LEZIONE SPECIALE:

25/03/2021

@ 12h00

("teoria dello scattering: perturbazioni indipendenti dal tempo")

[a lezione faremo quelle dipendenti dal tempo]

PERDITE DI E DI UNA PARTICELLA

- ionizzazione

$$\frac{dE}{dx} \qquad \frac{1}{\rho}\frac{dE}{dx}$$

LINEAR STOPPING POWER — MASS ST. P.

versione completa: Bethe-Bloch

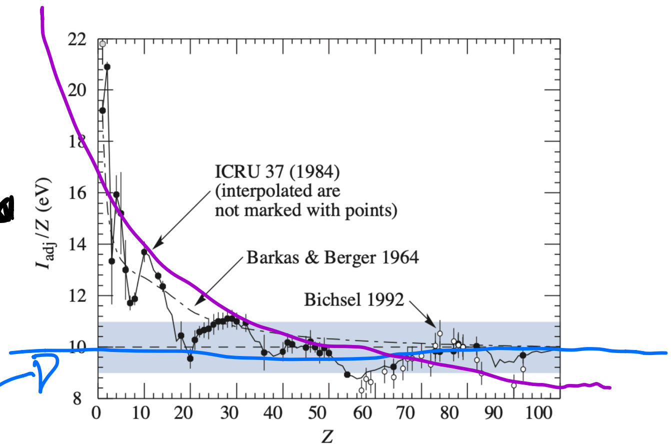
$$\frac{1}{\rho}\frac{dE}{dx} = \underbrace{C}_{0.307\ \mathrm{MeV}/(g/cm^2)} \left(\frac{Z}{A}\right)\left(\frac{z}{\beta}\right)^2\left[\frac{1}{2}\ln\frac{m_e c^2(\beta\gamma)^2 T_{max}}{\langle I\rangle^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} - \frac{K}{Z}\right]$$

DEL MEZZO

$$T_{MAX} = \frac{2m_e c^2(\beta\gamma)^2}{1+(m_e/M)^2 + 2m_e\gamma/M}$$

ci serve $\langle I \rangle$

$\frac{I}{Z}$



Z

$$\boxed{I \sim 10\,\text{eV} \cdot Z}$$

MASS STOPPING POWER

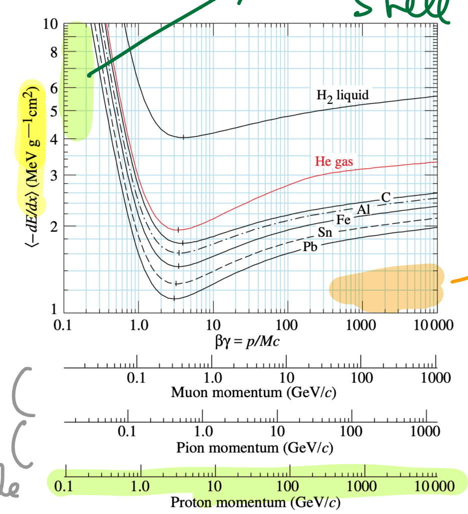
$\sim 1/\beta^2$

Correzione di shell

– il calcolo vale per $M \neq m_e$

$\frac{1}{\rho}\frac{dE}{dx}$ ← MeV ← cm

↑ g/cm³



] altrimenti $E'_{\parallel} = E_{\parallel}$ $\quad E'_{\perp} = \gamma E_{\perp}$

EFFETTO DENSITA'

$\rightarrow \beta\gamma = \frac{p}{Mc}$

$\beta = \frac{pc}{E}$

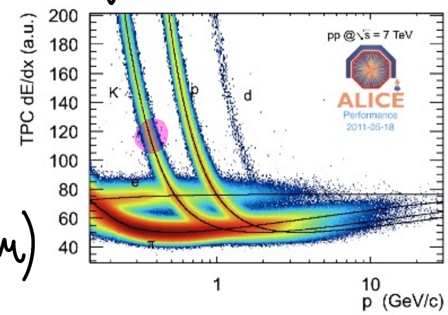
$\gamma = \frac{E}{Mc^2}$

ipotesi di particelle

$m_p \simeq 938\,\text{MeV}/c^2$

posso usare dE/dx per fare PARTICLE IDENTIFICATION (PID)

$\frac{dE}{dx}$ (MeV/cm) $\uparrow$



$\longrightarrow p$ delle particelle

SEZIONE D'URTO

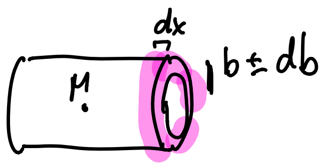
$$\Delta E = 2 m_e c^2 \left(\frac{z}{\beta}\right)^2 \cdot \left(\frac{r_e}{b}\right)^2$$

nel singolo urto con un e^-

$$b^2 = \dots \cdot \frac{1}{\Delta E}$$

$$d\sigma = 2\pi\, b\, db$$

$$|2b\,db| = \left| m_e c^2 \left(\frac{z}{\beta}\right)^2 r_e^2 \right| \frac{d(\Delta E)}{(\Delta E)^2}$$



$$d\sigma = 2\pi\, b\, db = 2\pi \cdot \frac{m_e c^2 z^2 r_e^2}{\beta^2} \cdot \frac{d(\Delta E)}{(\Delta E)^2}$$

$$\frac{d\sigma}{d(\Delta E)} \propto \frac{1}{(\Delta E)^2}$$

molto più probabile che nel singolo urto si perda poca energia

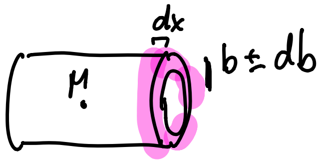
$$\frac{d\sigma}{d(\Delta E)}(\Delta E = 1) = A$$

$$\frac{d\sigma}{d(\Delta E)} \cdot (\Delta E = 4) = \frac{A}{16}$$

le collisioni con alto ΔE sono RARE

RANGE ("percorso residuo")

- CASO GENERALE di un percorso qualunque



$$dP = \delta P \, dN_B = \frac{\sigma}{S} \cdot (S \cdot dx \cdot n_B) = \sigma \cdot n_B \cdot dx$$

FLUSSO DI PARTICELLE INCIDENTI

$$d\phi = -\phi \, dP = -\sigma n_B \, dx \, \phi$$

$$\phi = \phi_0 \, e^{-\frac{x-x_0}{\lambda}} \qquad \lambda \equiv \frac{1}{\sigma n_B}$$

CAMMINO LIBERO MEDIO

$$\mu = \sigma n_B$$

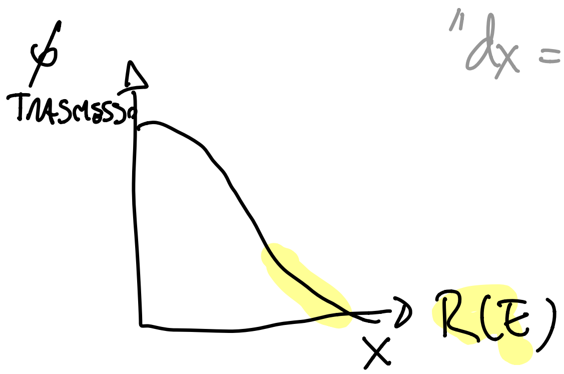
COEFF. DI ASSORBIMENTO

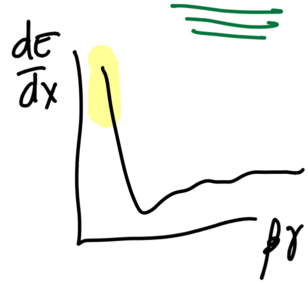
- RANGE

$$R(E) = \int_0^R dx = \int_0^E \left(\frac{dE}{dx}\right)^{-1} dE$$

CINETICA

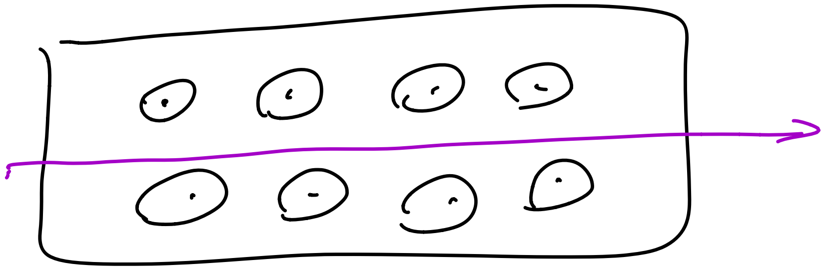
"$dx = \frac{dx}{dE} dE$"



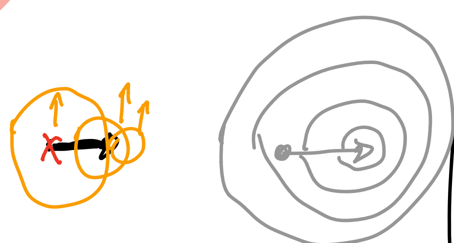


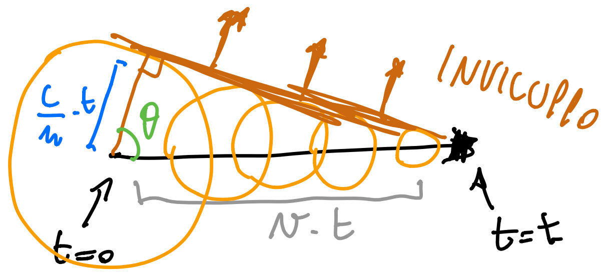
SI TRATTA SEMPRE DI VALORI MEDI

EFFETTO ČERENKOV



v "piccolo" | v "grande"





$$v < \frac{c}{n}$$

n] INDICE DI RIFRAZIONE DEL MEZZO

$\frac{c}{n}$: VELOCITÀ DELLA LUCE NEL MATERIALE ATTRAVERSATO

$$v > \frac{c}{n}$$

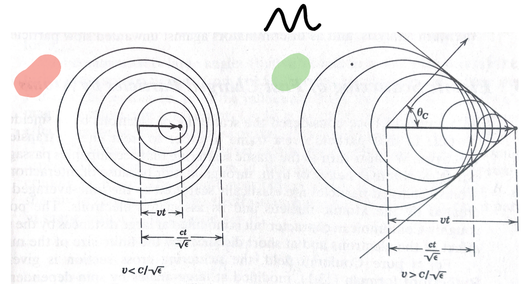
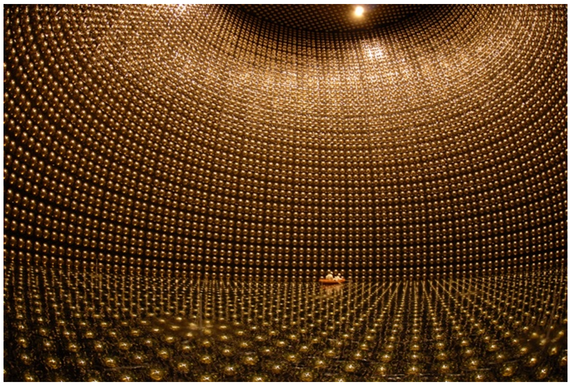
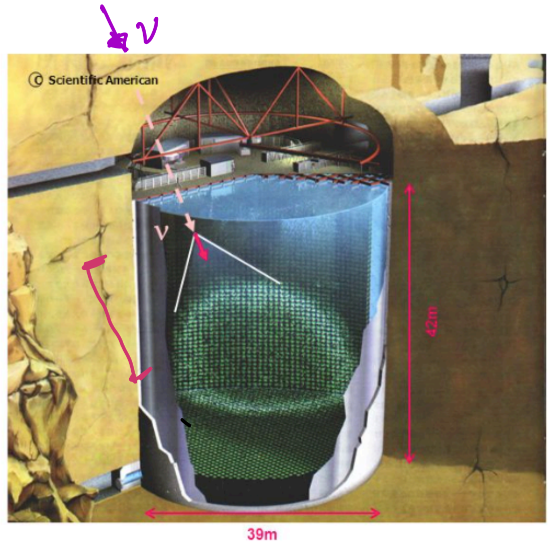


Figure 13.5 Cherenkov radiation. Spherical wavelets of fields of a particle traveli less than and greater than the velocity of light in the medium. For $v > c/\sqrt{\epsilon}$, an electromagnetic "shock" wave appears, moving in the direction given by the Cher angle θ_C.

$$\frac{c}{n} \cdot t = v \cdot t \cdot \cos\theta$$

$$\rightarrow \cos\theta = \frac{1}{\beta \cdot n(\hbar\omega)}$$





$$\underbrace{\frac{d^2 N}{\underbrace{dx}_{\text{distanza attraversata}} \cdot d\omega}}_{}$$

FOTONI

LA STIMO CON $\Longrightarrow$ $N(\omega) = \frac{E}{\hbar\omega}$

$$\frac{d^2 E}{dx\, d\omega} = \frac{z^2 e^2}{4\pi\varepsilon_0} \cdot \frac{\omega}{c^2}\left[1 - \frac{1}{\beta^2 n^2(\omega)}\right]$$

$$= \frac{z^2 e^2}{4\pi\varepsilon_0} \cdot \frac{\omega}{c^2}\left[\sin^2(\theta(\omega))\right]$$

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\hbar c}$$

$$\cos\theta = \frac{1}{\beta n(\omega)}$$

$$\frac{d^2 N}{dx\, d\omega} = \frac{z^2 \alpha}{c} \sin^2(\theta(\omega))$$

$$\frac{dN}{dx} = \frac{z^2\alpha}{c}\int_{\Delta\omega} d\omega\, \sin^2(\theta(\omega))$$

] IN CUI E' SENSIBILE IL RIVELATORE

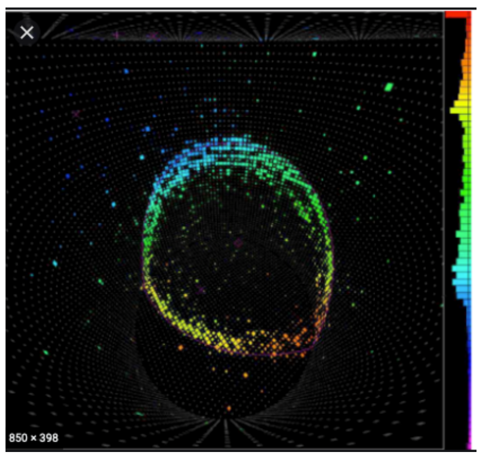
$$\simeq \frac{z^2\alpha}{c}\langle \sin^2(\theta(\omega))\rangle \cdot \Delta\omega$$

$$\simeq 700 \text{ fotoni} \times z^2 \sin^2\theta_c \text{ per cm}$$

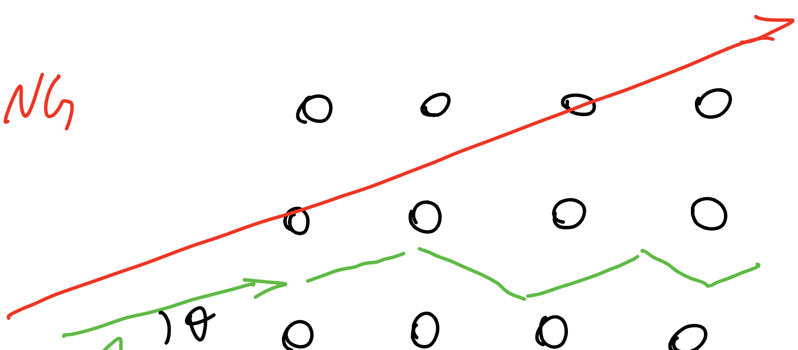
è un effetto PICCOLO di perdita di energia! $\simeq 10^{-3} \times$ IONIZZAZIONE

$E_\gamma \sim 4$ eV

$\Delta E \sim 2$ keV

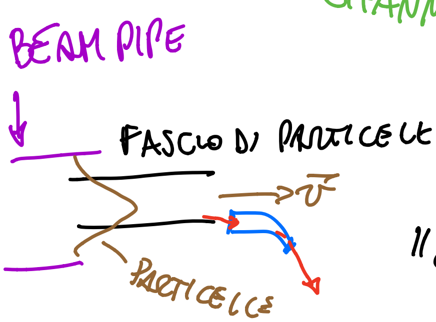


CHANNELING



CHANNELING

$f(\theta, \beta) \hookrightarrow \beta < 0.1$



"COLLIMAZIONE" di un fascio

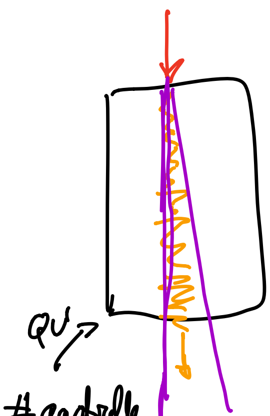
SCATTERING COULOMBIANO MULTIPLO COL NUCLEO DI TANTI NUCLEI

$(\text{Rutherford})^N$

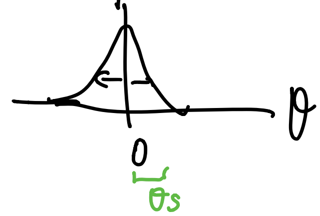
$$\frac{d\sigma}{d\Omega} = \frac{(zZe^2)^2}{(16\pi\varepsilon_0 \cdot T)} \cdot \frac{1}{\sin^4 \theta/2}$$

$P(\theta \sim 0)$ è $\underline{\underline{\text{alta}}}$

$\langle \theta \rangle = 0$ DOPO TANTI SCATTERING



NUMERO DI COLLISIONI IN ANGOLO SOLIDO $d\Omega$



$$d^2n = n_B\, dx\, d\sigma = \rho \cdot \frac{N_A}{A} dx$$

NUCLEI per cm^2

$$\times \frac{d\sigma}{d\Omega} \cdot d\Omega$$

Rutherford: $T \sim \frac{pv}{2}$ (non relativistico)

ISOTROPICA IN ϕ

$$= \rho \frac{N_A}{A} \cdot dx \cdot \frac{d\sigma}{d\Omega} \cdot 2\pi \cdot \theta \cdot d\theta$$

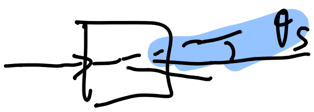
$\sin\theta \sim \theta$ per θ piccoli

$$= \rho \frac{N_A}{A} \cdot dx \left(\frac{zZe^2}{16\pi\varepsilon_0 \frac{pv}{2}} \right)^2 \cdot \frac{1}{(\theta/2)^4} \cdot 2\pi \cdot \theta \cdot d\theta$$

$$= \rho \frac{N_A}{A} \cdot dx \cdot \left(\frac{zZ}{4T}\right)^2 \cdot (r_e m_e c^2)^2 \cdot \frac{1}{\theta^3} \cdot 2\pi \cdot d\theta$$

$$\frac{e^2}{4\pi\varepsilon_0 r_e} = m_e c^2$$

$$= 8\pi r_e^2 \cdot \frac{(m_e c^2)^2}{p^2 v^2} \cdot z^2 Z^2 \cdot \rho \frac{N_A}{A} \cdot \frac{1}{\theta^3}\, d\theta\, dx$$



$$\langle \theta_s^2 \rangle = \int \Big(\theta^2 - \underbrace{\langle \theta \rangle^2}_{=0}\Big)\, dn$$

$$= \int \theta^2 \cdot \frac{d^2 n}{d\theta\, dx}\, d\theta\, dx$$

$$\equiv \int \theta_x^2\, dx$$

$$\theta_x^2 = 8\pi r_e^2 \cdot \rho \frac{N_A}{A} \cdot z^2 Z^2 \frac{(m_e c^2)^2}{p^2 v^2} \cdot \int \frac{1}{\theta}\, d\theta$$

$$b = \frac{A}{2} \cdot \frac{1}{\tan \theta/2}$$

b_{MIN} : diametro del nucleo

b_{MAX} : " dell'atomo