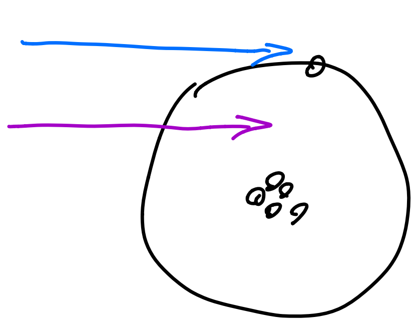
INTERAZIONE PARTICELLE - MATERIA

1) che può succedere?
2) con che P?
3) che posso misurare?



$$\frac{\Delta E}{\Delta x} \to \frac{dE}{dx}$$

ATOMO DI BOHR $\quad L = \hbar \cdot n$

es. idrogeno:

$$E_n = -\frac{1}{2n} \cdot \alpha^2 \cdot m_e c^2$$

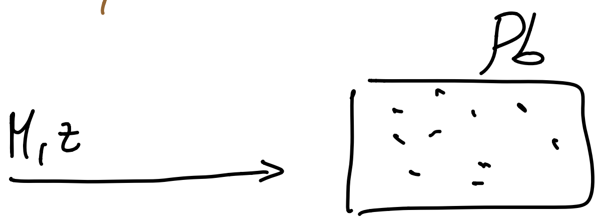
$$\alpha = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{\hbar c} \sim \frac{1}{137}$$

$$r_n = \frac{n^2 \cdot r_e}{\alpha^2}$$

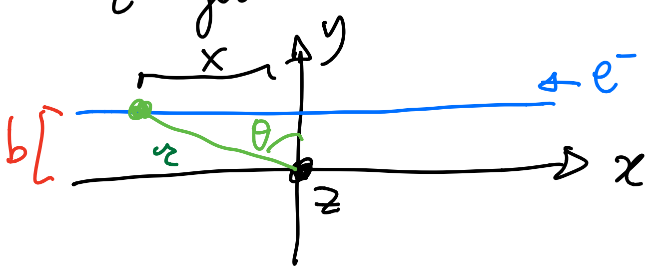
$$r_e \doteq \quad m_e c^2 = \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r_e}$$

$$\simeq 0.5 \cdot 10^{-10}\, m$$

dE/dx



assumiamo e^- lungo una retta "z" ferma:



$$\frac{x}{b} = \tan\theta$$

$$dx = b \cdot \frac{1}{\cos^2\theta}\, d\theta$$

ΔE? $\Delta E = \Delta E\,(b)$

Galileo : $\Delta E = \dfrac{\Delta p^2}{2m_e}$ $m \equiv m_e$

$$\Delta \vec{p} = \int_{-\infty}^{\infty} \vec{F} \cdot dt$$

$$\Delta p_{\parallel} = \int_{-\infty}^{\infty} \vec{F} \cdot \hat{x}\, dt = \int_{-\infty}^{\infty} \cdot F \sin\theta \,\frac{dx}{v}$$

$$= \int_{-\infty}^{\infty} \frac{z e^2}{4\pi\varepsilon_0} \cdot \frac{1}{x^2+b^2} \cdot \frac{x}{\sqrt{x^2+b^2}}\,\frac{dx}{v}$$

$v \equiv \text{cost}$ $r = \sqrt{x^2+b^2}$ $dr = \frac{1}{2}\, 2x \,\frac{1}{\sqrt{x^2+b^2}}$

$$= \frac{ze^2}{4\pi\varepsilon_0 v}\int_{-\infty}^{\infty}\frac{1}{r^2}\cdot dr = 0$$

$$\Delta P_\perp = \frac{ze^2}{4\pi\varepsilon_0 v}\cdot\int_{-\infty}^{\infty}\frac{1}{x^2+b^2}\cos\theta\,\frac{dx}{v}$$

$$dx = \frac{b}{\cos^2\theta}\,d\theta$$

$$= \frac{ze^2}{4\pi\varepsilon_0 v}\cdot\int_{-\pi/2}^{\pi/2}\frac{1}{x^2+b^2}\cdot\frac{\cos\theta}{\cos^2\theta}\,b\,d\theta$$

$$= \frac{ze^2}{4\pi\varepsilon_0 v}\cdot\frac{b^2}{b^2}\int_{-\pi/2}^{\pi/2}\underbrace{\frac{1}{x^2+b^2}}_{\cos^2\theta}\cdot\frac{1}{\cos\theta}\,b\,d\theta$$

$$= \frac{ze^2}{4\pi\varepsilon_0 v}\cdot\frac{1}{b^2}\cdot\int_{-\pi/2}^{\pi/2}\cos\theta\, b\, d\theta$$

$$= \underbrace{\frac{ze^2}{4\pi\varepsilon_0 b^2}}_{\text{FORZA COSTANTE}}\cdot\underbrace{\frac{2b}{v}}_{\text{TEMPO DI SCATTERING}} = \Delta P$$

$$\Delta E = \frac{\Delta p^2}{2m} = \Delta E(b) = \frac{z^2e^4\;4}{(4\pi\varepsilon_0)^2 b^2 v^2}\,\frac{1}{2m}$$

$$=$$

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c} \qquad r_e \doteq \; mc^2 = e^2\,\frac{1}{4\pi\varepsilon_0 r_e}$$

$$= \frac{z^2 e^4 \cdot 2}{(4\pi\varepsilon_0)^2 \cdot b^2 \cdot (\beta^2 c^2) \cdot m}$$

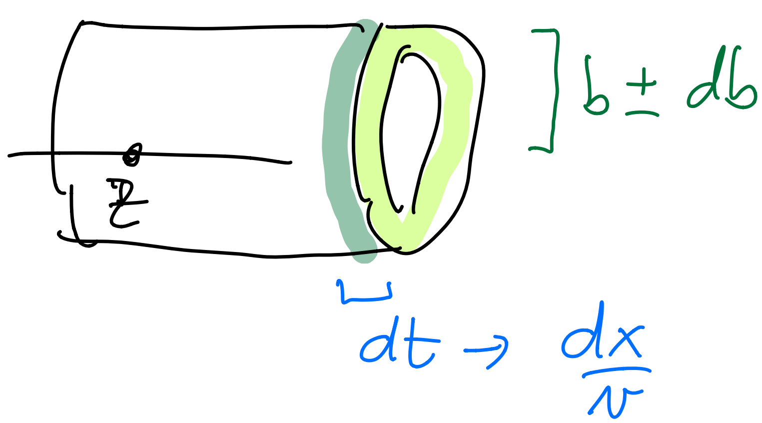
$$4\pi\varepsilon_0 = \frac{e^2}{r_e m c^2}$$

$$= \frac{z^2 e^4 \cdot 2}{(4\pi\varepsilon_0)^2\, b^2 \cdot \beta^2 \cdot \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r_e}}$$

$$= \frac{z^2 e^4 \cdot 2}{\frac{e^2}{r_e m c^2} \cdot b^2 \cdot \beta^2 \cdot e^2 \cdot \frac{1}{r_e}}$$

$$= 2 \cdot \left(\frac{z}{\beta}\right)^2 \left(\frac{r_e}{b}\right)^2 m c^2 = \Delta E(b)$$

SINGOLO URTO



quanti elettroni vedo dopo dt

(dopo tragitto dx?)

$$dN_e = n_e \cdot 2\pi b \cdot db \cdot dx$$

in caso urto $\Delta E = \Delta E(b)$

$$dE = dN_e \cdot \Delta E$$

$$\frac{d^2E}{dx\,db} = 2 \cdot \left(\frac{z}{\beta}\right)^2 \left(\frac{r_e}{b}\right)^2 mc^2 \frac{n_e 2\pi b\, db\, dx}{dx\, db}$$

$$= 4\pi \cdot \left(\frac{z}{\beta}\right)^2 r_e^2 \cdot mc^2 \cdot n_e \frac{1}{b}$$

$$\frac{dE}{dx} = \int_{b_{MIN}}^{b_{MAX}} db \frac{d^2E}{dx\,db} = 4\pi \left(\frac{z}{\beta}\right)^2 r_e^2 mc^2 \cdot n_e \cdot \log\left(\frac{b_{MAX}}{b_{MIN}}\right)$$

b_{MAX}

$$t_{SCAT} = \frac{2b}{v} < T_{\text{orbita elettrone}}$$

$$= \gamma \cdot \frac{1}{\nu}$$

ν freq. di rivoluz. dell'e^-

$$b < b_{MAX} = \frac{1}{2} \cdot \frac{v}{\nu}\gamma$$

$$= \frac{1}{2} \cdot \frac{\beta c}{\nu}\gamma$$

b_{MIN} $\quad \Delta p \Delta x > \hbar/2$

$$\Delta p \sim p$$

$$\Delta x > b_{MIN} = \frac{\hbar}{2} \cdot / m\gamma v$$

$$= \frac{\hbar}{2} \cdot / m\beta\gamma c$$

$$\frac{dE}{dx} = 4\pi\left(\frac{z}{\beta}\right)^2 r_e^2 mc^2 \cdot n_e \cdot \log\left(\frac{b_{MAX}}{b_{MIN}}\right)$$

$$= 4\pi\left(\frac{z}{\beta}\right)^2 r_e^2 mc^2 \cdot n_e \cdot \log\left(\frac{\beta\gamma c \cdot m\beta\gamma c}{\hbar\nu}\right)$$

$$= 4\pi\left(\frac{z}{\beta}\right)^2 \cdot r_e^2 \cdot n_e \cdot mc^2 \cdot \log\left(\frac{(\beta\gamma)^2 mc^2}{\hbar\nu}\right)$$

FORMULA DI BOHR $\qquad$ "MEDIA"

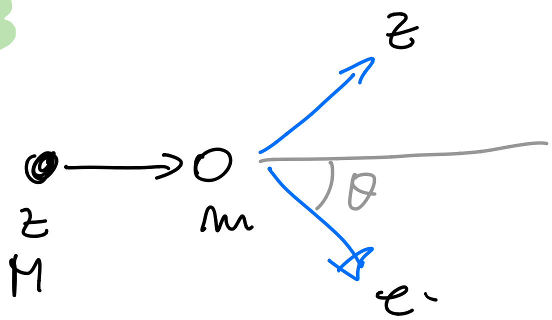
- M Q
- $v \sim c$

$$\Delta E(b) = 2mc^2 r_e^2 \left(\frac{z}{\beta}\right)^2 \cdot \frac{1}{b^2}$$

$$b = \ldots$$

$$b^2_{MAX} \xrightarrow{\text{corrisponde}} \Delta E_{MIN}$$

$$\log\left(\frac{b_{MAX}}{b_{MIN}}\right) = \frac{1}{2}\log\left(\frac{\Delta E_{MAX}}{\Delta E_{MIN}}\right)$$



MAX ENERGY TRANSFER $\cos\theta = 1$

$$|i\rangle \quad \underline{P} = (E, \vec{P})$$
$$\underline{P_e} = (m, \vec{0})$$

$\boxed{c = 1}$

$$E^2 = M^2 + p^2$$

$|f\rangle \quad \underline{P}' = (E', \vec{P}')$

$\underline{P_e} = (E_e', \vec{P}_e')$

θ: angolo fra $\vec{P}$ e $\vec{P}_e'$

$$\vec{P} + \vec{0} = \vec{P}' + \vec{P}_e'$$

$$E + m = E' + E_e'$$

$$E' = E + m - E_e'$$

$$E'^2 = M^2 + P'^2 = E^2 + m^2 + E_e'^2 + 2Em - 2mE_e' - 2EE_e'$$

$$P'^2 = (\vec{P} - \vec{P}_e')^2 = P^2 + P_e'^2 - 2P \cdot P_e' \cdot \cos\theta$$

$$M^2 + P^2 + P_e'^2 - 2PP_e'\cos\theta = E^2 + m^2 + E_e'^2 + 2mE - 2mE_e' - 2EE_e'$$

$$-2PP_e'\cos\theta = m^2 + m^2 + 2mE - 2mE_e' - 2EE_e'$$

$$-PP_e'\cos\theta = m(m+E) - E_e'(m+E)$$

$$P^2P_e'^2\cos^2\theta = (m+E)^2\left[m - E_e'\right]^2$$

$$= (m+E)^2\left[m^2 - 2mE_e' + E_e'^2\right]$$

$$P^2\left[E_e'^2 - m^2\right]\cos^2\theta = (m+E)^2\left[m^2 - 2mE_e' + E_e'^2\right]$$

$$E_e'^2\left[P^2\cos^2\theta - (m+E)^2\right]$$

$$+ E_e'\left[2m(m+E)^2\right]$$

$$+ \left[-P^2m^2\cos^2\theta - m^2(m+E)^2\right] = 0$$

$$E_e' = m + T_e' = -\left(2m(m+E)^2 \pm \sqrt{4m^2(m+E)^4 + 4\left(P^2\cos^2\theta - (m+E)^2\right) \times \left(P^2m^2\cos^2\theta + m^2(m+E)^2\right)}\right)$$

$$\times\left[\frac{1}{2\left(P^2\cos^2\theta - (m+E)^2\right)}\right]$$

IL MIO OBIETTIVO (voglio T_e' MAX)

$$= -\frac{2m(m+E)^2 \pm \sqrt{4m^2(m+E)^4 + 4m^2\left[\left(p^2\cos^2\theta - (m+E)^2\right)\left(p^2\cos^2\theta + (m+E)^2\right)\right]}}{2\left(p^2\cos^2\theta - (m+E)^2\right)}$$

$$= -\frac{2m(m+E)^2 \pm 2m\,p^2\cos^2\theta}{2\left(p^2\cos^2\theta - (m+E)^2\right)}$$

$$= m\,\frac{(m+E)^2 \pm p^2\cos^2\theta}{(m+E)^2 - p^2\cos^2\theta} = T_e' + m$$

$$T'_{e\,max} = mc^2\,\frac{(mc^2+E)^2 + p^2c^2}{(mc^2+E)^2 - p^2c^2} - mc^2$$

massimo a θ

$$E^2 = m^2c^4 + p^2c^2$$

$$E = M\gamma c^2$$

$$P = M\gamma v$$

$$= \frac{2m(\beta\gamma)^2 c^2}{1 + \frac{2m}{M}\cdot\gamma + \left(\frac{m}{M}\right)^2} = \Delta E_{MAX}$$

se $2m\gamma \ll M$

$$\Delta E_{MAX} \sim \underbrace{2m}_{\text{dell'elettrone}}(\beta\gamma)^2 c^2$$

ΔE_{MIN}

$$\Delta E_{MIN} = ?$$

$$\Delta E_{MIN} \sim \langle I \rangle \sim (\underbrace{10\ eV}_{13.6\ eV}) \cdot Z$$

Z del mezzo

$$C = 4\pi r_e^2 \underbrace{m}_{\text{dell'elettrone}} c^2 \cdot N_A = 0.310\ MeV/(g/cm^2)$$

$$\frac{dE}{dx} = \rho \cdot C \cdot \left(\frac{z}{\beta}\right)^2 \cdot \frac{Z}{A} \cdot \frac{1}{2}\log\left(\frac{mc^2 \cdot (\beta\gamma)^2}{\langle I \rangle}\right)$$

LINEAR STOPPING POWER

della particella

del mezzo $\sim 1/2$

FORMULA DI BETHÉ

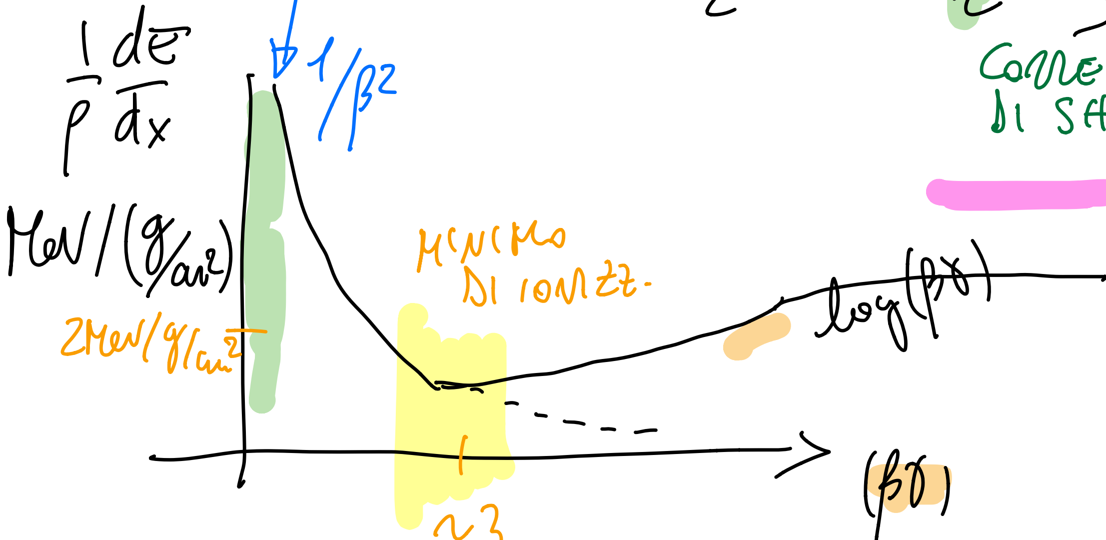
$$\frac{1}{\rho}\frac{dE}{dx}$$

MASS STOPPING POWER

BETHE-BLOCH

$$\frac{1}{\rho}\frac{dE}{dx} = C \cdot \left(\frac{z}{\beta}\right)^2 \left(\frac{Z}{A}\right)^2 \cdot \left[\frac{1}{2}\log\left(\frac{mc^2(\beta\gamma)^2 \cdot T_{MAX}}{\langle I \rangle^2}\right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{K}{Z}\right]$$

CORREZIONE DI SHELL



$$E_\perp \xrightarrow{v \sim c} \gamma E_\perp$$

$$\longrightarrow \log(\beta\gamma)$$