

Homework n.6

The squared energy and momentum of a particle are independently measured:

$$E^2 = 1010 \pm 17 \text{ eV}^2$$

$$P^2 = 1064 \pm 25 \text{ eV}^2$$

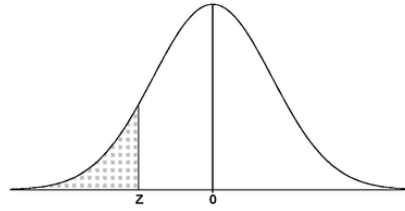
Put an upper limit on the squared mass

$$m^2 = E^2 - P^2$$

of the particle using:

- The classical frequentist approach
- The unified approach (Feldman Cousins) with the mean of the Gaussian constrained to be non-negative
(see Feldman and Cousins Phys.Rev.D 57 3873 (1998))
- The Bayesian approach (briefly comment the choice of the prior)

Cumulative Standard Normal Distribution Table



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0.00	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.50	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.60	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.70	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
-3.80	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001

*Note: z-values less than -3.89 produce a probability of zero.

Flip-flop problem: the frequentist unified approach (Feldman and Cousins PRD 57 3873 (1998))

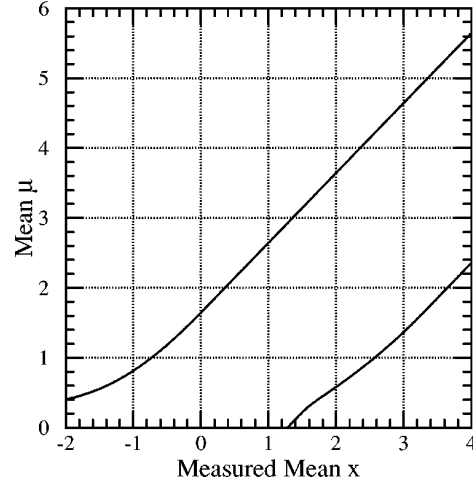


FIG. 10. Plot of our 90% confidence intervals for the mean of a Gaussian, constrained to be non-negative, described in the text.

TABLE X. Our confidence intervals for the mean μ of a Gaussian, constrained to be non-negative, as a function of the measured mean x_0 , for commonly used confidence levels. Italicized intervals correspond to cases where the goodness-of-fit probability (Sec. IV C) is less than 1%. All numbers are in units of σ .

x_0	68.27% C.L.	90% C.L.	95% C.L.	99% C.L.	x_0	68.27% C.L.	90% C.L.	95% C.L.	99% C.L.
-3.0	<i>0.00, 0.04</i>	<i>0.00, 0.26</i>	<i>0.00, 0.42</i>	<i>0.00, 0.80</i>	0.1	0.00, 1.10	0.00, 1.74	0.00, 2.06	0.00, 2.68
-2.9	<i>0.00, 0.04</i>	<i>0.00, 0.27</i>	<i>0.00, 0.44</i>	<i>0.00, 0.82</i>	0.2	0.00, 1.20	0.00, 1.84	0.00, 2.16	0.00, 2.78
-2.8	<i>0.00, 0.04</i>	<i>0.00, 0.28</i>	<i>0.00, 0.45</i>	<i>0.00, 0.84</i>	0.3	0.00, 1.30	0.00, 1.94	0.00, 2.26	0.00, 2.88
-2.7	<i>0.00, 0.04</i>	<i>0.00, 0.29</i>	<i>0.00, 0.47</i>	<i>0.00, 0.87</i>	0.4	0.00, 1.40	0.00, 2.04	0.00, 2.36	0.00, 2.98
-2.6	<i>0.00, 0.05</i>	<i>0.00, 0.30</i>	<i>0.00, 0.48</i>	<i>0.00, 0.89</i>	0.5	0.02, 1.50	0.00, 2.14	0.00, 2.46	0.00, 3.08
-2.5	<i>0.00, 0.05</i>	<i>0.00, 0.32</i>	<i>0.00, 0.50</i>	<i>0.00, 0.92</i>	0.6	0.07, 1.60	0.00, 2.24	0.00, 2.56	0.00, 3.18
-2.4	<i>0.00, 0.05</i>	<i>0.00, 0.33</i>	<i>0.00, 0.52</i>	<i>0.00, 0.95</i>	0.7	0.11, 1.70	0.00, 2.34	0.00, 2.66	0.00, 3.28
-2.3	0.00, 0.05	0.00, 0.34	0.00, 0.54	0.00, 0.99	0.8	0.15, 1.80	0.00, 2.44	0.00, 2.76	0.00, 3.38
-2.2	0.00, 0.06	0.00, 0.36	0.00, 0.56	0.00, 1.02	0.9	0.19, 1.90	0.00, 2.54	0.00, 2.86	0.00, 3.48
-2.1	0.00, 0.06	0.00, 0.38	0.00, 0.59	0.00, 1.06	1.0	0.24, 2.00	0.00, 2.64	0.00, 2.96	0.00, 3.58
-2.0	0.00, 0.07	0.00, 0.40	0.00, 0.62	0.00, 1.10	1.1	0.30, 2.10	0.00, 2.74	0.00, 3.06	0.00, 3.68
-1.9	0.00, 0.08	0.00, 0.43	0.00, 0.65	0.00, 1.14	1.2	0.35, 2.20	0.00, 2.84	0.00, 3.16	0.00, 3.78
-1.8	0.00, 0.09	0.00, 0.45	0.00, 0.68	0.00, 1.19	1.3	0.42, 2.30	0.02, 2.94	0.00, 3.26	0.00, 3.88
-1.7	0.00, 0.10	0.00, 0.48	0.00, 0.72	0.00, 1.24	1.4	0.49, 2.40	0.12, 3.04	0.00, 3.36	0.00, 3.98
-1.6	0.00, 0.11	0.00, 0.52	0.00, 0.76	0.00, 1.29	1.5	0.56, 2.50	0.22, 3.14	0.00, 3.46	0.00, 4.08
-1.5	0.00, 0.13	0.00, 0.56	0.00, 0.81	0.00, 1.35	1.6	0.64, 2.60	0.31, 3.24	0.00, 3.56	0.00, 4.18
-1.4	0.00, 0.15	0.00, 0.60	0.00, 0.86	0.00, 1.41	1.7	0.72, 2.70	0.38, 3.34	0.06, 3.66	0.00, 4.28
-1.3	0.00, 0.17	0.00, 0.64	0.00, 0.91	0.00, 1.47	1.8	0.81, 2.80	0.45, 3.44	0.16, 3.76	0.00, 4.38
-1.2	0.00, 0.20	0.00, 0.70	0.00, 0.97	0.00, 1.54	1.9	0.90, 2.90	0.51, 3.54	0.26, 3.86	0.00, 4.48
-1.1	0.00, 0.23	0.00, 0.75	0.00, 1.04	0.00, 1.61	2.0	1.00, 3.00	0.58, 3.64	0.35, 3.96	0.00, 4.58
-1.0	0.00, 0.27	0.00, 0.81	0.00, 1.10	0.00, 1.68	2.1	1.10, 3.10	0.65, 3.74	0.45, 4.06	0.00, 4.68
-0.9	0.00, 0.32	0.00, 0.88	0.00, 1.17	0.00, 1.76	2.2	1.20, 3.20	0.72, 3.84	0.53, 4.16	0.00, 4.78
-0.8	0.00, 0.37	0.00, 0.95	0.00, 1.25	0.00, 1.84	2.3	1.30, 3.30	0.79, 3.94	0.61, 4.26	0.00, 4.88
-0.7	0.00, 0.43	0.00, 1.02	0.00, 1.33	0.00, 1.93	2.4	1.40, 3.40	0.87, 4.04	0.69, 4.36	0.07, 4.98
-0.6	0.00, 0.49	0.00, 1.10	0.00, 1.41	0.00, 2.01	2.5	1.50, 3.50	0.95, 4.14	0.76, 4.46	0.17, 5.08
-0.5	0.00, 0.56	0.00, 1.18	0.00, 1.49	0.00, 2.10	2.6	1.60, 3.60	1.02, 4.24	0.84, 4.56	0.27, 5.18
-0.4	0.00, 0.64	0.00, 1.27	0.00, 1.58	0.00, 2.19	2.7	1.70, 3.70	1.11, 4.34	0.91, 4.66	0.37, 5.28
-0.3	0.00, 0.72	0.00, 1.36	0.00, 1.67	0.00, 2.28	2.8	1.80, 3.80	1.19, 4.44	0.99, 4.76	0.47, 5.38
-0.2	0.00, 0.81	0.00, 1.45	0.00, 1.77	0.00, 2.38	2.9	1.90, 3.90	1.28, 4.54	1.06, 4.86	0.57, 5.48
-0.1	0.00, 0.90	0.00, 1.55	0.00, 1.86	0.00, 2.48	3.0	2.00, 4.00	1.37, 4.64	1.14, 4.96	0.67, 5.58
0.0	0.00, 1.00	0.00, 1.64	0.00, 1.96	0.00, 2.58	3.1	2.10, 4.10	1.46, 4.74	1.22, 5.06	0.77, 5.68