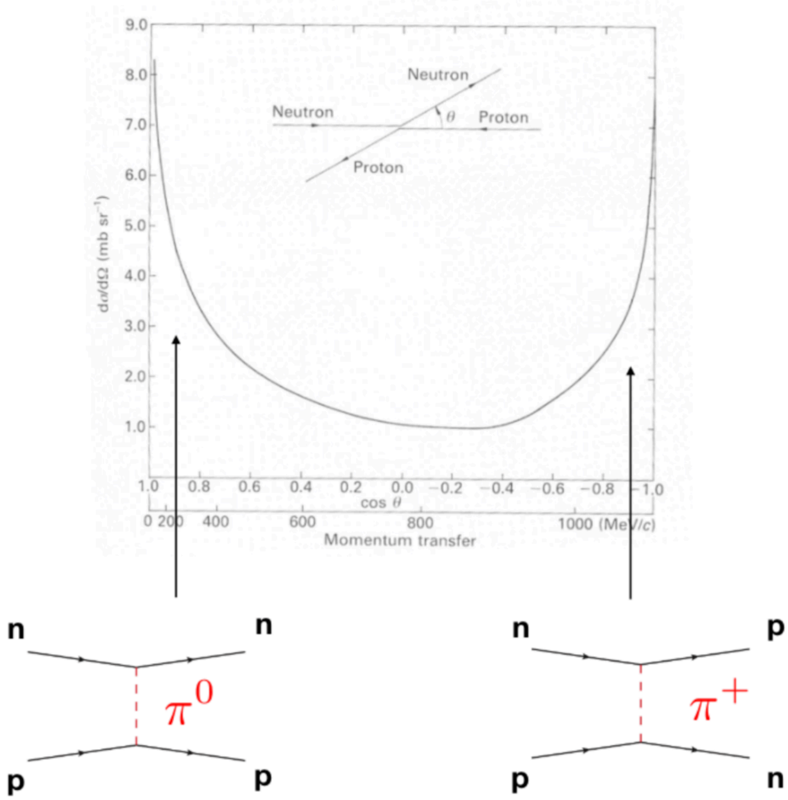


DECADIMENTO

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$|i\rangle \rightarrow |f\rangle \neq |i\rangle$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$H \rightarrow \gamma + \gamma$$

$$H \rightarrow Z\,Z$$

$$\hookrightarrow e^+ + e^-$$

$$\hookrightarrow \mu^+ + \mu^-$$

es $$A \rightarrow B + C$$

$$|i\rangle = |a\rangle$$

$$|f\rangle = a(t)|a\rangle + c(t)|b+c\rangle$$

$$i\hbar \frac{\partial}{\partial t} |a\rangle = H_{eff} |a\rangle$$

H_{eff} non può essere hermitiana

$$|a,t\rangle = a(t)\,|a\rangle$$
$$= e^{-i\frac{E_i t}{\hbar}}\,|a\rangle$$

$$P(a,t) = \langle a,t|a,t\rangle = \langle a,0|a,0\rangle$$

$$H_{eff} = H - i\,\frac{\Gamma}{2}$$

CONVENZIONE

H_{eff} NON è hermitiana

Γ HERMITIANO

autovalori di H_{eff}:

$$E = E_0 - i\,\Gamma/2$$

(x è 3D)

$$\psi(x,t) = \psi(x) \times e^{-\frac{i}{\hbar}E_0 t} \, e^{-\Gamma/2\hbar \, t}$$

E_0 energia dello stato iniziale

se è 1 particella $E_0 = mc^2$

nel suo sistema di riferimento

$$|\psi(x,t)|^2 = e^{-\Gamma/\hbar \, t}$$

$$\Gamma = \frac{\text{Probabilità}}{\text{tempo}} = \frac{2\pi}{t} |T_{fi}|^2 \rho(E_i)$$

Consideriamo N particelle

- probabilità decadimento di 1 particella non dipende dalle N−1
- prob. di decadimento è proprietà intrinseca della particella
- P/T non dipende da T

$$dP = \lambda\, dt$$

λ è un <u>rate</u>

$(\lambda = 1/\tau)$

Con N particelle

$$-dN = dP\, N(t)$$
$$= \lambda\, dt\, N(t)$$

$$\ln (N)_{t_0}^{t} = -\lambda (t - t_0)$$

$$N(t) = N(0)\, e^{-t\lambda}$$

$t_0 \equiv 0$

$N(0) = N_0$

$$\tau \equiv \langle t \rangle = \frac{\int_0^\infty t\, P(t)\, dt}{\int_0^\infty P(t)\, dt}$$

$$= \frac{\int_0^\infty t\, e^{-\lambda t}\, dt}{\int_0^\infty e^{-\lambda t}\, dt} = \frac{\int_0^\infty t\, e^{-\lambda t}\, dt}{-\frac{1}{\lambda}\left[e^{-\lambda t}\right]_0^\infty}$$

$$= \lambda \int_0^\infty t\, e^{-\lambda t}\, dt = \frac{1}{\lambda}$$

ATTIVITÀ : $A(t) \equiv N(t)\lambda$

$$= \frac{N(t)}{\tau} = \frac{N(0)\, e^{-t/\tau}}{\tau}$$

$$\text{Becquerel} \equiv \# \frac{\text{disintegrazioni}}{\text{secondo}}$$

TEMPO DI DIMEZZAMENTO

$$N(T_{1/2}) = \frac{N(0)}{2} = e^{-T_{1/2}} N(0)$$

$$T_{1/2} = \ln 2\, \tau$$

$$H \to \gamma + \gamma$$

$$\psi(x) \to \chi(E)\ ?$$

$P(E) = ?$

$$\chi(E) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, e^{+i\frac{E}{\hbar}t}\, \psi(x,t)$$

$$\psi(x, t<0) \equiv 0$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty dt\, e^{\frac{iE}{\hbar}t} \cdot e^{-\frac{i}{\hbar}(E_0 - i\Gamma/2)t} \psi(x)$$

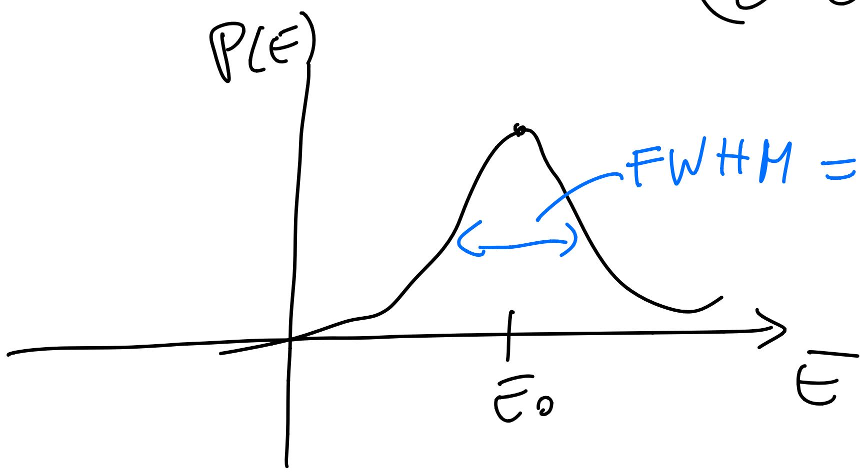
$$= \frac{1}{\sqrt{2\pi}} \int_0^\infty dt\, e^{\frac{i}{\hbar}(E-E_0)t} e^{-\frac{\Gamma}{2\hbar}t} \psi(x)$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{\frac{i}{\hbar}(E-E_0)t - \frac{\Gamma}{2\hbar}t}}{i/\hbar\,(E-E_0) - \Gamma/2\hbar} \right]_0^\infty \psi(x)$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{+i\hbar}{(E-E_0) + i\Gamma/2} \right] \psi(x)$$

$$= \chi(E)$$

$$P(E) = |\chi(E)|^2 \propto \frac{\hbar^2}{(E-E_0)^2 + (\Gamma/2)^2}$$

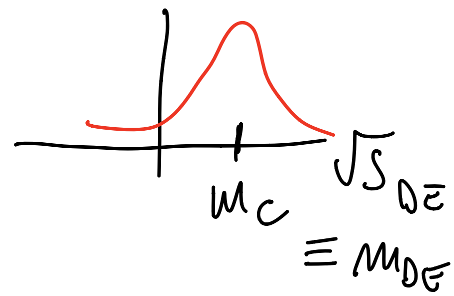


BREIT-
WIGNER

es $A \to B + C$
$\quad\quad\quad\quad\hookrightarrow D + E$

$|i\rangle : |A\rangle$
$|f\rangle : |B + D + E\rangle$

se calcolo $\sqrt{s_{DE}}$

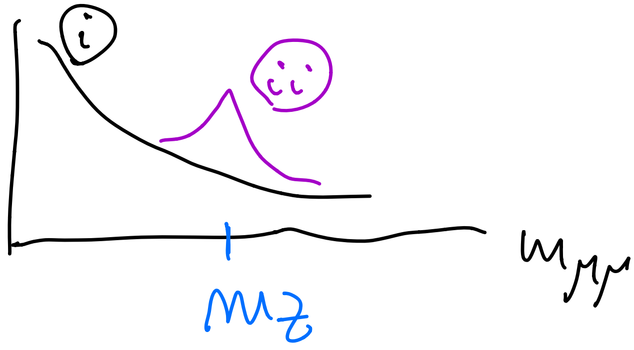


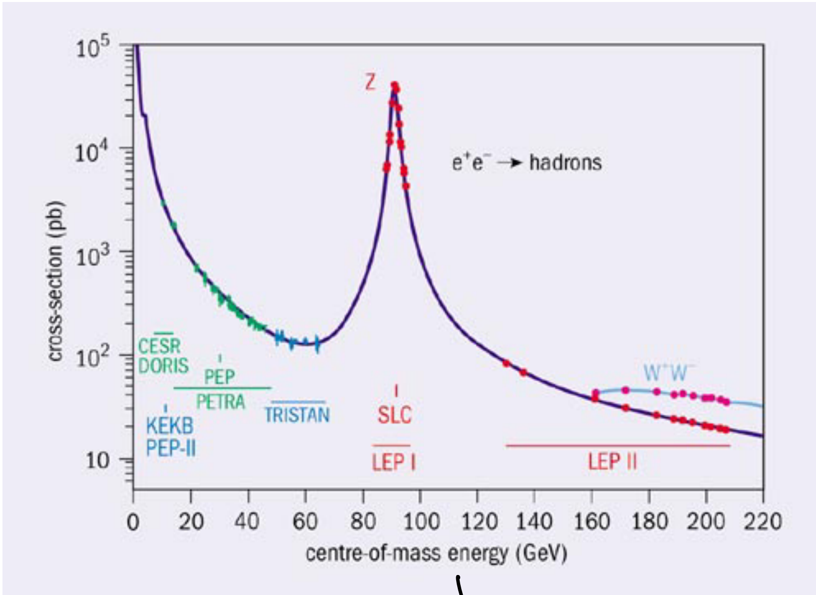
es $e^+ + e^- \to \mu^+ + \mu^-$ (i)

ma anche

$e^+ + e^- \to Z \to \mu^+ + \mu^-$ (ii)

1) MISURO $\underline{p}_{\mu^+}$, $\underline{p}_{\mu^-}$
2) CALCOLO $m_{\mu\mu}$





$e^+ + e^-$

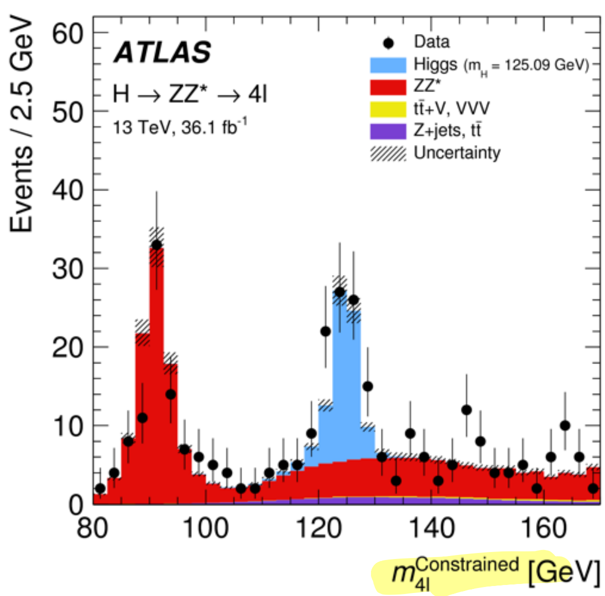
$\rightarrow$ particle - antiparticle

$\sqrt{s} = M_{ee} = M_{ab\,ab}$

$e^+ \longrightarrow \quad \longleftarrow e^-$

$E_{e^-} = E_{e^+} \equiv E$

$\sqrt{s} = 2E$



$p + p \rightarrow H + X$

$\hookrightarrow Z + Z$

$\hookrightarrow \mu\mu$

$\hookrightarrow \mu\mu$

$\rightarrow M_{\mu\mu\mu\mu}$

RADIOATTIVITÀ

α | β | γ

$$N(Z,A) \to N'(Z-2, A-4) + \alpha$$

$$N(Z,A) \to N'(Z+1, A) + e^- + \bar{\nu}_e$$

$$n \to p + \bar{\nu}_e + e^-$$

"β^-"

$$N^*(Z,A) \to N(Z,A) + \gamma$$

"β^+" $N(Z,A) \to N'(Z-1, A) + e^+ + \nu_e$

$p \to n + e^+ + \nu_e$ (NON PUÒ ACCADERE NEL VUOTO)

$m_\alpha \simeq 938\ \text{MeV} \times 4 \simeq 3.7\ \text{GeV}$

$T \simeq 200\ \text{MeV}$

$m_{e^+} = m_{e^-} = 511\ \text{keV}$

$T \sim 1\ \text{MeV}$

$m_\gamma = 0$

$T \sim 1\ \text{MeV}$

in generale

$$N \longrightarrow D_1 + D_2 \quad (+ \dots)$$

$$E_N^* = m_N c^2 = (m_1 c^2 + T_1) + (m_2 c^2 + T_2) + \dots$$

$Q \equiv$ energia di disintegrazione

$$\equiv (m_N - m_1 - m_2 \cdots) c^2$$

$$= T_1 + T_2 + \ldots$$

se $Q < 0$, il decadimento è proibito cinematicamente

esercizio dimostrare che (2 corpi)

$$T_1 = \frac{2 m_2 + Q}{2(m_1 + m_2 + Q)} \cdot Q$$

se passo da $|i\rangle$ senza particelle radioattive a uno $|f\rangle$ con particelle radioattive

es ATMOSFERA

$$^{14}_{7}N + \overbrace{n}^{\text{RAGGIO COSMICO}} \rightarrow p + {}^{14}_{6}C$$

SPALLAZIONE

(2) REATTORI NUCLEARI

$$\frac{dN}{dt} = -\lambda N + R$$

$(R>0)$
descrive l'arrivo
di particelle
radioattive

$$\frac{dN}{dt} = 0 \rightarrow N = R/\lambda$$

se R = costante

$$\frac{dN}{dt} = -\lambda \left(N - R/\lambda \right)$$

$$\frac{dN}{N - R/\lambda} = -\lambda dt$$

$$\log\left(\frac{N(t) - R/\lambda}{N(0) - R/\lambda}\right) = -\lambda(t-t_0)$$

$$N(t) = R/\lambda + e^{-\lambda(t-t_0)}\left(N(0) - R/\lambda\right)$$

$$S_1 \rightarrow S_2 \rightarrow S_3$$

$$\begin{cases} \dfrac{dN_1}{dt} = -\lambda_1 N_1 \\ \dfrac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1 \end{cases}$$

$$\frac{dN_3}{dt} = +\lambda_2 N_2$$

$$N_1(t) = N_1(0)\, e^{-\lambda_1 t}$$

$$\frac{dN_2(t)}{dt} = -\lambda_2 N_2 + \lambda_1 N_1(0)\, e^{-\lambda_1 t}$$

$$f'(t) = -f(t) \cdot a(t) + g(t)$$

$$f'(t) + f(t)\,a(t) = g(t)$$

risolvo

$$f'(t) + f(t)\,a(t) = 0$$

$$f(t) = K\, e^{-\int a(t)\,dt}$$

$$= K\, e^{-\lambda_2 t}$$

e quindi cerco soluzioni

$$f(t) = K(t)\, e^{-\lambda_2 t}$$

$$K'(t)\, e^{-\lambda_2 t} + (-\lambda_2)\, e^{-\lambda_2 t} K(t) + K(t)\, e^{-\lambda_2 t} (\lambda_2) = \lambda_1 N_1(0)\, e^{-\lambda_1 t}$$

$$K'(t)\, e^{-\lambda_2 t} = \lambda_1 N_1(0)\, e^{-\lambda_1 t}$$

$$K'(t) = \lambda_1 N_1(0)\, e^{-(\lambda_1 - \lambda_2)t}$$

$$K(t) = \lambda_1 N_1(0) \frac{1}{\lambda_2 - \lambda_1} e^{-(\lambda_1 - \lambda_2)t} + C$$

$$N_2(t) = k(t)\, e^{-\lambda_2 t}$$

$$= \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)\, e^{-\lambda_1 t} + C\, e^{-\lambda_2 t}$$

ottengo C da $N_2(0)$

$$\left.\begin{matrix} N_2(0) \\ N_1(0) \end{matrix}\right\} \text{le \underline{conosco}}$$

però $C = N_2(0) - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0) \left[e^{-\lambda_1 t} - e^{-\lambda_2 t} \right]$$

$$+ N_2(0)\, e^{-\lambda_2 t}$$

$$N_3(t) = \lambda_2 \int_0^t N_2(t')\, dt'$$

Se $\lambda_2 \gg \lambda_1$

(S_1 decade lento
S_2 decade "subito")

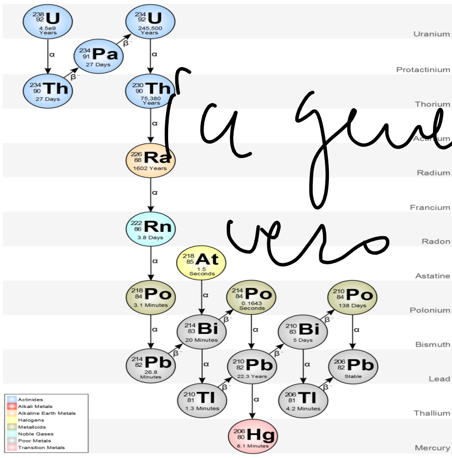
$$N_2(t) \simeq \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(0)\, e^{-\lambda_1 t}$$

$$\simeq \frac{\lambda_1}{\lambda_2} N_1(t)$$

EQUILIBRIO SECOLARE

$$\lambda_2 N_2(t) = \lambda_1 N_1(t)$$

attività uguali



La generale è ancora vero