Observables in EPP

Quantities to be measured in EPP

- ***Physics quantities*** (to be compared with theory expectations)
  - Cross-section
  - Branching ratio
  - Asymmetries
  - Particle Masses, Widths and Lifetimes
- ***Quantities related to the experiment*** (BUT to be measured to get physics quantities)
  - Efficiencies
  - Luminosity
  - Backgrounds

Cross-section - I

- Suppose we have done an experiment and obtained the following quantities for a given final state:
  - N_{cand}, N_b, ε, ϕ
- What is ϕ ? It is the "**flux**", something telling us how many collisions could take place per unit of time and surface.
  - Consider a "**fixed-target**" experiment (transverse size of the target >> beam dimensions):

$$\phi = \dot{N}_{proj} N_{tar} \delta x = \frac{\dot{N}_{proj} \rho \delta x}{A m_N} = \frac{\dot{N}_{proj} \rho (g/cm^3) N_A \delta x(cm)}{A}$$

  - Consider a "**colliding beam**" experiment

$$\phi = f_{coll} \frac{N_1 N_2}{4\pi \Sigma_X \Sigma_Y} = L$$

(head-on beams: N_1 and N_2 number of particles per beam, Σ_X, Σ_Y beam transverse gaussian areas, f_{coll} collision frequency) In this case we normally use the word "**Luminosity**". Flux or luminosity are measured in: **cm^{-2}s^{-1}**

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  - Consider a "**colliding beam**" experiment

=> $$\phi = f_{coll} \frac{N_1 N_2}{4\pi \Sigma_X \Sigma_Y} \frac{1}{n_b} = L \qquad L = f_{coll} n_b \frac{n_1 n_2}{4\pi \Sigma_X \Sigma_Y}$$

with
n_1, n_2 number of particles per bunch
n_b number of bunches

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Cross-section - II

- In any case, the rate of events due to final state X is:

$$\dot{N}_X = \phi \sigma_X$$

- **σ_X is the cross-section**, having the dimension of an area.
  - it doesn't depend on the experiment but on the process only
  - can be compared to the theory
  - for a given $\boldsymbol{\sigma_X}$, the higher is $\boldsymbol{\phi}$, the larger the event rate
  - given an initial state, for every final state X you have a specific cross-section
  - the "**total cross-section**" is obtained by adding the cross-sections for all possible final states: *the cross-section is an **additive** quantity*.
  - The unit is the "**barn**". 1 barn = 10^{-24} cm^2.

Cross-section - III

- Suppose we have taken data for a time ***Δt***: the total number of events collected will be:

$$N_X = \sigma_X \times \int_{\Delta t} \phi \, dt$$

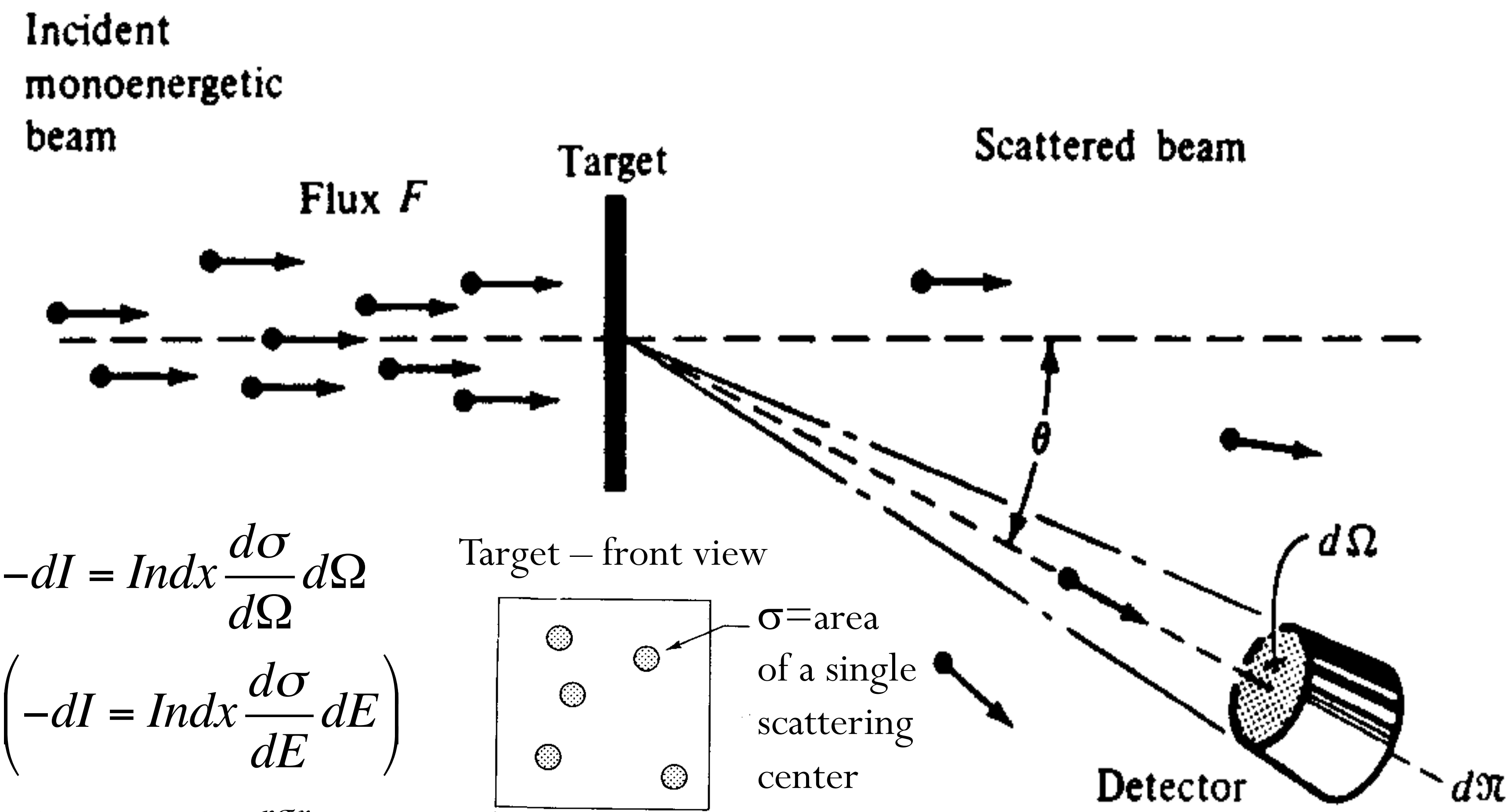
The flux integral over time is the ***Integrated Flux*** or (in case of colliding beams) ***Integrated Luminosity***. Integrated luminosity is measured in: **b^{-1}**

- How can we measure this cross-section ?

$$\sigma_X = \frac{N_X}{\int \phi dt} = \frac{1}{\int \phi dt} \frac{N_{cand} - N_b}{\varepsilon}$$

- Sources of uncertainty: we apply the uncertainty propagation formula. We assume no correlations btw the quantities in the formula (L_{int} = integral of flux)

$$\left(\frac{\sigma(\sigma_X)}{\sigma_X}\right)^2 = \left(\frac{\sigma(L_{\text{int}})}{L_{\text{int}}}\right)^2 + \left(\frac{\sigma(\varepsilon)}{\varepsilon}\right)^2 + \frac{\sigma^2(N_{cand}) + \sigma^2(N_b)}{(N_{cand} - N_b)^2}$$



Target – front view

σ=area of a single scattering center

$$-dI = Indx\frac{d\sigma}{d\Omega}d\Omega$$

$$\left(-dI = Indx\frac{d\sigma}{dE}dE\right)$$

$$I(x) = I(0)e^{-n\sigma x}$$

$$\sigma = \int_{4\pi}\frac{d\sigma}{d\Omega}d\Omega$$

$$\left(\sigma = \int_{0}^{E_{MAX}}\frac{d\sigma}{dE}dE\right)$$

x=target thickness

n = density of scattering centers

I = beam intensity

Branching ratio measurement - I

- Given an unstable particle ***a***, it can decay in several (say *N*) final states, $k=1,\dots,N$. If Γ is the ***total width*** of the particle ($\Gamma=1/\tau$ with τ particle lifetime), for each final state we define a "***partial width***" in such a way that

$$\Gamma = \sum_{k=1}^{N} \Gamma_k$$

- The ***branching ratio*** of the particle *a* to the final state *X* is

$$B.R.(a \to X) = \frac{\Gamma_X}{\Gamma}$$

- To measure the B.R. the same analysis as for a cross-section is needed. In this case we need the number of decaying particles $\boldsymbol{N_a}$ (not the flux) to normalize:

$$B.R.(a \to X) = \frac{N_{cand} - N_b}{\varepsilon} \frac{1}{N_a}$$

Branching ratio measurement - II

- Sometimes the normalization is done relative to another process of known B.R. (relative measurement):

$$\frac{B.R.(a \rightarrow X)}{B.R.(a \rightarrow Y)} = \left(\frac{N_{cand,X} - N_{b,X}}{N_{cand,Y} - N_{b,Y}} \right) \left(\frac{\varepsilon_Y}{\varepsilon_X} \right)$$

Differential cross-section - I

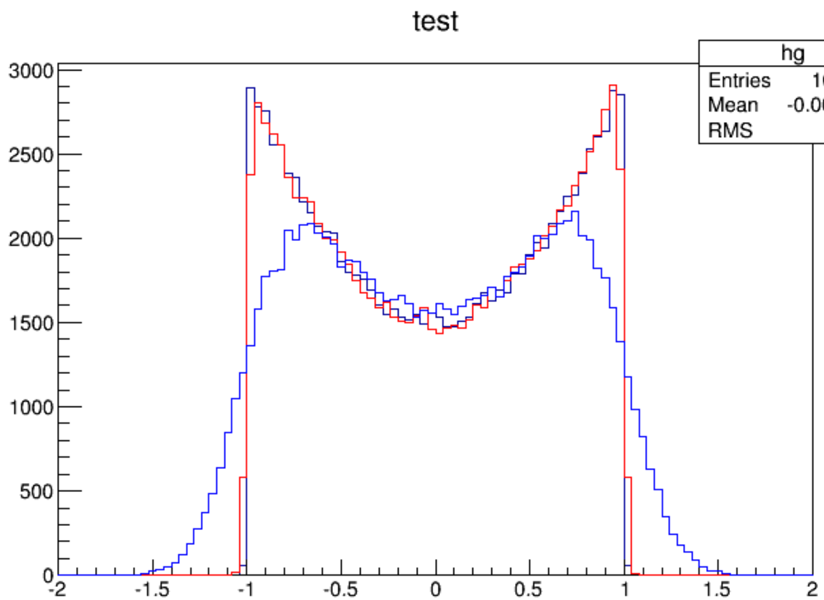
- If we want to consider only final states with a given kinematic configuration (momenta, angles, energies,…) and give the cross-section as a function of these variables
- Experimentally we have to divide in bins and count the number of events per bin.
- Example: differential cross-section vs. scattering angle

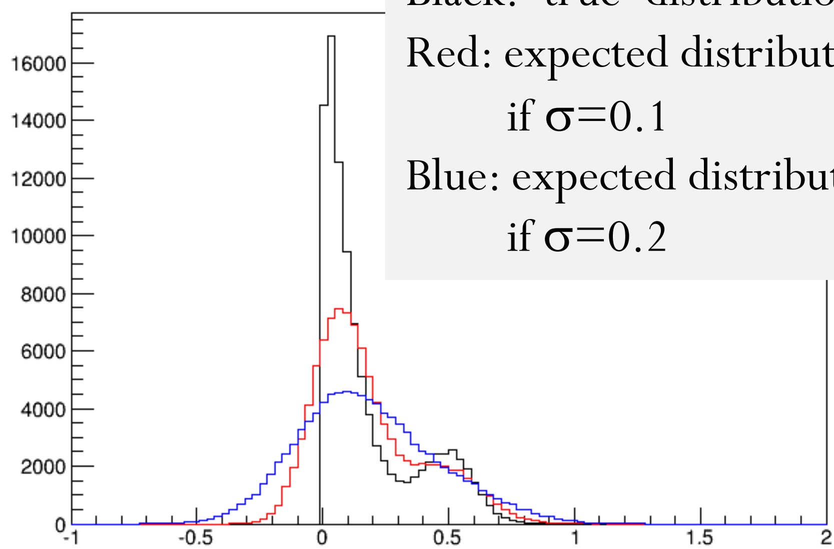
$$\left(\frac{d\sigma}{d\cos\theta}\right)_i = \frac{1}{\int \phi dt}\left(\frac{N^i_{cand} - N^i_b}{\varepsilon_i}\right)\frac{1}{\Delta\cos\theta_i}$$

- NB: N_{cand}, N_b and ε as a function of θ are needed.

Differential cross-section - II

- Additional problems appear.
  - Efficiency is required per bin (can be different for different kinematic configurations).
  - Background is required per bin (as above).
  - The migration of events from one bin to another is possible:





Black: "true" distribution

Red: expected distribution if $\sigma=0.1$

Blue: expected distribution if $\sigma=0.2$

Folding – Unfolding - II

- In case there is a substancial migration of events among bins (resolution larger than bin size), this affects the comparison btw exp.histo ($\boldsymbol{n_i^{exp}}$) and theory ($\boldsymbol{n_i^{th}}$). This can be solved in two different ways:
  - **Folding** of the theoretical distribution: the theoretical function $\boldsymbol{f^{th}(x)}$ is "smeared" through a smearing matrix $\boldsymbol{M}$ based on our knowledge of the resolution; $\boldsymbol{n_i^{th} \rightarrow n'^{th}_i}$

$$n_i'^{th} = \sum_{j=1}^{N} n_j^{th} M_{i,j}$$

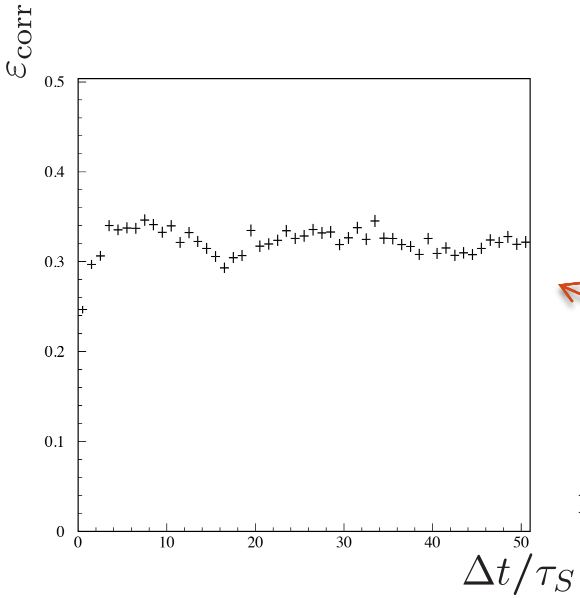
$$n_i^{th} = \int_{x_i}^{x_{i+1}} dx f^{th}(x)$$

  - **Unfolding** of the experimental histogram: $\boldsymbol{n_i^{exp} \rightarrow n'^{exp}_i}$. Very difficult procedure, mostly unstable, inversion of $\boldsymbol{M}$ required

$$n_i'^{\exp} = \sum_{j=1}^{N} n_j^{\exp} M_{i,j}^{-1}$$

Theory parameters

$$I_j(\vec{q}) = \int_{(j-1)\bar{\Delta}t}^{j\bar{\Delta}t} d(\Delta t) \overbrace{\int_{\Delta t}^{\infty} I(t_1, t_2; \vec{q})\, d(t_1 + t_2)}^{I^{theory}(\Delta t;\vec{q})}$$



efficiency

normalization

$$n_i = N\left(\sum_j s_{ij}\,\epsilon_j\, I_j(\vec{q})\right) + \overbrace{N^{\mathrm{reg}} I_i^{\mathrm{reg}} + N^{4\pi} I_i^{4\pi}}^{\text{Background contributions}}$$

Smearing matrix

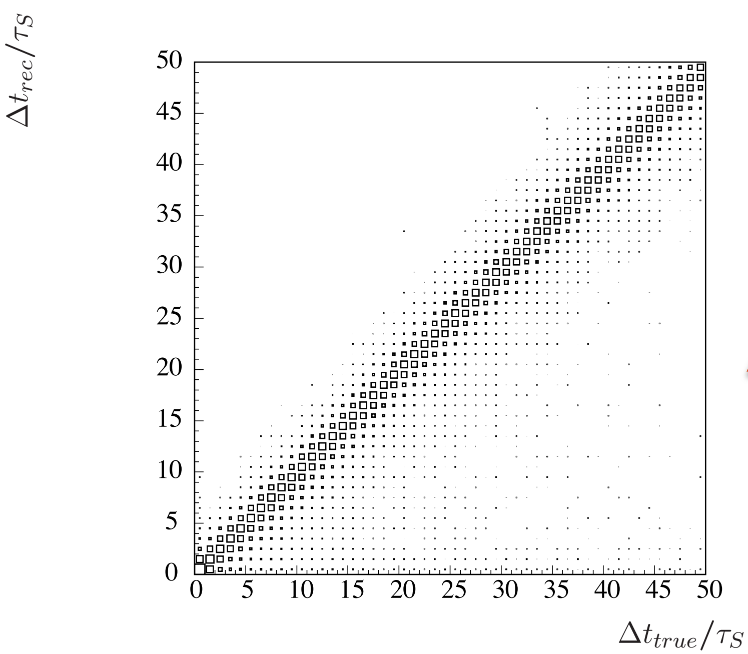
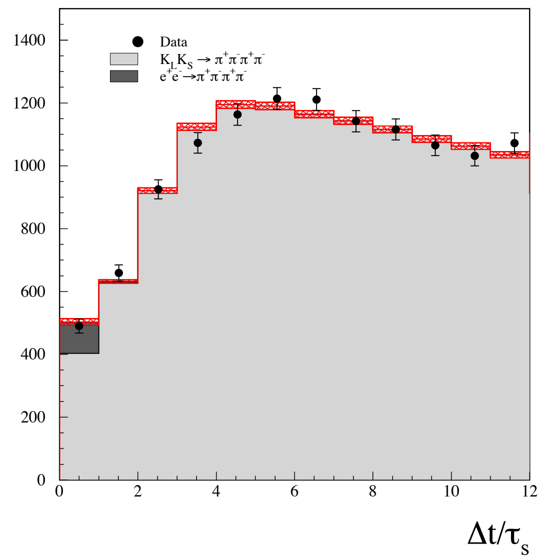


Fig. 6: *Smearing matrix*

Data: points

MC: histo n_i



Asymmetry measurement - I

- A very useful and powerful observable:

$$\mathrm{A} = \frac{N^+ - N^-}{N^+ + N^-}$$

- It can be "charge asymmetry", Forward-Backward asymmetry",…
  - Independent from the absolute normalization
  - (+) and (-) could have different efficiencies, but most of them could cancel:

$$\mathrm{A} = \frac{N^+/\varepsilon^+ - N^-/\varepsilon^-}{N^+/\varepsilon^+ + N^-/\varepsilon^-}$$

- Statistical error (**N**=**N**$^+$+**N**$^-$) :

$$\sigma(\mathrm{A}) = \frac{1}{\sqrt{N}}\sqrt{1-\mathrm{A}^2}$$

Asymmetry measurement - II

Let us assume $\varepsilon^+ = \varepsilon^-$: the efficiencies cancel out in the asymmetry.

The statistical uncertainty on the asymmetry can be evaluate using a binomial model where $N = N^+ + N^-$, $n = N^+$, $f^+ = n/N$, so that $\mathcal{A} = 2f^+ - 1$. We get:

$$\sigma^2(\mathcal{A}) = 4\sigma^2(f^+) = 4\frac{f^+(1-f^+)}{N} \tag{87}$$

but, since

$$f^+ = \frac{1+\mathcal{A}}{2} \tag{88}$$

we have also

$$\sigma(\mathcal{A}) = 2\sqrt{\frac{(1+\mathcal{A})/2(1-(1+\mathcal{A})/2)}{N}} = \frac{2}{\sqrt{N}}\sqrt{\frac{1+\mathcal{A}}{2}\frac{1-\mathcal{A}}{2}} = \frac{1}{\sqrt{N}}\sqrt{1-\mathcal{A}^2} \tag{89}$$

The uncertainty on the asymmetry goes as the inverse of the square root of the total number of events. The same result is obtained by assuming independent poissonian fluctuations for N^+ and N^-.

$$\sigma(\mathrm{A}) = \frac{1}{\sqrt{N}}\sqrt{1-\mathrm{A}^2}$$