Not only event counting

- Once the candidate sample is obtained many quantities can be measured (particle properties, e.g. particle mass).
- BUT in most cases they are obtained from a **FIT** to a data distribution. So, you divide events in bins and extract the quantity as a ***fit parameter*** ➔ the event counting is still one major source of uncertainty ➔ the uncertainty on the parameter depends on the statistics $\approx \sqrt{N_i}$.
- Example:
  - Measure the mass of a "imaginary" particle of M=5 GeV.
  - Mass spectrum, gaussian peak over a uniform background
  - FIT in three different cases: 10^3, 10^4 and 10^5 events selected

Mass uncertainty due to statistics

Observations:

- → Poissonian uncertainty on each bin
- → Reduce bin size for higher statistics
- → Fit function = A+B*Gauss(M)
- → Free parameters: A,B,M (fixed width)
- → The fit is good for each statistics

Results

$N=10^3$ events:

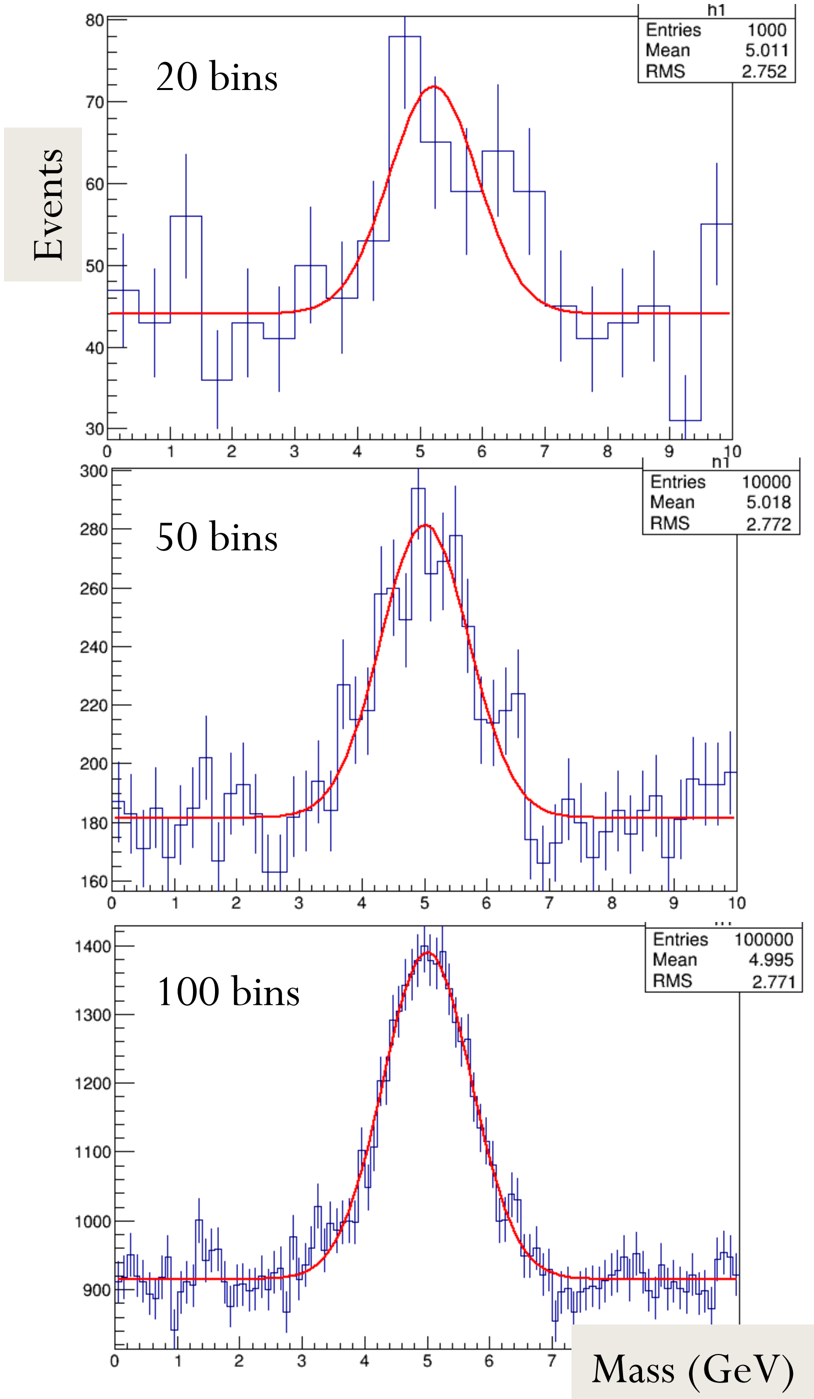
Mass = 5.22±0.22 GeV, χ^2 = 28 / 18 dof

$N=10^4$ events:

Mass = 5.01±0.06 GeV, χ^2 = 38 / 48 dof

$N=10^5$ events:

Mass = 5.02±0.02 GeV, χ^2 = 83 / 98 dof



Where could be a systematic uncertainty here ?

- Absolute mass scale: this can be measured using a candle of known mass. Not always it is available. e.g. Z for the Higgs mass at the LHC.
- Mass resolution: in most cases the width of the peak is given by the experimental resolution that sometimes is not perfectly gaussian, giving rise to possible distortion to the curve.
- Physics effects: knowledge of the line-shape, interference with the background…
- In general: M = central value ± stat.uncert. ± syst.uncert.

$\eta \to \pi^+\pi^-\pi^0$ decay

The light quark masses: study of $\eta \to \pi^+\pi^-\pi^0$decay

$\eta \to \pi\pi\pi$ decay ⇒ Isospin violation

e.m. strongly suppressed, induced dominantly by the strong interaction associated with the u-d quark mass difference

$$X = \sqrt{3}\frac{T_{\pi^+} - T_{\pi^-}}{Q_\eta}$$

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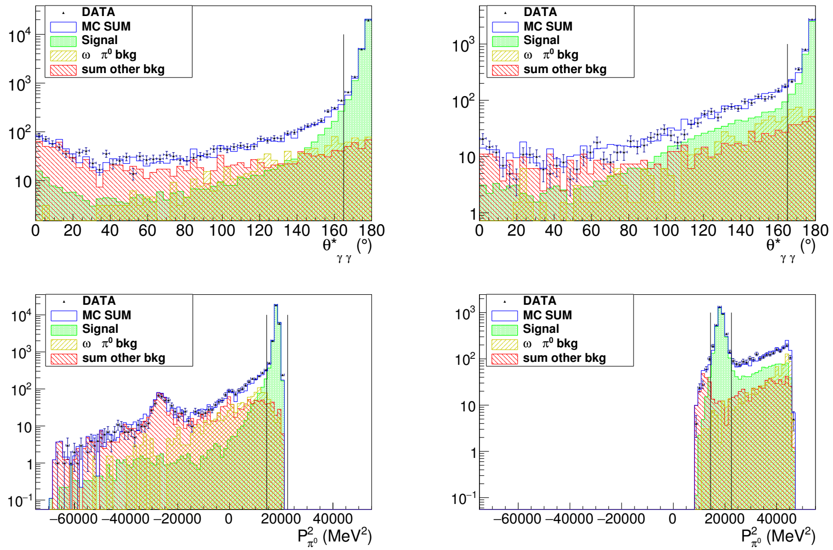
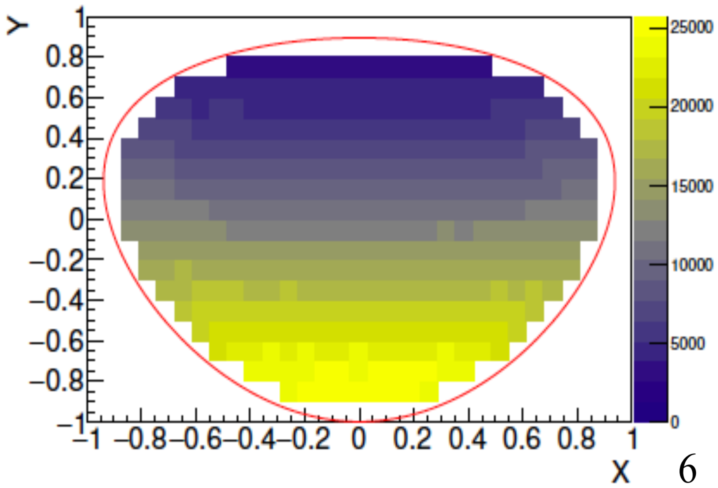


Fit to the Dalitz Plot

$$|A(X,Y)|^2 \propto 1 + aY + bY^2 + cX + dX^2 + eXY + fY^3 + \ldots$$



6

$\chi^2/\text{dof} = 360/365 \qquad p = 56\%$

$$a = -1.095 \pm 0.003^{+0.003}_{-0.002}$$

$$b = +0.145 \pm 0.003 \pm 0.005$$

$$d = +0.081 \pm 0.003^{+0.006}_{-0.005}$$

$$f = +0.141 \pm 0.007^{+0.007}_{-0.008}$$

$$g = -0.044 \pm 0.009^{+0.012}_{-0.013}$$

syst. error ($\times 10^4$)	Δa	Δb	Δd	Δf	Δg
EGmin	± 6	± 12	± 10	± 5	± 16
BkgSub	± 8	± 7	± 11	± 6	± 38
BIN	± 17	± 13	± 9	± 36	± 44
$\theta_{+\gamma}, \theta_{-\gamma}$ cut	$^{+0}_{-1}$	$^{+0}_{-2}$	$^{+2}_{-2}$	$^{+3}_{-0}$	$^{+3}_{-2}$
Δt_e cut	$^{+6}_{-11}$	$^{+12}_{-1}$	$^{+18}_{-1}$	$^{+3}_{-8}$	$^{+26}_{-54}$
$\Delta t_e - \Delta t_\pi$ cut	± 0	$^{+0}_{-1}$	$^{+3}_{-1}$	± 0	$^{+2}_{-1}$
$\theta^*_{\gamma\gamma}$ cut	$^{+14}_{-5}$	$^{+2}_{-1}$	$^{+21}_{-12}$	$^{+5}_{-25}$	$^{+26}_{-38}$
MM	$^{+8}_{-10}$	$^{+46}_{-43}$	$^{+49}_{-45}$	$^{+57}_{-62}$	$^{+100}_{-92}$
ECL	± 0	± 8	± 6	± 9	± 12
TOTAL	$^{+26}_{-25}$	$^{+52}_{-48}$	$^{+59}_{-50}$	$^{+69}_{-77}$	$^{+123}_{-129}$

c, e param. are C-violating, consistent with zero

KLOE-2 JHEP 05(2016)019

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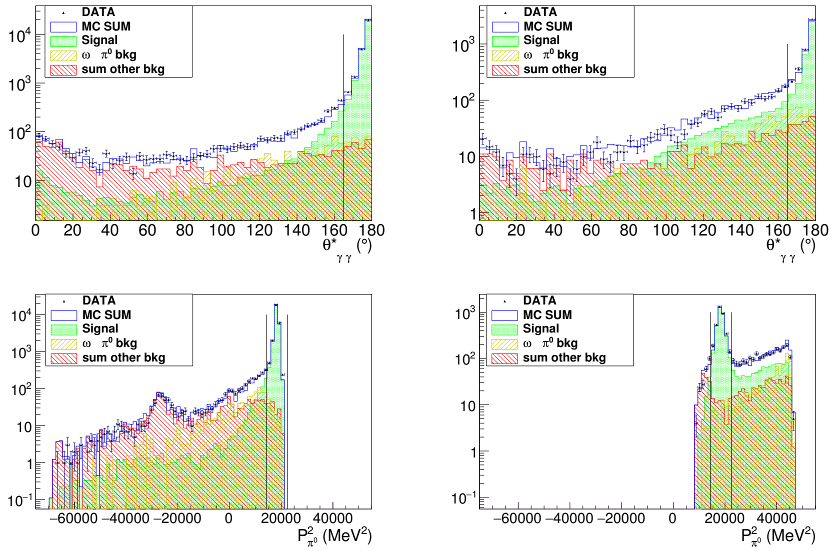

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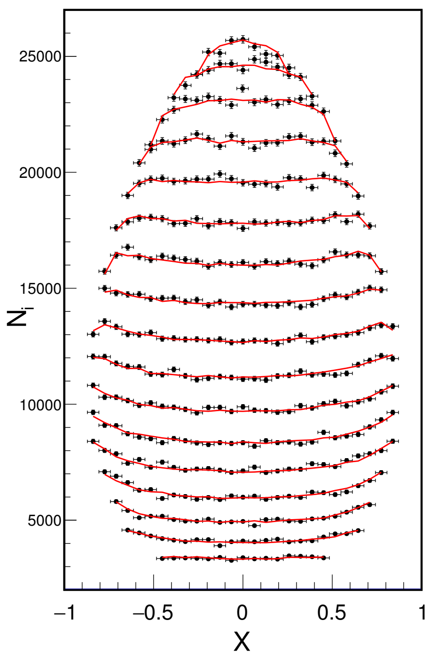

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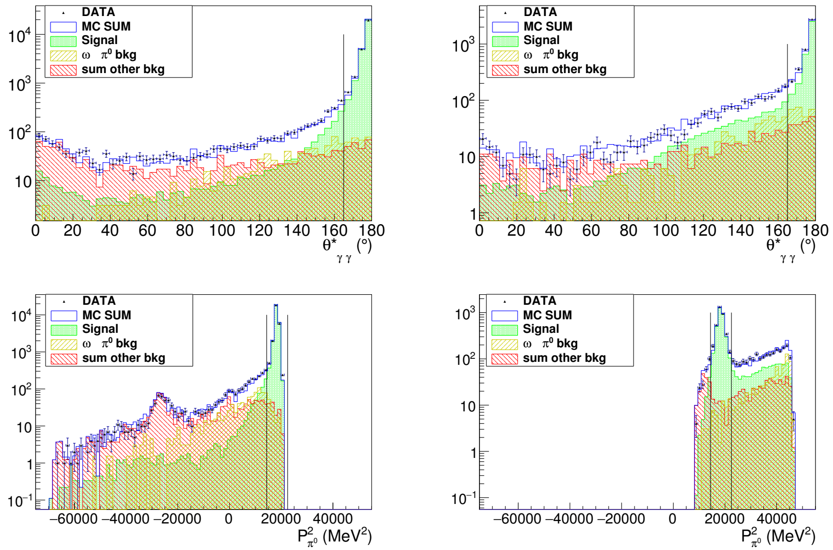

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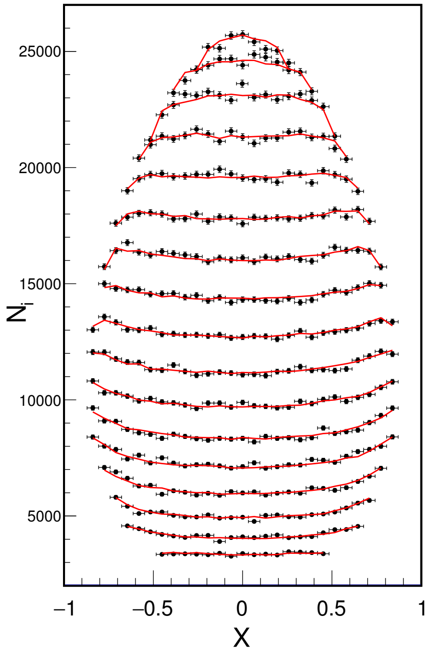

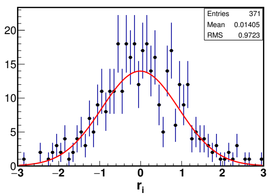

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Uncertainty combination

central value ± **stat.uncert.** ± **syst.uncert.**

Can we combine stat. and syst. ? If yes how ?

The two uncertainties might have different probability meaning: typically one is a gaussian 68% C.L., the other is a "maximum" uncertainty, so in general it is better to hold them separate.

If needed better to add in quadrature rather than linearly.

Comments on multivariate methods:

The emphasis is often on controlling systematic uncertainties between the modeled training data and Nature to avoid false discovery.

Although many classifier outputs are "black boxes", a discovery at 5σ significance with a sophisticated (opaque) method will win the competition if backed up by, say, 4σ evidence from a cut-based method.

Summarizing

- Steps of an PP experiment (assuming the accelerator and the detector are there):
  - Design of a **trigger**
  - Definition of an offline **selection**
  - **Event counting** and **normalization** (including **efficiency** and **background** evaluation)
  - **Fit** of "candidate" distributions
- Uncertainties
  - Statistical due to Poisson fluctuations of the event counting
  - Statistical due to binomial fluctuations in the efficiency measurement
  - Systematic due to non perfect knowledge of detector effects.

Proposed exercises

. We have designed an event selection chain based on the simulation in such a way that at the end of the selection 25% of the selected events are *signal* events and 75% are *background* events. How many total candidates do we need to collect in order to observe the signal with at least 5 st.dev. significance ?

:. The expected rate of neutrinos interacting in our detector is 0.23×10^{-2} evts/day, and the average efficiency for the detection of such interactions is 43.2%. Evaluate the probability to detect at least a neutrino in the first 24h, in the first year and in the first 10 years of operation.

. In the 2011+2012 LHC dataset (corresponding to about 25 fb^{-1}), a sample of 2.24×10^5 $t\bar{t}$ events has been collected. We know that $\sigma(pp \to t\bar{t} + X)$ is 177 ± 5 pb. How large was the efficiency for $t\bar{t}$ events assuming no background ?

Quantities to measure in EPP

Quantities to measure in EPP

- ***Physics quantities*** (to be compared with theory expectations)
  - Cross-section
  - Branching ratio
  - Asymmetries
  - Particle Masses, Widths and Lifetimes
- ***Quantities related to the experiment*** (BUT to be measured to get physics quantities)
  - Efficiencies
  - Luminosity
  - Backgrounds

Cross-section - I

- Suppose we have done an experiment and obtained the following quantities for a given final state:
  - N_{cand}, N_b, ε, ϕ
- What is ϕ ? It is the "**flux**", something telling us how many collisions could take place per unit of time and surface.
  - Consider a "**fixed-target**" experiment (transverse size of the target >> beam dimensions):

$$\phi = \dot{N}_{proj} N_{tar} \delta x = \frac{\dot{N}_{proj} \rho \delta x}{A m_N} = \frac{\dot{N}_{proj} \rho (g/cm^3) N_A \delta x(cm)}{A}$$

  - Consider a "**colliding beam**" experiment

$$\phi = f_{coll} \frac{N_1 N_2}{4\pi \Sigma_X \Sigma_Y} = L$$

(head-on beams: N_1 and N_2 number of particles per beam, Σ_X, Σ_Y beam transverse gaussian areas, f_{coll} collision frequency) In this case we normally use the word "**Luminosity**". Flux or luminosity are measured in: **$cm^{-2}s^{-1}$**

Cross-section - II

- In any case, the rate of events due to final state *X* is:

$$\dot{N}_X = \phi\sigma_X$$

- **σ_X is the cross-section**, having the dimension of a surface.
  - it doesn't depend on the experiment but on the process only
  - can be compared to the theory
  - for a given $\boldsymbol{\sigma_X}$, the higher is $\boldsymbol{\phi}$, the larger the event rate
  - given an initial state, for every final state *X* you have a specific cross-section
  - the "**total cross-section**" is obtained by adding the cross-sections for all possible final states: *the cross-section is an **additive** quantity*.
  - The unit is the "**barn**". 1 barn = 10^{-24} cm^2.

Cross-section - III

- Suppose we have taken data for a time $\boldsymbol{\Delta t}$: the total number of events collected will be:

$$N_X = \sigma_X \times \int_{\Delta t} \phi \, dt$$

The flux integral over time is the ***Integrated Flux*** or (in case of colliding beams) ***Integrated Luminosity***. Integrated luminosity is measured in: **b^{-1}**

- How can we measure this cross-section ?

$$\sigma_X = \frac{N_X}{\int \phi dt} = \frac{1}{\int \phi dt} \frac{N_{cand} - N_b}{\varepsilon}$$

- Sources of uncertainty: we apply the uncertainty propagation formula. We assume no correlations btw the quantities in the formula (L_{int} = integral of flux)

$$\left(\frac{\sigma(\sigma_X)}{\sigma_X}\right)^2 = \left(\frac{\sigma(L_{\text{int}})}{L_{\text{int}}}\right)^2 + \left(\frac{\sigma(\varepsilon)}{\varepsilon}\right)^2 + \frac{\sigma^2(N_{cand}) + \sigma^2(N_b)}{(N_{cand} - N_b)^2}$$

Branching ratio measurement

- Given an unstable particle ***a***, it can decay in several (say N) final states, k=1,…,N. If $\mathbf{\Gamma}$ is the ***total width*** of the particle ($\mathbf{\Gamma}$=1/$\boldsymbol{\tau}$ with $\boldsymbol{\tau}$ particle lifetime), for each final state we define a "***partial width***" in such a way that

$$\Gamma = \sum_{k=1}^{N} \Gamma_k$$

- The ***branching ratio*** of the particle a to the final state X is

$$B.R.(a \to X) = \frac{\Gamma_X}{\Gamma}$$

- To measure the B.R. the same analysis as for a cross-section is needed. In this case we need the number of decaying particles $\boldsymbol{N_a}$ (not the flux) to normalize:

$$B.R.(a \to X) = \frac{N_{cand} - N_b}{\varepsilon} \frac{1}{N_a}$$

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- Sometimes the normalization is done relative to another process of known B.R. (relative measurement)

$$\frac{B.R.(a \to X)}{B.R.(a \to Y)} = \left(\frac{N_{cand,X} - N_{b,X}}{N_{cand,Y} - N_{b,Y}} \right) \left(\frac{\varepsilon_Y}{\varepsilon_X} \right)$$

Differential cross-section - I

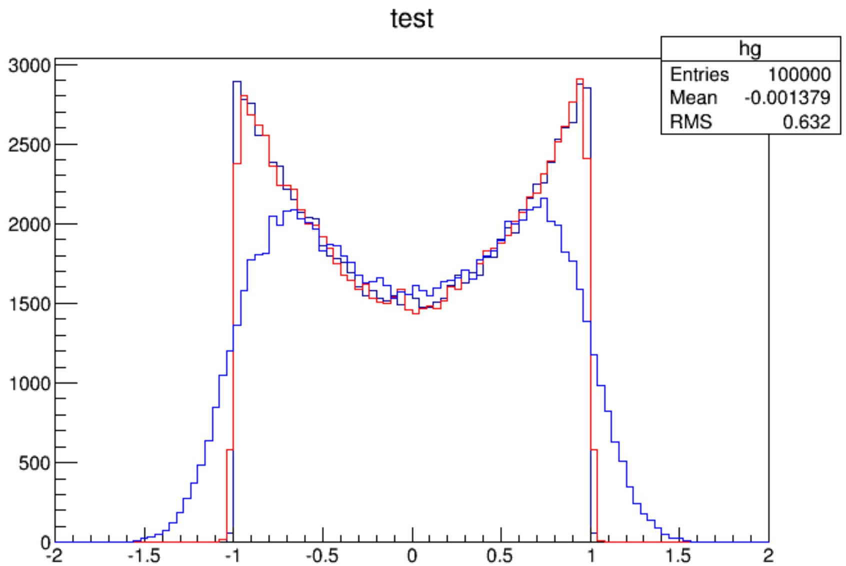
- If we want to consider only final states with a given kinematic configuration (momenta, angles, energies,…) and give the cross-section as a function of these variables
- Experimentally we have to divide in bins and count the number of events per bin.
- Example: differential cross-section vs. scattering angle

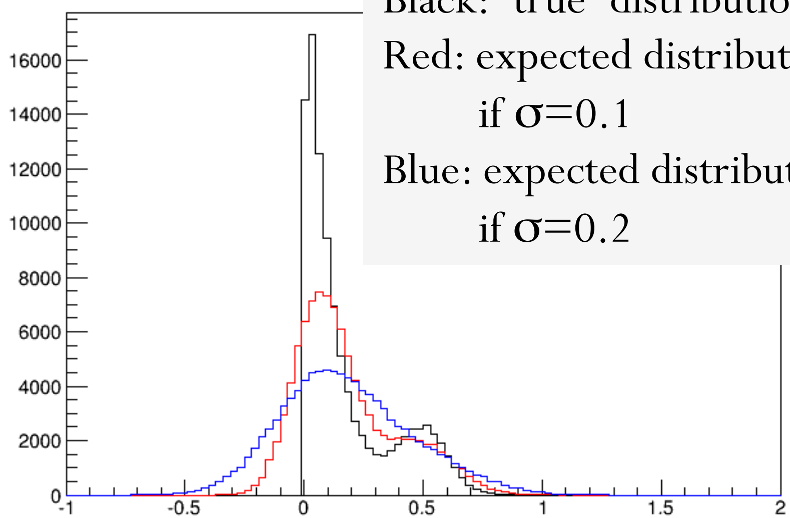
$$\left(\frac{d\sigma}{d\cos\theta}\right)_i = \frac{1}{\int \phi dt}\left(\frac{N^i_{cand} - N^i_b}{\varepsilon_i}\right)\frac{1}{\Delta\cos\theta_i}$$

- NB: N_{cand}, N_b and ε as a function of θ are needed.

Differential cross-section - II

- Additional problems appear.
  - Efficiency is required per bin (can be different for different kinematic configurations).
  - Background is required per bin (as above).
  - The migration of events from one bin to another is possible:





Black: "true" distribution
Red: expected distribution if σ=0.1
Blue: expected distribution if σ=0.2

Folding - Unfolding

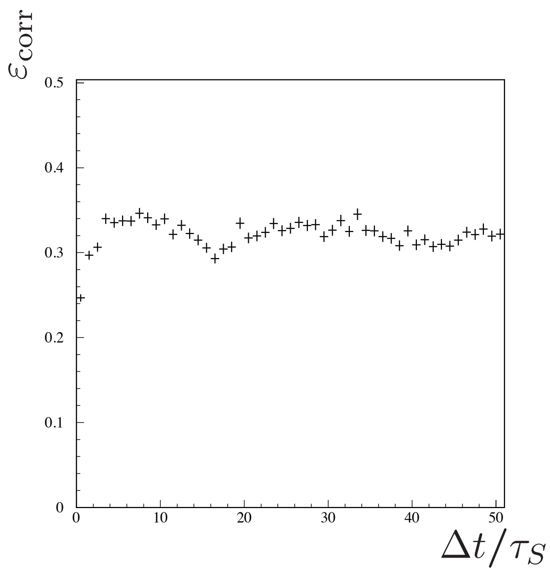
- In case there is a substancial migration of events among bins (resolution larger than bin size), this affects the comparison btw exp.histo ($\boldsymbol{n_i^{exp}}$) and theory ($\boldsymbol{n_i^{th}}$). This can be solved in two different ways:
  - **Folding** of the theoretical distribution: the theoretical function $\boldsymbol{f^{th}(x)}$ is "smeared" through a smearing matrix $\boldsymbol{M}$ based on our knowledge of the resolution; $\boldsymbol{n_i^{th} \rightarrow n'^{th}_i}$

$$n_i'^{th} = \sum_{j=1}^{N} n_j^{th} M_{i,j}$$

$$n_i^{th} = \int_{x_i}^{x_{i+1}} dx f^{th}(x)$$

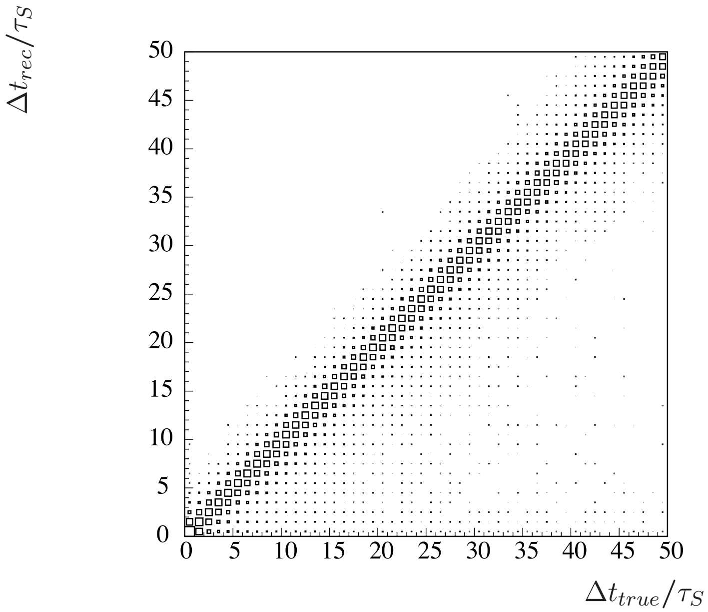
  - **Unfolding** of the experimental histogram: $\boldsymbol{n_i^{exp} \rightarrow n'^{exp}_i}$. Very difficult procedure, mostly unstable, inversion of $\boldsymbol{M}$ required

$$n_i'^{\exp} = \sum_{j=1}^{N} n_j^{\exp} M_{i,j}^{-1}$$



$$I_j(\vec{q}) = \int_{(j-1)\bar{\Delta}t}^{j\bar{\Delta}t} d(\Delta t) \int_{\Delta t}^{\infty} I(t_1, t_2; \vec{q})\, d(t_1 + t_2)$$

$$n_i = N \left(\sum_j s_{ij}\, \epsilon_j\, I_j(\vec{q}) \right) + N^{\text{reg}} I_i^{\text{reg}} + N^{4\pi} I_i^{4\pi}$$



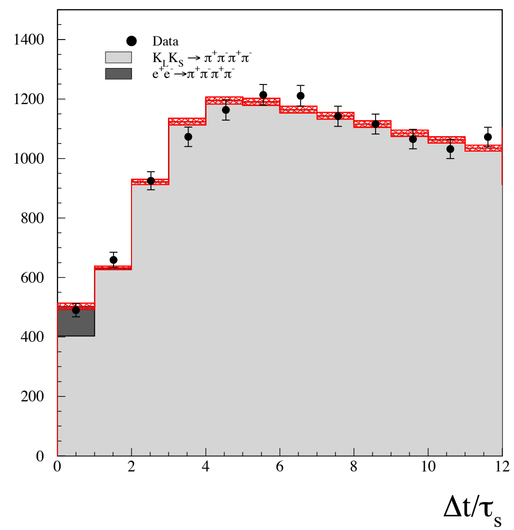


Fig. 6: *Smearing matrix*

Asymmetry measurement

- A very useful and powerful observable:

$$A = \frac{N^+ - N^-}{N^+ + N^-}$$

- It can be "charge asymmetry", Forward-Backward asymmetry",…
  - Independent from the absolute normalization
  - (+) and (-) could have different efficiencies, but most of them could cancel:

$$A = \frac{N^+/\varepsilon^+ - N^-/\varepsilon^-}{N^+/\varepsilon^+ + N^-/\varepsilon^-}$$

- Statistical error ($\mathbf{N}=\mathbf{N}^+ + \mathbf{N}^-$) :

$$\sigma(A) = \frac{1}{\sqrt{N}}\sqrt{1 - A^2}$$

Asymmetry measurement

The statistical uncertainty on the asymmetry can be evaluate using a binomial model where $N = N^+ + N^-$, $n = N^+$, $f^+ = n/N$, so that $\mathcal{A} = 2f^+ - 1$. We get:

$$\sigma^2(\mathcal{A}) = 4\sigma^2(f^+) = 4\frac{f^+(1-f^+)}{N} \tag{87}$$

but, since

$$f^+ = \frac{1+\mathcal{A}}{2} \tag{88}$$

we have also

$$\sigma(\mathcal{A}) = 2\sqrt{\frac{(1+\mathcal{A})/2(1-(1+\mathcal{A})/2)}{N}} = \frac{2}{\sqrt{N}}\sqrt{\frac{1+\mathcal{A}}{2}\frac{1-\mathcal{A}}{2}} = \frac{1}{\sqrt{N}}\sqrt{1-\mathcal{A}^2} \tag{89}$$

The uncertainty on the asymmetry goes as the inverse of the square root of the total number of events. The same result is obtained by assuming independent poissonian fluctuations for N^+ and N^-.

$$\sigma(\mathrm{A}) = \frac{1}{\sqrt{N}}\sqrt{1-\mathrm{A}^2}$$