CLs method

7.4. **A modified frequentist approach: the CL_s method.** Now we consider a method, developed in the last years and applied in many analyses especially from LHC experiments, including the search for the Higgs boson. It is the **modified frequentist** approach to the problem of setting upper/lower limits in search experiments.

n_i events and expected events $y_i = \mu s_i + b_i$

Signal strength $\mu = \dfrac{\sigma}{\sigma_{th}}$ Theory expectation μ=1

$$L(\underline{n}/\mu, \underline{\theta}) = \prod_{i=1}^{M} \frac{(\mu s_i + b_i)^{n_i} e^{-(\mu s_i + b_i)}}{n_i!}$$

CLs method

Add histogram of control regions, mj , background enriched

$E[m_j] = u_j(\underline{\theta})$ depending on the nuisance parameters (and not on μ)

$$L(\underline{n}/\mu, \underline{\theta}) = \prod_{i=1}^{M} \frac{(\mu s_i + b_i)^{n_i} e^{-(\mu s_i + b_i)}}{n_i!} \prod_{j=1}^{K} \frac{u_j^{m_j} e^{-u_j}}{m_j!}$$

CLs method

Define the test statistics

$$q_\mu = -2 \ln \frac{L(\mu, \hat{\hat{\underline{\theta}}})}{L(\hat{\mu}, \hat{\underline{\theta}})}$$

profile likelihood ratio

symbols: $\hat{\mu}$ and $\hat{\underline{\theta}}$ are the best values of the parameters obtained by maximizing L; $\hat{\hat{\underline{\theta}}}$ are the values of the nuisance parameters obtained by maximizing L at μ fixed. The test

7.4.2. *Discovery.* In order to falsify a null hypothesis H_0 we need to test the background-only hypothesis. This can be done by using the test statistics q_0, that is eq. 207 for $\mu = 0$

$$q_0 = -2 \ln \frac{L(0, \hat{\hat{\underline{\theta}}})}{L(\hat{\mu}, \hat{\underline{\theta}})} \tag{210}$$

If we call q_0^{obs} the value of q_0 obtained using the data, we can easily define a p-value

$$p_0 = \int_{q_0^{obs}}^{\infty} f(q_0/0) dq_0 \tag{211}$$

that, for what we have seen in the previous paragraph, is essentially a χ^2 test. If p_0 is below the defined limit we falsify the hypothesis and we have done the discovery.

CLs method

The Wilks theorem (see sect.5) has the consequence that under general hypotheses and in the large sample limit, since q_μ is a likelihood ratio, the pdf $f(q_\mu/\mu)$ has a χ^2 distribution with 1 degree of freedom. In particular the distribution of q_0 for a sample of purely background simulated events has a χ^2_1 pdf. It is interesting to notice that a χ^2_1 variable is essentially the square of a standard gaussian variable:

$$\chi^2_1 = \left(\frac{x-\mu}{\sigma}\right)^2 \tag{208}$$

so that its square root is a standard gaussian variable. This allows to use the quantity

$$\sqrt{q_0} = \sqrt{-2\ln\frac{L(0,\hat{\hat{\underline{\theta}}})}{L(\hat{\mu},\hat{\underline{\theta}})}} \tag{209}$$

as a measure, in number of standard deviation, of the agreement of the data with the null hypothesis. Such a quantity is used in many circumstances to define the statistical significance that can be reached by an experiment to reject the background-only hypothesis. The "score function" defined by eq.59 is an application of this formula.

SIGNIFICANCE $\quad Z_A = \sqrt{2(S+B)\ln\left(1+\frac{S}{B}\right)-2S}$

Have we really observed the final state X ? - I

- We need a criterium to say ok, we have seen the signal or our data are compatible with the background.
- Which statistical uncertainty have we on N_X ?
  - Assume a Poisson statistics to describe N_{cand} negligible uncertainty on ε. We call (using more "popular" symbols):
  - $N = N_{cand}$
  - $B = N_b$
  - $S = N - B = N_x$

$$\left(\frac{\sigma(N_X)}{N_X}\right)^2 = \frac{\sigma^2(N)+\sigma^2(B)}{S^2} = \frac{N+\sigma^2(B)}{S^2}$$

$$\frac{N_X}{\sigma(N_X)} = \frac{S}{\sigma(S)} = \frac{S}{\sqrt{N+\sigma^2(B)}} = \frac{S}{\sqrt{S+B}}$$

Additional assumption: $\sigma^2(B) << N$

σ(S)/S is the relative uncertainty on S, its inverse is "how many st.devs. away from 0" → S/√B when low signals on top of large bck

Have we really observed the final state X ? - II

- This quantity is the "significance" of the signal. The higher is $S/\sigma(S) = S/\sqrt{S+B}$, the larger is the number of std.dev. away from 0 of my measurement of S (SCORE FUNCTION)
  - $S/\sqrt{S+B} < 3$ probably I have not osserved any signal (my candidates can be simply a fluctuation of the background)
  - $3 < S/\sqrt{S+B} < 5$ probably I have observed a signal (probability of a background fluctuation very small), however no definite conclusion, more data needed.➔ ***evidence***
  - $S/\sqrt{S+B} > 5$ observation is accepted. ➔ ***observation***
- NB1: All this is "conventional" it can be discussed
- NB2: $S/\sqrt{S+B}$ is an approximate figure, it relies on some assumptions *(see previous slide)*.

The New $s/\sqrt{b}$

The new $s/\sqrt{b}$

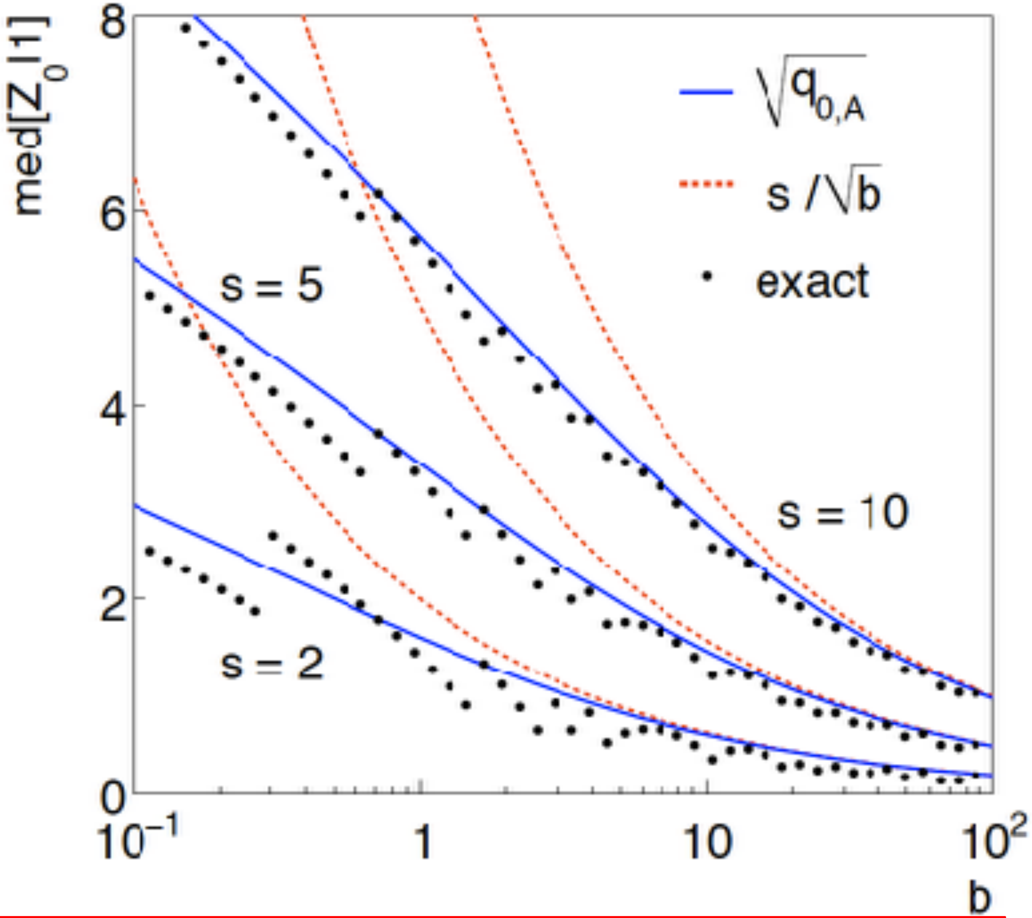
$$Z_A = \sqrt{q_{0,A}}$$

$$\mathrm{med}[Z_0|1] = \sqrt{q_{0,\mathrm{A}}} = \sqrt{2\left((s+b)\ln(1+s/b) - s\right)}$$

$$Z_A = \sqrt{q_{0,A}} \xrightarrow{s/b \ll 1} \frac{s}{\sqrt{b}} + O(s/b)$$

The New $s/\sqrt{b}$

$s/\sqrt{b}$?



The new $s/\sqrt{b}$

$$\mathrm{med}[Z_0|1] = \sqrt{q_{0,\mathrm{A}}} = \sqrt{2\left((s+b)\ln(1+s/b) - s\right)}$$

CLs method

7.4.3. *Signal exclusion: CL_{s+b}.* We consider now how the test statistics shown in eq. 207 can be used for the exclusion of a given theory. Eq. 207 is rewritten with $\mu = 1$[36]

$$q_1 = -2 \ln \frac{L(1, \hat{\hat{\underline{\theta}}})}{L(\hat{\mu}, \hat{\underline{\theta}})} \tag{212}$$

The lower is q_1, the more compatible the data are with the theory, and the less compatible the data are with the pure background expectations. The pdf of q_1 can be evaluated starting from MC samples, either generated with $\mu = 1$ or for samples of pure background events generated with $\mu = 0$. We call respectively $f(q_1/1)$ and $f(q_1/0)$ the two pdf's. A graphical example of these pdf's is shown in Figure 22. The separation between the two pdf's determines the capability to discriminate the searched model with respect to the background[37].

CLs method

Test statistics distribution

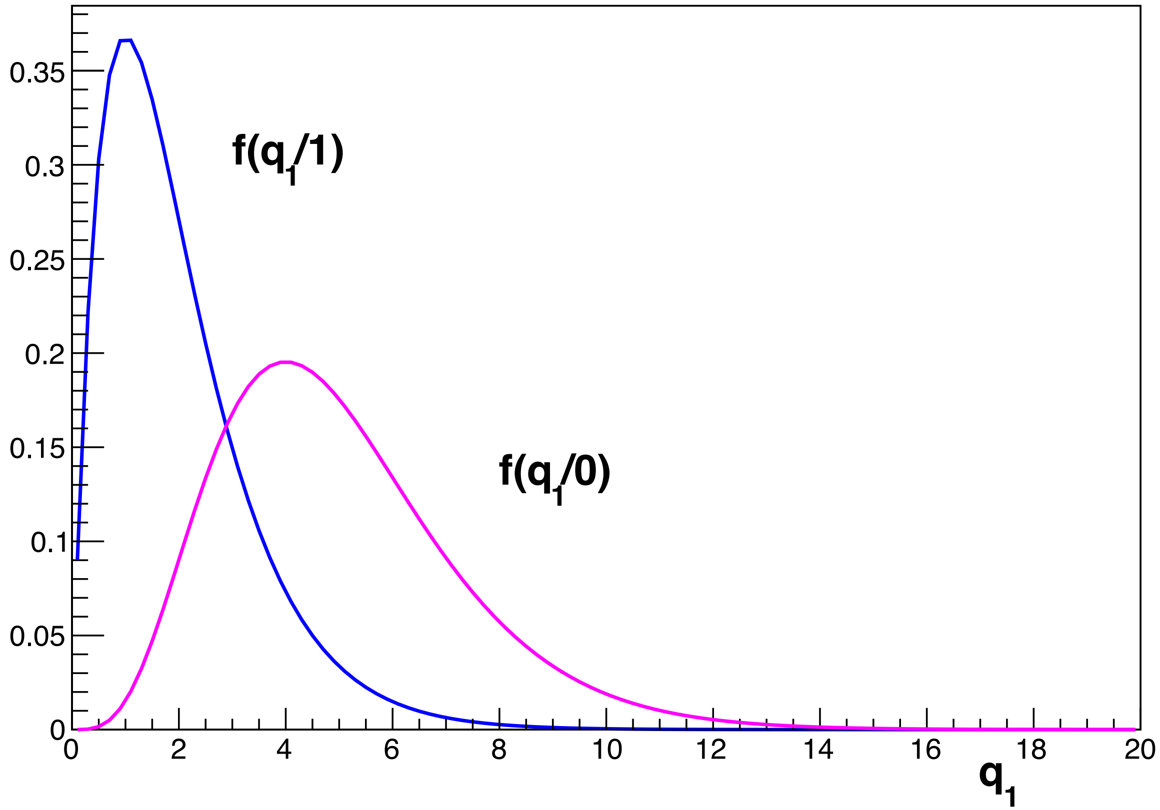


FIGURE 22. Example of q_1 distributions in the two hypotheses, namely $\mu = 1$ and $\mu = 0$. The separation between the two distributions indicate the capability to discriminate the two hypotheses.

CLs method

evaluate the **sensitivity** of the experiment.

define $\tilde{q}_1$ as the **median** of the $f(q_1/0)$ function[3]

expected

$$CL_{s+b}^{exp} = \int_{\tilde{q}_1}^{\infty} f(q_1/1) dq_1$$

CLs method

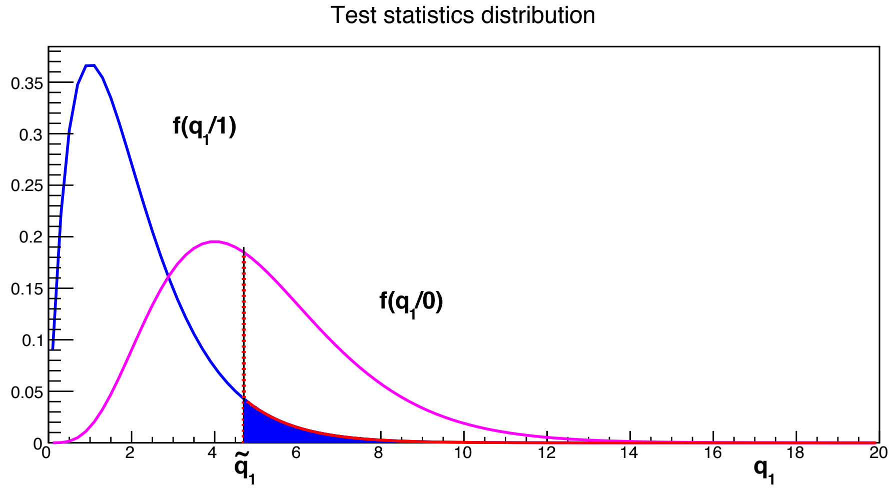


FIGURE 23. For the same example of alternative hypotheses shown in Fig. 22: construction of CL_{s+b}^{exp} (upper plot) and of CL_{s+b}^{obs} (lower plot). In both cases the CL is given by the blue area. In the upper plot the median q_1 from background experiments is indicated as $\tilde{q}_1$; in the lower plot the q_1 obtained by data is indicated as q_1^{obs}.

However, we have determined the median CL only. In actual background-only experiments, we will have background fluctuations, in such a way that q_1 values will be obtained distributed according to $f(q_1/0)$. So we can evaluate an interval of confidence levels, by repeating the procedure above for two positions of q_1 , $\tilde{q}_1^{(1)}$ and $\tilde{q}_1^{(2)}$ such that respectively:

$$\int_{-\infty}^{\tilde{q}_1^{(1)}} f(q_1/0)dq_1 = \frac{1-\beta}{2} \tag{214}$$

$$\int_{-\infty}^{\tilde{q}_1^{(2)}} f(q_1/0)dq_1 = \frac{1+\beta}{2} \tag{215}$$

with e.g. $\beta = 68.3\%$ to have a gaussian one-std.deviation interval. Confidence levels are then evaluated applying eq. 213 to $\tilde{q}_1^{(1)}$ and $\tilde{q}_1^{(2)}$.

CLs method

Observation

(216)
$$CL_{s+b}^{obs} = \int_{q_1^{obs}}^{\infty} f(q_1/1)dq_1$$

and this is the **observed** confidence level. If it is below, say 5% we exclude the signal at 95% CL.

CLs method

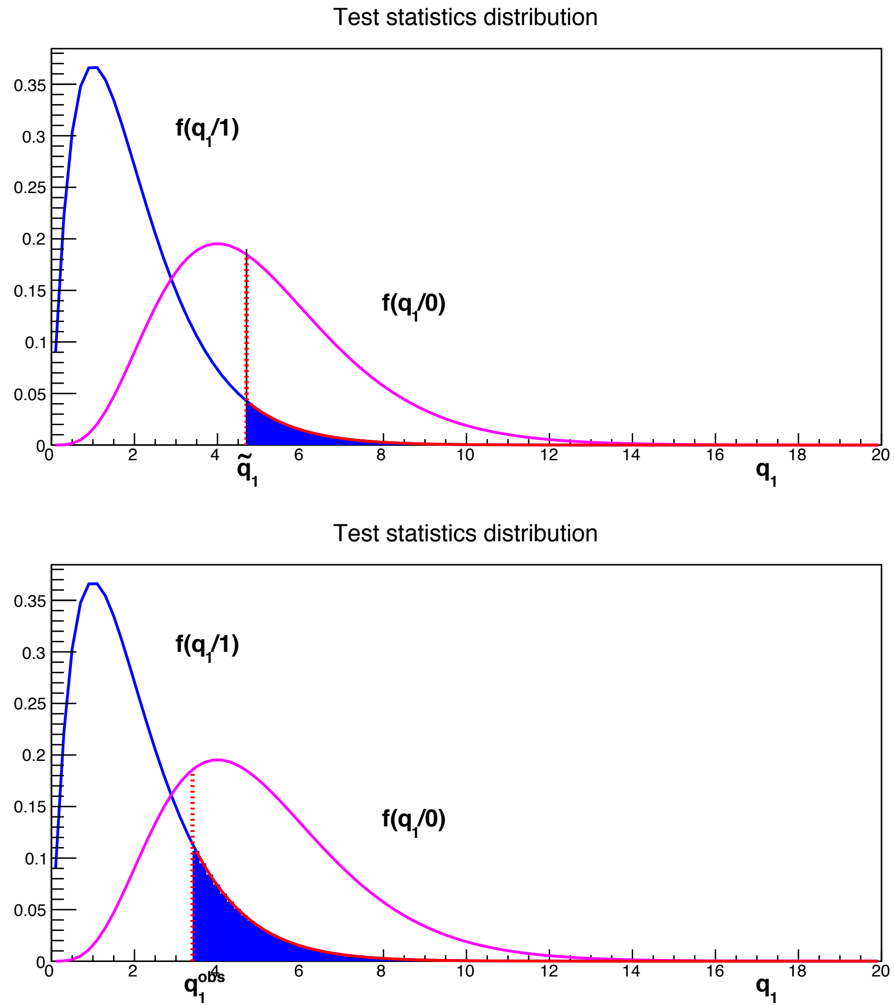


FIGURE 23. For the same example of alternative hypotheses shown in Fig. 22: construction of CL_{s+b}^{exp} (upper plot) and of CL_{s+b}^{obs} (lower plot). In both cases the CL is given by the blue area. In the upper plot the median q_1 from background experiments is indicated as $\tilde{q}_1$; in the lower plot the q_1 obtained by data is indicated as q_1^{obs}.

CLs method

7.4.4. *Signal exclusion: CL_s.* A problem in the procedure outlined in the previous section has been put in evidence, and a correction to it, the so called modified frequentist approach has been proposed. We discuss now this method, also called CL_s method that is now widely employed for exclusion of new physics signals.

Let's consider the situation shown in Figure 24 where the two pdf's $f(q_1/0)$ and $f(q_1/1)$ have a large overlap signaling a small sensitivity. If we evaluate in this situation CL_{s+b}^{exp} we find a large value, so that we do not expect to be sensitive to exclusion. However what happens if q_1^{obs} is the one shown in the same Figure ? The observed CL_{s+b}^{obs} is well below 5% and the signal has to be excluded at 95% CL. But, are we sure that we have to exclude it ? In the same Figure the quantity CL_b^{obs} is reported:

$$CL_b^{obs} = \int_{q_1^{obs}}^{\infty} f(q_1/0) dq_1 \tag{217}$$

that is also very small in this case. Apparently the signal is small and the background "under-fluctuates", so that q_1^{obs} is scarcely compatible with the signal+background hypothesis but also with the background-only hypothesis. So, we are excluding the signal, essentially because the background has fluctuated.

In order to avoid this somehow unmotivated exclusion, the CL_s procedure has been defined. The idea is to use, as confidence level, the CL_s quantity, either expected or

CLs method

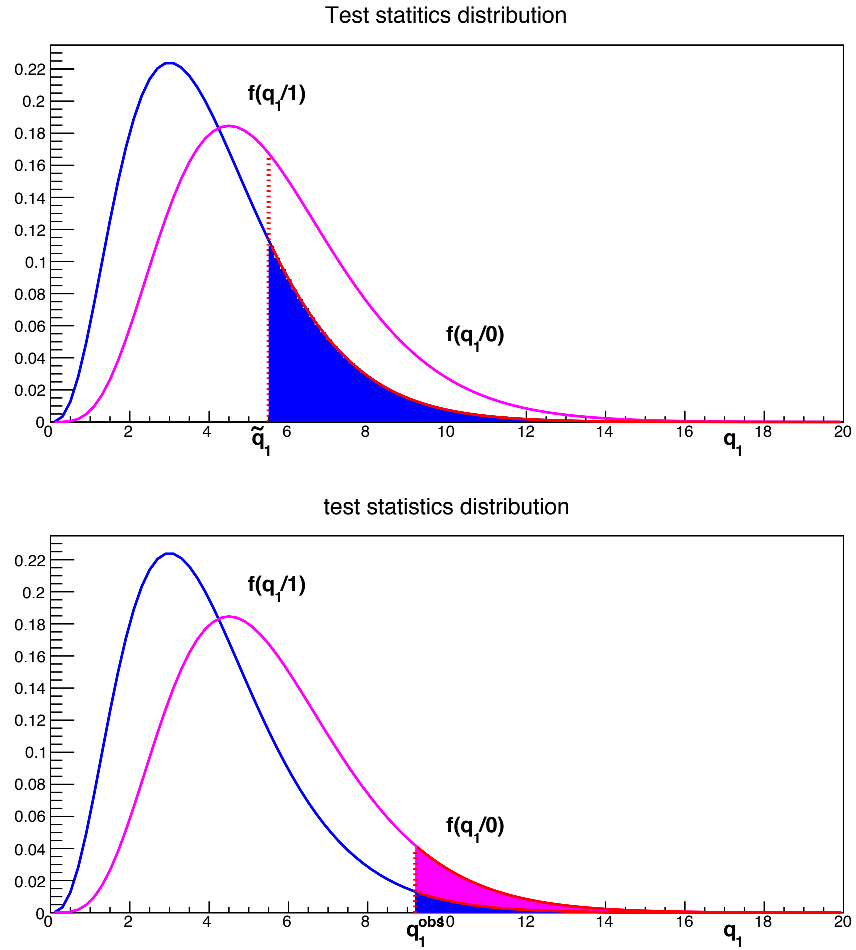


FIGURE 24. Same construction of Fig. 23 for a situation where the discrimination between the two hypotheses is particularly poor and the overlap between the two distributions is high. The CL_{s+b}^{exp} is high (upper plot) but for a particular experiment with a under fluctuation of the background the CL_{s+b}^{obs} can be small in such a way to reject the signal hypothesis (lower plot). In the lower plot the magenta area shows CL_b^{obs} from which CL_s is built. In this case using the CL_s prescription rather than the CL_{s+b} one the signal is not rejected.

CLs method

$$CL_s = \frac{CL_{s+b}}{CL_b}$$

The CL_s method is also said **modified frequentist** approach. In fact, the confidence interval obtained in this way has not the coverage properties required by the ”orthodox” frequentist paradigm. So if we build a confidence interval with a CL_s of α, the coverage is in general larger than α, so that the Type-I errors are less than $1 - \alpha$.

Basic Definitions: POWER

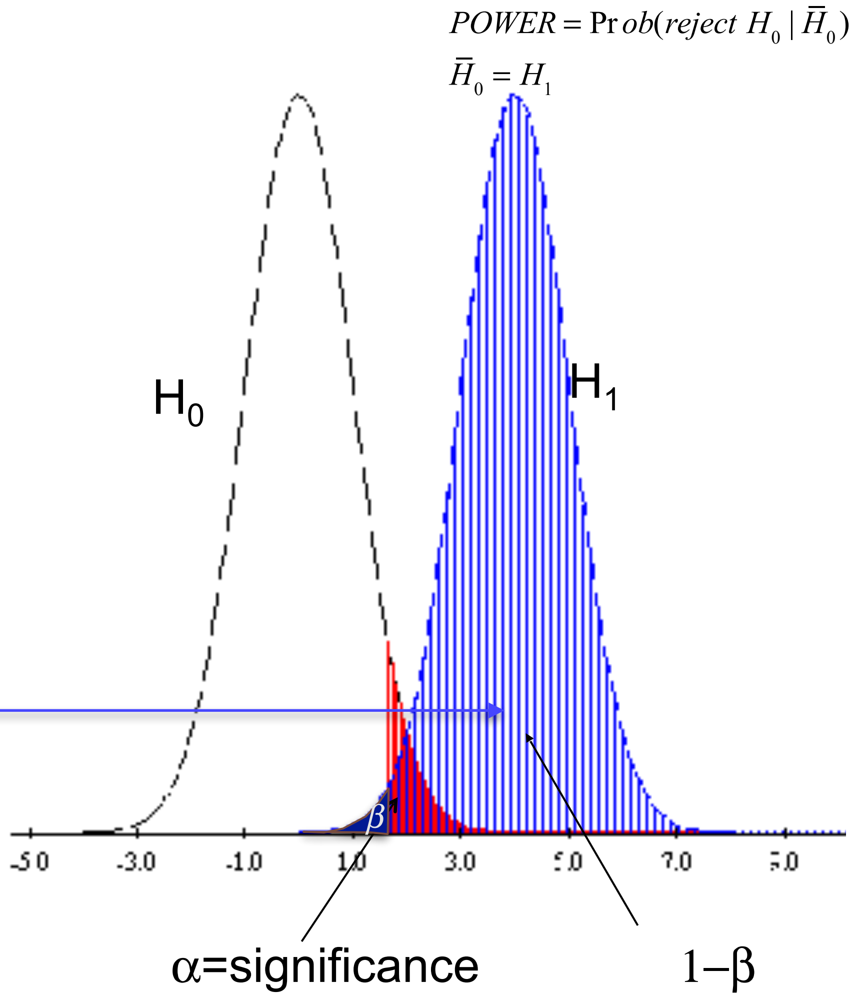
- $\alpha = \Pr ob(reject\ H_0 \mid H_0)$
- The POWER of an hypothesis test is the probability to reject the null hypothesis when it is indeed wrong (the alternate analysis is true)
- $POWER = \Pr ob(reject\ H_0 \mid \bar{H}_0)$

  $\beta = Prob(accept\ H_0 \mid \bar{H}_0)$

  $1-\beta = Prob(reject\ H_0 \mid \bar{H}_0)$

  $\bar{H}_0 = H_1$

  $1-\beta = Prob(reject\ H_0 \mid H_1)$
- The power of a test increases as the rate of type II error decreases



CLs

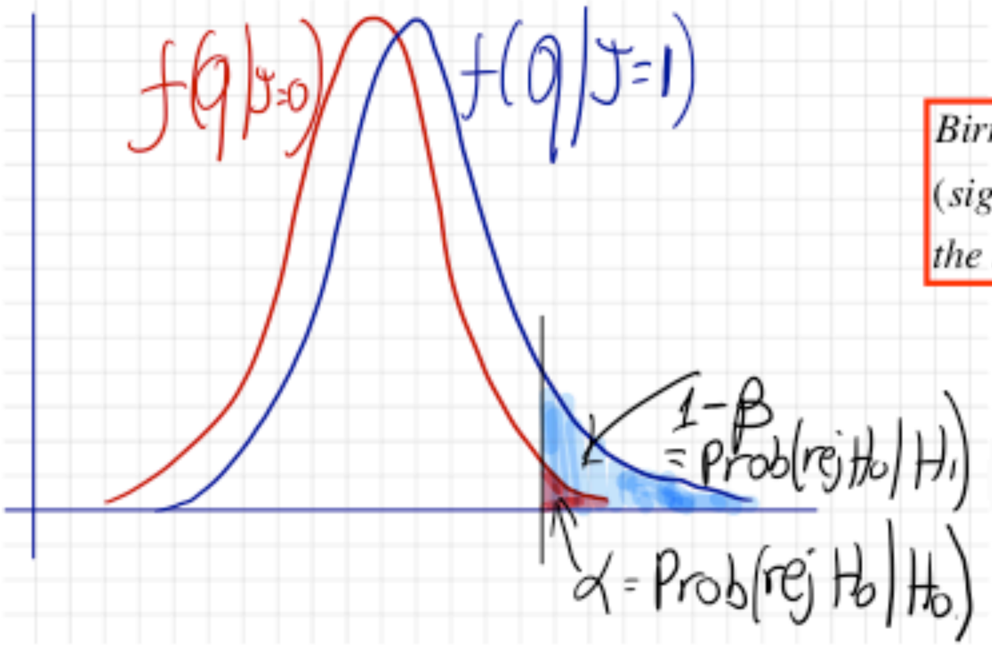
Birnbaum (1977)

"A concept of statistical evidence is not plausible unless it finds

'strong evidence for H_1 as against H_0'

with small probability (α) when H_0 is true,

and with much larger probability ($1-\beta$) when H_1 is true."



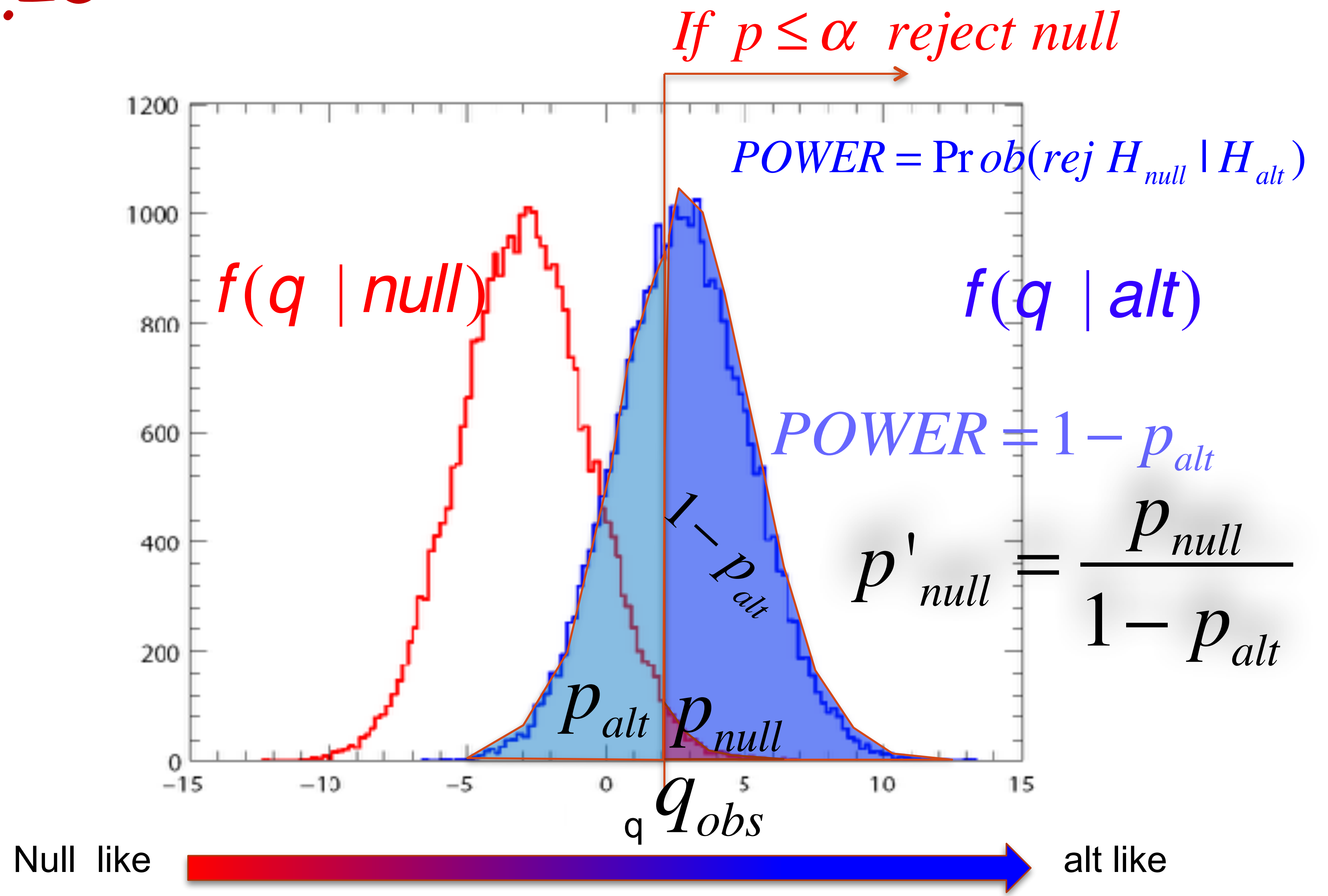
Birnbaum (1962) *suggested that* $\alpha / 1-\beta$ (*significance / power*) *should be used as a measure of the strength of a statistical test, rather than* α *alone*

$$p = 5\% \rightarrow p' = 5\% / 0.155 = 32\%$$

$$p' \equiv CL_S$$

$$p'_\mu = \frac{p_\mu}{1-p_0}$$

CLs



$If\ p \leq \alpha\ reject\ null$
$POWER = \Pr ob(rej\ H_{null} \mid H_{alt})$
$f(q \mid null)$
$f(q \mid alt)$
$POWER = 1 - p_{alt}$
$1 - p_{alt}$
$p'_{null} = \frac{p_{null}}{1 - p_{alt}}$
p_{alt}
p_{null}
1200
1000
800
600
400
200
0
−15
−10
−5
0
5
10
15
q
q_{obs}
Null like
alt like

CLs method

Determination of $\mu=\mu*$ such that CLs = 1-α => confidence level α.
Repeat the previous analysis for a generic μ

$$CL_{s+b}^{(\mu)} = \int_{q_\mu^{obs}}^{\infty} f(q_\mu/\mu) dq_\mu$$

$$CL_b^{(\mu)} = \int_{q_\mu^{obs}}^{\infty} f(q_\mu/0) dq_\mu$$

$$CL_s^{(\mu)} = \frac{CL_{s+b}^{(\mu)}}{CL_b{}^{(\mu)}}$$

By increasing μ, $CL_s^{(\mu)}$ decreases, and the value μ^* such that $CL_s^{(\mu^*)} = 1-\alpha$ is the upper limit on μ at the required confidence level α.

p-value – testing the null hypothesis

When testing the b hypotheis (null=b), it is custom to set

$\alpha = 2.9\ 10^{-7}$

→ if $p_b < 2.9\ 10^{-7}$ the b hypothesis is rejected

→Discovery

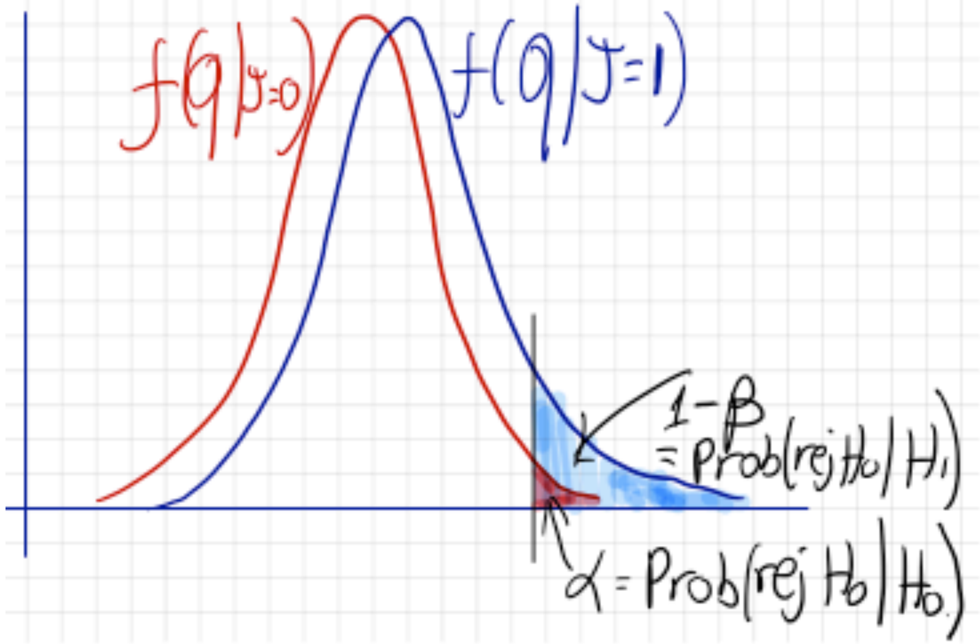
When testing the s+b hypothesis (null=s+b), set α =5%
if p_{s+b}<5% the signal hypothesis is rejected at the 95% Confidence Level (CL)

→ Exclusion

CLs

Birnbaum (1962) *suggested that* $\alpha / 1-\beta$ (*significance / power*)*should be used as a measure of the strength of a statistical test ,rather than* α *alone*

$$p = 5\% \rightarrow p' = 5\% / 0.155 = 32\%$$

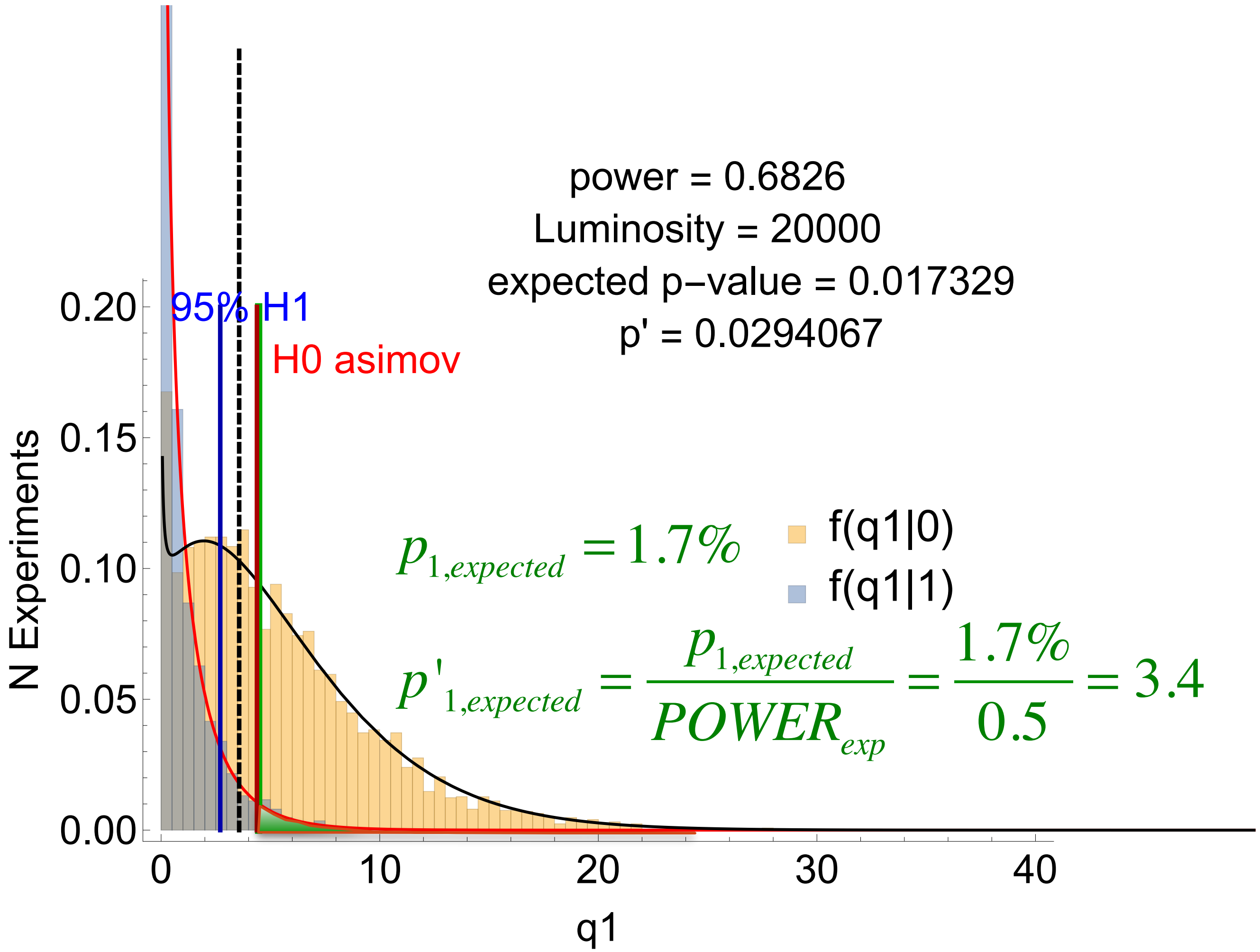


$$p' \equiv CL_S$$

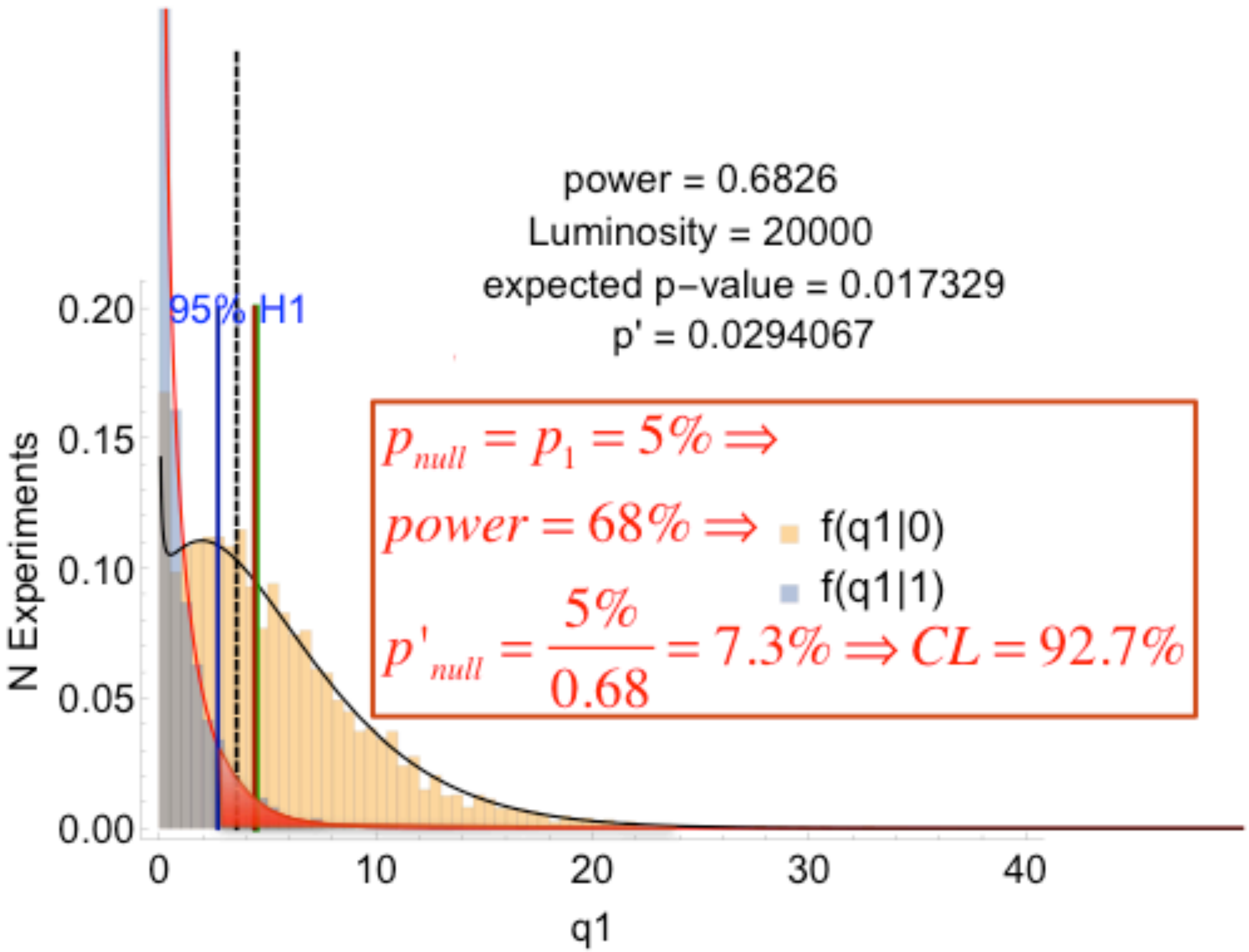
$$p'_\mu = \frac{p_\mu}{1-p_0}$$

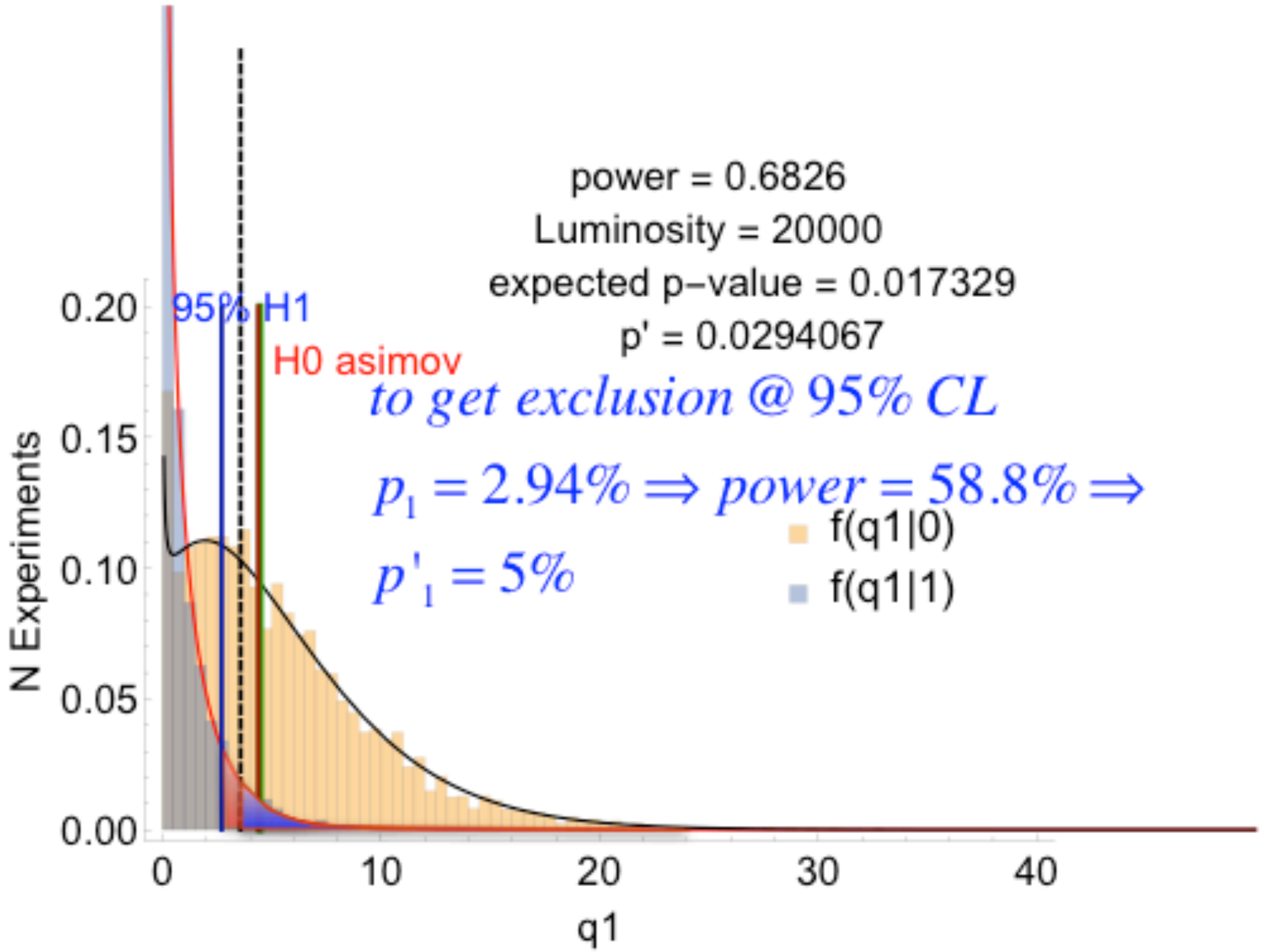
The CLs method
Was brought into
HEP By Alex Read (2002)
A.L. Read,
Presentation of search results:
The CL(s) technique,
"J.\ Phys.\ G {\bf 28}, 2693 (2002).

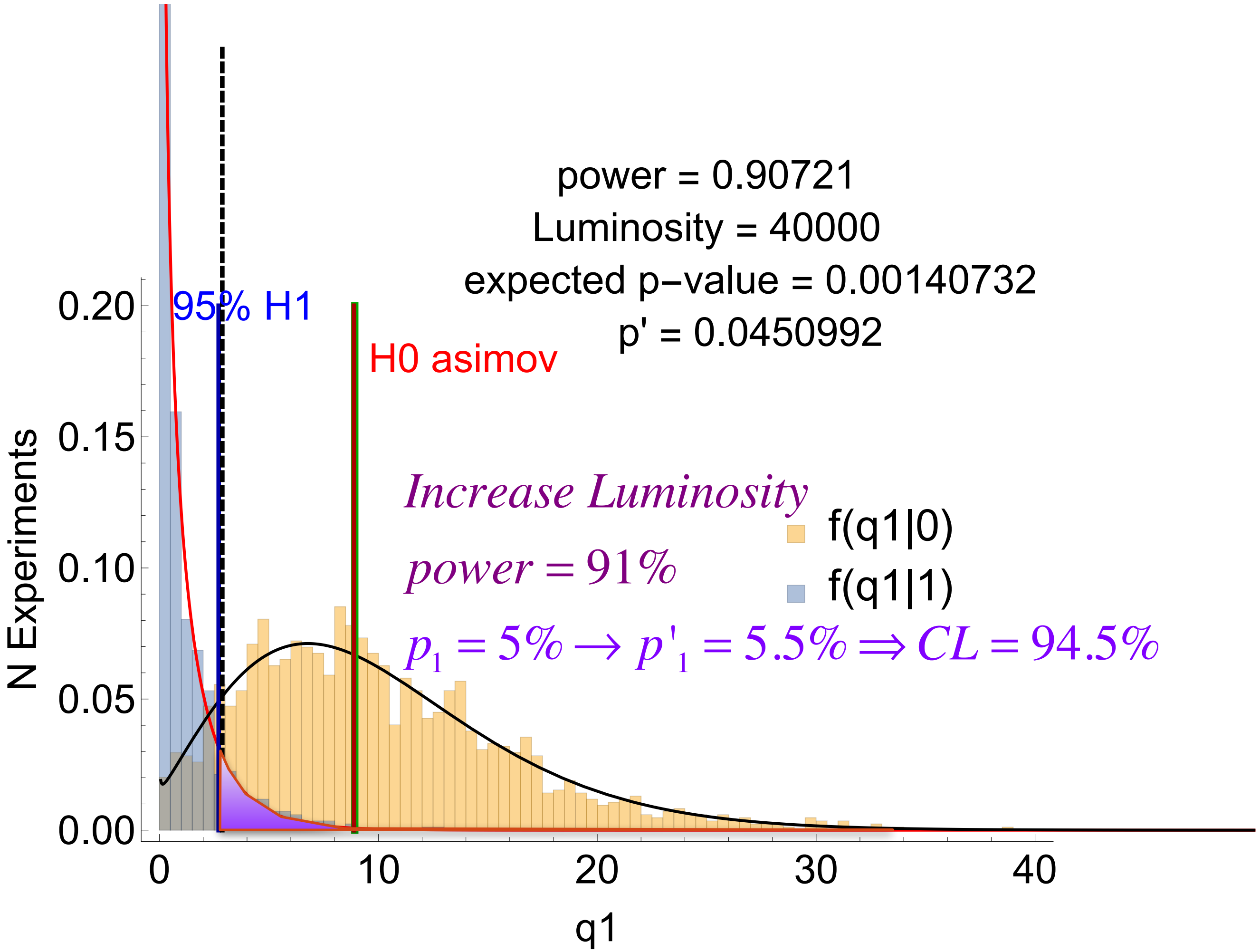
Birnbaum was re-discovered later
By O. Vitells



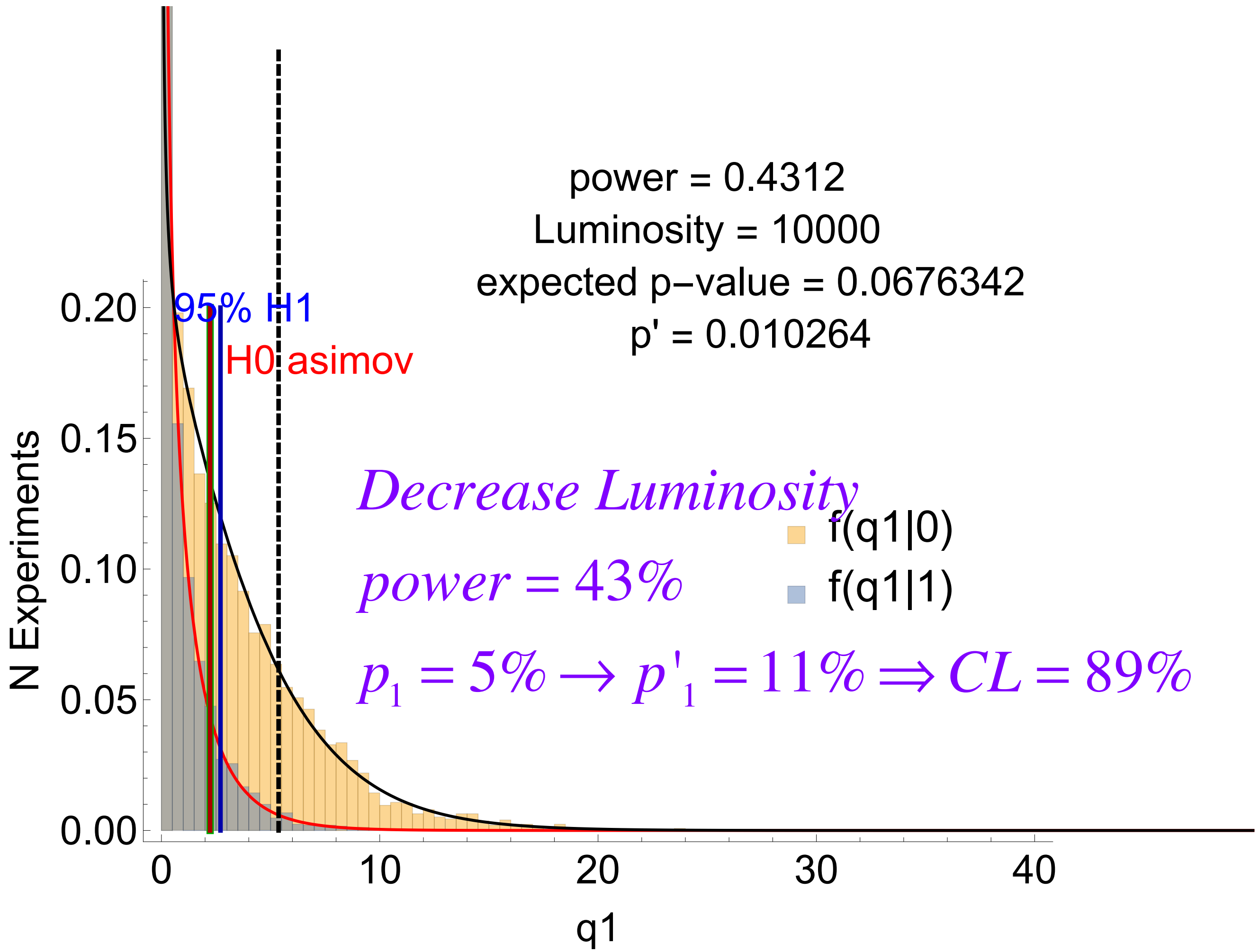
power = 0.6826
Luminosity = 20000
expected p−value = 0.017329
p' = 0.0294067
95% H1
H0 asimov
$p_{1,expected} = 1.7\%$
$p'_{1,expected} = \frac{p_{1,expected}}{POWER_{exp}} = \frac{1.7\%}{0.5} = 3.4$
f(q1|0)
f(q1|1)
N Experiments
q1

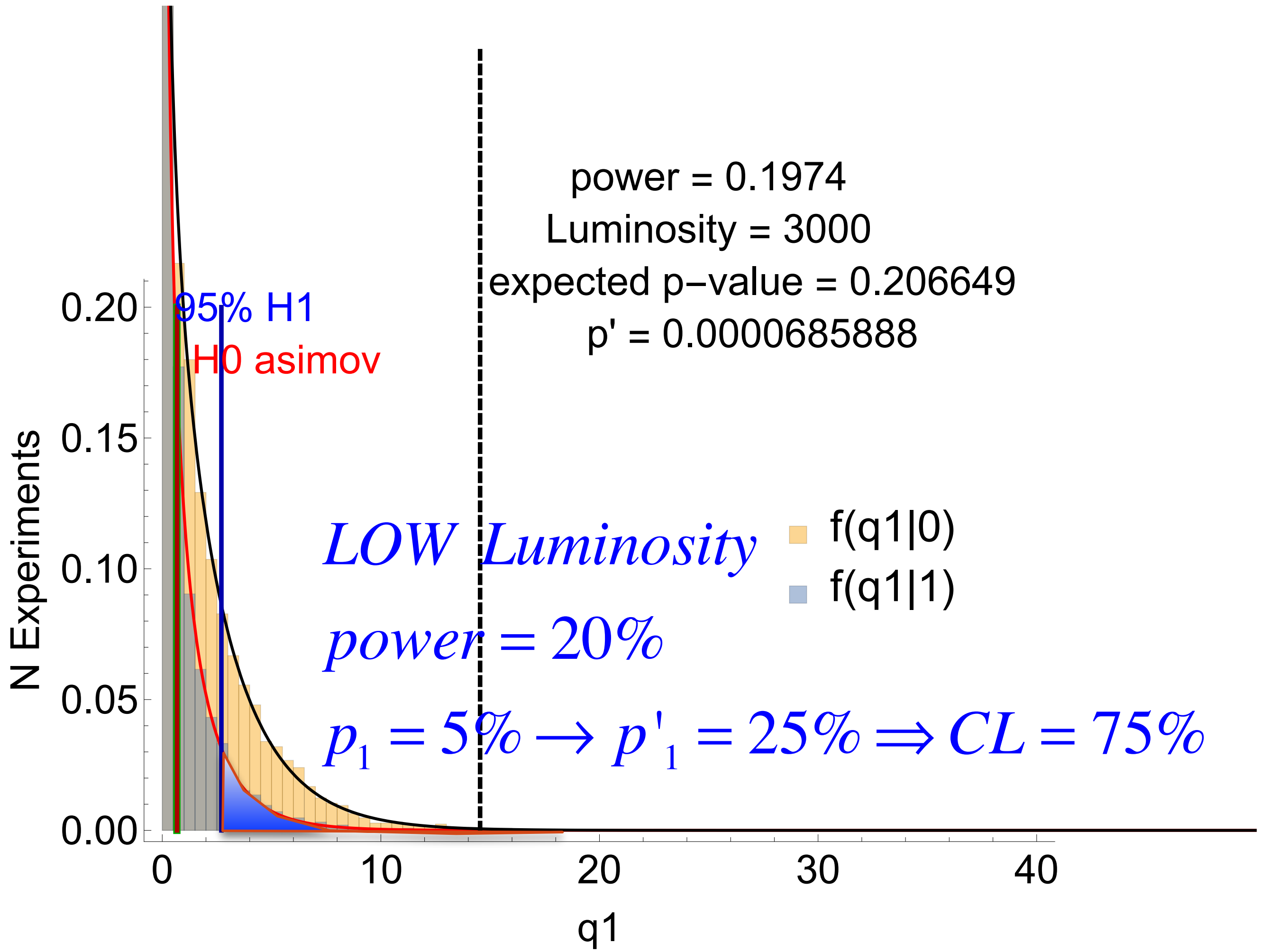

power = 0.6826
Luminosity = 20000
expected p−value = 0.017329
p' = 0.0294067
95%
H1
$p_{null} = p_1 = 5\% \Rightarrow$
$power = 68\% \Rightarrow$
$p'_{null} = \frac{5\%}{0.68} = 7.3\% \Rightarrow CL = 92.7\%$
f(q1|0)
f(q1|1)
N Experiments
q1
0.00
0.05
0.10
0.15
0.20
0
10
20
30
40


power = 0.6826
Luminosity = 20000
expected p−value = 0.017329
p' = 0.0294067
95% H1
H0 asimov
to get exclusion @ 95% CL
$p_1 = 2.94\% \Rightarrow power = 58.8\% \Rightarrow$
$p'_1 = 5\%$
f(q1|0)
f(q1|1)
N Experiments
q1
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40



power = 0.90721
Luminosity = 40000
expected p−value = 0.00140732
p' = 0.0450992
95% H1
H0 asimov
Increase Luminosity
power = 91%
$p_1 = 5\% \to p'_1 = 5.5\% \Rightarrow CL = 94.5\%$
f(q1|0)
f(q1|1)
N Experiments
0.20
0.15
0.10
0.05
0.00
0
10
20
30
40
q1

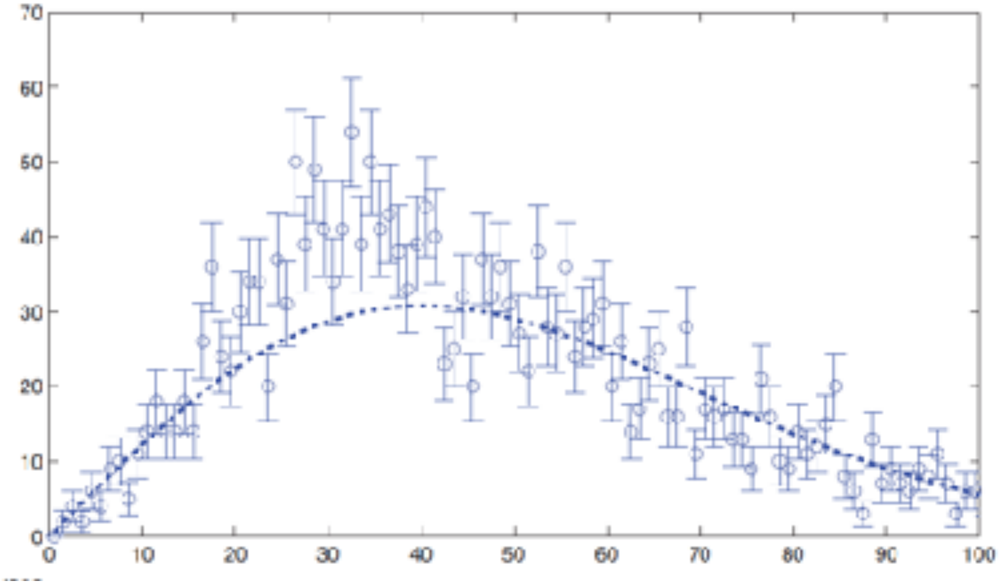

power = 0.4312
Luminosity = 10000
expected p−value = 0.0676342
p' = 0.010264
95% H1
H0 asimov
Decrease Luminosity
power = 43%
p1 = 5% → p'1 = 11% ⇒ CL = 89%
f(q1|0)
f(q1|1)
N Experiments
q1
0.00
0.05
0.10
0.15
0.20
0
10
20
30
40


power = 0.1974
Luminosity = 3000
expected p−value = 0.206649
p' = 0.0000685888
95% H1
H0 asimov
LOW Luminosity
power = 20%
$p_1 = 5\% \to p'_1 = 25\% \Rightarrow CL = 75\%$
f(q1|0)
f(q1|1)
N Experiments
q1
0.00
0.05
0.10
0.15
0.20
0
10
20
30
40


Look Elsewhere Effect
E.G., O. Vitells "Trial factors for the look elsewhere effect in high energy physics",
Eur. Phys. J. C 70 (2010) 525

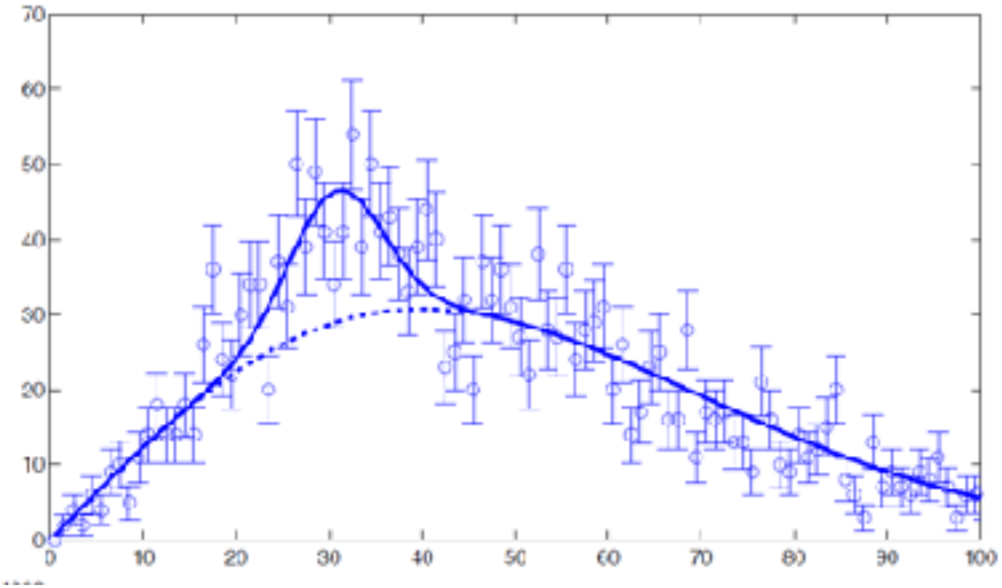
Look Elsewhere Effect

- Is there a signal here?



Look Elsewhere Effect

- Looks like a signal at m=30
- What is its significance?



Test the BG hypothesis
At m=30

$$q_0(\theta) = \begin{cases} -2\log\dfrac{L(\mu=0)}{L(\hat{\mu},\theta)} \\ 0 \end{cases}$$

$$q_{fix,obs} = -2\ln\frac{L(b)}{L(\hat{\mu}s(m=30)+b)}$$

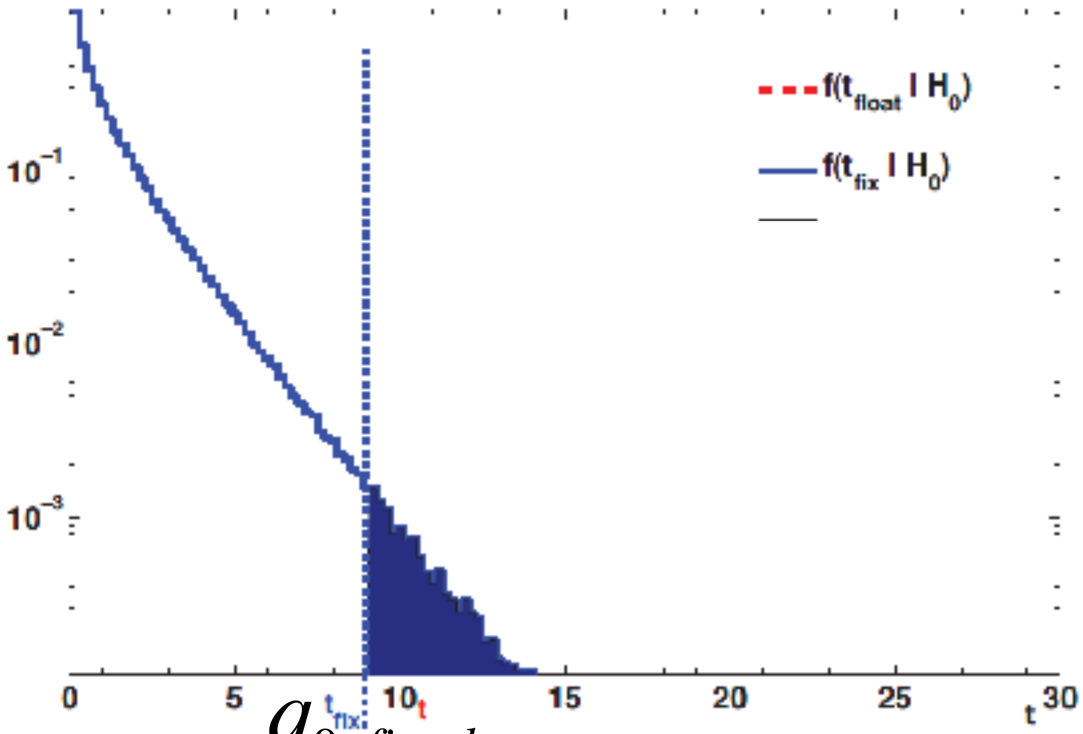
$$Z = \sqrt{q_{0,fix,obs}}$$

Look Elsewhere Effect

$$q_{0,fix} = -2ln\frac{L(\mu=0)}{L(\hat{\mu}s(30)+b)}$$

$$f(q_{0,fix} \mid H_0) \sim \chi^2$$

$$p_{fix} = \int_{q_{fix,obs}}^{\infty} f(q_0 \mid H_0)dq_0$$



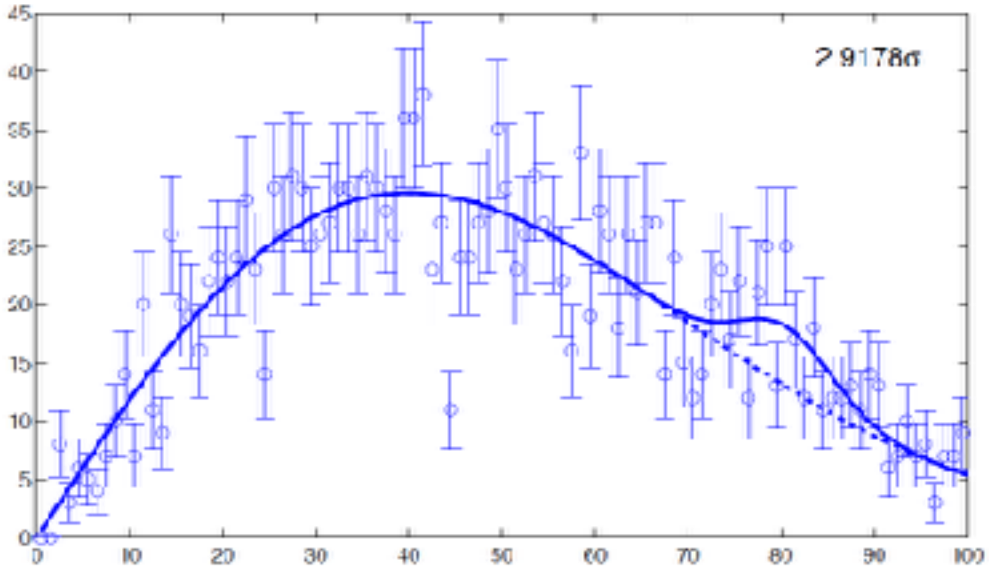
p_{fix} *answers the question :*

What is the probability to have a fluctuation

as or bigger than the observed one?

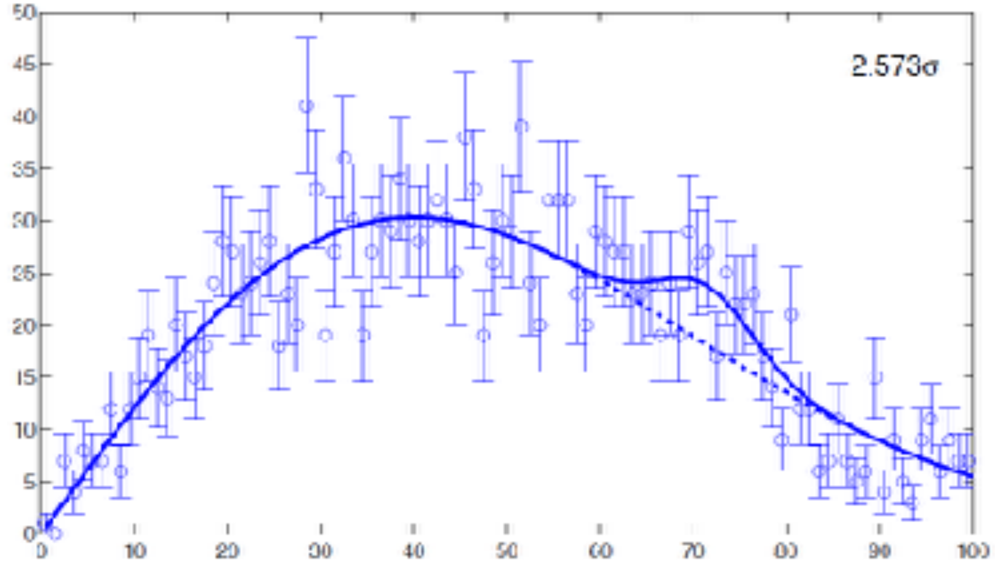
Look Elsewhere Effect

- Would you ignore this signal, had you seen it?



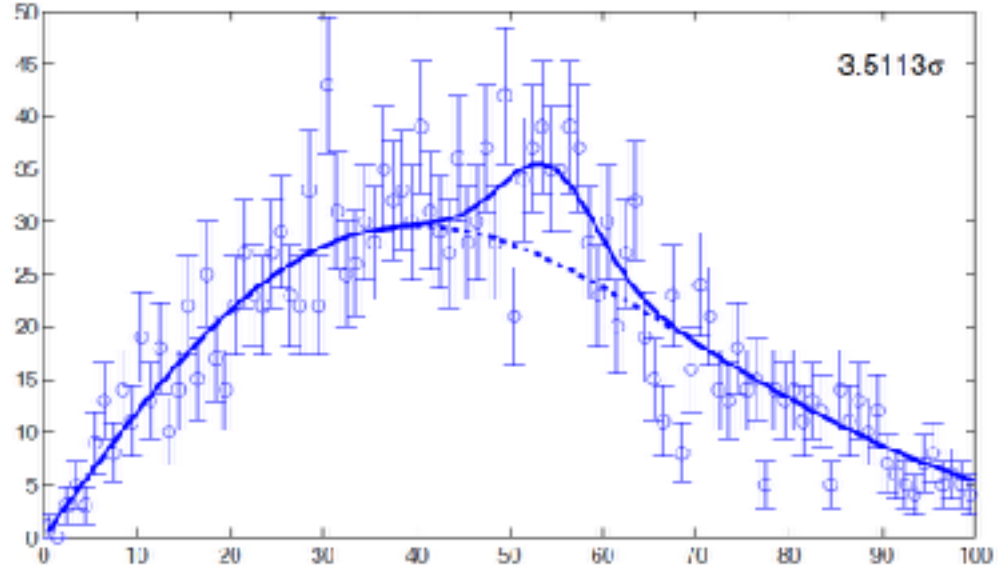
Look Elsewhere Effect

- Or this?



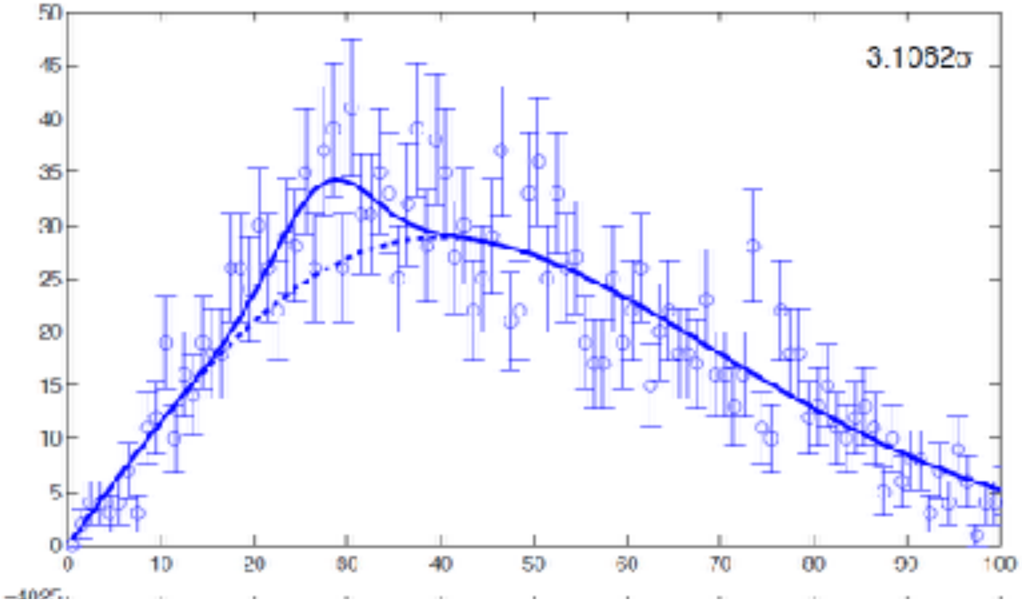
Look Elsewhere Effect

- Or this?



Look Elsewhere Effect

- Or this?
- Obviously NOT!
- ALL THESE "SIGNALS" ARE BG FLUCTUATIONS



The right question :

What is the probability to have a fluctuation as or bigger than the observed one *ANYWHERE in the mass search range*?

It is reasonable to think that, if ΔM is the mass range and σ_M is the experimental mass resolution[39] the enhancement LEE will be:

$$LEE = \frac{p_0^{global}}{p_0^{local}} \sim \frac{\Delta M}{\sigma_M} \tag{222}$$

In fact the mass range can be considered as given by a number $\Delta M/\sigma_M$ of independent observations.

More specifically, if q_0 is used as test statistics for the particle discovery, this quantity will be a function of the mass $q_0(m)$. Given a specified CL α corresponding to a threshold c on q_0, the Look-Elsewhere enhancement, also called **trial factor** is defined as:

$$LEE = \frac{p(q_0^{max}(m) > c)}{p(q_0(m) > c)} \tag{223}$$

where $q_0^{max}(m)$ is the maximum value of the test statistics in the full explored range.

More specifically, if q_0 is used as test statistics for the particle discovery, this quantity will be a function of the mass $q_0(m)$. Given a specified $CL\ \alpha$ corresponding to a threshold c on q_0, the Look-Elsewhere enhancement, also called **trial factor** is defined as:

$$LEE = \frac{p(q_0^{max}(m) > c)}{p(q_0(m) > c)} \tag{223}$$

where $q_0^{max}(m)$ is the maximum value of the test statistics in the full explored range.

A generally accepted estimate is

$$LEE = \frac{1}{3}\frac{\Delta M}{\sigma_M} Z_{fix} \tag{224}$$

where Z_{fix} is the local "significance" in number of gaussian standard deviations of the assumed threshold $Z_{fix} \sim \sqrt{c}$. This becomes equal to eq. 222 for $Z_{fix} = 3$, that is for a 3 std. deviation local signal.

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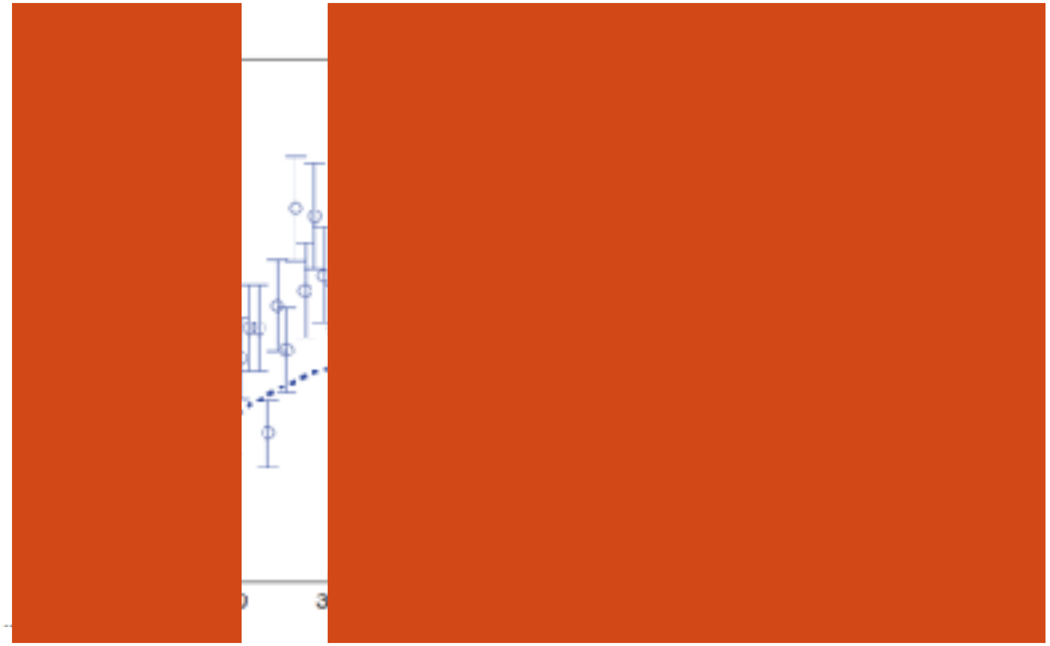
Let's consider a resonance search on a 100 GeV wide mass range where a 3σ signal is found at a given mass, with a resolution of 2 GeV. If we apply eq. 224 we get a trial of 50, so that: $p_0^{local} = 1.34 \times 10^{-3} \rightarrow p_0^{global} = 6.7\%$. On the other hand, in case of a 5σ local effect, the trial is 80 but $p_0^{local} = 2.86 \times 10^{-7} \rightarrow p_0^{global} = 2.3 \times 10^{-5}$. This explains why, in the search for an unknown particle, a 5σ effect is normally required, a 3σ one not being considered sufficient.

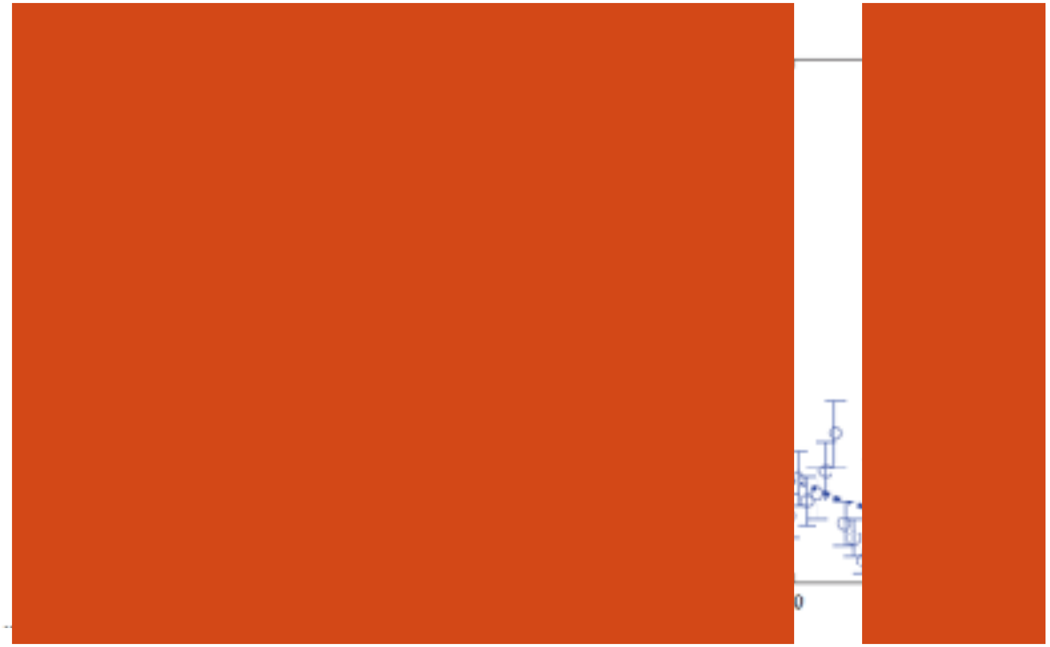
Sliding Window

$$q_0 = -2ln\frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



Sliding Window

$$q_0 = -2ln\frac{L(\mu = 0)}{L(\hat{\mu}s(m)+b)}$$



Sliding Window

- Assuming the signal can be only at one place
- pick the one with the MAXIMUM SIGNIFICANCE

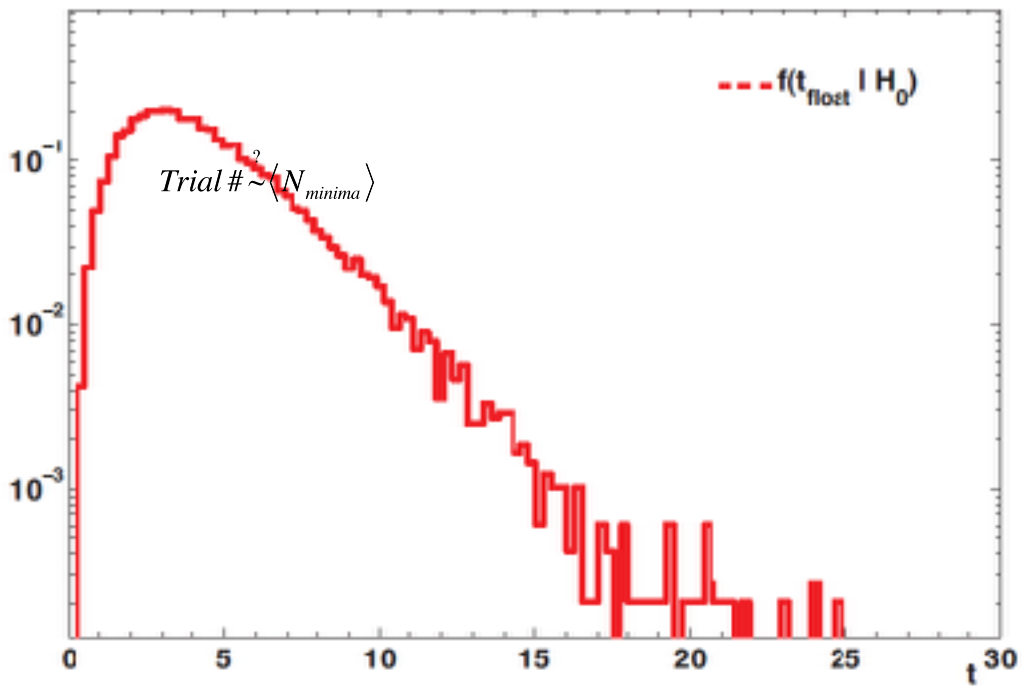
$$q_{0,float} = \max_{m}\left(q_0(m)\right)$$

Look Elsewhere Effect

- The distribution $f(q_{float}|H_0)$ does not follow a chi-squared with 2dof because the mass parameter is not defined under the null hypothesis

$$\exists m_{fix} \quad q_0(\hat{m}) \geq q_0(m_{fix})$$

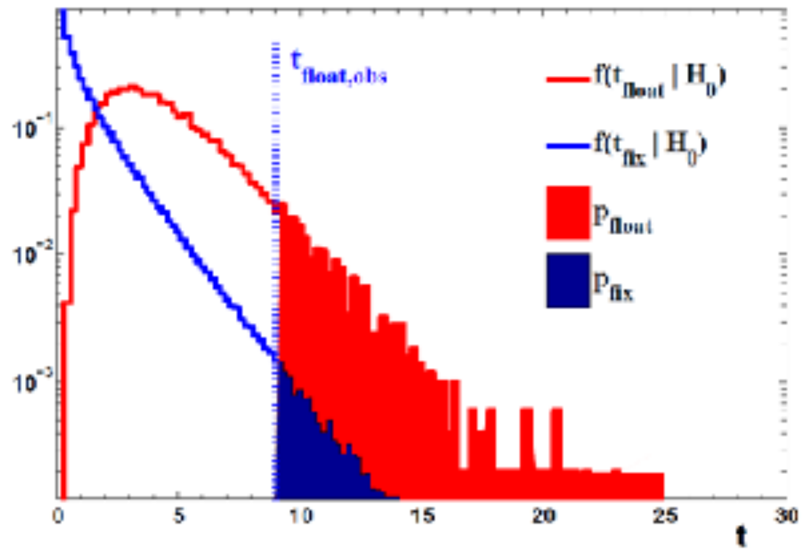
The χ^2_1 distribution is pushed to the right



trial#

- Assume a maximal local fluctuation at mass $\hat{m} = 30$
- The observed q_0 is given by

$$q_{0,obs} = -2ln\frac{L(\mu=0)}{L(\hat{\mu}s(m)+b)}$$



$$p_{fix} = \int_{q_{0,obs}}^{\infty} f(q_{0,fix} | H_0) dq_{0,fix}$$

$$p_{float} = \int_{q_{0,obs}}^{\infty} f(q_{0,float} | H_0) dq_{0,float}$$

$$\boxed{trial\# = \frac{p_{float}}{p_{fix}}}$$

$$p_0^{obs} = \int_{q_0^{obs}}^{\infty} f(q_0/0)dq_0$$

$$p_0^{exp} = \int_{\tilde{q}_0}^{\infty} f(q_0/0)dq_0$$

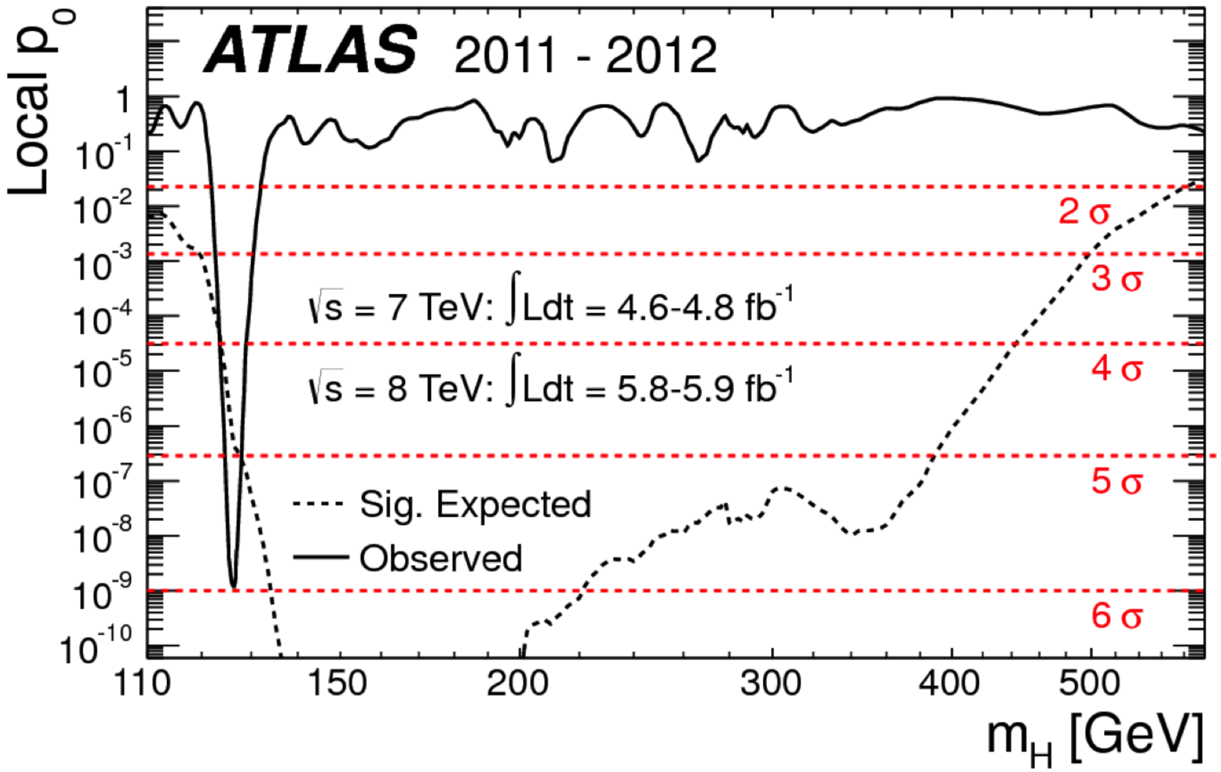


FIGURE 25. Discovery plot. Observed (solid) and expected (dashed) local p_0s as a function of the Higgs mass. The corresponding gaussian significance is shown in the right hand scale. At M_H=125 GeV a large and narrow fluctuation is observed. The probability that the background only can give rise to an equal or larger fluctuation than the one observed, is of order 10^{-9} and corresponds to slightly less than 6 gaussian standard deviations. The observed fluctuation is larger than the one expected for a Standard Model Higgs boson. (taken from ATLAS collaboration, Phys.Lett. B716 (2012) 1-29)

Local significance

$$p_0 = \int_Z^\infty G(x/0,1)dx$$

significance Z	p_0
1	15.8%
2	2.27%
3	1.34×10^{-3}
4	3.16×10^{-5}
5	2.86×10^{-7}
6	9.87×10^{-10}
7	1.28×10^{-12}

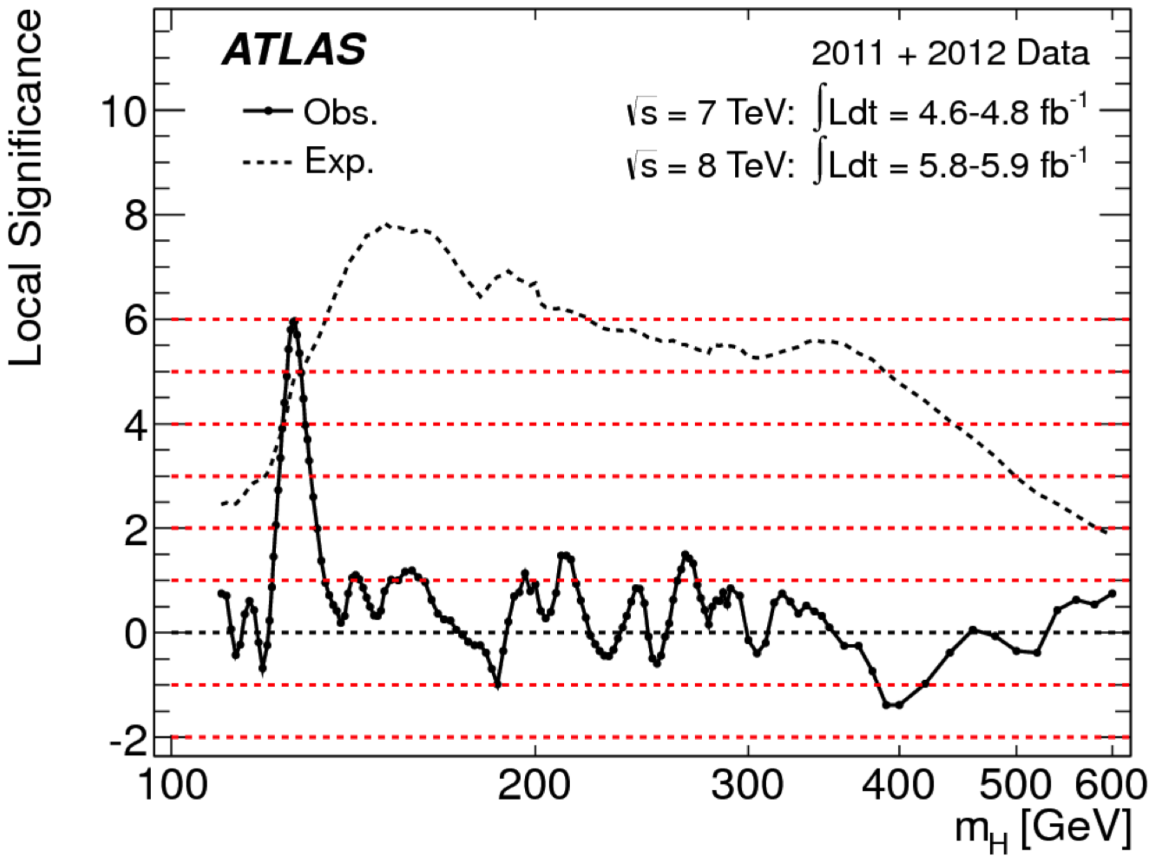


FIGURE 26. Same as figure 25 but expressed in terms of significance, namely in number of gaussian standard deviations. (taken from ATLAS collaboration, Phys.Lett. B716 (2012) 1-29)

At M_H=125 GeV a significance of 5.9σ is observed.

Global significance is 5.1σ if we consider the full explored mass range $110\div600$ GeV.

$$CL_s^{obs} = \frac{\int_{q_1^{obs}}^{\infty} f(q_1/1)dq_1}{\int_{q_1^{obs}}^{\infty} f(q_1/0)dq_1}$$

$$CL_s^{exp} = \frac{\int_{\tilde{q}_1}^{\infty} f(q_1/1)dq_1}{\int_{\tilde{q}_1}^{\infty} f(q_1/0)dq_1}$$

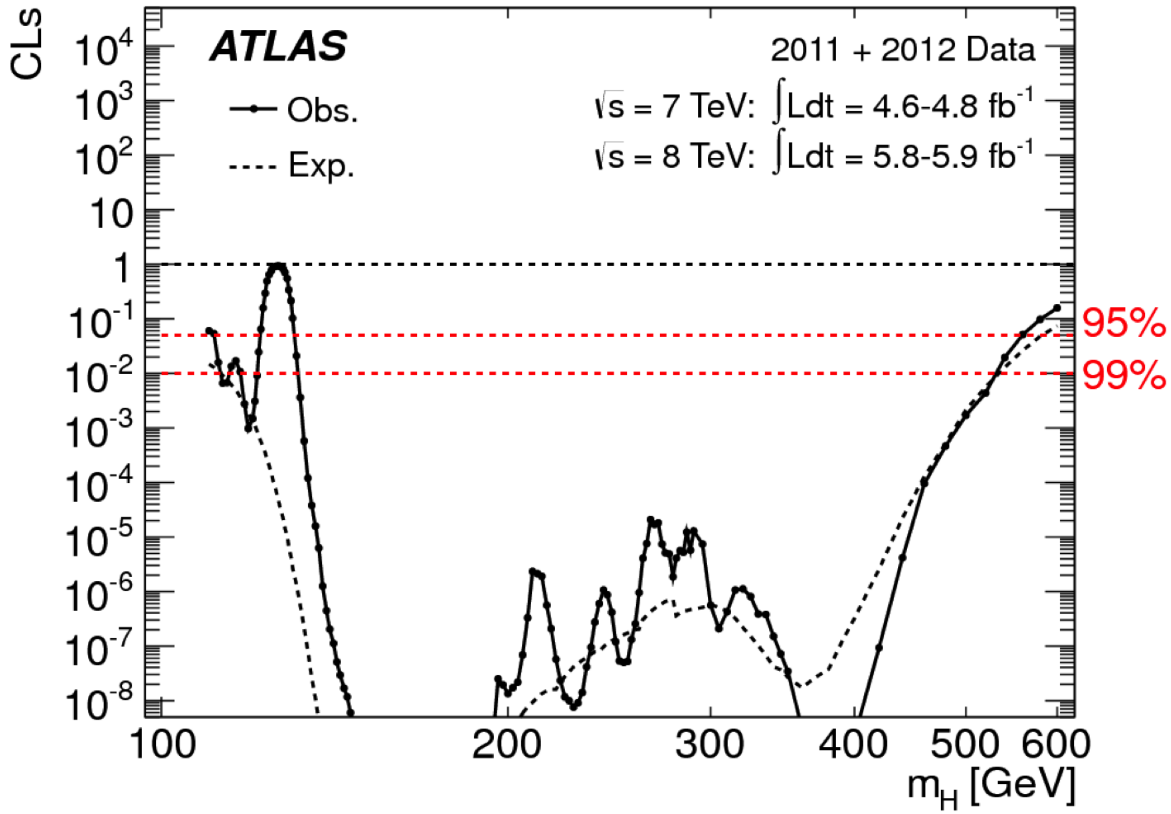


FIGURE 27. Exclusion plot. The CL_s is plotted vs. M_H. For all the masses where CL_s is below a fixed confidence level (95% and 99% are explicitly indicated in the plot), the Standard Model signal is excluded at that CL. Using a 95% limit only the region around 125 GeV is not excluded. (taken from ATLAS collaboration, Phys.Lett. B716 (2012) 1-29)

This is the first exclusion plot, since all the values of M_H with a CL_s below e.g. 5% are excluded at the 95% CL. Almost the full mass range considered by the experiment is excluded apart from the region around the signal.

$$CL_s^{obs} = \frac{\int_{q_1^{obs}}^{\infty} f(q_1/1)dq_1}{\int_{q_1^{obs}}^{\infty} f(q_1/0)dq_1}$$

$$CL_s^{exp} = \frac{\int_{\tilde{q}_1}^{\infty} f(q_1/1)dq_1}{\int_{\tilde{q}_1}^{\infty} f(q_1/0)dq_1}$$

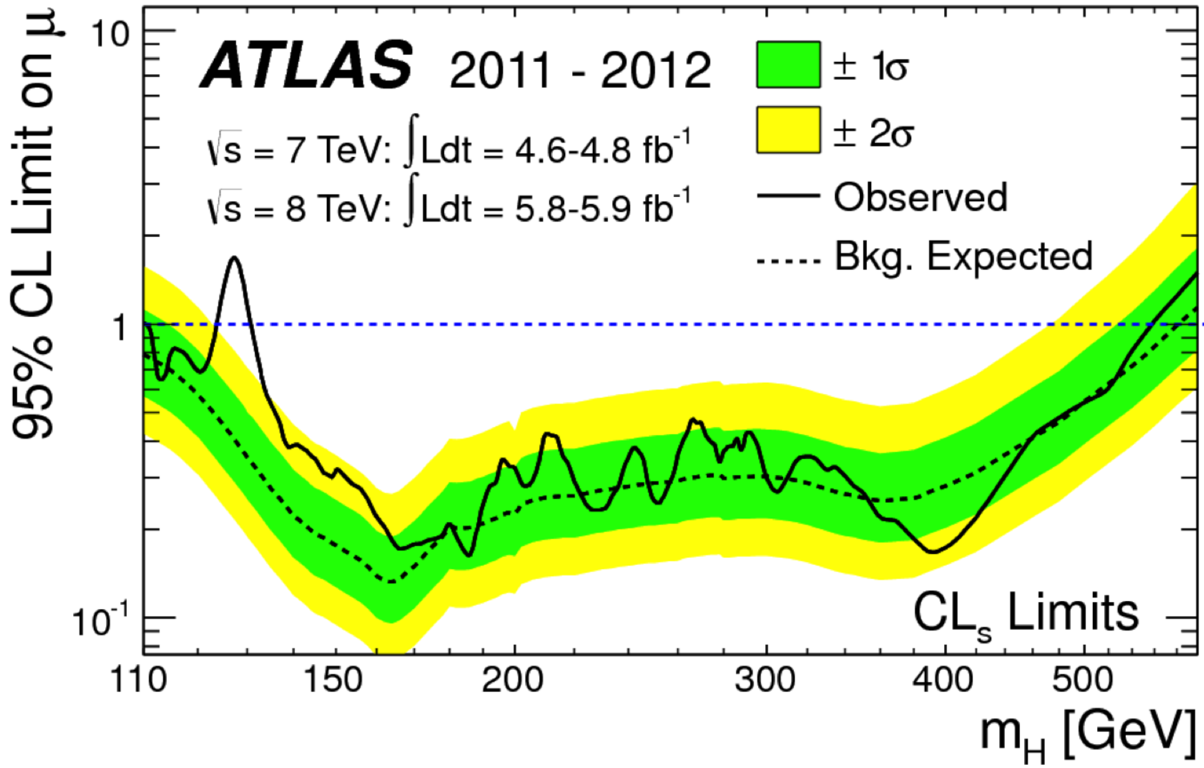


FIGURE 28. Exclusion "Brazilian plot". Observed (solid) and expected(dashed) 95% CL upper limits on the signal strength μ as a function of M_H. ± 1 (green) and ± 2 (yellow) std.deviations bands are also shown for the expected limit. (taken from ATLAS collaboration, Phys.Lett. B716 (2012) 1-29)

7.6.4. *Upper limits on μ.* Figure 28 shows the upper limit on μ as a function of M_H. The solid line shows the observed 95% upper limit on μ, that is that value of μ for which the observed value of CL_s (given by eq. 228) is equal to 5%. The dashed line shows the expected 95% upper limit, based on the median value of q_1 (according to eq. 229).

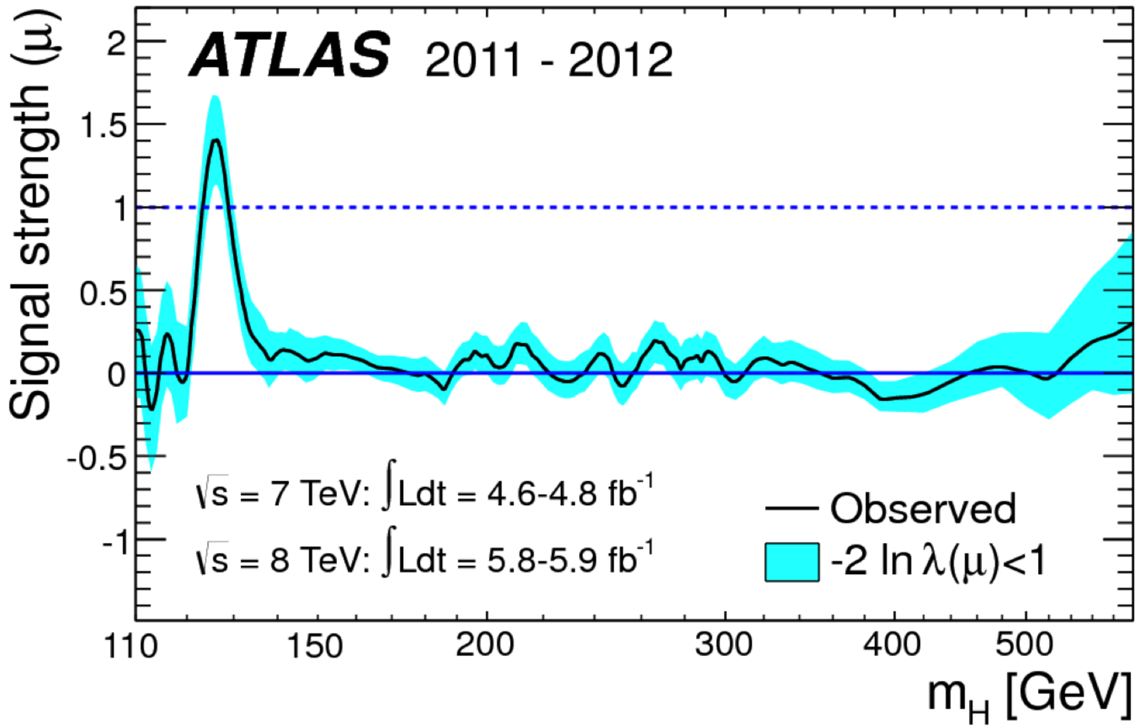


FIGURE 29. Best estimate of the signal strength with a confidence interval of 1 std.deviation as a function of M_H. For all the excluded region, the result is compatible with 0. In the signal region $\hat{\mu}$ deviates from the Standard Model expected value of 1 by slightly more than 1 st. deviation.(taken from ATLAS collaboration, Phys.Lett. B716 (2012) 1-29)

7.6.5. *Signal Strength.* Figure 29 shows the best value of the signal strength μ as a function of M_H. For each mass value, the profile likelihood ratio (eq. 207) is minimized with respect to μ, and a central confidence interval with a probability content of 68.3%