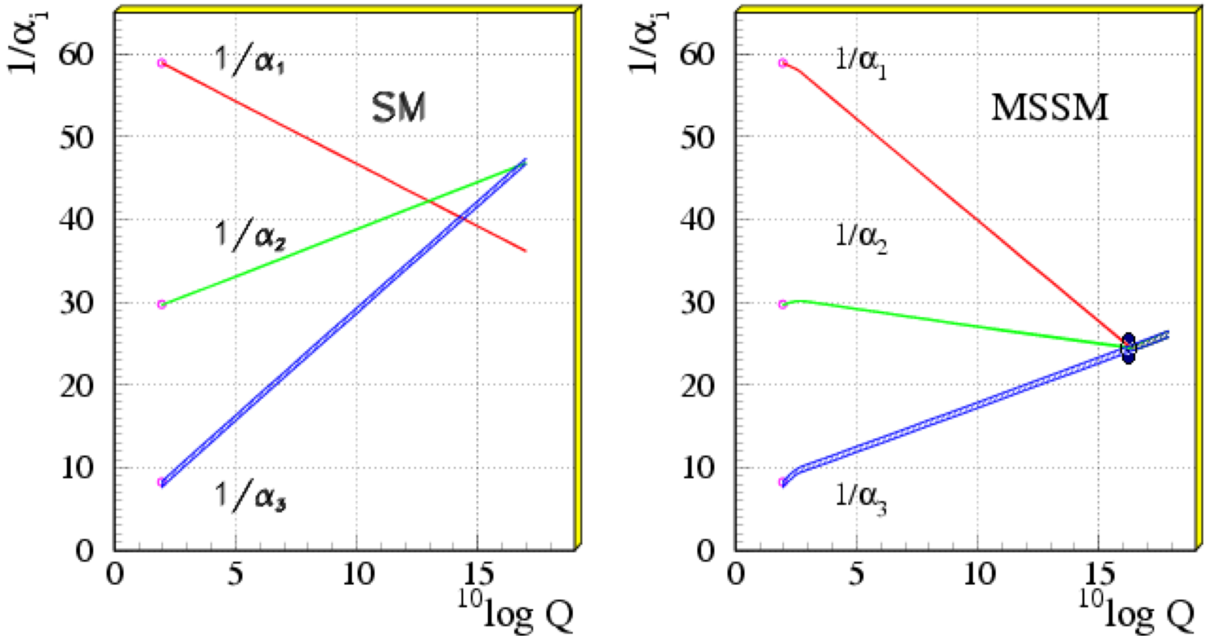
Energy scales in the ∞ly small - VI

- Grand Unification Scale. From the observation that weak, em and strong coupling constants are "running" coupling constants, if we plot them vs. q^2 we get:



Around 10^{16} GeV meeting point ??

Energy scales in the ∞ly small - VII

- Why LHC is concentrate on the O(TeV) scale ?
- There is an intermediate scale around the TeV. It is motivated by the "naturalness" – "fine tuning" – "hierarchy" problem connected to the properties of the Higgs Mass.

Mass parameter in the SM lagrangian

$$m_H^2 \sim \boxed{-2\mu^2} + \boxed{\frac{g^2}{(4\pi)^2} M^2}$$

Quantum corrections

- The Higgs mass $\boldsymbol{m_H}$ is UV sensitive (its value depends on quantum corrections)

- $\boldsymbol{M}$ is the scale up to which we have the UV theory.

- If no other scale is there btw Higgs and Planck, $\boldsymbol{M}=\boldsymbol{M_{Planck}}$, so that strong cancellations are needed between $-2\boldsymbol{\mu}^2$ and $\boldsymbol{g}^2\boldsymbol{M}^2/(4\pi)^2$ to give the observed Higgs Mass

- This is un-natural..

- If $\boldsymbol{M} \approx$ O(TeV) all becomes natural, e.g. MSSM, Technicolor,…

$$\Delta \gtrsim \left(\frac{m_{\mathrm{NP}}}{0.5\ \mathrm{TeV}}\right)^2$$

Energy scales in the ∞ly small - Summary

quantity	value	Energy
Bohr radius	0.53×10^{-10} m (0.5 Å)	3.7 keV
Electron Compton wavelength	3.86×10^{-13} m (386 fm)	0.51 MeV
Electron classical radius	2.82×10^{-15} m (2.8 fm)	70 MeV
Proton radius – QCD confinement scale	0.82×10^{-15} m (0.8 fm)	240 MeV
Fermi scale (electroweak scale)	7×10^{-19} m	250 GeV
“New Physics” scale		1 TeV
GUT Scale		10^{16} GeV
Planck scale	1.62×10^{-35} m	1.2×10^{19} GeV

The TeV scale is the maximum reachable with the present accelerator technology

Probability/Frequency of a final state:
the cross-section and the decay width

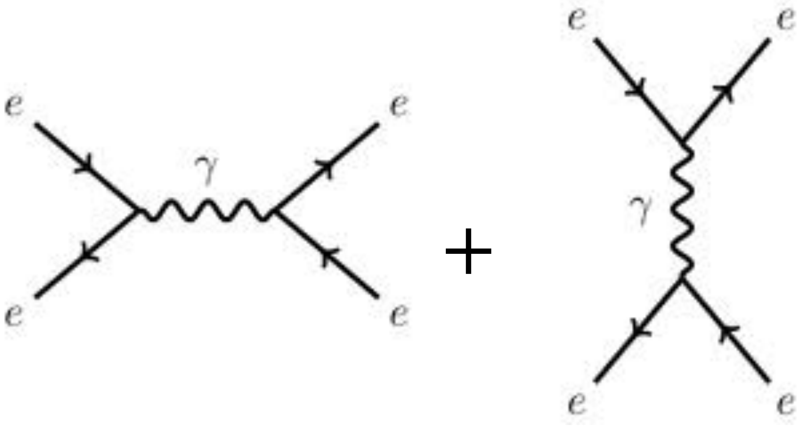
- The **cross-section** measures the “probability” of a given final state in a collision (actual definition will be in a later lecture). It is a $[L]^2$.
- The **decay width** and the **branching ratio** measure the “probability” of a given final state in a decay. The decay width is the inverse of the lifetime so that it is a $[T]^{-1}$. The branching ratio is an adimensional quantity
- If we include **cross-sections** and **decay widths**, we enter in the quantum field theories where the normalized Planck constant enters in the game.
- In the “natural system” the units are

$$\hbar = c = 1$$

  - **cross-section** is a $(\text{length})^2$ so an $(\text{energy})^{-2}$.
  - **decay width** is a $(\text{time})^{-1}$ so an (energy)
  - $1\ \text{GeV}^{-2} = 3.88\text{x}10^{-4}$ barn $(1\ \text{b} = 10^{-24}\ \text{cm}^2 = 100\ \text{fm}^2)$

Cross-section scales

- Relation between an experimental cross-section and the theory (same applies for branching ratios)



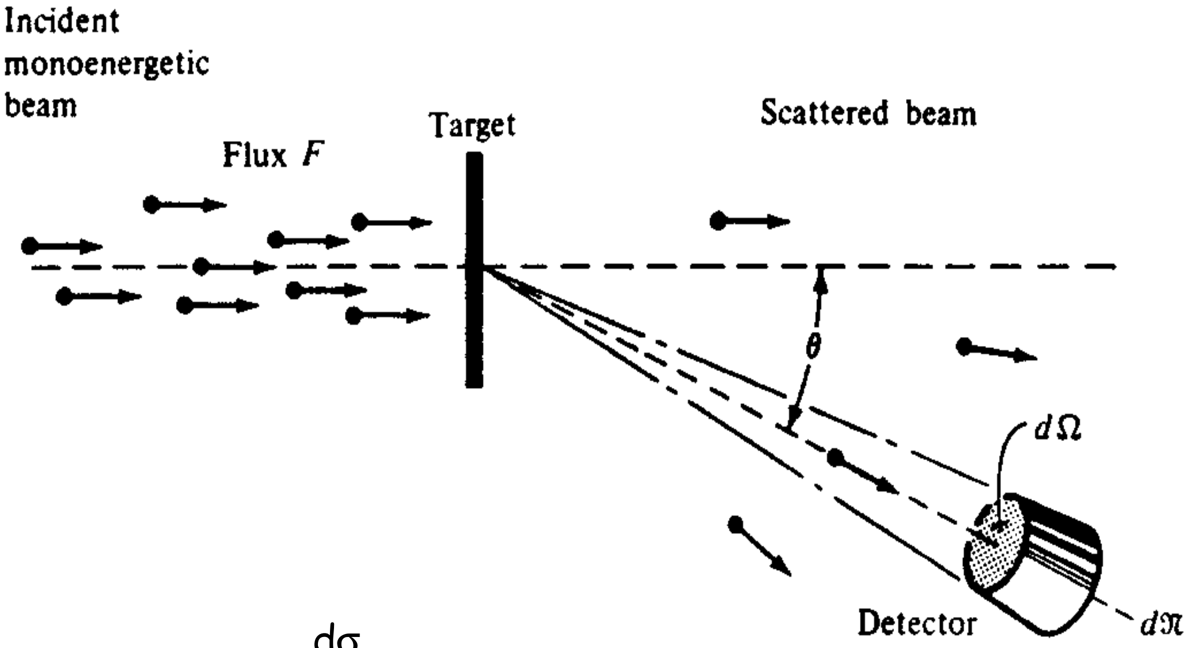
$$\sigma = \int \left| \qquad + \qquad \right|^2 d\varphi$$

Two ingredients in the theory calculations:

→ dynamics (amplitude from lagrangian, Feynman diagrams… mainly the coupling constants);

→ phase space ***dϕ***

NB: the integration on the phase space DEPENDS in general on the experiment details (accessible kinematic region) → **Montecarlo**



$$\sigma(\theta) = \frac{d\sigma}{d\Omega}$$

$$dN = I \cdot n \cdot x \cdot \frac{d\sigma}{d\Omega} d\Omega$$

$$N = \int_{4\pi} I \cdot n \cdot x \cdot \frac{d\sigma}{d\Omega} d\Omega = I \cdot n \cdot x \cdot \int_{4\pi} \frac{d\sigma}{d\Omega} d\Omega = I \cdot n \cdot x \cdot \sigma_{tot}$$

$$\sigma_{tot} = \int_{4\pi} \frac{d\sigma}{d\Omega} d\Omega$$

At colliders:
$N=L\sigma$

Cross-section order of magnitude estimates

- Based on dimensional arguments and few numbers (neglects phase-space and more…)
  - Electromagnetic processes: $e^+e^- \rightarrow \mu^+\mu^-, \gamma\gamma$
  - Weak processes: νN scattering
  - Hadron strong interaction scattering: pp scattering

α	**1/137**
$\mathbf{G_F}$	$\mathbf{10^{-5}\ GeV^{-2}}$
$\mathbf{r_p}$	**1 fm**
$\mathbf{1\ GeV^{-2}}$	$\mathbf{3.88 \times 10^{-4}\ b}$

$$\sigma\left(e^+e^- \to \mu^+\mu^-,\gamma\gamma\right) \approx \frac{\alpha^2}{s}$$

$$\sigma\left(\nu e \to \nu e\right) \approx G_F^2 2m_e E_\nu$$

$$\sigma\left(pp\right) \approx \pi r_p^2$$

$S=(1\ GeV)^2$	$S=(100\ GeV)^2$
20 nb	2 pb
40 fb	4 pb
30 mb	30 mb

Experimentally:

$\sigma(\nu e- => \nu e-) \sim 10^{-41}$ cm^2 x E_ν (GeV) = 0.01 fb x E_ν (GeV)

E_ν neutrino energy in laboatory

$S=2m_e E_\nu = 2*0.000511*E_\nu(\mathrm{GeV})$ GeV^2

$=> E_\nu(\mathrm{GeV}) \sim 1000 * s\ (\mathrm{GeV}^2)$

$\sigma(\nu e- => \nu e-)(s=1\ \mathrm{GeV}^2) \sim 10$ fb

$\sigma(\nu e- => \nu e-)(s=100\ \mathrm{GeV}^2) \sim 1$ pb

LifeTime (or Width) of a particle vs. theory

- As for the cross-section the value depends on two ingredients:
  - Decay type (weak, em, strong) through decay matrix element
  - Volume of the available phase space
- The Width Γ is an additive quantity: you have to add the ***partial widths*** of the single decays to get the ***total width***
- Useful formulas: two-body decay phase-space (rest system)

$$\Gamma = \frac{1}{8\pi}\frac{p}{M^2}|\mathfrak{M}|^2.$$

NB Dimensions: If Γ is [E] ➔ |M| is also [E]

$$|\vec{p}_1| = |\vec{p}_2| = \frac{[(M^2-(m_1+m_2)^2)(M^2-(m_1-m_2)^2)]^{1/2}}{2M},$$

Width (LifeTime) order of magnitude estimates

- The amplitude square has the dimensions of E^2.
  - Weak → $|\mathrm{Ampl}|^2 \approx G_F^2 \times M^6$
  - E.m. → $|\mathrm{Ampl}|^2 \approx \alpha^2 \times M^2$
  - Strong → $|\mathrm{Ampl}|^2 \approx \alpha_s(M)^2 \times M^2$
- Examples of estimates (wrong by factor ≈10 maximum):

Interaction	**Decay**	**Phase space (MeV^{-1})**	**$\|\mathrm{Ampl}\|^2$ (MeV^2)**	**Γ (MeV)**	**τ (s)**
Weak	$\pi^\pm \to \mu^\pm \nu$	6.0×10^{-5}	6.0×10^{-10}	3.6×10^{-14}	1.8×10^{-8} **(2.6×10^{-8})**
e.m.	$\pi^0 \to \gamma\gamma$	1.5×10^{-4}	0.97	1.4×10^{-4}	4.6×10^{-18} **(8.5×10^{-17})**
strong	$\rho^0 \to \pi^+\pi^-$	2.4×10^{-5}	6.0×10^5	13 **(150)**	5.0×10^{-23}

	Lifetime τ	**Width Γ**
Weak decays		
K_S, K_L	0.89564×10^{-10} s, 5.116×10^{-8} s	
$K^{\pm}$	1.2380×10^{-8} s	
Λ	2.632×10^{-10} s	
B-hadrons	$\approx 10^{-12}$ s	
Muon	2.2×10^{-6} s	
Tau-lepton	2.9×10^{-13} s	
Top-quark	$\approx 5 \times 10^{-25}$ s	2 GeV
e.m. decays		
π^0	8.52×10^{-17} s	8 eV
η	$\approx 10^{-19}$ s	1.30 keV
Strong decays		
J/ψ		92.9 keV
Υ		54.02 keV
ρ		149.1 MeV
ω		8.49 MeV
ϕ		4.26 MeV
Δ		114 ÷ 120 MeV

Recap - fundamental interactions

- Electromagnetic interaction:
  - Can be studied at all energies with "moderate" cross-sections;
  - Above O(100 GeV) becomes electro-weak
- Weak interactions:
  - At low energies it can be studied using decays of "stable" particles – large lifetimes and small cross-sections;
  - Above O(100 GeV) becomes electro-weak
- Strong interactions:
  - At low energy (below 1 GeV) "hadronic physics" based on confinement: no fundamental theory available by now
  - At high energies (above 1 GeV) QCD is a good theory: however since partons are not directly accessible, only "inclusive" quantities can be measured and compared to theory. Importance of simulations to relate partonic quantities to observables.

Comparison between beam possibilities

- Electrons:
  - Clean, point-like, fixed (almost) energy, but large irradiation due to the low mass. “Exclusive” studies are possible (all final state particles are reconstructed and a complete kinematic analysis can be done)
  - ➔ e^+e^- colliders less for energy frontier,mostly for precision measurements
- Protons:
  - Bunch of partons with momentum spectrum, but low irradiation. “Inclusive” studies are possible. A complete kinematic analysis is in general not possible (only in the transverse plane it is to first approximation possible)
  - ➔ highest energies are “easily” reachable, high luminosity are reachable but problems in the interpretation of the results; very “demanding” detectors and trigger systems.
- Anti-protons:
  - Difficult to obtain high intensities and high luminosity but no problems with energies, same problems of protons (bunch of partons)
- ➔ p-antip limited by luminosity, e^+e^- limited by energy BUT perfect for precision studies, pp good choice for energy frontier

Implications for experiments:

- You need high energy for
  - Probe electro-weak scales, get closer to higher scales
  - Enlarge the achieveble mass spectrum (particle discoveries)
- You need high beam intensity and large/dense targets or high efficiency detectors
  - To access low probability phenomena
- You need high resolution detectors
  - To improve particle discrimination especially for rare events.

End of the Introduction

- Present prospects of Elementary Particle experiments:
  - ENERGY frontier → LHC, HL-LHC, ILC, TLEP,….
  - INTENSITY frontier → flavour-factories, fixed target,…
  - SENSITIVITY frontier → detectors for dark matter, neutrinos,..
- The general idea is to measure quantities for which you have a clear prediction from the Standard Model, and a hint that a sizeable correction would be present in case of "New Physics".