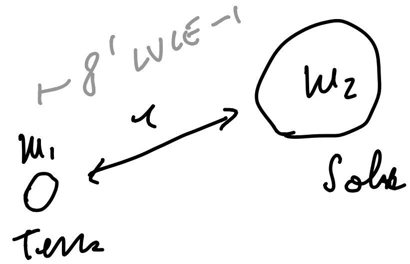
PREVIOUSLY ON FNSN 1

PARTICELLE

INTERAZIONI



$$\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$$

NO INT. A DISTANZA

MQ

MR

TEORIA DEI CAMPI

- scoperta elettrone
- scoperta radioattività

PARTICELLE

- sono localizzate nello spazio

$p, E \ldots \qquad E^2 = p^2 + m^2 \quad (c=1)$

- posso crearle e si annichilano

$$A + B \to C + D + E + F \ldots$$

es $$p + p \to \pi^+ + \pi^0 + \pi^- + \ldots$$

SCATTERING

es $e^+ + e^- \to \mu^+ + \mu^-$

- alcune di esse sono instabili

$$A \to B + C + \dots$$

es $n \to p + e + \bar{\nu}_e$

CONTROESEMPIO: e^- ELEMENTARE

p NON ELEMENTARE

$$p \to n + \text{altro}$$

NON PUÒ ACCADERE

$\sqrt{s} = m_p = m_n + \varepsilon$

MA $m_p < m_n$

- hanno gradi di libertà

  - es: SPIN

- esistono antiparticelle

  è la particella che ha lo stesso massa, stesso spin, ma es carica elettrica opposta (es) e^- $m_{e^+} = m_{e^-}$

e^+ spin sempre $1/2$

(es) γ spin 1, massa 0] È LA SUA STESSA ANTIPARTIC.

TEORIA DELLO SCATTERING DIP. DAL TEMPO

$$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$$

Particelle libere

$$P = -i\hbar\nabla$$

$$H_0 = \frac{P^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2$$

$$\varphi_k = \varphi_k(\vec{x},t) = \underbrace{\varphi_k(\vec{x})}_{\text{SPAZIO}}\, \underbrace{e^{-\frac{i}{\hbar}E_k t}}_{\text{TEMPO}}$$

$$H_0\varphi_k(\vec{x},t) = E_k\,\varphi_k(\vec{x},t)$$

$$\int_V d^3x\, \varphi_k^*(\vec{x},t)\,\varphi_l(\vec{x},t) = \delta_{kl}$$

1 PARTICELLA PER VOLUME $\mathcal{V}$

caso generale

$$H = H_0 + V(\vec{x}, t)$$

$$H\psi = E\psi$$

$$\psi(\vec{x}, t) = \sum_k a_k(t) \overbrace{\varphi_k(\vec{x}, t)}^{\text{SOLUZ. DI } H_0}$$

$\hbar = 1$

$$i \frac{\partial \psi}{\partial t} = i \sum_k \left[\frac{\partial a_k(t)}{\partial t} \varphi_k(\vec{x}, t) + a_k(t) \frac{\partial \varphi_k(\vec{x}, t)}{\partial t} \right]$$

$$= i \sum_k \left[\frac{\partial a_k(t)}{\partial t} \varphi_k(\vec{x}) \cdot e^{-iE_k t} + a_k(t) \cdot \varphi_k(\vec{x}) (-iE_k) e^{-iE_k t} \right]$$

$=$

$$H\psi = (H_0 + V(\vec{x}, t))\psi$$

$$= \sum_k a_k(t) \cdot (E_k) \cdot \varphi_k(\vec{x})\, e^{-iE_k t}$$

$H_0 \varphi = E_k \varphi$

$$+ \sum_k a_k(t) \cdot V(\vec{x}, t) \cdot \varphi_k(x)\, e^{-iE_k t}$$

È SEMPRE $H\psi = i\frac{\partial \psi}{\partial t}$

$$i \sum_k \frac{\partial a_k(t)}{\partial t} \varphi_k(\vec{x})\, e^{-iE_k t} = \sum_k a_k(t) \cdot V(\vec{x}, t)\, \varphi_k(x)\, e^{-iE_k t}$$

tipicamente

$|i\rangle$, $|f\rangle$ LIBERA

$$|f\rangle = \varphi_f(\vec{x})\, e^{-iE_f t}$$

assumo $V(\vec{x}, t) = V(\vec{x})$

proiettiamo su $\langle f|$

per avere $\frac{\partial a_f}{\partial t}$ = bla bla bla

$$i \sum_k \varphi_f^*(\vec{x})\, e^{iE_f t} \frac{\partial a_k(t)}{\partial t}\, \varphi_k(\vec{x})\, e^{-iE_k t}$$

$$= \sum_k \varphi_f^*(\vec{x}) \cdot e^{iE_f t} \cdot a_k(t) \cdot V(\vec{x},t)\, \varphi_k(\vec{x})\, e^{-iE_k t}$$

$$\int_V d^3x\, \varphi_k^* \varphi_\ell = \delta_{k\ell}$$

$\delta_{k\ell}$ impone $E_f = E_i$

$$\int_V d^3x \,\dots = i \cdot \frac{\partial a_f(t)}{\partial t} = \sum_k \int d^3x\, \varphi_f^*(\vec{x}) \cdot V(\vec{x},t)\, \varphi_k(\vec{x}) \times a_k(t) \cdot e^{i(E_f - E_k)t}$$

$$= \sum_k a_k(t) \cdot \underbrace{\langle f | V | k \rangle}_{\text{"}T_{fk}\text{"}} e^{-i(E_k - E_f)t}$$

IPOTESI $\quad |i\rangle \quad |f\rangle \quad$ iniz. - finale

$$a_i(t) \equiv 1 \qquad \forall t$$

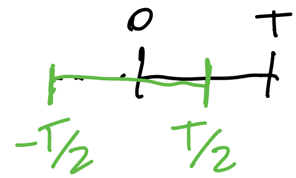
$$a_{k\neq i}(t) \equiv 0$$

$$i \frac{\partial a_f(t)}{\partial t} = \langle f | V | i \rangle\, e^{-i(E_i - E_f)t}$$

$$\frac{\partial a_f(t)}{\partial t} = -i\, T_{fi}\, e^{-i(E_i - E_f)t}$$

$$a_f(t) = -i\, T_{fi} \int dt\; e^{-i(E_i - E_f)t}$$

PROB. DI TRANS.



$$P_{fi} = P_{fi}(T) = |a_f(T)|^2$$

$t \to t - T/2$

$$= |T_{fi}|^2 \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt'\; e^{-i(E_i-E_f)t} \cdot e^{i(E_i-E_f)t'}$$

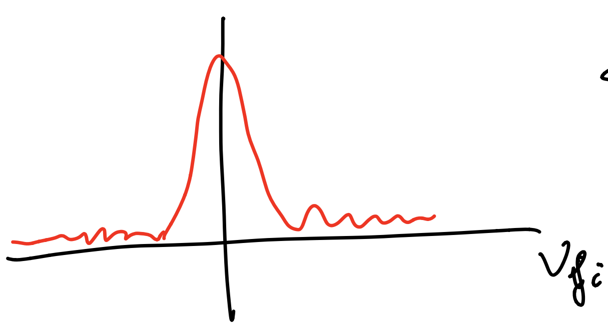
$$\int_{-T/2}^{T/2} dt\, e^{-i(E_f-E_i)t} = \frac{e^{i(E_f-E_i)T/2}}{i(E_f-E_i)} - \frac{e^{-i(E_f-E_i)T/2}}{i(E_f-E_i)} = 2 \cdot \frac{\sin\left((E_f-E_i)\frac{T}{2}\right)}{E_f-E_i} = \frac{2\sin(\nu_{fi}\cdot T/2)}{\nu_{fi}}$$

VOGLIO IL RATE DI TRANSIZIONE

$$\nu_{fi} = \frac{E_f - E_i}{\hbar}$$

$$d\Gamma_{fi} = \frac{P_{fi}}{T} = \frac{1}{T}|T_{fi}|^2 \int_{-T/2}^{T/2} \frac{2\sin(\nu_{fi} T/2)}{\nu_{fi}} \left(\text{prendo } \lim_{T\to\infty}\right) \times e^{i(E_i-E_f)t'}\, dt'$$

$$= \frac{1}{T}|T_{fi}|^2 \cdot \frac{4\sin^2(\nu_{fi}\cdot T/2)}{\nu_{fi}^2}$$



se c'è un continuo di stati con energie $[E_f, E_f + dE_f]$

$$\lim_{T\to\infty} \frac{1}{T} \cdot |T_{fi}|^2 \cdot \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt' \, e^{i(E_f-E_i)t} \times e^{-i(E_f-E_i)t'}$$

$$\times \, dn$$

QUANTI STATI ACCESSIBILI CI SONO FRA E_f ed $E_f + dE_f$

APPLICO

$$\lim_{T\to\infty} \int_{-T/2}^{T/2} e^{ik(x-x_0)} dk = 2\pi \, \delta(x-x_0)$$

E OTTENGO

$$= d\Gamma_i = |T_{fi}|^2 \cdot \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} dt' \frac{dn}{dE_f} \cdot dE_f \cdot 2\pi$$

$$= |T_{fi}|^2 \cdot 2\pi \cdot \left.\frac{dn}{dE_f}\right|_{E_f=E_i} dE_f$$

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \cdot \left.\frac{dn}{dE_f}\right|_{E_f=E_i}$$

$$= \frac{2\pi}{\hbar} \cdot |T_{fi}|^2 \rho(E_i)$$

REGOLA D'ORO DI FERMI

ORDINE SUCCESSIVO

assumo $\quad a_i(t) \cong 1$

$a_{k\neq i}(t) \simeq 0 \quad$ (prima era $\equiv 0$)

$$i\frac{\partial a_f(t)}{\partial t} = \sum_k a_k(t) \langle f|V|k\rangle e^{-i(E_k-E_f)t}$$

$$= \langle f|V|i\rangle e^{-i(E_i-E_f)t} + \sum_{k\neq i} a_k(t) \langle f|V|k\rangle e^{-i(E_k-E_f)t}$$

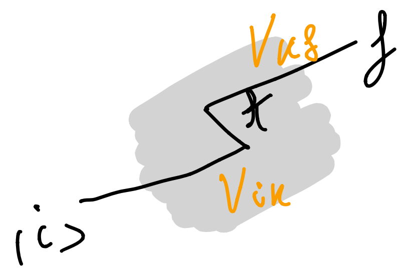
IPOTIZZO

$$a_k(T) \equiv -i\,T_{ki}\int_0^T e^{i(E_k-E_i)t}\,dt$$

$$= -i\langle k|V|i\rangle \frac{e^{i(E_k-E_i)T}}{i(E_k-E_i)}$$

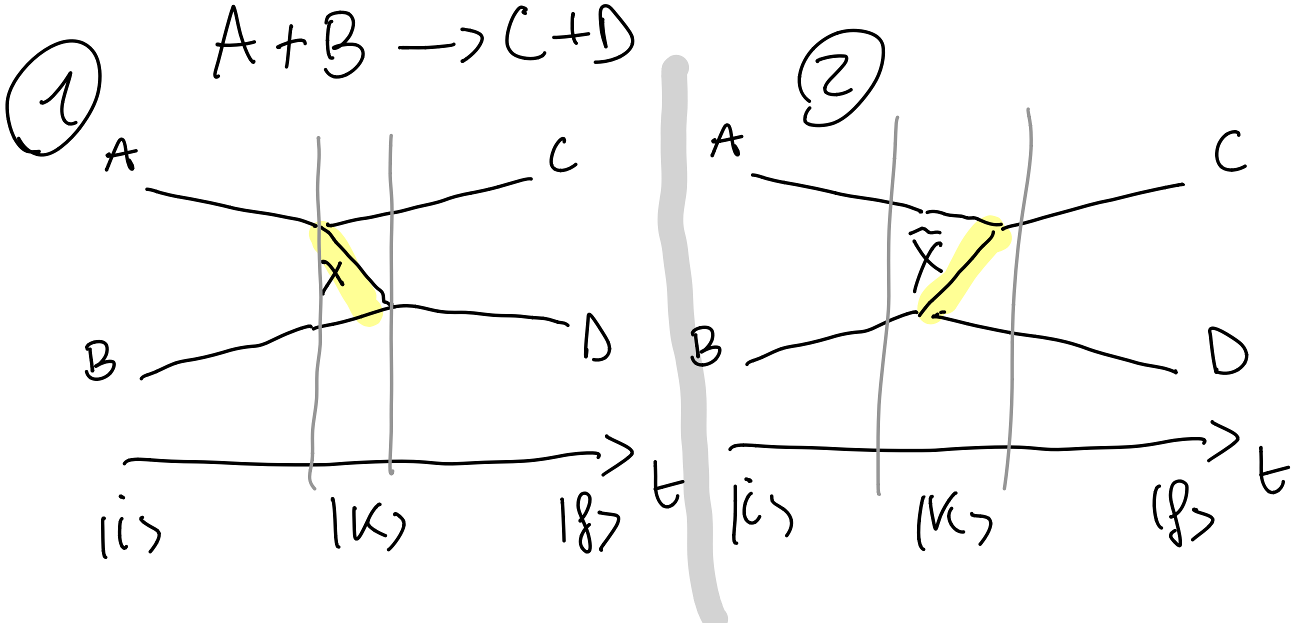
$$\frac{\partial a_f(t)}{\partial t} = -i\left[\langle f|V|i\rangle + \sum_{k\neq i}\frac{\langle f|V|k\rangle\langle k|V|i\rangle}{E_k-E_i}\right] \times e^{-i(E_i-E_f)t}$$

$$= -i \, T_{fi}^{(\text{al 2° ordine})} \cdot e^{-i(E_i - E_f)t}$$



CAPIAMO!

$A + B \longrightarrow C + D$



$|i\rangle$: $A + B$

$|k\rangle$: $(X + C) + B$ $\qquad$ $|k\rangle = (X + D) + A$

$|f\rangle$: $C + D$

X e $\tilde{X}$ sono le proprie antip.

così $m_X = m_{\tilde{X}}$

altra assunzione: scambio di impulso

(1) $\vec{P}_A = \vec{P}_X + \vec{P}_C$