Homework n.1

Evaluate numerically and plot graphically the convolution integral of the energy spectrum f(E) with a gaussian resolution g(E) defined below.

$f(E) = a_1 f_1(E)+a_2 f_2(E)+a_3 f_3(E)$

$f_1(E)=1/E$ for $0.1 < E < 0.9$ MeV
$f_1(E)=0$ for $E<0.1$ or $E> 0.9$ MeV

$f_2(E)=G(\mu=1\ \text{MeV}, \sigma=0.01\ \text{MeV})$

$f_3(E)=G(\mu=1.3\ \text{MeV}, \sigma=0.01\ \text{MeV})$

$a_1 = 1.5$
$a_2 = 1$
$a_3 = 2$

$g(E)=G(E,\sigma)$ with $\sigma/E = 5\%/\sqrt{E(\text{MeV})}$

Homework n.2

At the end of the event selection looking for a signal $e^+e^- \Rightarrow X$ we get

- candidate events: Ncand = 1590;
- background events: Nb=640±60 (evaluated from side-bands);

The efficiency is $\varepsilon = 0.246$ with negligible uncertainty

a) Evaluate the number of signal events N_X and its relative uncertainty.

b) We want to reduce the uncertainty on the signal and apply a rejection on the background. Assuming an uncertainty of 10% on the background evaluation (after rejection), which rejection factor is needed on Nb above to obtain $\sigma(N_X)/N_X < 3.5\ \%$?

Given the formula:

$$\frac{|\eta_{+-}|^2}{|\eta_{00}|^2} = \frac{\left[\dfrac{BR(K_L \to \pi^+\pi^-)}{BR(K_S \to \pi^+\pi^-)}\right]}{\left[\dfrac{BR(K_L \to \pi^0\pi^0)}{BR(K_S \to \pi^0\pi^0)}\right]} \cong 1 + 6\Re e\left(\frac{\varepsilon'}{\varepsilon}\right)$$

Show that :

$$\delta \Re e\left(\frac{\varepsilon'}{\varepsilon}\right)_{stat} = \frac{1}{6}\frac{1}{\sqrt{(2/3)N_L^0}}$$

with N^0_L number of counts $K_L \to \pi^0\pi^0$.

1) In which approximation does the formula hold?

The relationship between branching ratios $BR_{S,L}$ and counts $N_{S,L}$ is given by:

$$N_{S,L}^{\pm,0} = N_{S,L}^{\pm,0}(obs) - Bck_{S,L}^{\pm,0} = N_{KK} \cdot \rho_{S,L}(tag) \cdot BR_{S,L}^{\pm,0} \cdot \langle \rho_{S,L}^{\pm,0} \rangle \cdot \iint_{FV} g(l-l')I(l)\,dl\,dl'$$

with:

- $N_{S,L}^{\pm,0}(obs)$ is the number of observed decays into $\pi^+\pi^-$, $\pi^0\pi^0$
- N_{KK} is the total number of produced $K_S K_L$ pairs
- $\rho_{S,L}(tag)$ is the tagging efficiency
- $BR_{S,L}^{\pm,0}$ is the branching ratio of the decay $K_{S,L} \Rightarrow \pi^+\pi^-$, $\pi^0\pi^0$
- $\langle \rho_{S,L}^{\pm,0} \rangle$ is the average detection efficiency for the decays $K_{S,L} \Rightarrow \pi^+\pi^-$, $\pi^0\pi^0$
- $\iint_{FV} g(l-l')I(l)\,dl\,dl'$ is the convolution integral of an exponential decay intensity $I(l)=\exp(-l/l_{S,L})$ with the resolution $g(l-l')$ on the decay length l, and integrated on the fiducial volume FV
- $Bck_{S,L}^{\pm,0}$ is the contribution of background events

$$N_L^0 = \underbrace{3\,\mu b}_{\sigma_{e^+e^-\to\phi}} \cdot \underbrace{\mathcal{L}}_{\int L dt} \cdot \underbrace{0.66}_{\rho_L(tag)} \cdot \underbrace{0.34}_{BR(\phi\to K_S K_L)} \cdot \underbrace{10^{-3}}_{BR_L^0} \cdot \underbrace{\left(e^{-30/350} - e^{-150/350}\right)}_{\text{fiducial volume}}$$

2) Given an integrated luminosity of $\mathcal{L}=10^4$ pb^{-1}, which rejection factor of the background $K_L \to 3\pi^0$ (on the signal $K_L \to 2\pi^0$) is necessary to have an uncertainty $\delta(\mathrm{Re}(\varepsilon'/\varepsilon))<3\times10^{-4}$ and assuming to know the background with a 20% precision?