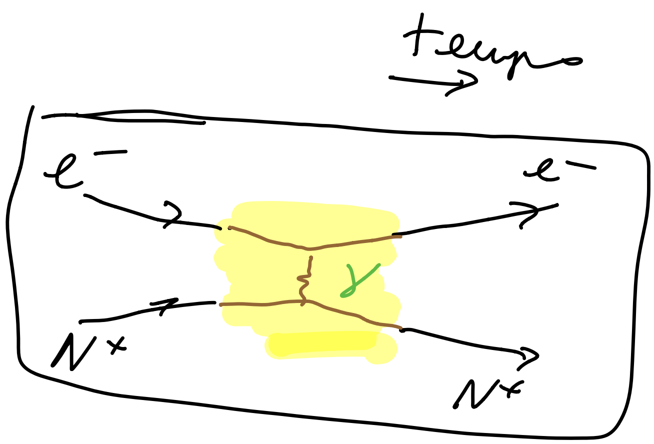


FORZA / INTERAZIONE	MEDIATORE
GRAVITAZIONALE	gravitone
EM.	fotone
FORTE	gluoni (8)
DEBOLE	W^+, Z, W^-

$$\left.\begin{matrix} ct \\ x \\ y \\ z \end{matrix}\right\} \quad [\ ] = [L]$$

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}$$

CONTRAVARIANTE

$$= \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$A \cdot B$$

$$(X^\mu) \cdot (X^\mu) = X^0 X^0 - X^1 X^1 - X^2 X^2 - X^3 X^3$$

vettore covariante

$$X_\mu = \begin{pmatrix} ct \\ -x \\ -y \\ -z \end{pmatrix}$$

$$\sum_{\mu=0}^{3} X^\mu \cdot X_\mu = X^0 X^0 - X^1 X^1 \dots$$

convenzione

$$\mu, \nu, \rho, \sigma \in \{0, 1, 2, 3\}$$

$$i, j, k, l \dots \in \{1, 2, 3\}$$

$$\sum_{\mu=0}^{3} X^\mu X_\mu \equiv X^\mu X_\mu$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$S = \int_0^{\Delta t} \mathcal{L}\, dt$$

- particella libera

Galileo $\quad \mathcal{L} = T = \frac{1}{2} m v^2$

Lorentz $\quad \mathcal{L} = ?$

S invariante di Lorentz

$$S = \int \mathcal{L}\, dt$$

$$= \int d\tau \cdot \alpha$$

costante

$$d\tau = \frac{ds}{c} = \frac{dt}{\gamma}$$

$$\lim_{v\to 0} \mathcal{L} = \lim_{v\to 0} \frac{\alpha}{\gamma}.$$

$$\mathcal{L} = \frac{\alpha}{\gamma} = \alpha\sqrt{1-\frac{v^2}{c^2}} \simeq \alpha\left(1-\frac{1}{2}\frac{v^2}{c^2}\right)$$

$$= \mathcal{L}_{\text{classica}} = \frac{1}{2}mv^2$$

a meno di una costante,

$$\frac{1}{2}mv^2 = -\frac{1}{2}\frac{v^2}{c^2}\alpha$$

$$\alpha = -mc^2$$

$$\mathcal{L} = -mc^2\sqrt{1-v^2/c^2}$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}$$

$\dot{q} \rightarrow \{v_x, v_y, v_z\}$

$q \rightarrow \{x, y, z\}$

$$\mathcal{L} = \mathcal{L}(q, \dot{q}, t) = \mathcal{L}(x_i, \dot{x}_i, t)$$

$$= \mathcal{L}(\dot{x}_i)$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right] = \frac{\partial \mathcal{L}}{\partial x_i} = 0 \rightarrow P_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \text{cost}$$

$$P_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \frac{\partial\left[-mc^2\sqrt{1-\frac{\dot{x}_i\dot{x}^i}{c^2}}\right]}{\partial \dot{x}_i}$$

$$= -mc^2 \frac{\partial\left(\sqrt{1-\frac{\dot{x}_i\dot{x}^i}{c^2}}\right)}{\partial \dot{x}_i}$$

$$= -mc^2 \frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-1/2} \cdot \left(-\frac{2\dot{x}_i}{c^2}\right)$$

$$= m\dot{x}_i\gamma$$

$$\vec{p} = m\gamma\vec{v}$$

$$H \equiv \sum_{i=1}^{3} p\dot{q} - \mathcal{L}$$

$$= \sum \frac{\partial \mathcal{L}}{\partial \dot{x}_i} \cdot \dot{x}_i - \mathcal{L}$$

$$= \sum m\gamma v_i \cdot v_i - \left(-mc^2\sqrt{1-v^2/c^2}\right)$$

$$= m\gamma v^2 + mc^2/\gamma$$
$$= m\gamma (v^2 + c^2/\gamma^2)$$
$$= m\gamma c^2 (v^2/c^2 + 1/\gamma^2)$$
$$= m\gamma c^2$$
$$= H = E$$

$$\gamma^2 = \frac{1}{1-\beta^2}$$
$$\beta = v/c$$

$$E = m\gamma c^2$$
$$p = |\vec{p}| = m\gamma v$$

$$p^\mu = \left(\frac{E}{c}, p^1, p^2, p^3\right) \rightarrow p^{\mu\prime} = \dots$$

$$x^\mu \quad ct \quad x^1 \; x^2 \; x^3$$

LORENTZ

$$x^{\mu\prime} = \dots$$

INVARIANTE:
$$p_\mu p^\mu = \frac{E}{c}\cdot\frac{E}{c} - p_x^2 - p_y^2 - p_z^2$$
$$= \frac{E^2}{c^2} - p^2$$
$$= \frac{(m\gamma c^2)^2}{c^2} - (m\gamma v)^2$$
$$= m^2\gamma^2 c^2 - m^2\gamma^2 v^2$$

$$= m^2 \gamma^2 c^2 \left(1 - v^2/c^2\right)$$
$$= m^2 c^2$$
$$|p^\mu| \equiv \sqrt{p_\mu p^\mu} = m c$$

$$E^2 = m^2 c^4 + p^2 c^2$$
$$\left(E = \sqrt{m^2 c^4 + p^2 c^2}\right)$$
$$\equiv \sqrt{m^2 + p^2}$$

$c = 1$ $\quad$ $\hbar c = 1$ $\quad$ NATURAL UNITS

E in eV

$$1 eV = 1.6 \cdot 10^{-19}\, C \cdot V$$
$$= 1.6 \cdot 10^{-19}\, J$$

$$m_p \cdot c^2 = 1.6 \cdot 10^{-27}\, kg \cdot \left(3 \cdot 10^8\, m/s\right)^2$$
$$= 938 \cdot MeV$$

$$[m] = MeV/c^2$$

perché l'energia si conserva?

l'impulso " " ?

$\vec{L}$ " " ?

E : traslazione temporale

p : traslazione spaziale

$\vec{L}$: rotazione

TEOREMA DI NOETHER

$$\mathcal{L} = -mc^2 \sqrt{1 - \frac{\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2}{c^2}}$$

invarianza spaziale

$$\frac{\partial \mathcal{L}}{\partial x_i} = 0$$

$$\frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right] = \frac{\partial \mathcal{L}}{\partial x_i} = 0$$

$$p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}_i} = \text{costante}$$

invarianza sotto trasl. temporale

$$\frac{\partial L}{\partial t} = 0$$

$$\frac{dL}{dt} = \frac{\partial L}{\partial t} + \sum_{i=1}^{3}\left[\frac{\partial L}{\partial x_i}\cdot\frac{\partial x_i}{\partial t} + \frac{\partial L}{\partial \dot{x}_i}\cdot\frac{\partial \dot{x}_i}{\partial t}\right]$$

$$= \frac{\partial L}{\partial t} + \sum_{i=1}^{3}\left[\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i}\right)\cdot\frac{\partial x_i}{\partial t} + \frac{\partial L}{\partial \dot{x}_i}\cdot\frac{\partial \dot{x}_i}{\partial t}\right]$$

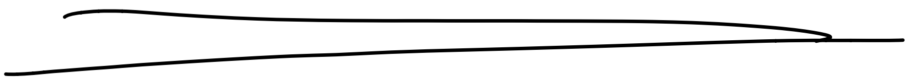
$$= \frac{\partial L}{\partial t} + \frac{d}{dt}\left[\sum_{i=1}^{3}\left[\frac{\partial L}{\partial \dot{x}_i}\cdot\dot{x}_i\right]\right]$$

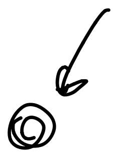
$$-\frac{\partial L}{\partial t} = \overbrace{0}^{\text{inv. sotto trasl. temporale}} = -\frac{dL}{dt} + \frac{d}{dt}\sum^{3}\left[\frac{\partial L}{\partial \dot{x}_i}\cdot\dot{x}_i\right]$$

$$= -\frac{d}{dt}\left[L - \sum_{i=1}^{3}\frac{\partial L}{\partial \dot{x}_i}\cdot\dot{x}_i\right]$$

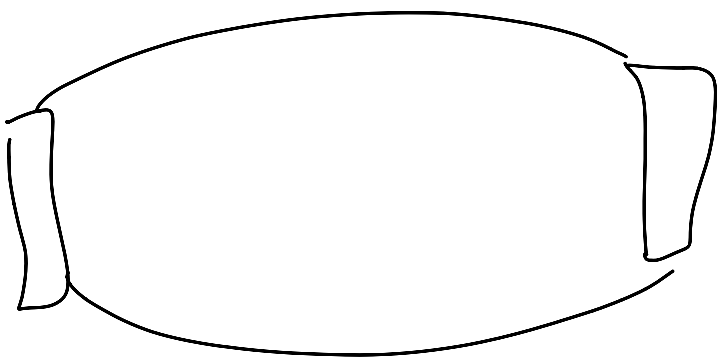
$$= -\frac{d}{dt}\left[-H\right] = \frac{d}{dt}\left[H\right]$$

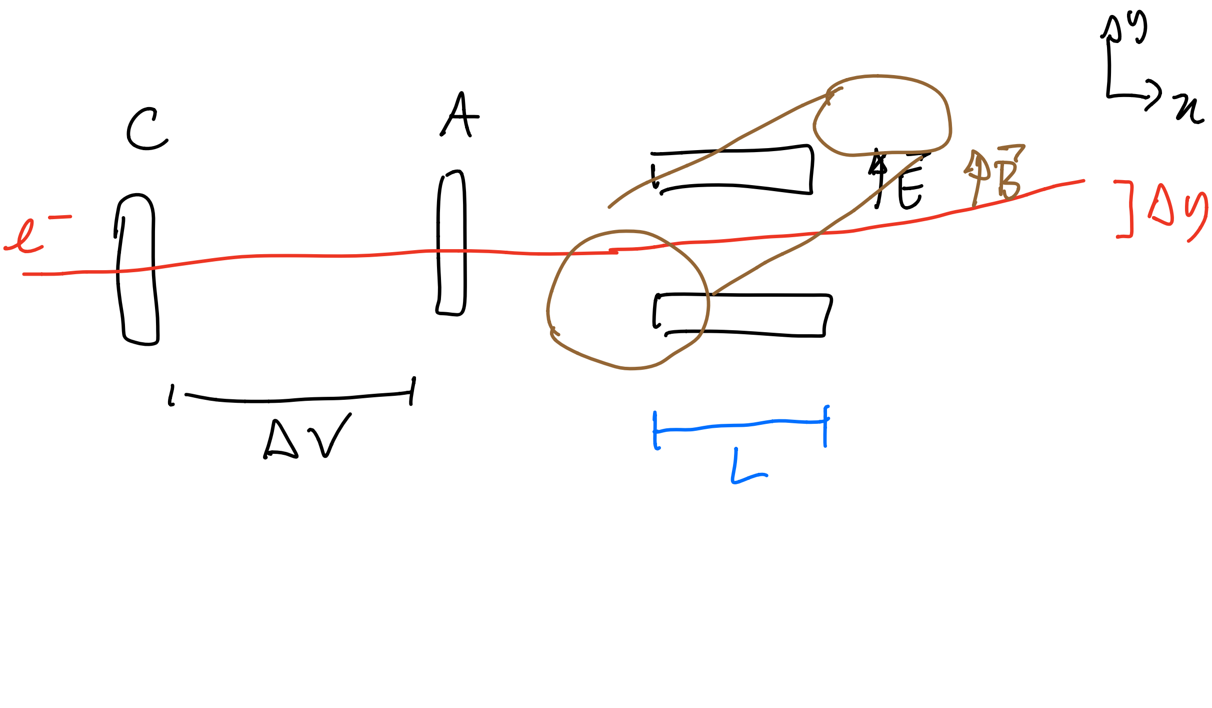
A OGNI SIMM. CONTINUA DI $\mathcal{L}$
corrisponde UNA GRANDEZZA
CONSERVATA





elettrone





$$\vec{F} = q\vec{E} = qE\,\hat{y}$$

$$\vec{F} = q\vec{v}\times\vec{B} = qvB\,\hat{y}$$

solo con $\vec{E}$:

$$\Delta y = \frac{1}{2}\,\frac{q}{m}\cdot E\,\frac{L^2}{v_x^2}$$

IMPONGO $\vec{B}$ tale che

$$qE = qv_xB \longrightarrow v_x$$

$$\longrightarrow q/m$$