Data l'espressione:

$$\frac{|\eta_{+-}|^2}{|\eta_{00}|^2} = \frac{\dfrac{[BR(K_L \to \pi^+\pi^-)]}{[BR(K_S \to \pi^+\pi^-)]}}{\dfrac{[BR(K_L \to \pi^0\pi^0)]}{[BR(K_S \to \pi^0\pi^0)]}} \cong 1 + 6\Re e\left(\frac{\varepsilon'}{\varepsilon}\right)$$

Dimostrare che :

$$\delta\Re e\left(\frac{\varepsilon'}{\varepsilon}\right)_{stat} = \frac{1}{6}\frac{1}{\sqrt{(2/3)N_L^0}}$$

con N^0_L numero di conteggi $K_L \to \pi^0\pi^0$.
In quale approssimazione vale la formula?

La relazione fra $BR^{\pm,0}_{S,L}$ e $N^{\pm,0}_{S,L}$ è data da:

$$N^{\pm,0}_{S,L} = N^{\pm,0}_{S,L}(obs) - Bck^{\pm,0}_{S,L} = N_{KK} \cdot \rho_{S,L}(tag) \cdot BR^{\pm,0}_{S,L} \cdot \langle \rho^{\pm,0}_{S,L} \rangle \cdot \iint_{FV} g(l-l')I(l)\,dl\,dl'$$

dove:

- $N^{\pm,0}_{S,L}(obs)$ e' il numero effettivamente osservato di decadimenti $\pi^+\pi^-, \pi^0\pi^0$;
- N_{KK} e' il numero totale di coppie K_S, K_L prodotte;
- $\rho_{S,L}(tag)$ e' l'efficienza di identificazione;
- $BR^{\pm,0}_{S,L}$ e' il "branching ratio" corrispondente al decadimento $K_{S,L} \to \pi^+\pi^-, \pi^0\pi^0$;
- $\langle \rho^{\pm,0}_{S,L} \rangle$ e' l'efficienza media di rivelazione dei decadimenti $K_{S,L} \to \pi^+\pi^-, \pi^0\pi^0$;
- $\iint_{FV} g(l-l')I(l)\,dl\,dl'$ rappresenta la convoluzione dell'intensita' $I(l) = e^{-l/l_{S,L}}$ dei

decadimenti con la risoluzione sperimentale g(l-l') sul cammino di decadimento l, integrata sul volume fiduciale del rivelatore.

- $Bck^{\pm,0}_{S,L}$ e' il contributo degli eventi di fondo.

$$N^0_L = \underbrace{3\mu b}_{\sigma_{e^+e^-\to\phi}} \cdot \underbrace{\mathcal{L}}_{\int L dt} \cdot \underbrace{0.66}_{\rho_L(tag)} \cdot \underbrace{0.34}_{BR(\phi\to K_S K_L)} \cdot \underbrace{10^{-3}}_{BR^0_L} \cdot \underbrace{\left(e^{-30/350} - e^{-150/350}\right)}_{fiducial\ volume}$$

Data una luminosita' integrata di $\mathcal{L}=10^4$ pb^{-1} qual'e' il fattore di reiezione del fondo $K_L \to 3\pi^0$ (sul segnale $K_L \to 2\pi^0$) necessario per avere un errore su $Re(\varepsilon'/\varepsilon)<3\times10^{-4}$ assumendo di conoscere il fondo con una precisione del 20%?

Soluzione

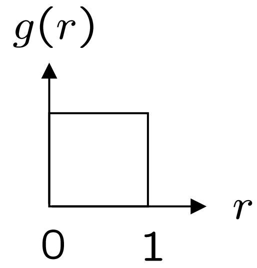
1) A parita' di luminosita', i decadimenti del K_L in due pioni sono molto piu' rari di quelli corrispondenti del K_S.
Quindi l'errore statistico e' dominato dai conteggi sul K_L.
Trascurando l'incertezza sulle efficienze e la sottrazione del fondo, considerando che $N^0_L \sim 2 N^{\pm}_L$ si ottiene la formula data.

2) Si generalizza la formula precedente ad includere l'errore derivante dalla sottrazione del fondo nei conteggi N^0_L (attenzione: tenere separati i contributi di N^0_L e $N^{\pm}_L$). Considerando che
$N(K_L \text{-> } 3\pi^0) \sim 200\ N^0_L$ e che $N_B = N(K_L \text{-> } 3\pi^0)/R$
date le condizioni indicate si ottiene per il fattore di reiezione: R>25800.

The Monte Carlo method

What it is: a numerical technique for calculating probabilities and related quantities using sequences of random numbers.

The usual steps:



(1) Generate sequence $r_1, r_2, \ldots, r_m$ uniform in [0, 1].

(2) Use this to produce another sequence $x_1, x_2, \ldots, x_n$ distributed according to some pdf $f(x)$ in which we're interested (x can be a vector).

(3) Use the x values to estimate some property of $f(x)$, e.g., fraction of x values with $a < x < b$ gives $\int_a^b f(x)\,dx$.

→ MC calculation = integration (at least formally)

MC generated values = 'simulated data'

→ use for testing statistical procedures

Random number generators

Goal: generate uniformly distributed values in [0, 1].

Toss coin for e.g. 32 bit number... (too tiring).

→ 'random number generator'

= computer algorithm to generate $r_1, r_2, \ldots, r_n$.

Example: multiplicative linear congruential generator (MLCG)

$n_{i+1} = (a\, n_i) \bmod m$, where

n_i = integer

a = multiplier

m = modulus

n_0 = seed (initial value)

N.B. mod = modulus (remainder), e.g. 27 mod 5 = 2.

This rule produces a sequence of numbers $n_0, n_1, \ldots$

Random number generators (2)

The sequence is (unfortunately) periodic!

Example (see Brandt Ch 4): $a = 3$, $m = 7$, $n_0 = 1$

$$
\begin{aligned}
n_1 &= (3 \cdot 1) \bmod 7 = 3 \\
n_2 &= (3 \cdot 3) \bmod 7 = 2 \\
n_3 &= (3 \cdot 2) \bmod 7 = 6 \\
n_4 &= (3 \cdot 6) \bmod 7 = 4 \\
n_5 &= (3 \cdot 4) \bmod 7 = 5 \\
n_6 &= (3 \cdot 5) \bmod 7 = 1
\end{aligned}
$$

← sequence repeats

Choose a, m to obtain long period (maximum = $m - 1$); m usually close to the largest integer that can represented in the computer.

Only use a subset of a single period of the sequence.

Random number generators (3)

$r_i = n_i/m$ are in [0, 1] but are they 'random'?

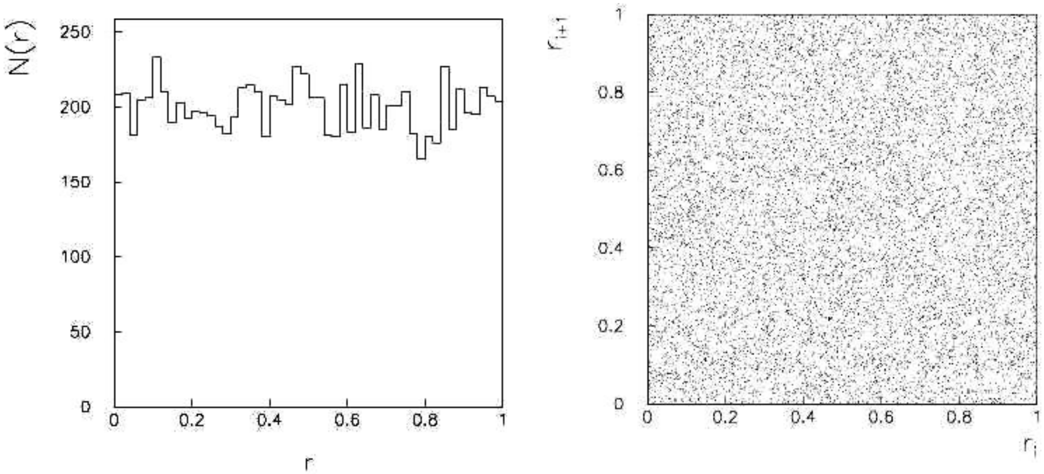
Choose a, m so that the r_i pass various tests of randomness:

uniform distribution in [0, 1],

all values independent (no correlations between pairs),

e.g. L'Ecuyer, Commun. ACM **31** (1988) 742 suggests

$a = 40692$
$m = 2147483399$

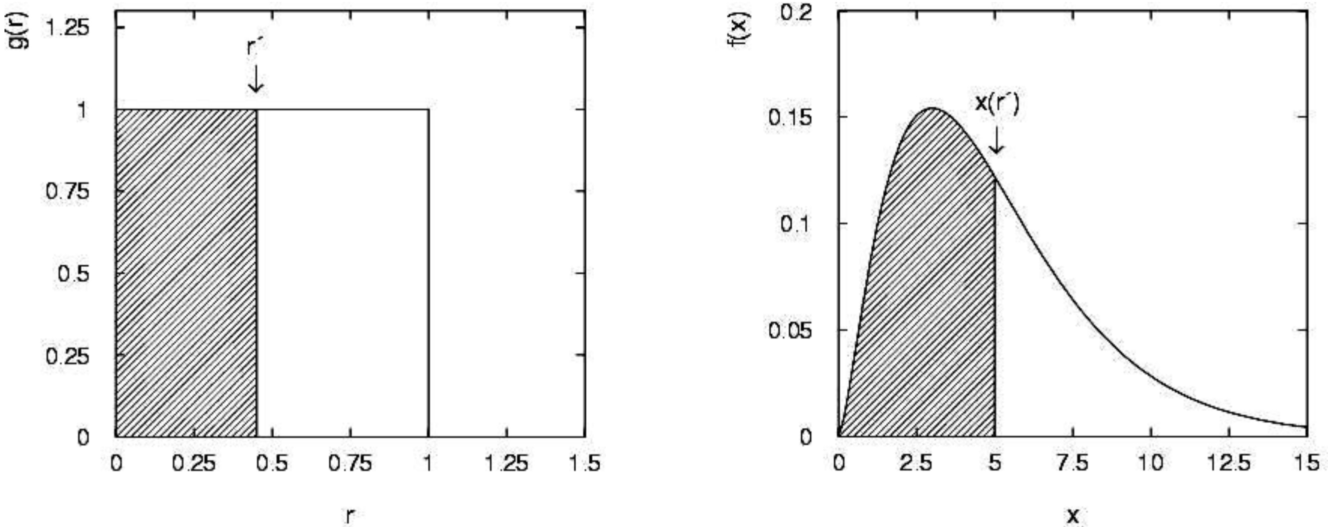


Far better generators available, e.g. `TRandom3`, based on Mersenne twister algorithm, period = $2^{19937} - 1$ (a "Mersenne prime").

See F. James, Comp. Phys. Comm. 60 (1990) 111; Brandt Ch. 4

The transformation method

Given $r_1, r_2, \ldots, r_n$ uniform in [0, 1], find $x_1, x_2, \ldots, x_n$ that follow $f(x)$ by finding a suitable transformation $x(r)$.



Require: $P(r \le r') = P(x \le x(r'))$

i.e. $\int_{-\infty}^{r'} g(r)\, dr = r' = \int_{-\infty}^{x(r')} f(x')\, dx' = F(x(r'))$

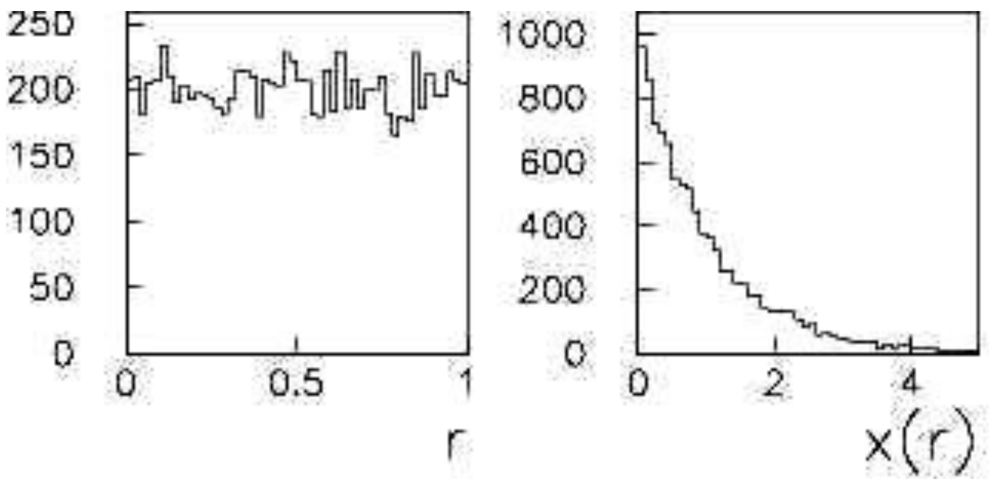
That is, **set $F(x) = r$ and solve for $x(r)$.**

Example of the transformation method

Exponential pdf: $f(x;\xi) = \frac{1}{\xi} e^{-x/\xi} \quad (x \geq 0)$

Set $\int_0^x \frac{1}{\xi} e^{-x'/\xi}\, dx' = r$ and solve for $x(r)$.

$\rightarrow \quad x(r) = -\xi \ln(1-r) \qquad (x(r) = -\xi \ln r$ works too.)



1. Generation of random,
   Pseudo-random numbers
2. Random variable r uniformly distributed between 0 and 1
3. Sampling of a discrete random variable
   Example:
   A discrete random variable x with 3 values, x1, x2, x3 with probabilities P1, P2 and P3 respectively (Σ Pi=1).
   Extract y=r
   if $0<y<P1$ => $x=x1$
   if $P1<y<(P1+P2)$ => $x=x2$
   if $(P2+P3)<y<1$ => $x=x3$
4. Sampling of a continuous random variable x with arbitrary pdf f(x)
   Extract y=r
   $x=F^{-1}(y)$ with $y=F(x)=\int_0^x f(x')dx'$
   Example:
   $f(x)=1/(b-a)$ => $x=a+(b-a)r$
   $f(\theta)=\sin\theta/2$ => $\cos\theta = 1-2r$ => $\theta=\text{acos}(1-2r)$
   $f(x)=\mu\exp(-\mu x)$ => $x=-\ln(1-r)/\mu$ => $x=-\ln(r)/\mu$