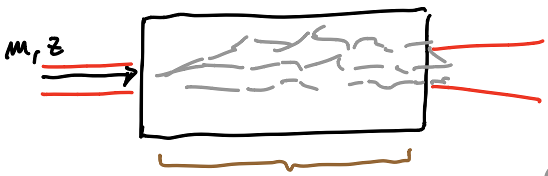
MEANWHILE IN GOTHAM

SCATTERING MULTIPLO

↳ COULOMBIANO, COL NUCLEO

⟹ SEQUENZA DI TANTI SCATTERING ELASTICI — "$\frac{dE}{dx} = 0$"



} il fascio di particelle ne esce "allargato"

Sezione d'urto differenziale in $d\Omega$:

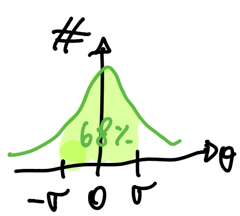
$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{16\pi\varepsilon_0 T}\right)^2 \cdot \frac{1}{\sin^4 \theta/2}$$

- $d\sigma/d\Omega$: SINGOLO SCATTERING RUTHERFORD
- z: della particella
- Z: MEZZO
- T: ENERGIA CINETICA DELLA PARTICELLA
- θ: ANGOLO DI SCATTERING

→ angolo più probabile: $\theta = 0°$ (da $\frac{d\sigma}{d\Omega} \propto \frac{1}{\theta^4}$) → $\langle\theta\rangle = 0$

TANTI SCATTERING → θ seguirà una GAUSSIANA (T.C.L.)

ci chiediamo la STD DEV di θ



$$\langle\theta^2\rangle = \int dn \left(\theta^2 - \langle\theta\rangle^2\right) = \int dn\, \theta^2 \qquad (\langle\theta\rangle^2 = 0)$$

$$dn = n_B \cdot dx \cdot d\sigma = \rho \frac{N_A}{A} dx \frac{d\sigma}{d\Omega} d\Omega$$

(n_B: BERSAGLI (ATOMI) per cm^2; $\frac{d\sigma}{d\Omega}$: RUTHERFORD)

$$d\Omega = \sin\theta\, d\theta\, d\phi \simeq \theta\, d\theta \cdot 2\pi$$

($\sin\theta \simeq \theta$: CI ASPETTIAMO θ PICCOLI; 2π: POTENZIALE CENTRALE → ISOTROPIA)

CI ASPETTIAMO θ PICCOLI ($\langle\theta\rangle = 0$ ...) →

$$\simeq \rho \frac{N_A}{A} dx \left(\frac{zZe^2}{16\pi\varepsilon_0 T}\right)^2 \cdot \frac{1}{(\theta/2)^4} \cdot 2\pi\,\theta\, d\theta$$

$\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_e} = m_e c^2$

$$= \rho \frac{N_A}{A} dx \cdot \left(\frac{zZ}{4T}\right)^2 \cdot (m_e c^2 r_e)^2 \cdot \frac{16}{\theta^3} \cdot 2\pi\, d\theta$$

$T = \frac{p v}{2}$ — TRATTAZIONE CLASSICA

$$= 8\pi \rho \frac{N_A}{A} \left(\frac{zZ}{p v}\right)^2 (m_e c^2 r_e)^2 \cdot \frac{1}{\theta^3} d\theta\, dx$$

perciò

INTEGRO PRIMA IN $d\theta$ E POI IN dx

$$\langle\theta^2\rangle = \int dn\, \theta^2 = \int \frac{dn}{d\theta dx} d\theta\, dx\, \theta^2 \equiv \int \theta_s^2\, dx$$

con

$$\theta_s^2 = 8\pi \rho \frac{N_A}{A} \left(\frac{zZ}{p v}\right)^2 (m_e c^2 r_e)^2 \int_{\theta_{MIN}}^{\theta_{MAX}} \frac{1}{\theta} d\theta$$

devo usare la relazione fra θ e b

$$b = \frac{A}{2} \cdot \frac{1}{\tan \theta/2}$$

$A \equiv \frac{zZe^2}{4\pi\varepsilon_0 T}$

[CF. LEZIONE 3 E BIBLIOGRAFIA SUL DIARIO DEL CORSO]

$$\theta_{MAX} \longleftrightarrow b_{MIN}$$

$$\log \frac{\theta_{MAX}}{\theta_{MIN}} = \log \frac{\frac{1}{b_{MIN}}}{\frac{1}{b_{MAX}}} = \log \frac{b_{MAX}}{b_{MIN}}$$

- $r_{ATOMO} \simeq \left(\frac{r_e}{\alpha^2}\right) \cdot Z^{-1/3}$ (THOMAS-FERMI)
- $r_{NUCLEO} \simeq 1.2\, fm \cdot A^{1/3} \simeq r_e/2 \cdot A^{1/3}$

$$\frac{b_{MAX}}{b_{MIN}} = \frac{2}{\alpha^2} \cdot Z^{-1/3} A^{-1/3} = \frac{2}{\alpha^2} \cdot \left(\frac{Z}{A}\right)^{1/3} \cdot Z^{-2/3}$$

$\alpha \sim \frac{1}{137}$

$$= \left[\frac{\sqrt{2}}{\alpha} \left(\frac{Z}{A}\right)^{1/6} Z^{-1/3} \right]^2 \simeq \left[183\, Z^{-1/3} \right]^2$$

$$\theta_S^2 = 8\pi \rho \frac{N_A}{A} \left(\frac{zZ}{pv}\right)^2 (m_e c^2 r_e)^2 \int_{\theta_{MIN}}^{\theta_{MAX}} \frac{1}{\theta} d\theta$$

$$= 8\pi \rho \frac{N_A}{A} \cdot \left(\frac{zZ}{pv}\right)^2 (m_e c^2 r_e)^2 \cdot 2 \cdot \log\left(183\, Z^{-1/3}\right)$$

$$= \left[\frac{4\pi \cdot (m_e c^2)^2}{\alpha} \right] \left[\alpha\, 4 \cdot \rho \cdot \frac{N_A}{A} \cdot Z^2 \cdot \log\left(183\, Z^{-1/3}\right) \right]$$

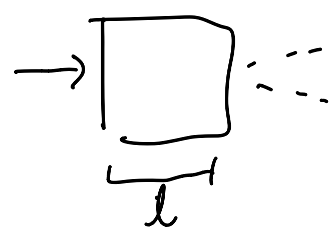
$$\times \left(\frac{z}{pv}\right)^2$$

dipende dalla particella

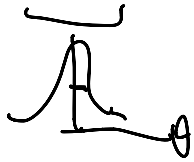
dipende dal materiale → $\frac{1}{X_0} \cdot \rho$

$$\simeq (21\ \text{MeV})^2 \equiv E_S^2$$

$$\langle \theta^2 \rangle = \int_0^{\ell} \theta_S^2\, dx$$



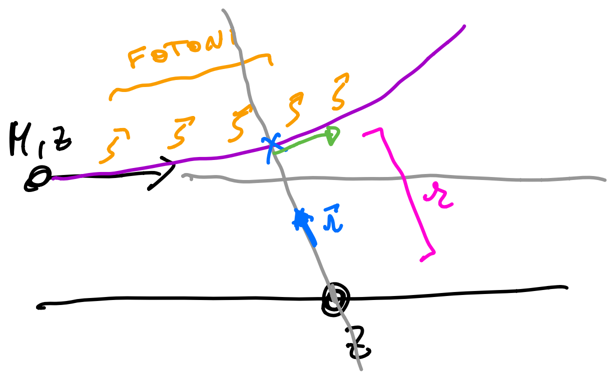
$$\sqrt{\langle \theta^2 \rangle} = z \cdot \frac{E_S}{pv} \cdot \sqrt{\frac{\ell \rho}{X_0}}$$



$$= z \cdot \frac{E_S}{pv} \sqrt{\frac{x}{X_0}}$$

LUNGHEZZA DI RADIAZIONE

$x \equiv \ell\, \rho$



BREMSSTRAHLUNG

RADIAZIONE DI FRENAMENTO
(IRRAGGIAMENTO)

O': solidale con M

- $E'_\perp = \gamma E_\perp$
- nel punto blu $\vec{v} \perp \vec{r}$, $\vec{a} \parallel \vec{r}$

$$M a' = \gamma \cdot \frac{zZe^2}{4\pi\varepsilon_0} \cdot \frac{1}{r^2}$$

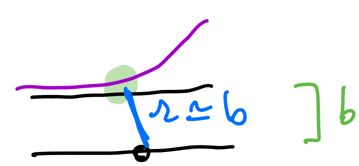
potenza irraggiata (LARMOR)

$$W' = \frac{dE'}{dt'} = \frac{q^2 \cdot a'^2}{6\pi\varepsilon_0 c^3} = \frac{(ze)^2}{6\pi\varepsilon_0 c^3} \cdot \frac{\gamma^2}{M^2} \cdot \frac{(zZe^2)^2}{(4\pi\varepsilon_0)^2} \cdot \frac{1}{r^4}$$

$$= \gamma^2 \cdot \frac{2}{3} \cdot \frac{1}{(4\pi\varepsilon_0)^3} \cdot \frac{z^4 \cdot Z^2 e^6}{M^2 r^4 c^3}$$

TEMPO DI SCATTERING

$$\Delta t = \gamma \Delta t' = \frac{2 \cdot b}{v}$$

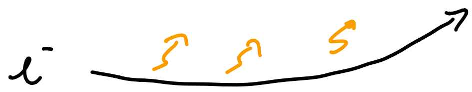


$\rightarrow r \simeq b$

$$\Delta t' = \frac{1}{\gamma} \frac{2b}{v}$$

$$\Delta E' = \int W' dt' \simeq W' \cdot \Delta t' = \frac{4}{3} \gamma \cdot \frac{z^4 Z^2 e^6}{(4\pi\varepsilon_0)^3 \cdot M^2 \cdot r^3} \cdot \frac{1}{v c^3}$$

($\Delta E'$: NEL SINGOLO SCATTERING)



VOGLIAMO SAPERE LO SPETTRO ENERGETICO DEI FOTONI EMESSI

$$\frac{dE}{d\omega} = \frac{dE'}{d\omega'} = \frac{\Delta E'}{\omega'} =$$

$\omega = 2\pi\nu = \left(\frac{1}{\gamma}\frac{2b}{v}\right)^{-1} \cdot 2\pi$

$E \propto \omega \to$ trasformano allo stesso modo (Lorentz)

$$= \frac{8}{3}\frac{z^4 Z^2 e^6}{(4\pi\varepsilon_0)^3} \cdot \frac{1}{r^2 M^2 \cdot c^3 v^2} \cdot \frac{1}{2\pi}$$

$\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{r_e} = m_e c^2$

$$= \frac{8}{3} \cdot z^4 \cdot Z^2 \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r^2 \cdot M^2 \cdot \beta^2 c^5} \cdot \frac{1}{2\pi}$$

$$= \frac{8}{3} z^4 Z^2 \cdot (r_e m_e c^2)^2 \cdot \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r^2 M^2 \beta^2 c^5} \cdot \frac{1}{2\pi}$$

$$= 8/3 \cdot r_e^2 z^4 Z^2 \left(\frac{m_e}{M}\right)^2 \cdot (\hbar c \alpha) \cdot \frac{1}{r^2 \cdot \beta^2 \cdot c} \cdot \frac{1}{2\pi}$$

$$= \frac{dE}{d\omega} = \frac{dE'}{d\omega'}$$

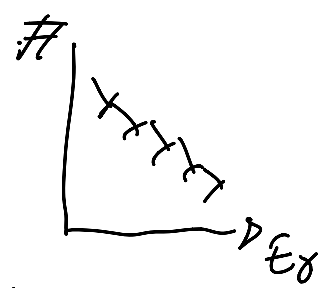
emetto fotoni con $E_\gamma = \hbar\omega$

NEL SINGOLO SCATT. "dE" $= n E_\gamma$

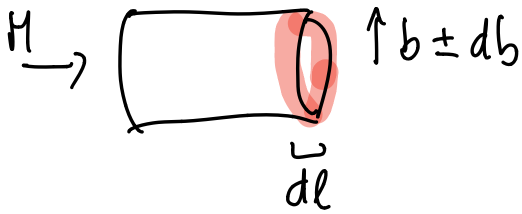
$$dn = dn(E_\gamma) = \frac{1}{E_\gamma} \cdot dE(E_\gamma)$$

SPETTRO

$$\frac{dn}{dE_\gamma} = \frac{1}{E_\gamma} \cdot \frac{dE}{dE_\gamma} = \frac{1}{E_\gamma} \cdot \frac{dE}{d(\hbar\omega)}$$



↳ FOTONI NEL SINGOLO SCATTERING



HO N SCATTERING dopo dl

$$\frac{dN}{dE_\gamma} = \rho \cdot \frac{N_A}{A} \cdot 2\pi b \cdot db \cdot dl \cdot dn/dE_\gamma$$

$$= \frac{1}{E_\gamma} \cdot \frac{8}{3} r_e^2 \cdot z^4 \cdot Z^2 \left(\frac{m_e}{M}\right)^2 \cdot \alpha \cdot \frac{1}{z^2\beta^2} \cdot \frac{1}{2\pi}$$

$$\times \rho \cdot \frac{N_A}{A} \cdot 2\pi b \cdot db \cdot dl$$

$$\frac{d^2N}{dE_\gamma \cdot dl} = \int db \, \frac{1}{E_\gamma} \cdot \frac{8}{3} r_e^2 \cdot z^4 \cdot Z^2 \cdot \left(\frac{m_e}{M}\right)^2 \cdot \alpha \cdot \frac{1}{z^2\beta^2}$$

$$\cdot \rho \frac{N_A}{A} \cdot 2\pi b$$

$b \simeq r$

$$= \overbrace{C}^{\text{TERMINI}} \cdot \int \frac{1}{b} db = C \cdot \ln\left(\frac{b_{MAX}}{b_{MIN}}\right)$$

USO LO STESSO RAGIONAMENTO DELLO SCATT. MULTIPLO

$X \equiv \rho \cdot l$

$$\frac{dE}{dx} = \int_0^E \frac{d^2N}{dE_\gamma \cdot dx} E_\gamma \, dE_\gamma$$

ENERGIA PERSA DALLA PARTICELLA = "$N \cdot E_\gamma$"

se fsup che E e b siano INDIPENDENTI

$$= \frac{z^4}{\beta^2} \cdot \left(\frac{m_e}{M}\right)^2 \cdot 4 r_e^2 \cdot \alpha \cdot \frac{N_A}{A} \cdot Z^2 \cdot \log(183 Z^{-1/3})$$

$$\times E \quad \int_0^E dE_\gamma$$

tenendo conto dello schermo

$$\frac{dE}{dx} = 4 r_e^2 \alpha \cdot \frac{N_A}{A} \cdot Z^2 \left[\log(183 Z^{-1/3}) + \frac{1}{18}\right] E$$

$$= \frac{E}{X_0}$$

ENERGIA DOPO UN TRATTO X ↓ ENERGIA INIZIALE ↓

$$E = e^{-X/X_0} E_0$$

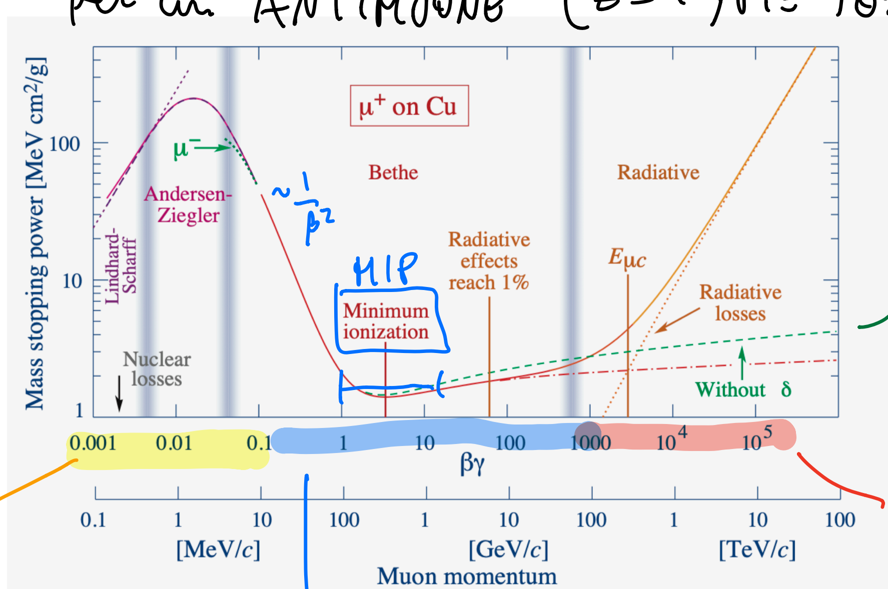
$$W' \propto a'^2 \propto \frac{1}{M^2}$$ → (QUASI) solo gli elettroni fanno bremsstrahlung

$$m_e = 511 \text{ keV}/c^2$$

$$m_\mu = 105.6 \text{ MeV}/c^2$$

EFFETTO DOMINANTE per e^-

per un ANTIMUONE ($z=1$, $M = 105.6 \text{ MeV}/c^2$)



$\frac{1}{\rho}\frac{dE}{dx}$ ↑

$\sim \frac{1}{\beta^2}$

MIP

BETHE-BLOCH SE NON CI FOSSE EFF. DENSITÀ

→ $\beta\gamma$

Calcoli complicati

BETHE-BLOCH

BREM

→ p_μ

ENERGIA CRITICA $E_c \simeq \dfrac{800\,MeV}{Z+1.2}$

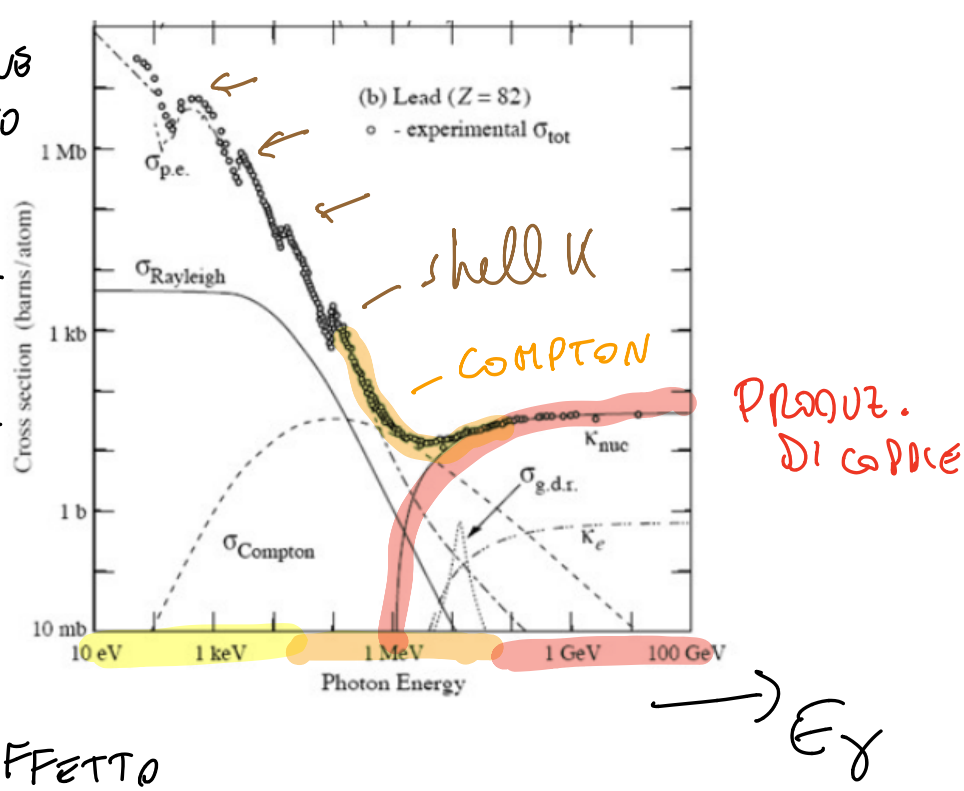
PER ELETTRONI

DOMINA BETHE-BLOCH | DOMINA BREM

E_c → E

FOTONI: come perdono energia?

SEZIONE D'URTO DI INTERAZ. FOTONE-MATERIA



- EFFETTO FOTOELETTRICO ($E_\gamma \in [I, 100\,keV]$) — I del mezzo
- COMPTON ($E \in [100\,keV, 2m_e \sim 1\,MeV/c^2]$)

- PRODUZIONE DI COPPIE "$\gamma \to e^+ e^-$"

- FOTOELETTRICO: $\sigma \propto Z^5 \cdot \frac{1}{E_\gamma^3}$

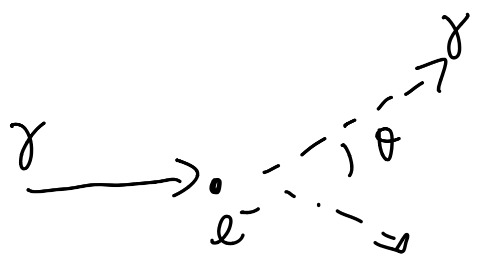
$$\gamma + A \to A^+ + e^-$$

- COMPTON

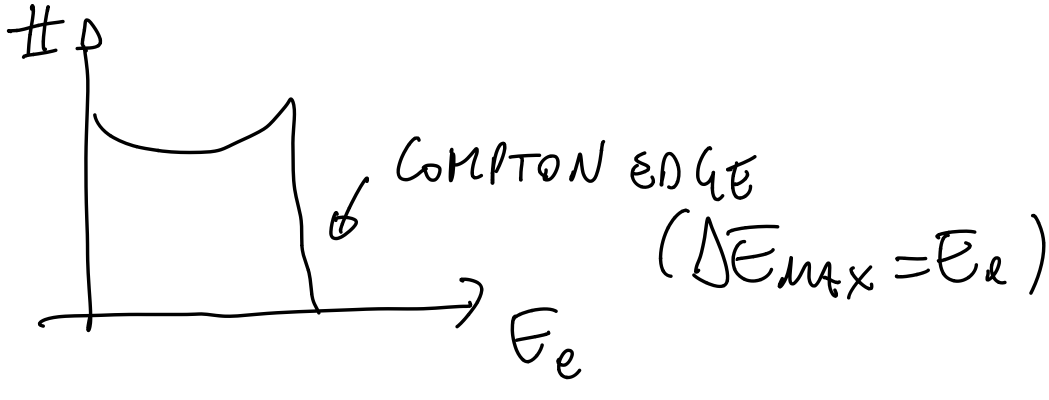
$$\gamma + e^- \to \gamma + e^-$$

scattering elastico

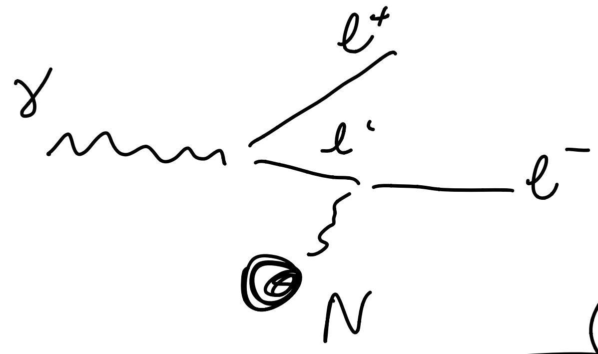
$$\Delta E = E - E' = E - \frac{E}{1 + \frac{E}{m}(1 - \cos\theta)}$$



$$\Delta E_{MAX} = \frac{2E/m}{1 + 2E/m}$$



— PRODUZIONE DI COPPIE $e^- e^+$



$$\boxed{\gamma \to e^- + e^+}$$

NO!

(esercizio: verificare che il processo sopra è impossibile, usando VS)

SI

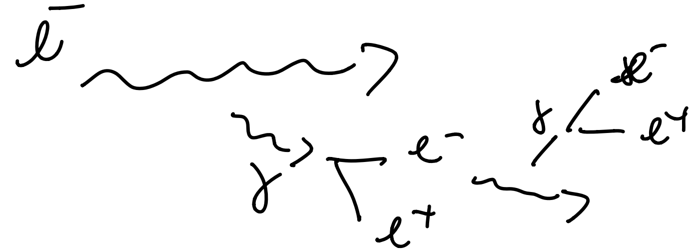
$$\boxed{\gamma + N \to e^- + e^+ + N}$$

$m_{e^+} = m_{e^-} = 511 \ \mathrm{keV}/c^2$

importante quando è cinematicamente permesso, cioè

$$E_\gamma \sim 2m_e$$

POSSO AVERE BREM ⟺ PRODUZ. DI COPPIE



SCIAMI ELETTROMAGNETICI