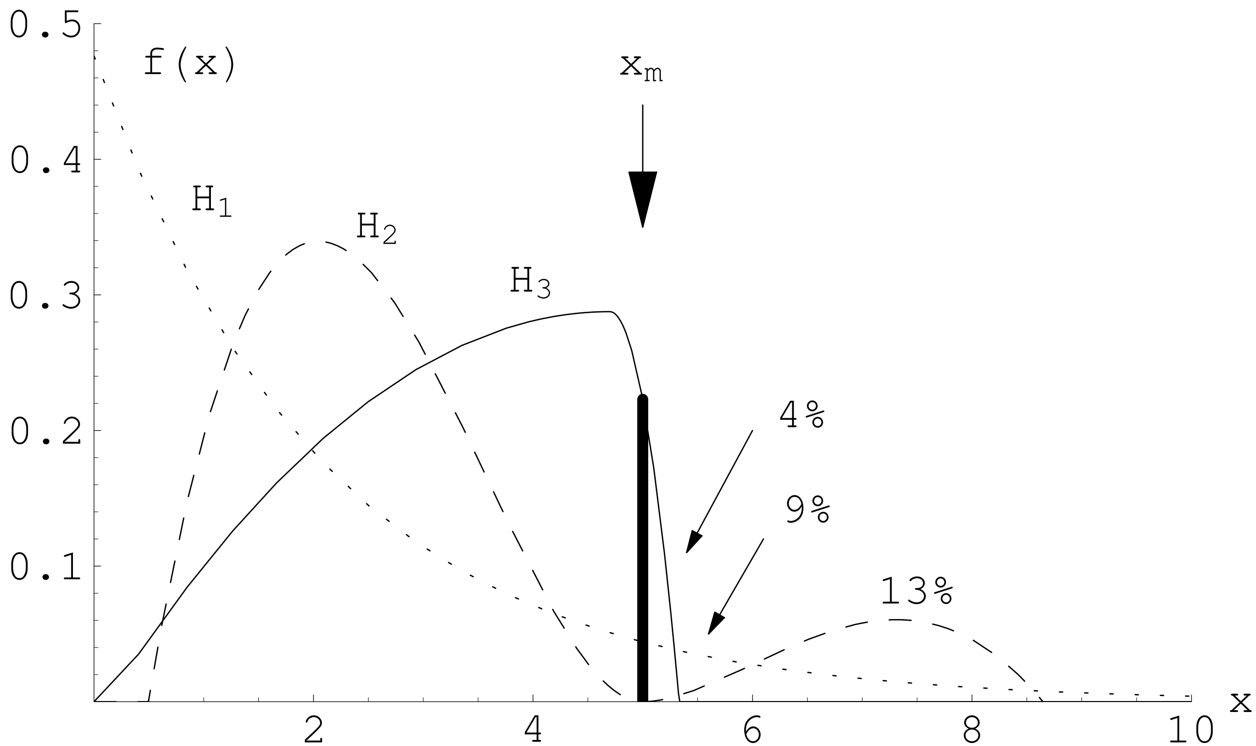

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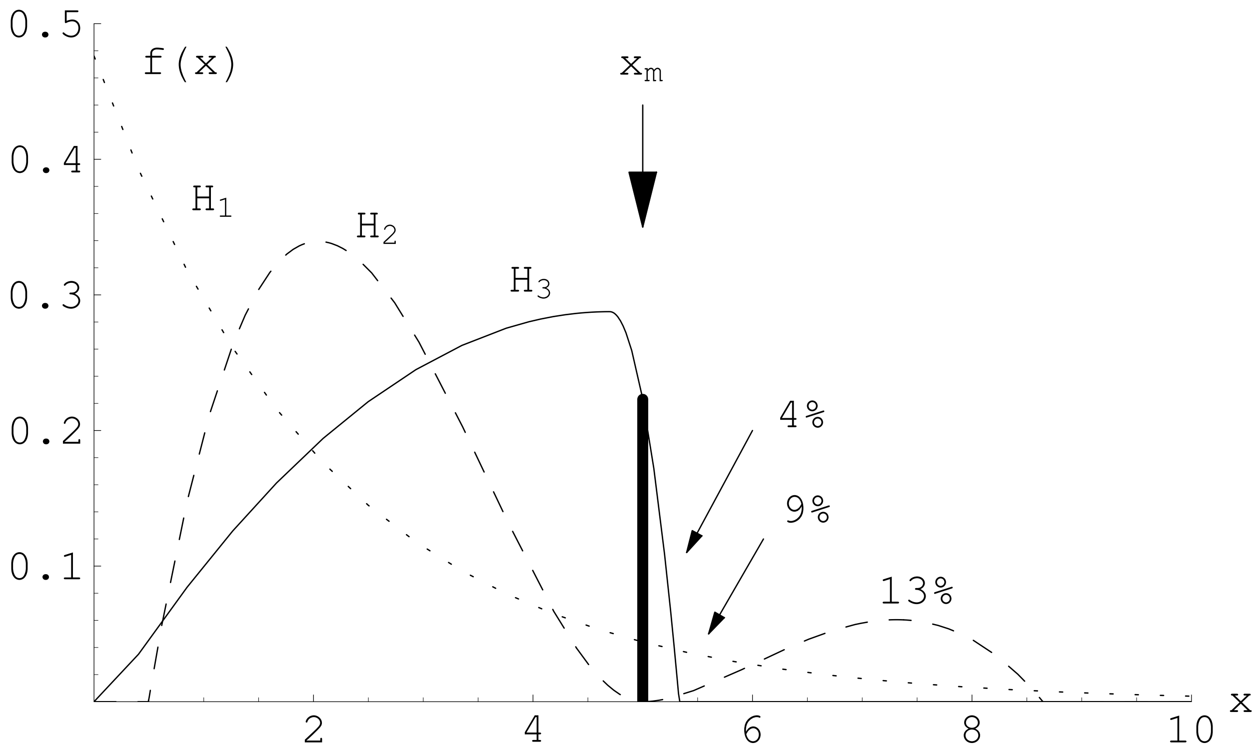

In particular, the hypothesis H_2 is (truly) falsified (impossible!), although it yields the largest 'p-value', or 'probability of the tail(s)'

An irrilevant experiment

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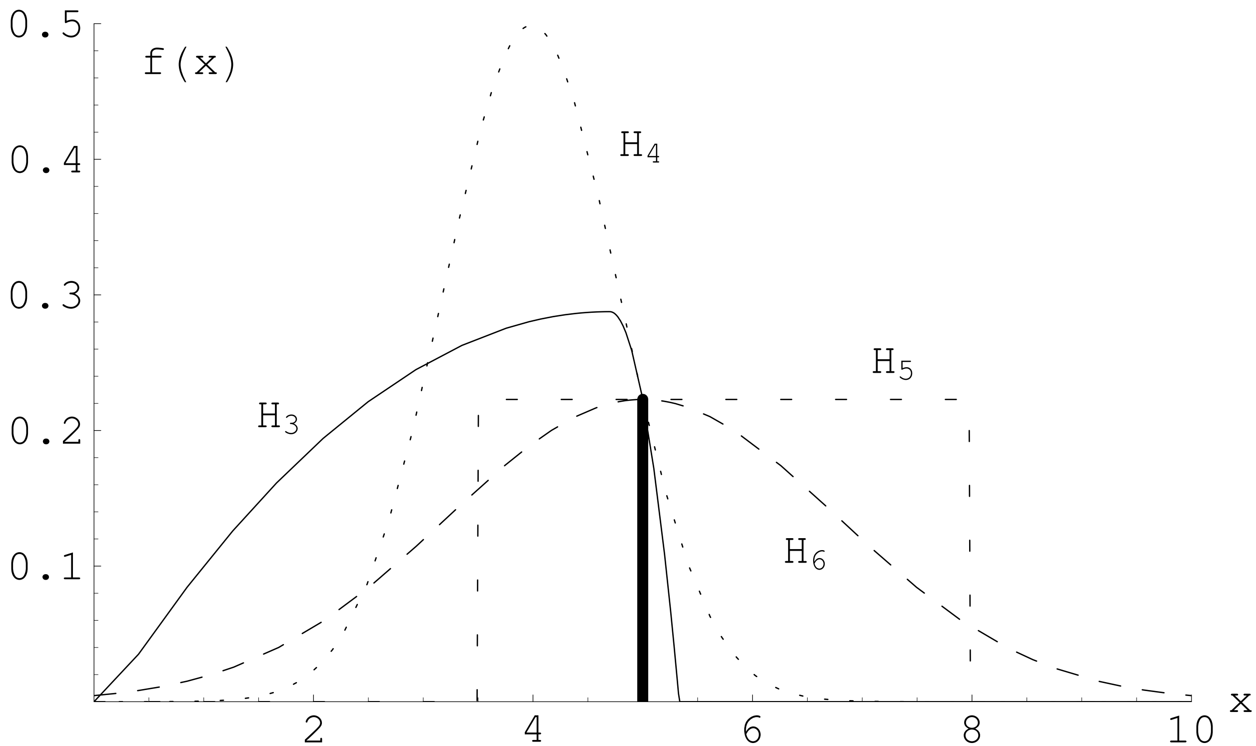

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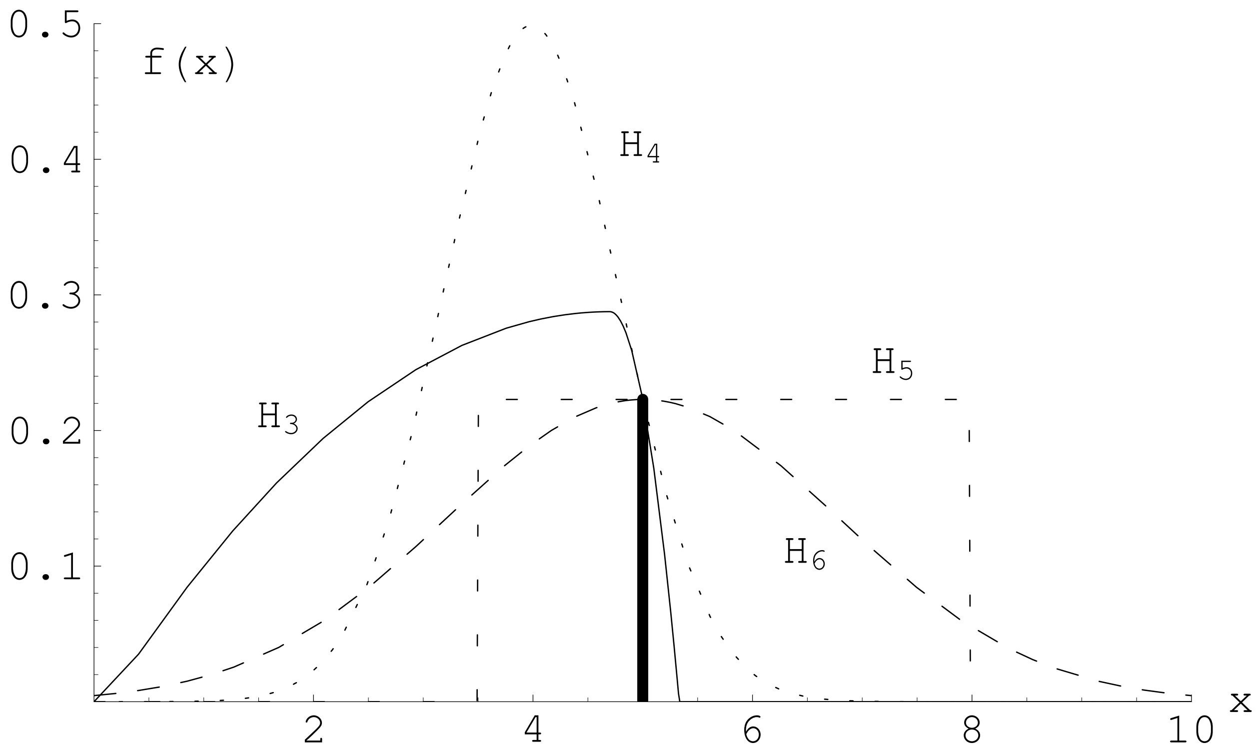

$$P(x_m \,|\, H_3) = P(x_m \,|\, H_4) = P(x_m \,|\, H_5) = P(x_m \,|\, H_6)$$

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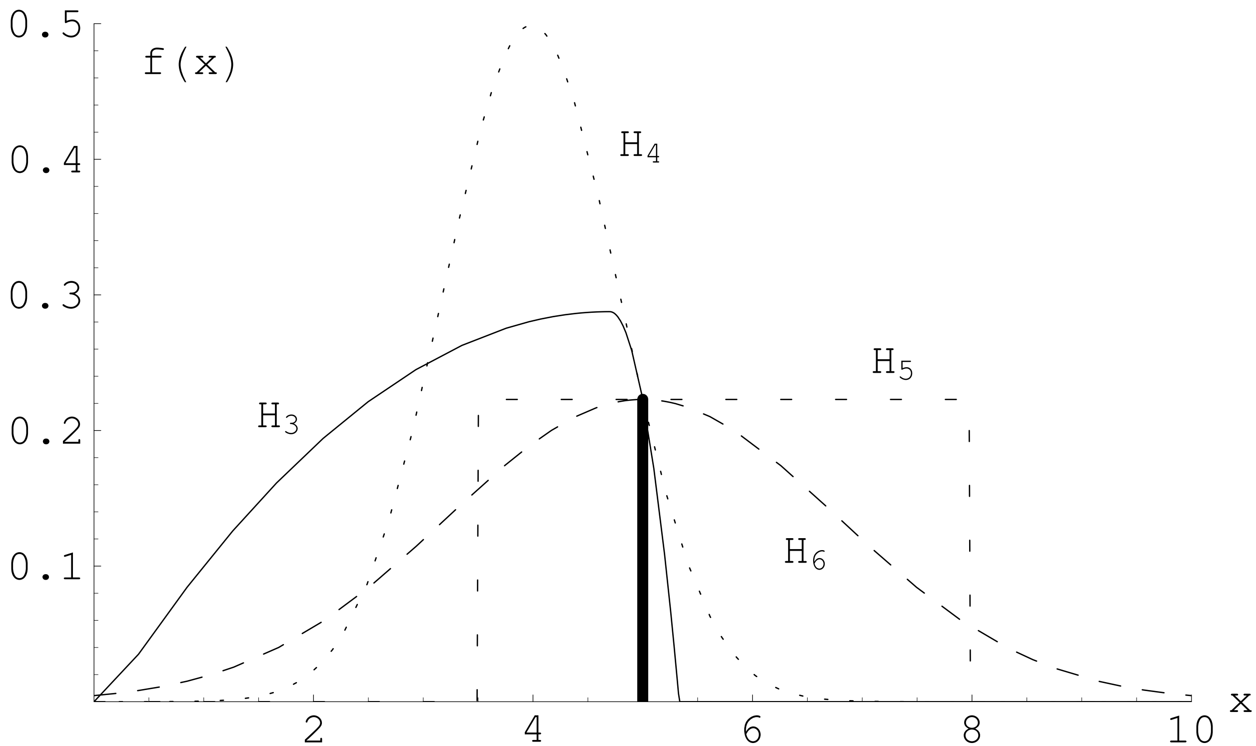

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$\rightarrow$ we mantain our opinions about H_i

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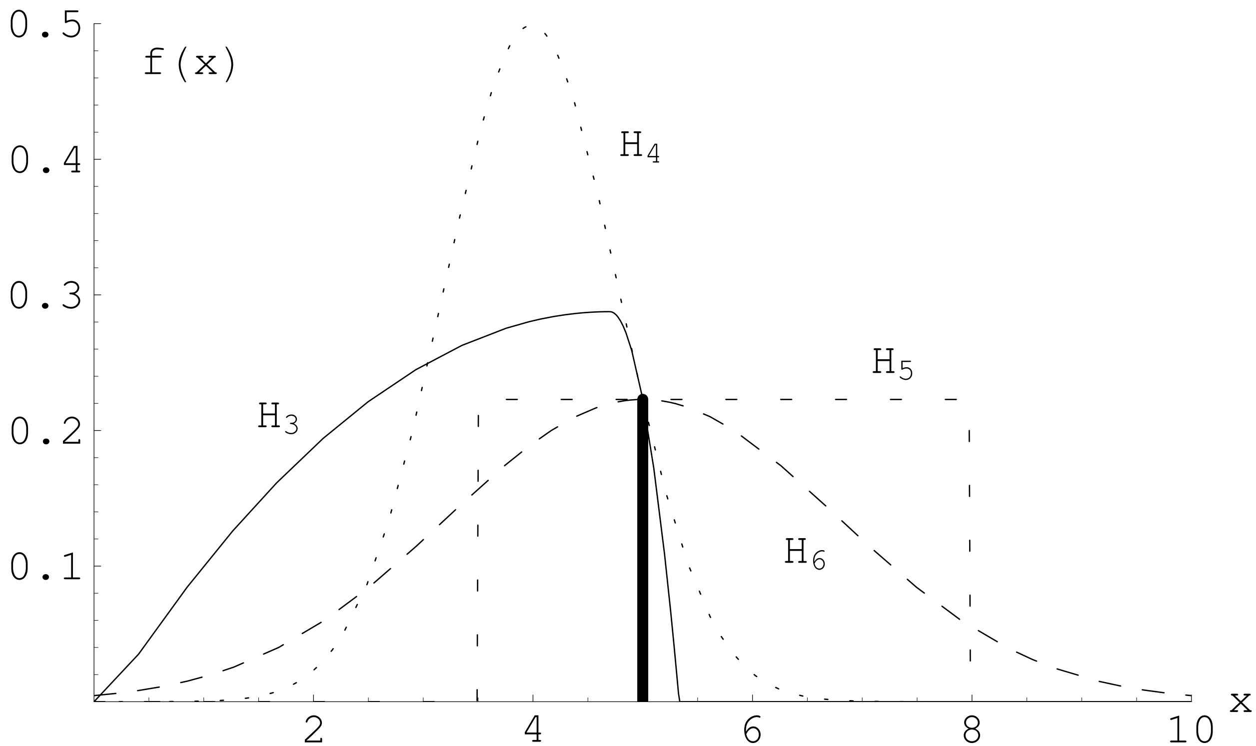

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⇒ *The experimental result is irrelevant!*

⇒ *...no matter what the different the p-values are!*

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Which p-value?...

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$\longrightarrow$ the test variable ('θ') is absolutely arbitrary:

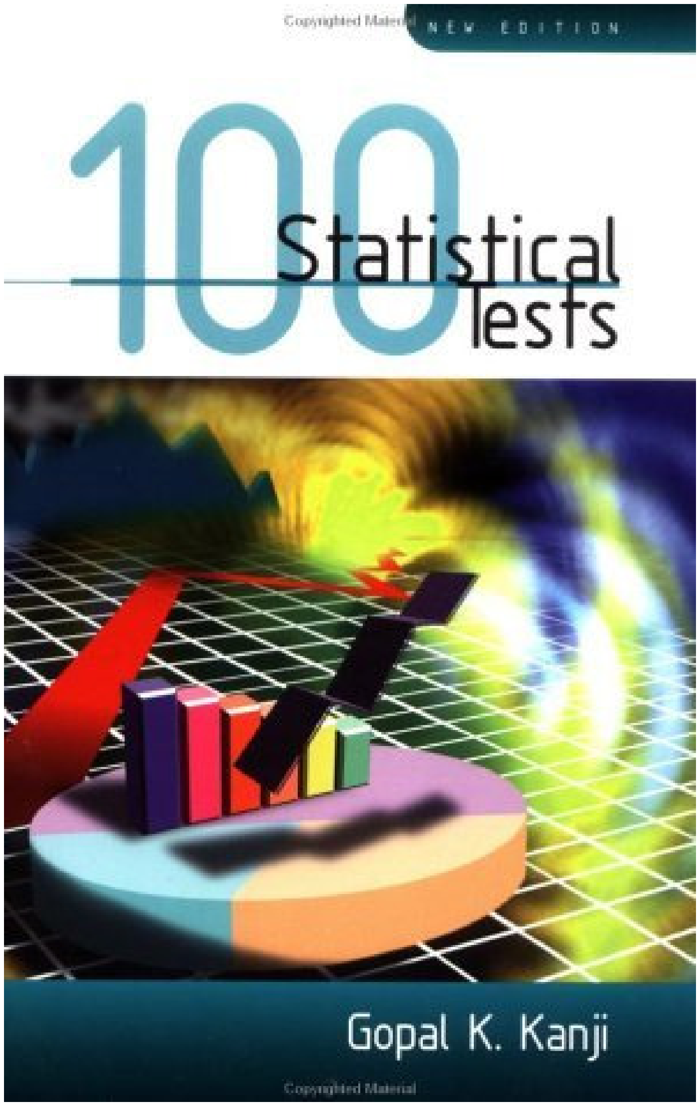
$$\theta = \theta(\mathbf{x})$$

$$\rightarrow f(\theta) \quad \text{[p.d.f]}$$

Experiment: $\rightarrow \theta_{mis} = \theta(\mathbf{x}_{mis})$

p-value $= P(\theta \geq \theta_{mis})$ ('one tail')

Which p-value?...



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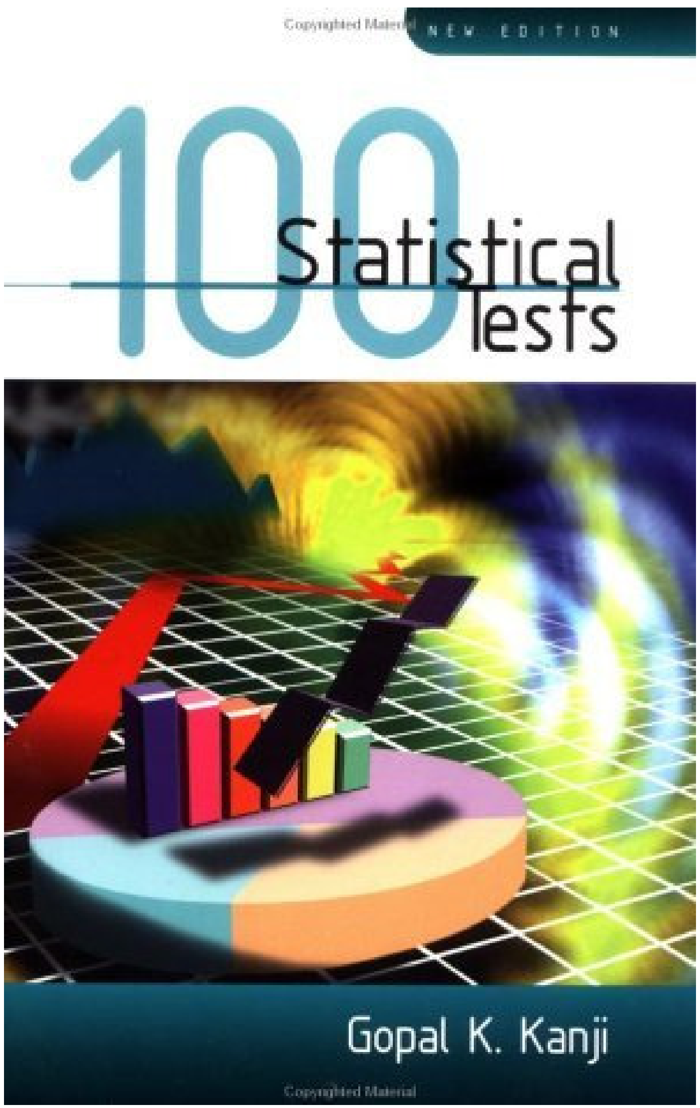

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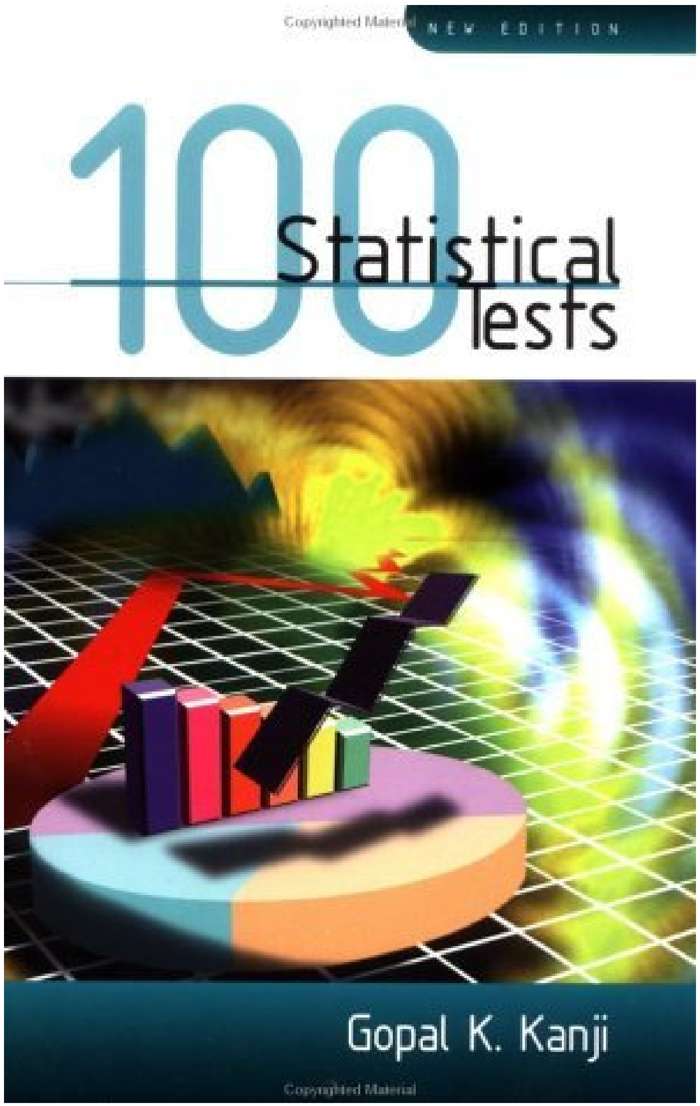
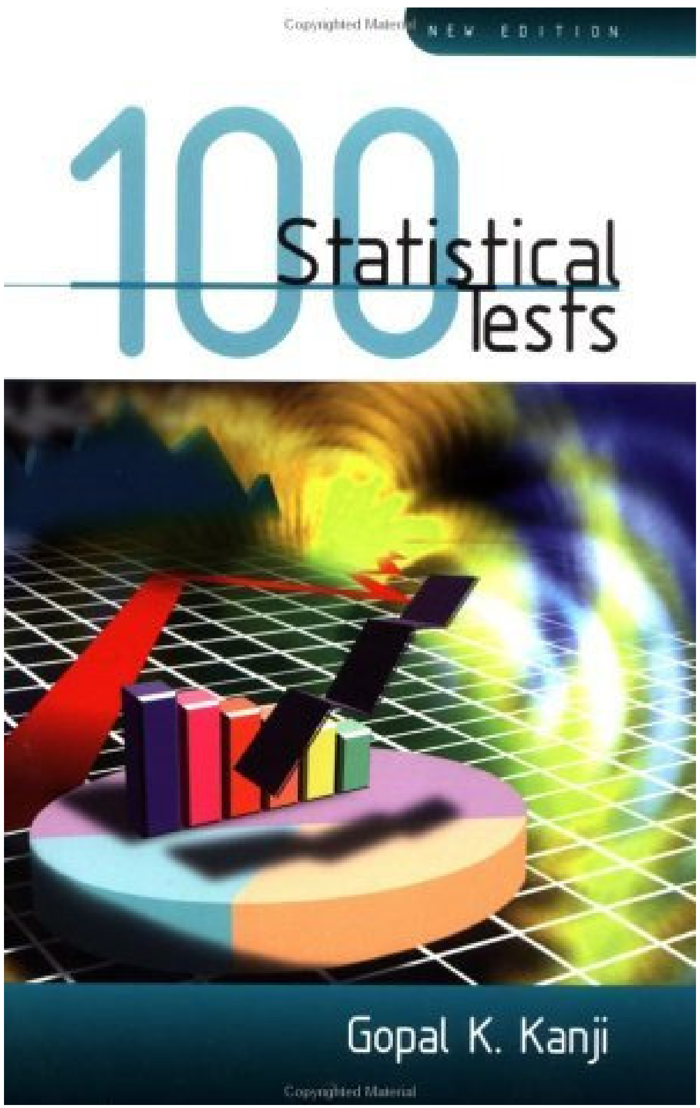


- far from exhaustive list,
- with arbitrary variants:

Which p-value?. . .



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$\Rightarrow$ practitioner chose the one that provide the result they like better:
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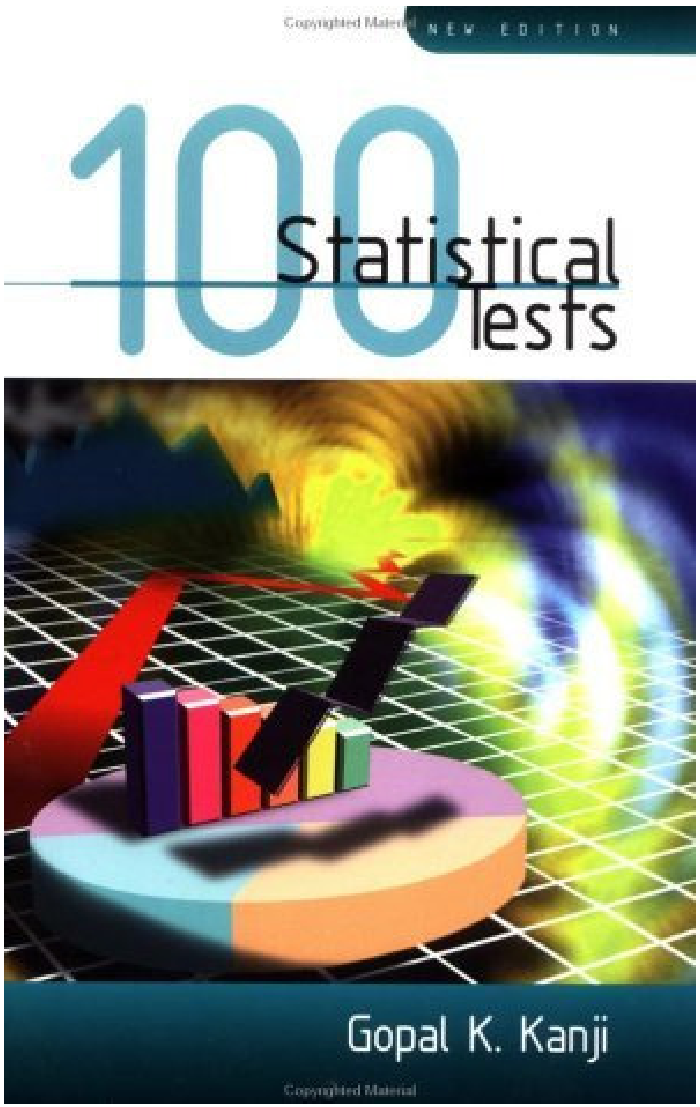

- far from exhaustive list,
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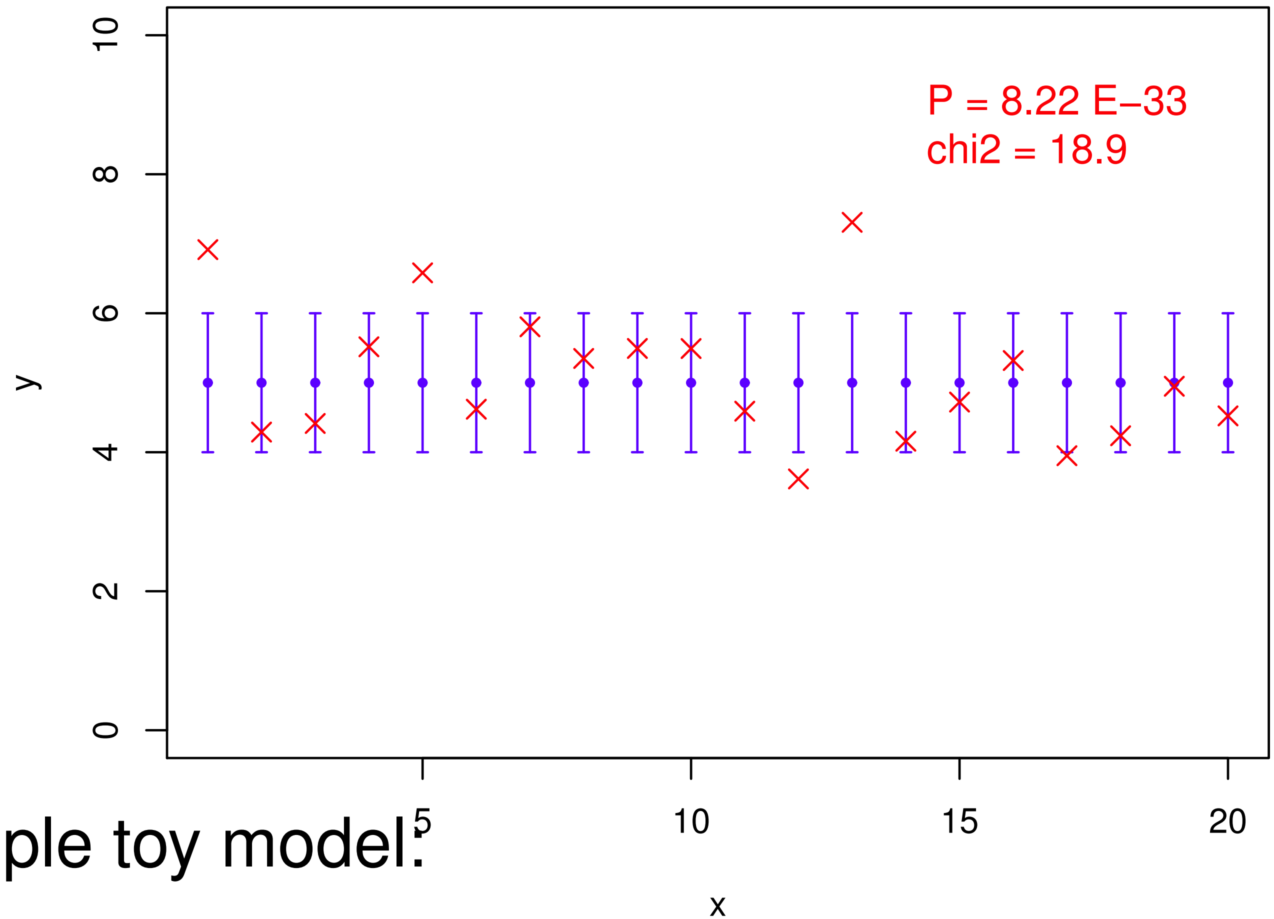
⇒ practitioner chose the one that provide the result they like better:
→ *like if you go around until "someone agrees with you"*

- personal **'golden rule'**: "the more exotic is the name of the test, the less I believe the result", because I'm pretty shure that several 'normal' tests have been descarded in the meanwhile...

χ^2... the mother of all p-values

Theory Vs experiment *(bars: expectation uncertainty)*:



Very simple toy model:

- True value of y: 5, independently of x (a.u.);
- Gaussian instrumental error with $\sigma = 1$.

Probability of the data sample

$P = 8.22 \times 10^{-33}$ is the probability of the 'configuration' of experimental points:

- obtained multiplying the probability of each point (independent measurements):

$$P = \prod_i P_i$$

where

$$P_i = \int_{y_{m_i}-\Delta y/2}^{y_{m_i}+\Delta y/2} f(y)dy$$

- as seen, P_i depends on the 'resoluzion' Δy (instrumental 'discretization'):

$$\rightarrow \text{we use} \quad \Delta y = \frac{1}{10}\,\sigma$$

‘Distance’ Experiment-theory: χ^2

The costruction of the χ^2 is very popular (usually in first lab. courses – ‘Fisichetta’):

$$\chi^2 = \sum_i \left(\frac{y_{m_i} - y_{th_i}}{\sigma_i}\right)^2 \rightarrow \sum_i \left(\frac{y_{m_i} - y_0}{\sigma}\right)^2$$

$$\begin{aligned}
\chi^2 &\sim \Gamma(\nu/2, 1/2) && [\rightarrow \nu = 20] \\
\text{E}[\chi^2] &= \nu && [\rightarrow 20] \\
\text{Var}[\chi^2] &= 2\nu && [\rightarrow 40] \\
\text{Std}[\chi^2] &= \sqrt{2\nu} && [\rightarrow 6.3] \\
\Rightarrow & \quad \boxed{\chi^2 = 20 \pm 6}
\end{aligned}$$

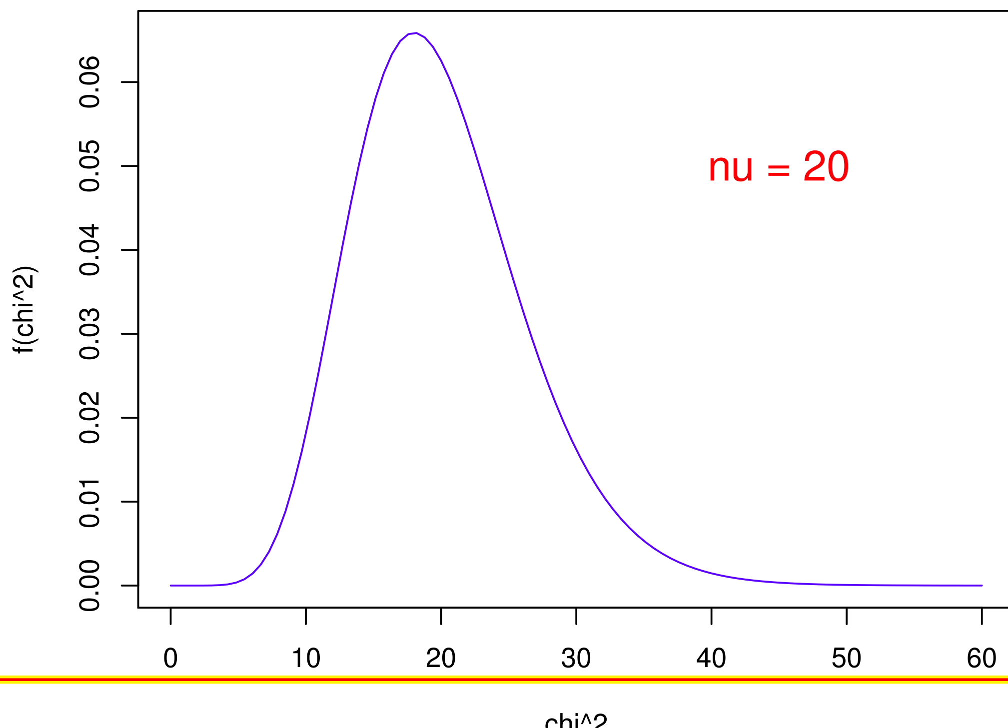
Our expectations about χ^2

$$\mathrm{E}[\chi^2] = \nu \qquad [\to 20]$$

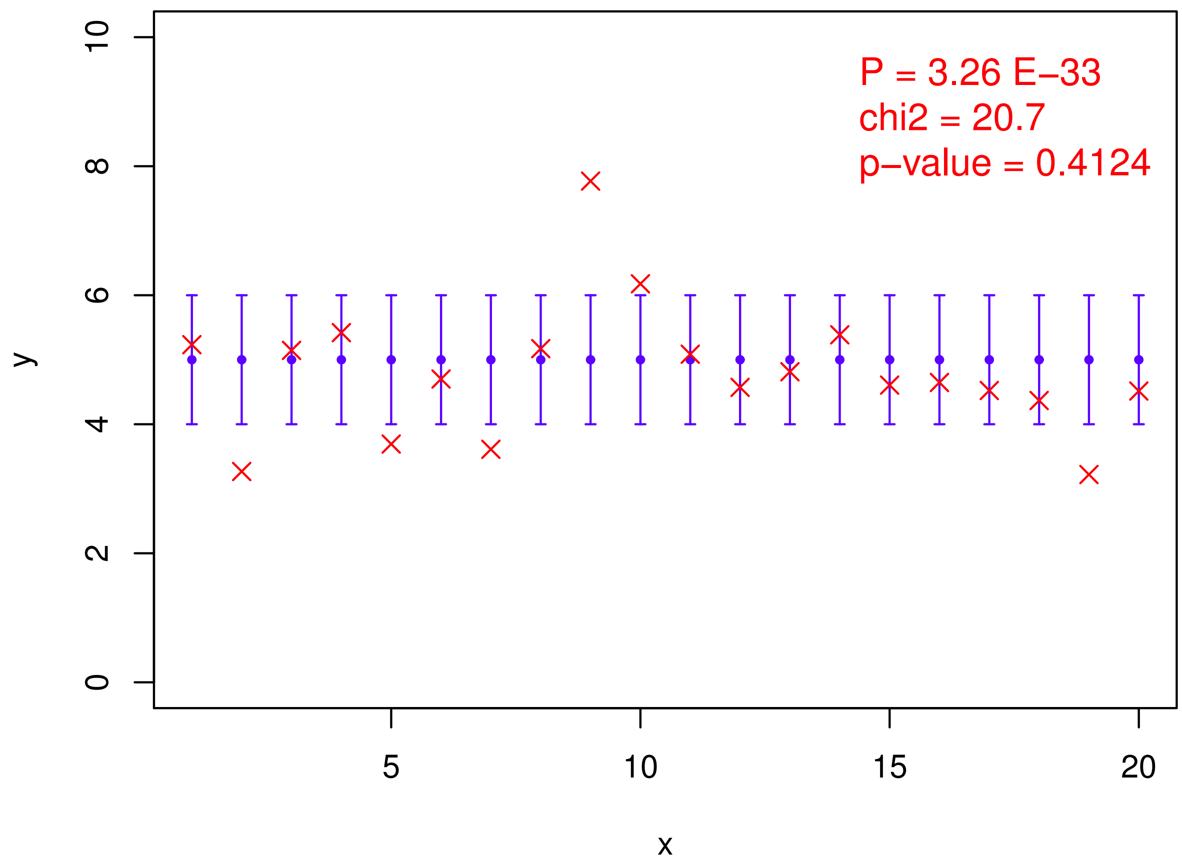
$$\mathrm{Std}[\chi^2] = \sqrt{2\nu} \qquad [\to 6.3]$$

$$\Rightarrow \quad \boxed{\chi^2 = 20 \pm 6}$$

[mode: 18]



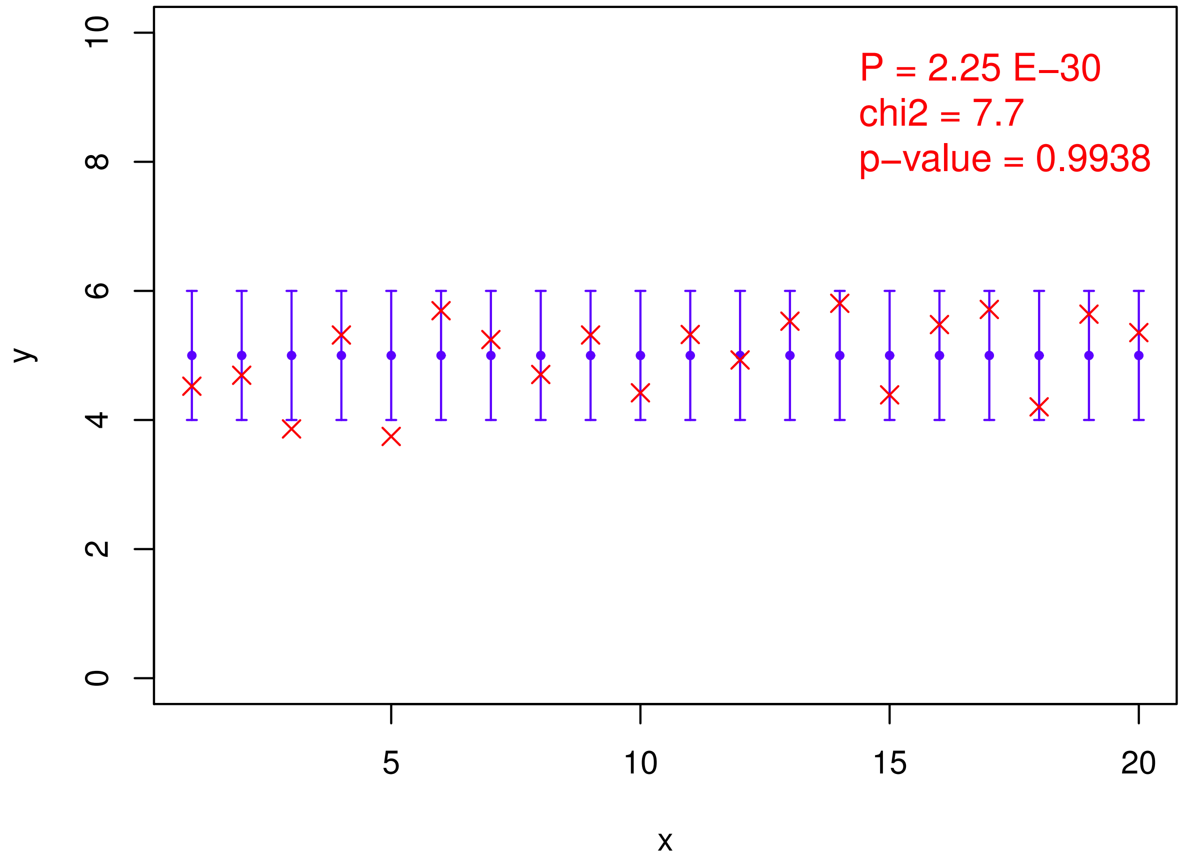
Some examples



In the average.

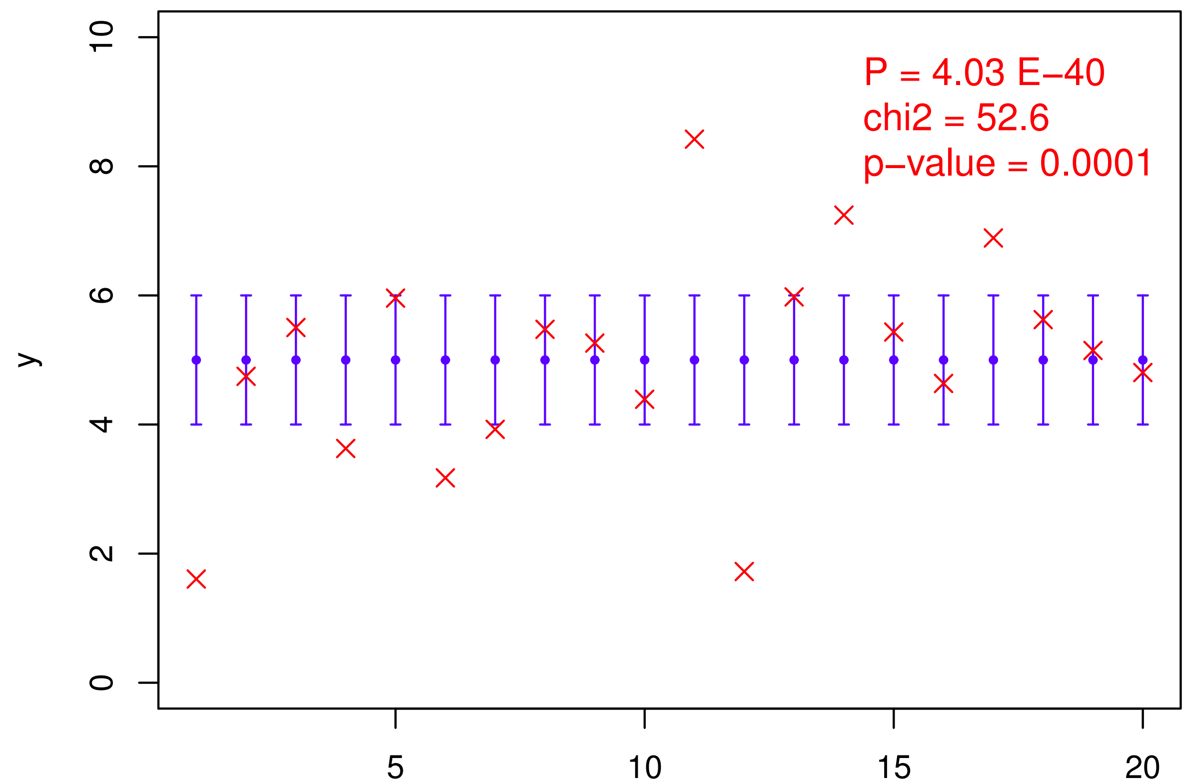
(but someone could see the points forming a 'constellation'. . .)

Some examples



Too good?

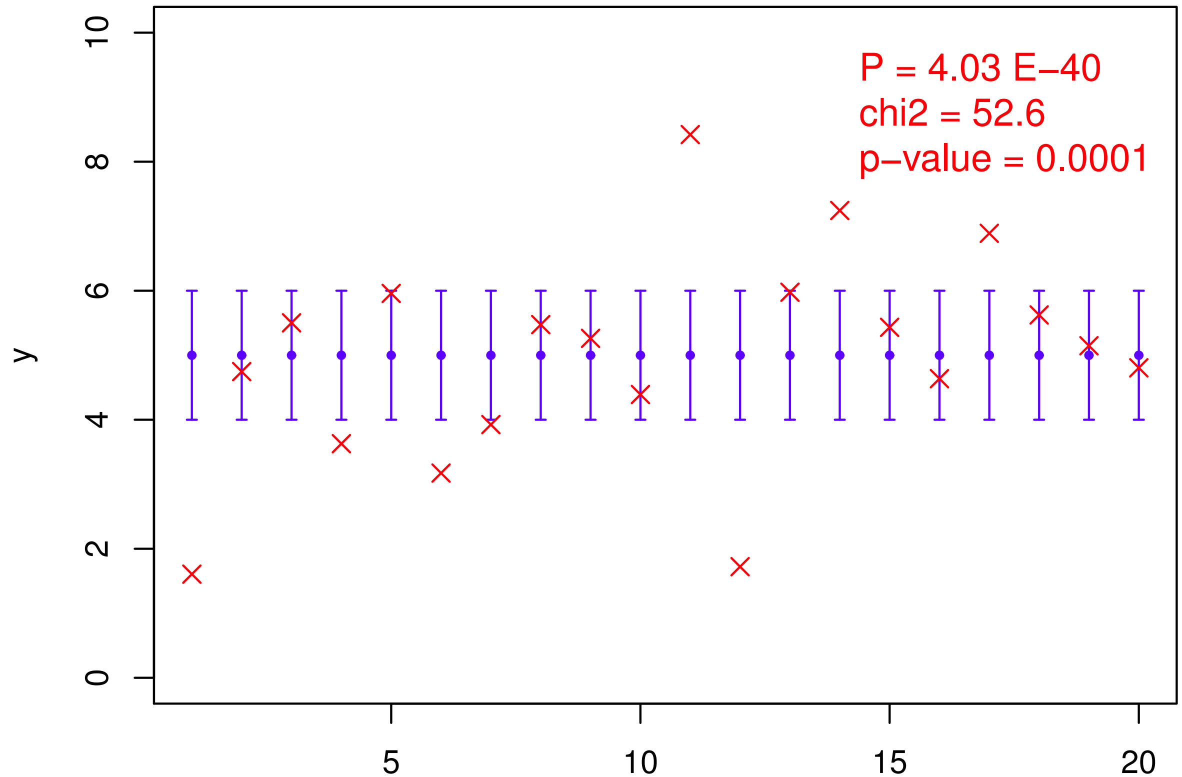
Some examples



$\chi^2 = 52.6$, with a p-value $= 0.93 \times 10^{-4}$

At limit?

Some examples

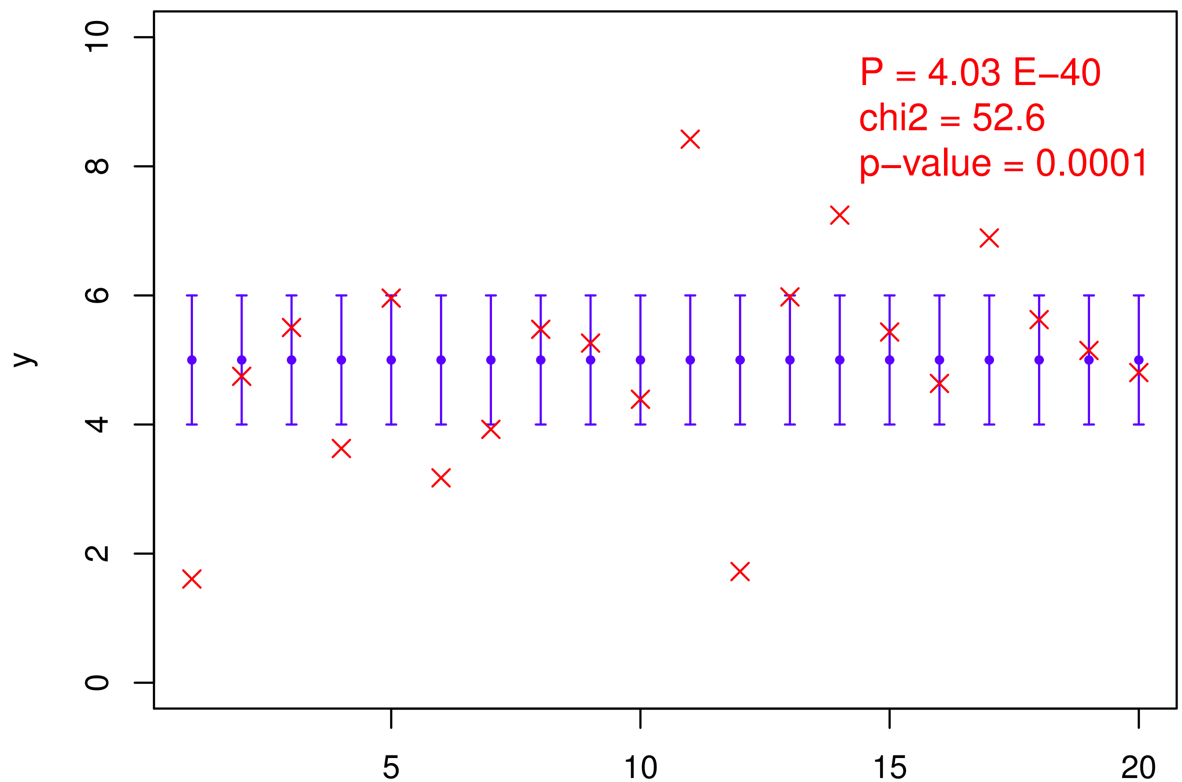


$\chi^2 = 52.6$, with a p-value $= 0.93 \times 10^{-4}$

At limit? Just come out at the first time (October 9, 13:01)

```
while(chi2.ym() < 38) source("chi2_1.R")
```

Some examples



Note: χ^2_{mis} 52.6 is 5.1σ from its expectation [$\frac{52.6-20}{\sqrt{40}} = 5.1$]

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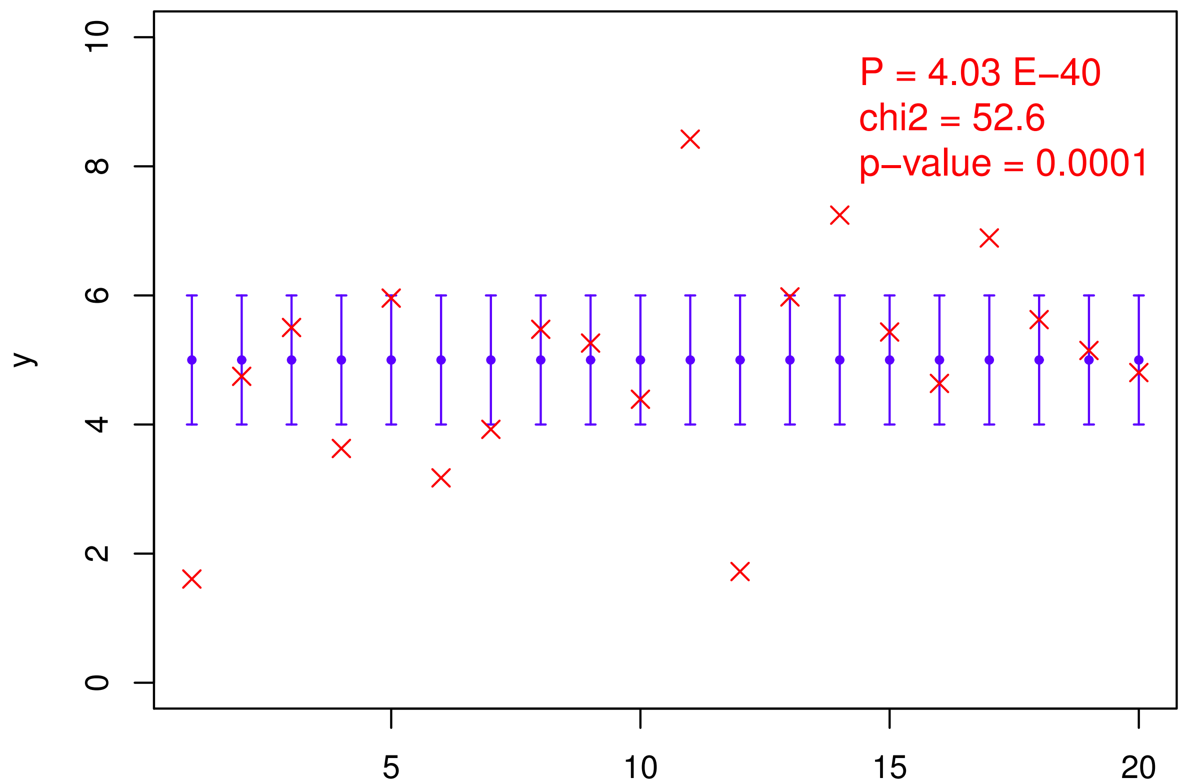

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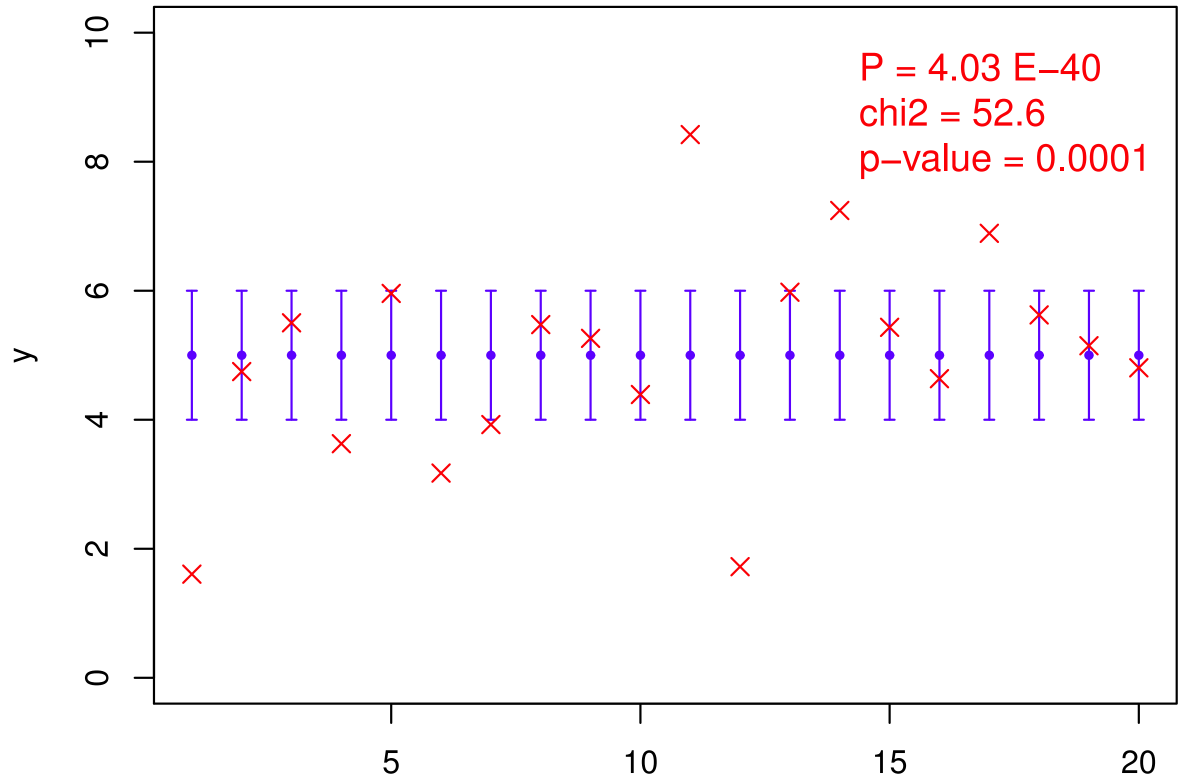

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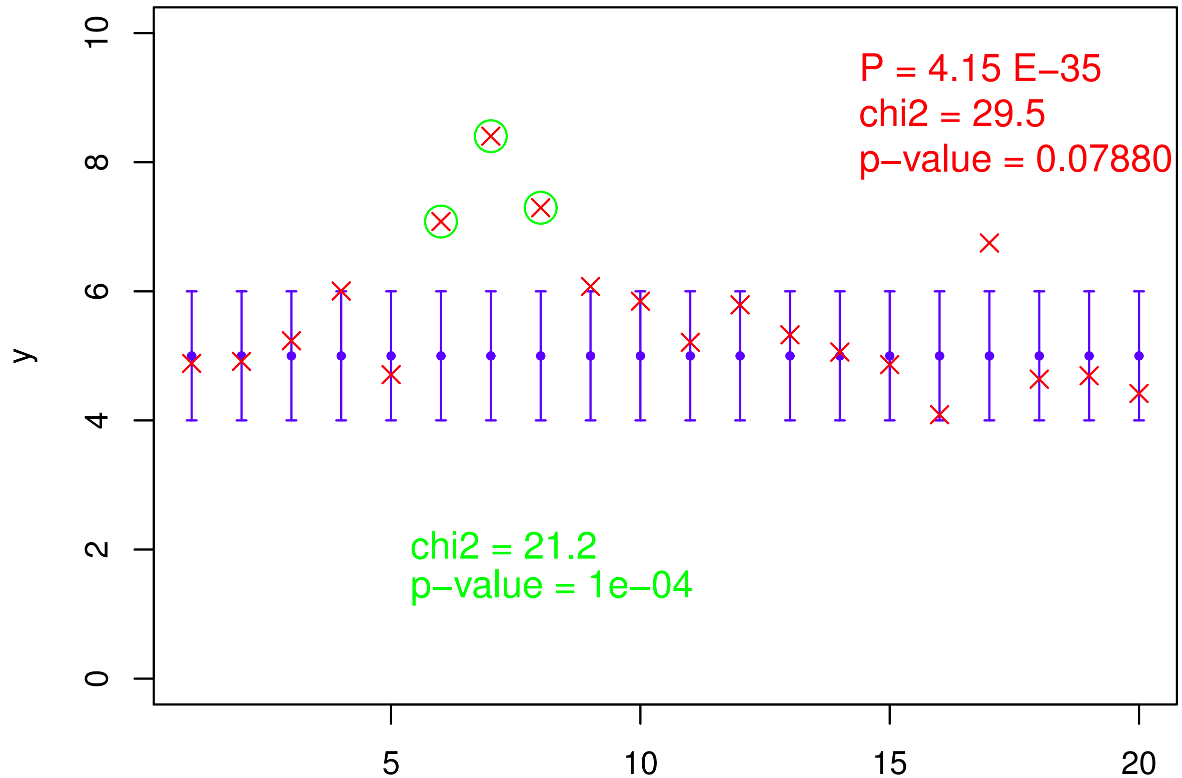
(as if there were already not enough confusion. . .)

The art of χ^2

Sometimes the χ^2 test does not give "the wished result"



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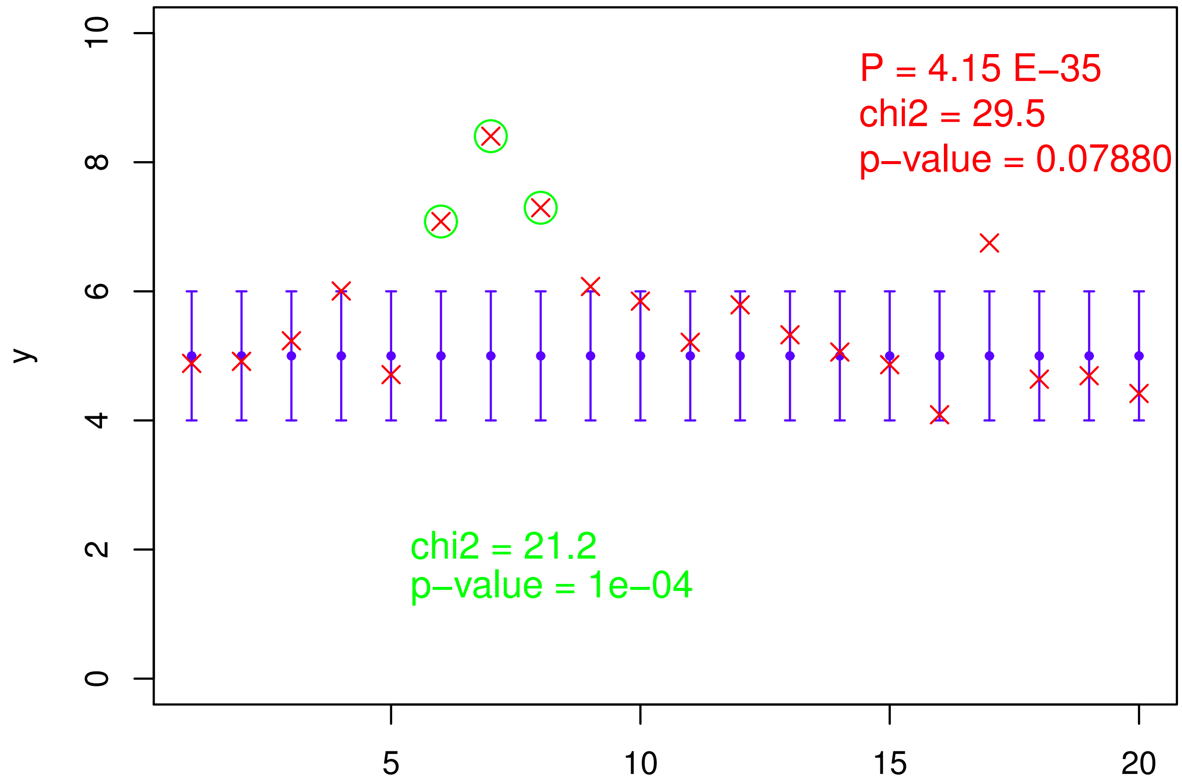

Then it is calculated in the 'suspicious region'

$\Rightarrow$ If we add the two side points, χ^2 becomes 22.2.

$\Rightarrow$ But with 5 points we had got a p-value of 5×10^{-4}

p-value: what they are

p-value:

- Probability of the tail(s) of a ‘test variable’ (a “statistic”):

$$P(\theta \geq \theta_{mis}) = \int_{\theta_{mis}}^{\infty} f(\theta \,|\, H_0)\, d\theta$$

$$P[(\theta \geq \theta_{mis}) \cap (\theta \leq (\theta^c)_{mis})] = 1 - \int_{(\theta^c)_{mis}}^{\theta_{mis}} f(\theta \,|\, H_0)\, d\theta$$

- θ is an arbitrary function of the data.
- …and often of a subsample of the data.
- $f(\theta \,|\, H_0)$ is obtained ‘somehow’, analitically, numerically, or by Monte Carlo methods.

p-value: what they <u>are not</u>

- What we wanted:
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$\Rightarrow$ BUT the p-value do not provide this:

$$P(\theta \geq \theta_{mis} \,|\, H_0) \not\Longleftrightarrow P(H_0 \,|\, \theta_{mis})$$

$\Rightarrow$ Although they are erroneously confused with this!

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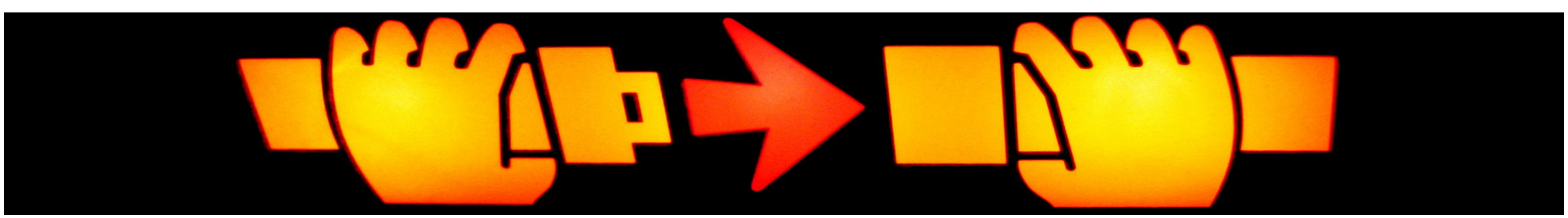
Tight seat belts!



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3. The p-value is not the probability of falsely rejecting the null hypothesis.

   . . .

7. . . .

The 5 sigma Higgs!

July 2012

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`http://www.roma1.infn.it/~dagos/badmath/#added`

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  - 'Mismatch' between our natural way of thinking and the statistics theory:
  - $P(H_0 \,|\, \text{data}) \longleftrightarrow P(\theta \geq \theta_{mis} \,|\, H_0)$

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- The 'classical' framework of hypothesis tests misses – because explicitly forbitten! – the foundamental thing we need in our game:
- It is enough get rid of '900 statisticians (the 'frequentists') and reload 'serious guys',
  $\rightarrow$ restart from Laplace, together with Gauss, Bayes, etc.,

Beliefs and bets

Recover the natural concept of probability

- “how much I am confident in something”
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"The usual touchstone, whether that which someone asserts is merely his persuasion – or at least his subjective conviction, that is, his firm belief – is betting. It often happens that someone propounds his views with such positive and uncompromising assurance that he seems to have entirely set aside all thought of possible error. A bet disconcerts him. Sometimes it turns out that he has a conviction which can be estimated at a value of one ducat, but not of ten. For he is very willing to venture one ducat, but when it is a question of ten he becomes aware, as he had not previously been, that it may very well be that he is in error." (Kant)

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- “I am rationally ready to change my opinion”
- “. . . but more unlikelly hypotheses initially were, the stronger evidence is needed, possible provided (independently) by several persons I trust”

Laplace's "Bayes Theorem"

"The greater the probability of an observed event given any one of a number of causes to which that event may be attributed, the greater the likelihood of that cause {given that event}.

$$P(C_i \,|\, E) \propto P(E \,|\, C_i)$$

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$$P(C_i \,|\, E) \;=\; \frac{P(E \,|\, C_i)\, P(C_i)}{\sum_j P(E \,|\, C_j)\, P(C_j)}$$

"This is the fundamental principle (*) of that branch of the analysis of chance that consists of reasoning *a posteriori* from events to causes"

(*) In his "Philosophical essay" Laplace calls 'principles' the 'fondamental rules'.

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Note: denominator is just a normalization factor.

$$\Rightarrow \qquad P(C_i\,|\,E) \;\propto\; P(E\,|\,C_i)\,P(C_i)$$

Most convenient way to remember Bayes theorem

Laplace's teaching

$$\frac{P(H_0\,|\,\text{data})}{P(H_1\,|\,\text{data})} = \frac{P(\text{dati}\,|\,H_0)}{P(\text{dati}\,|\,H_1)} \times \frac{P(H_0)}{P(H_1)}$$

- We should possible use the data, rather then the test variables 'θ' (χ^2 etc);
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- If $P(\text{data}\,|\,H_i) = 0$, it follows $P(H_i\,|\,\text{data}) = 0$:
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  $\Rightarrow$ falsification (the 'serious' one) is a corollary of the theorem, rather than a principle.
- There is no conceptual problem with the fact that $P(\text{dati}\,|\,H_1) \to 0$ (e.g. 10^{-37}), provided the ratio $P(\text{dati}\,|\,H_0)/P(\text{dati}\,|\,H_1)$ is not undefined.

But statistical tests do work!

Someone would object that p-values and, in general, ‘hypothesis tests’ *usually* do work!

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- Certainly! I agree!
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- But now we are also able to explain the reason.

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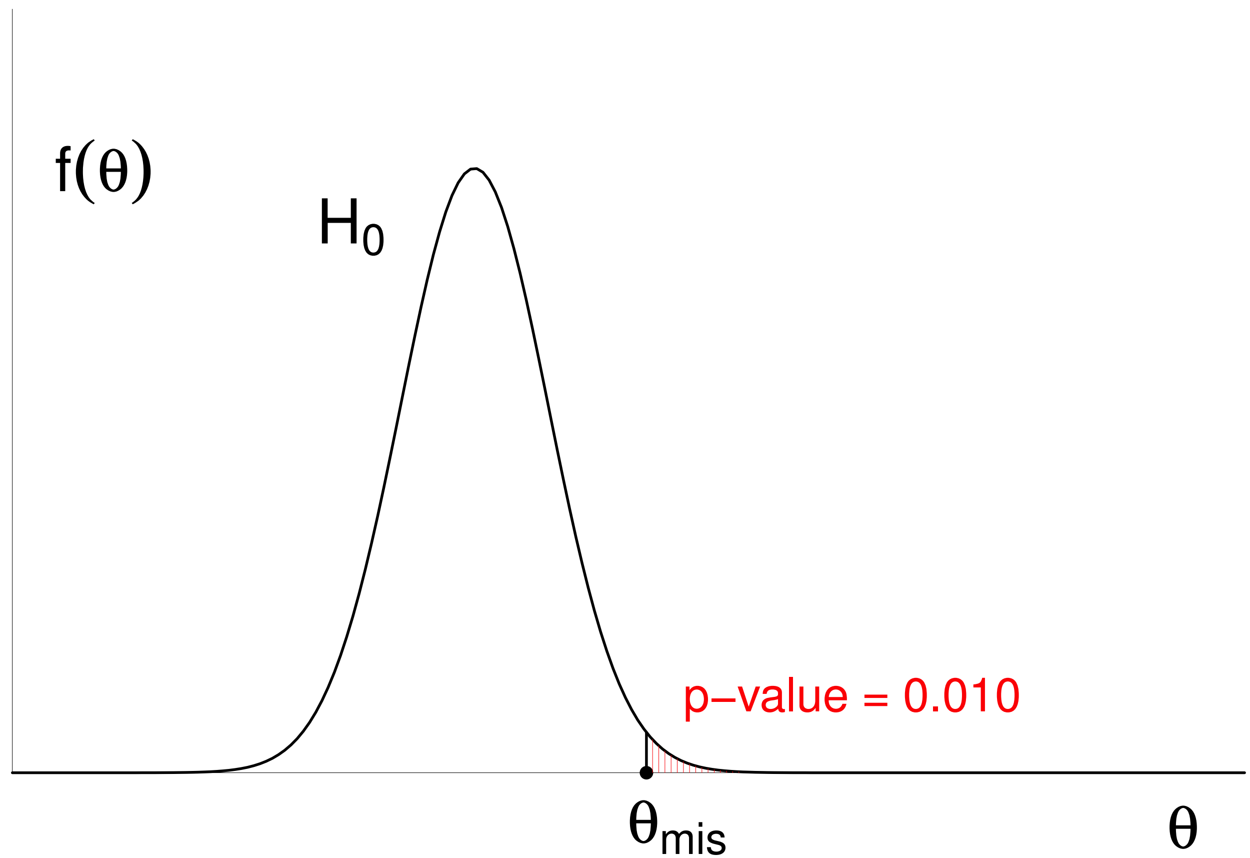

Why should the observation of θ_{mis} should diminish our confidence on H_0?

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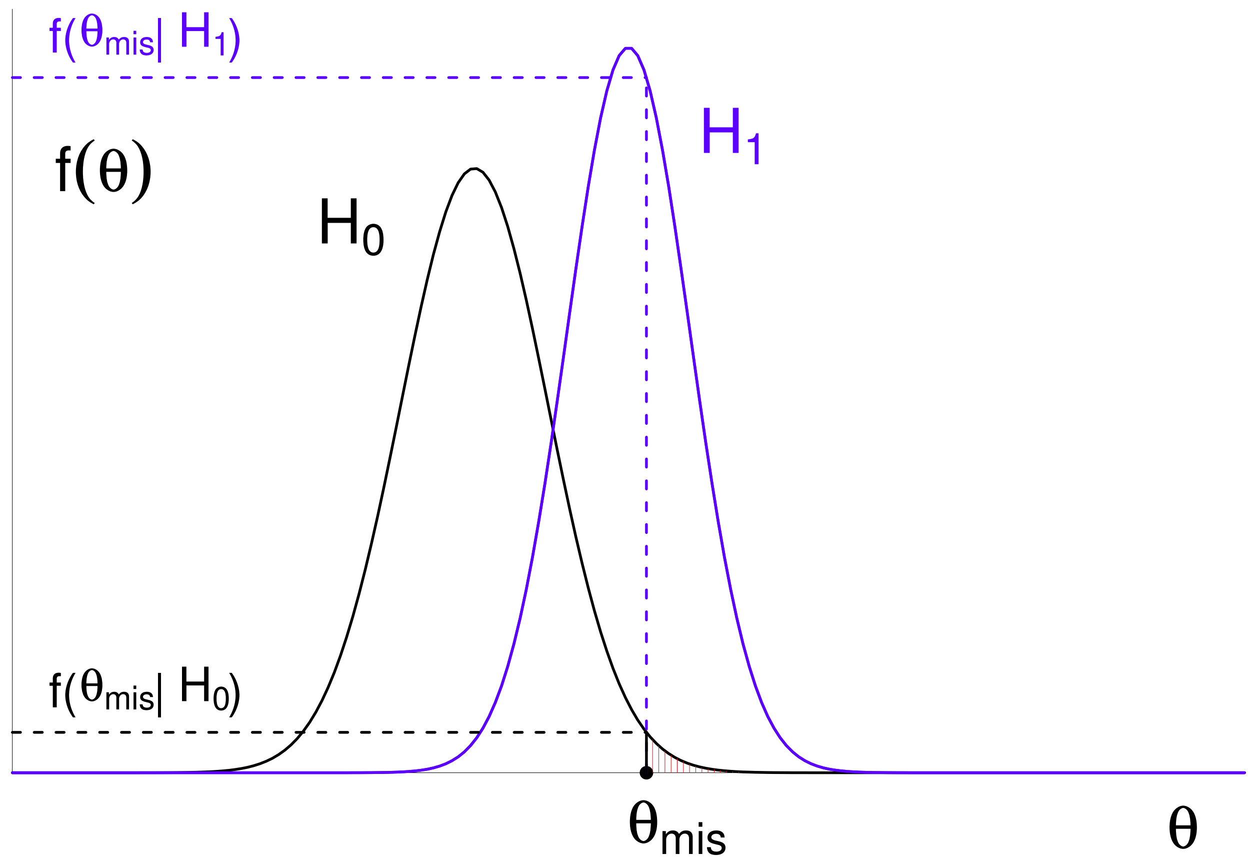

Because *often* we give *some chance* to a possible alternative hypothesis H_1, even if we are not able to exactly formulate it.

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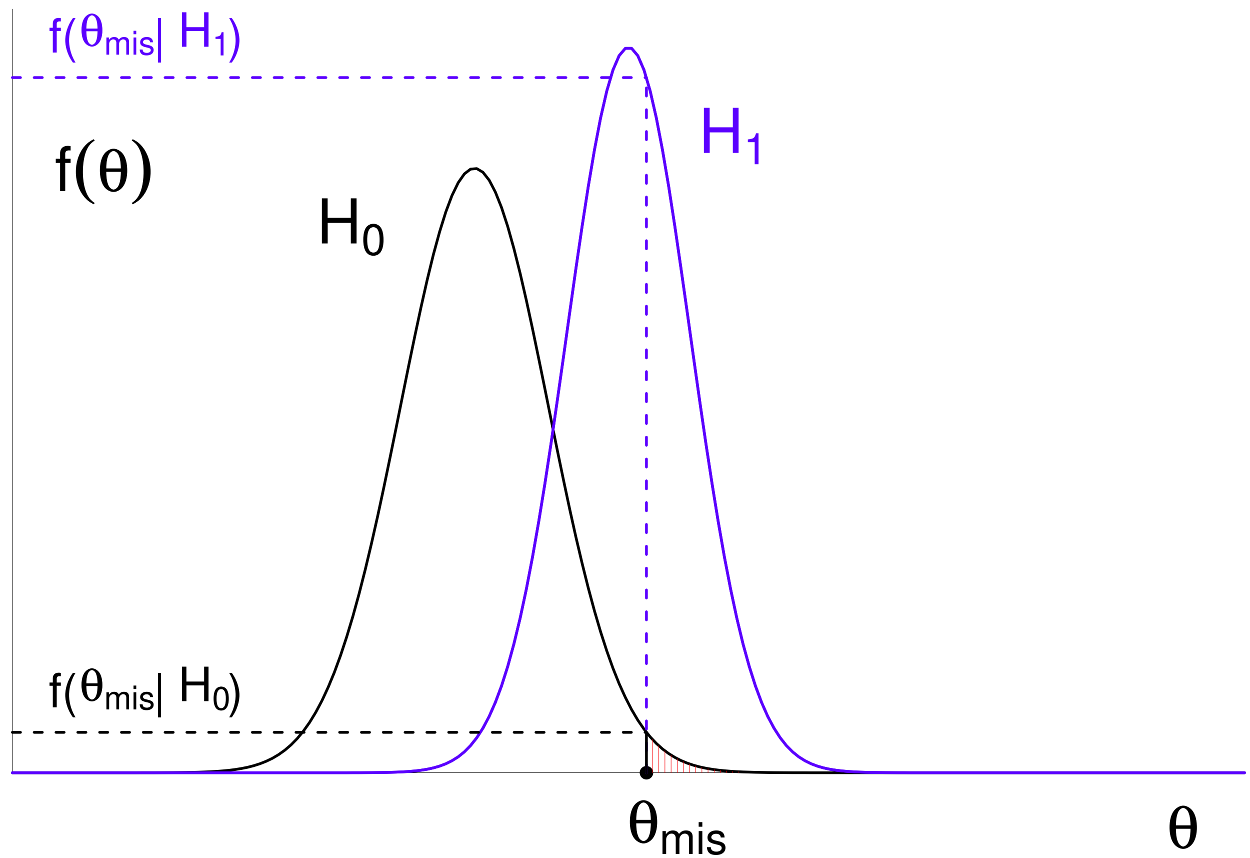

Indeed, what really matters is not the area to the right of θ_{mis}. What matters is the ratio of $f(\theta_{mis}\,|\,H_1)$ to $f(\theta_{mis}\,|\,H_0)$!
$\Rightarrow$ to a 'small' area it corresponds a 'small' $f(\theta_{mis}\,|\,H_0)$.

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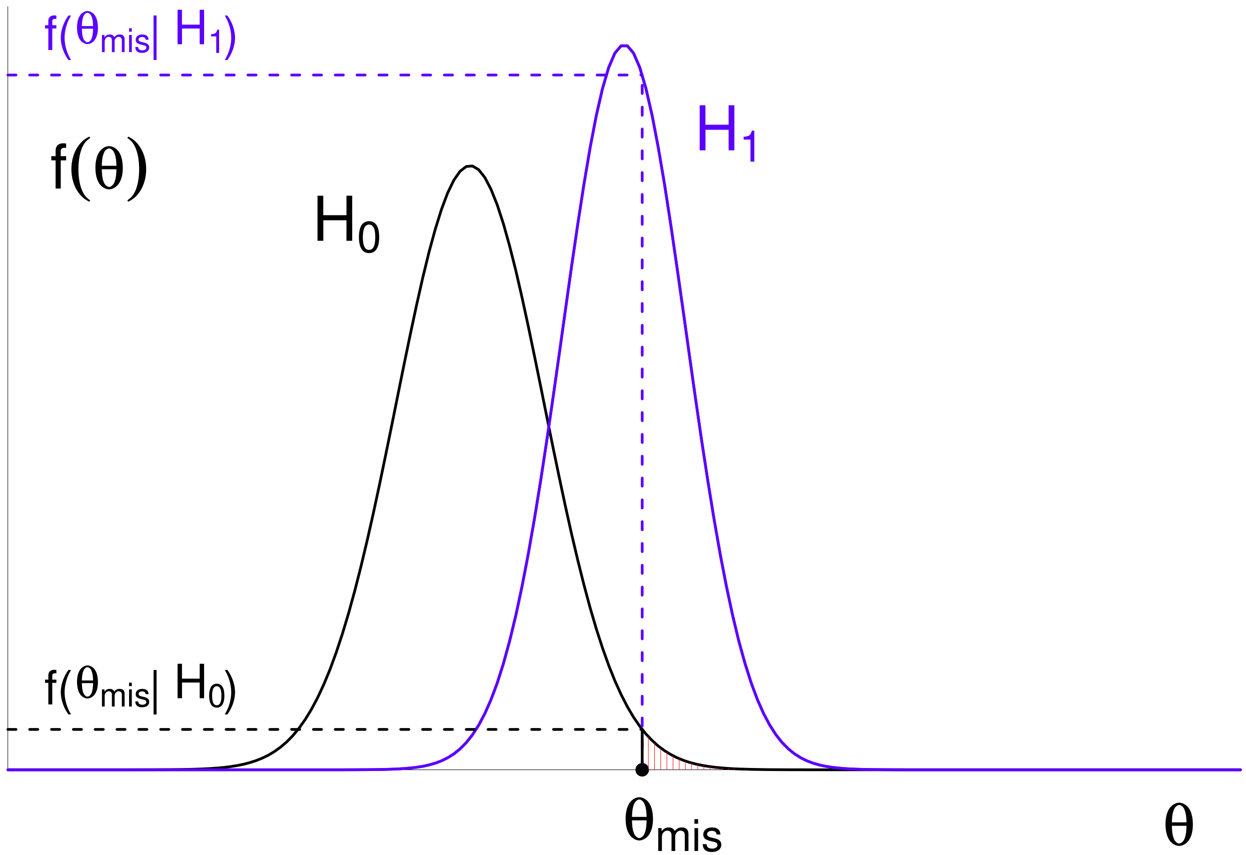

But is the alternative hypothesis H_1 is unconcievable, or hardly believable, the 'smalleness' of the area is irrelevant

Sensational announcements Vs sound Physics

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Don't get confused by sigma's and 'strange significances' that do not tell you how how much to believe in the claim.

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$\rightarrow$ The excess is surely a particle only if it is the Higgs!

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  (the diagrams entering R.C. are essentially the same the produce the Higgs in the final state!)
- Physics is something SERIOUS! (not a statistician’s toy)

Conclusions of Part 1

Philip Ball (Guardian, 23 dicembre 2011)
(`http://www.guardian.co.uk/commentisfree/2011/de`

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⇒ *He has finally won both bets!*