Beta distribution

$X \sim \text{Beta}(r, s))$:

$$f(x \,|\, \text{Beta}(r,s)) = \frac{1}{\beta(r,s)} x^{r-1}(1-x)^{s-1} \qquad \begin{cases} r,\ s > 0 \\ 0 \le x \le 1\,. \end{cases} \tag{4.54}$$

The denominator is just for normalization, i.e.

$$\beta(r,s) = \int_0^1 x^{r-1}(1-x)^{s-1}\,\mathrm{d}x\,.$$

Indeed this integral defines the beta function, resulting in

$$\beta(r,s) = \frac{\Gamma(r)\,\Gamma(s)}{\Gamma(r+s)}\,.$$

Since the beta distribution is not very popular among physicists, but very interesting for inferential purposes as *conjugate* distribution of the binomial, we show in Fig. 4.1 the variety of shapes that it can assume depending on the parameters r and s. Expected value and variance are:

$$\text{E}(X) = \frac{r}{r+s} \tag{4.55}$$

$$\text{Var}(X) = \frac{rs}{(r+s+1)\,(r+s)^2}\,. \tag{4.56}$$

If $r > 1$ and $s > 1$ the mode is unique, equal to $(r-1)/(r+s-2)$.

Triangular distribution

A convenient distribution for a rough description of subjective uncertainty on the value of influence quantities ('systematic effects') is given by the *triangular* distribution. This distribution models beliefs which decrease linearly in either side of the maximum (x_0) up to $x_0 + \Delta x_+$ on the right side and $x_0 - \Delta x_-$ on the left side (see Fig. 8.1). Expected value and variance are given by

$$\text{E}(X) = x_0 + \frac{\Delta x_+ - \Delta x_-}{3} \tag{4.57}$$

$$\sigma^2(X) = \frac{\Delta^2 x_+ + \Delta^2 x_- + \Delta x_+ \Delta x_-}{18}\,. \tag{4.58}$$

In the case of a *symmetric triangular* distribution ($\Delta x_+ = \Delta x_- = \Delta x$)

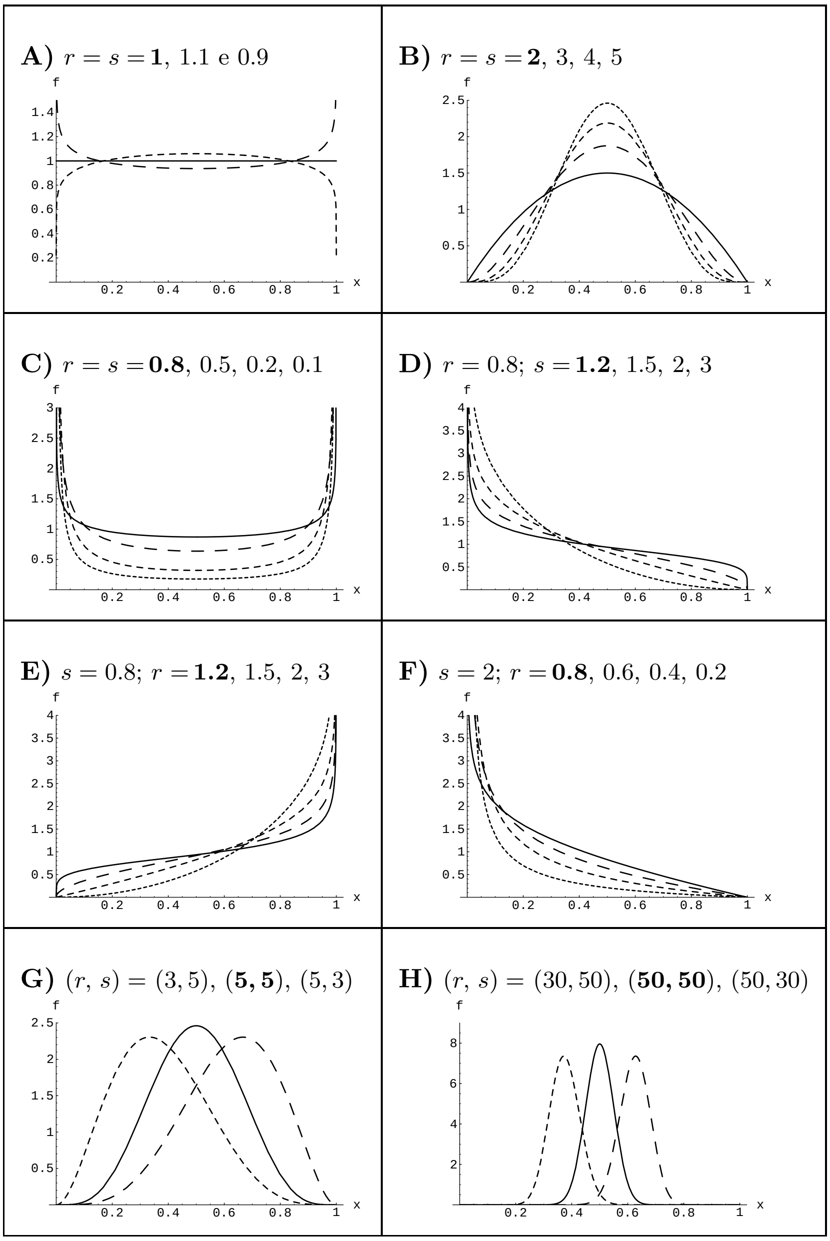

Fig. 4.1 Examples of Beta distributions for some values of r and s. The parameters in bold refer to continuous curves.