Prodigit: 427620

ES $\quad {}^{238}U \qquad \tau \sim 10^9$ anni

5 mg $^{238}UO_2$

contiene 4.41 mg di ^{238}U

$$^{238}U \to \alpha + X \quad \text{al } 100\%$$

$$BR(^{238}U \to \alpha + X) = 100\%$$

Misuro l'attività ($Bq \equiv$ disint. al secondo)

16.9 cps

efficienza di conteggio 0.315

$$A = ? = \frac{16.9}{0.315}\ Bq$$

$$A(t) = \lambda \cdot N(t) = \frac{1}{\tau} N(t)$$

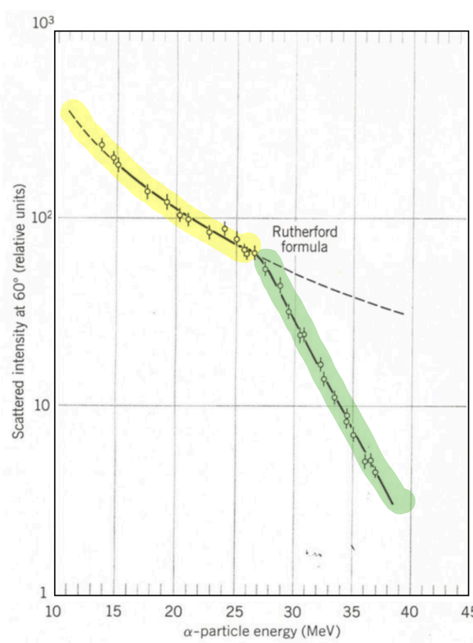
misura $N(t)$

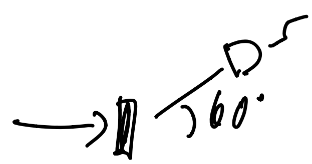
$$N(t) = \frac{4.41 \text{ mg}}{238 \text{ g/mol}} \cdot 6 \cdot 10^{23} \frac{1}{\text{mol}}$$

$$\tau = 4.5 \cdot 10^{9} \text{ anni}$$

$$\lambda N = \lambda' N' = \ldots$$

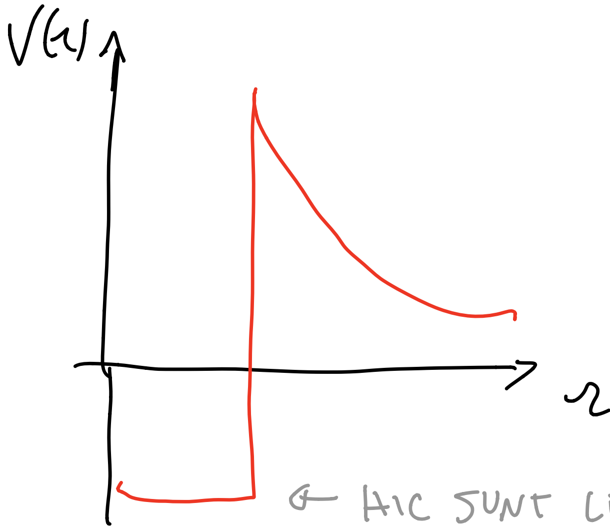
$\alpha + X \to \alpha + X$





ES $^{238}U \rightarrow {}^{234}Th + \alpha$

MODELLO DI GAMOW



← HIC SUNT LEONES

$Q \sim 5\,MeV$

$$T_\alpha = Q\,\frac{2M_{Th} + Q}{2(M_{Th} + Q + m_\alpha)}$$

$$\simeq 98\%\; Q \;\longrightarrow\; \text{REGIME NON RELATIVISTICO}$$

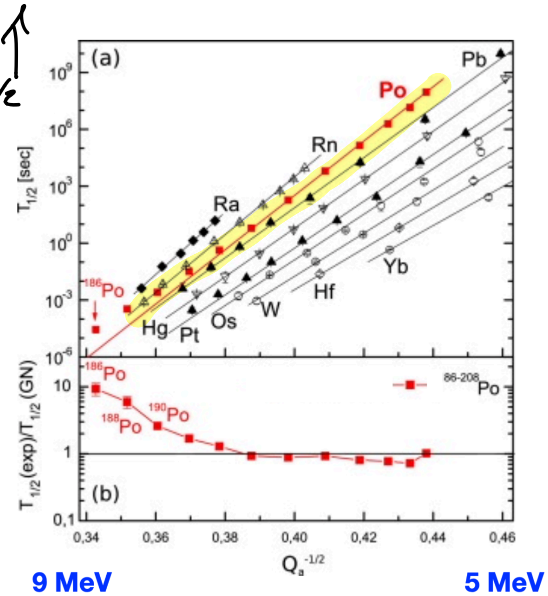
decadimento $\ln T_{1/2} \uparrow$

τ, Q

GEIGER-NUTTAL

legge empirica

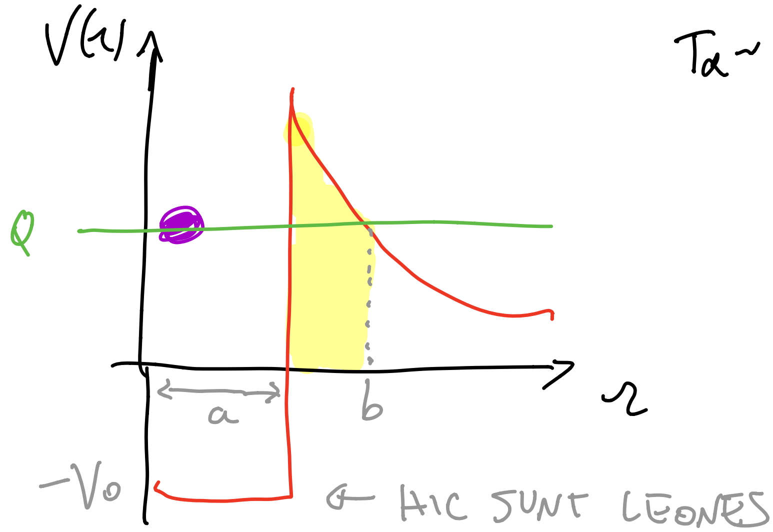
$$\ln \tau = A + \frac{B}{\sqrt{Q}}$$



9 MeV **5 MeV**

$[Q] = [MeV]^{-\frac{1}{2}}$

$\overrightarrow{\sqrt{Q}}$



$$T_\alpha \sim Q = \frac{1}{2}mv^2 + V$$

$$= \frac{1}{2}mv^2 - V_0$$

$$v = \sqrt{\frac{2(Q+V_0)}{m}}$$

$$f = \frac{v}{2a}$$

$V_0 \sim 30$ MeV

$Q \sim 5$ MeV

$A \sim 200$

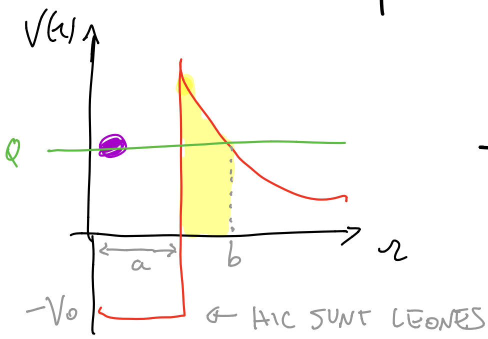
$v \sim 0.1\,c$

$f \sim 3 \cdot 10^{21}$ Hz

^{238}U: $\tau \sim 10^9$ anni

$\rightsquigarrow 10^{36}$ tentativi

$$\lambda = \frac{1}{\tau} = f \cdot P_{TUNNELING}$$

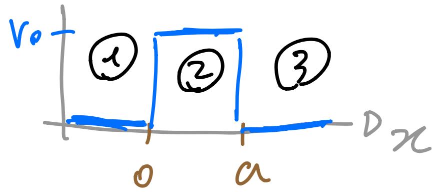


$\rightarrow$

APPROSSIMO V(r) tra a e b come tanti gradini

BARRIERA RETTANGOLARE

a, V_0 sono $\neq$ da a, V_0 del caso completo



$$i\hbar \frac{\partial \psi}{\partial t} = H\psi = (H_0 + V)\psi$$

$$\psi(x,t) = e^{-i\frac{E}{\hbar}t}\,\psi_E(x)$$

$$[H_0 + V(x)]\,\psi_E(x) = E\,\psi_E(x)$$

$$\tilde{H}_0 = \frac{P^2}{2m} = -\frac{\hbar^2}{2m}\nabla^2 = -\frac{\hbar^2}{2m}\frac{\partial}{\partial x^2}$$

1, 3 $\qquad \frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\,\psi_E = 0 \qquad K^2 = \frac{2mE}{\hbar^2}$

2 $\qquad \frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_0)\,\psi_E = 0 \qquad -K_0^2 = \frac{2m}{\hbar^2}(E - V_0)$

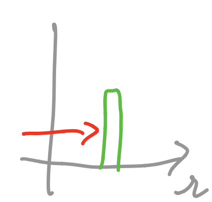
$$K_0 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$$

CI INTERESSA IL CASO $E < V_0$

1 $\quad \psi_E(x) = A \cdot e^{iKx} + B \cdot e^{-iKx}$

2 $\quad \psi_E(x) = C \cdot e^{K_0 x} + D\, e^{-K_0 x}$

3 $\quad \psi_E(x) = F\, e^{iKx}$



FATTORE DI TRASMISSIONE $\quad T \equiv \left|\frac{F}{A}\right|^2$

$\underline{x=0}$ continuità di ψ

$$A + B = C + D$$

continuità di ψ'

$$A\, iK + B(-iK) = C K_0 - K_0 D$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + K_0/iK & 1 - \frac{K_0}{iK} \\ 1 - \frac{K_0}{iK} & 1 + \frac{K_0}{iK} \end{pmatrix} \begin{pmatrix} C \\ D \end{pmatrix}$$

$\underline{x=a}$ $$C e^{K_0 a} + D e^{-K_0 a} = F e^{iKa}$$

$$K_0 C e^{K_0 a} - K_0 D e^{-K_0 a} = iK F e^{iKa}$$

risolvo $$\begin{pmatrix} C \\ D \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{-K_0 a}(1 + i K/K_0) \\ e^{K_0 a}(1 - iK/K_0) \end{pmatrix} F e^{iKa}$$

$$T = \left|\frac{F}{A}\right|^2 = \frac{1}{1 + \left(\frac{(K^2 - K_0^2)^2}{4K^2 K_0^2} + 1\right) \sinh^2(K_0 a)}$$

$K = \sqrt{\frac{2mE}{\hbar^2}}$

$K_0 = \sqrt{\frac{2(V_0 - E)m}{\hbar^2}}$

$$= \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(K_0 a)}$$

$$\sinh^2(K_0 a) = \left(\frac{e^{K_0 a} - e^{-K_0 a}}{2}\right)^2$$

$$= \frac{e^{2K_0 a} + e^{-2K_0 a} - 2}{4}$$

$$\simeq \frac{1}{4} e^{2K_0 a}$$ ipotizzo $K_0 \gg a$

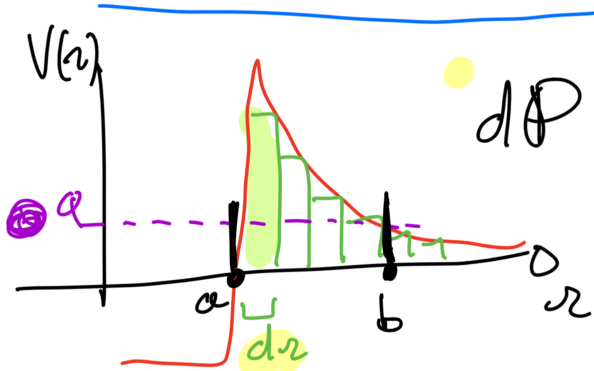
$$T \simeq \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \cdot \frac{1}{4} e^{2K_0 a}}$$

$$= \frac{16\, E(V_0 - E)}{V_0^2} e^{-2K_0 a}$$

ipotizzo $K_0 \sim K$

$$T \sim 4 e^{-2K_0 a}$$

$$= 4 e^{-2a \cdot \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}}$$



$$dP = 4 e^{-2\, dr \sqrt{\frac{2m(V(r) - q)}{\hbar^2}}}$$

$$P = \prod dP = 4 e^{-2\int_a^b dr \sqrt{\frac{2m(V(r)-Q)}{\hbar^2}}}$$

$$= 4 e^{-2G}$$

↳ FATTORE DI GAMOW

$$V(\vec{r}) = V(r) = \frac{2(Z-2)e^2}{4\pi\varepsilon_0 r} = \frac{2\alpha\hbar c(Z-2)}{r}$$

$$Q = \frac{2\alpha\hbar c(Z-2)}{b}$$

$$b = \frac{2\alpha\hbar c(Z-2)}{Q}$$

$$G = \int_a^b dr \sqrt{\frac{2m}{\hbar^2}(V(r)-Q)}$$

$$= \sqrt{\frac{2m}{\hbar^2}} \int_a^b \sqrt{2\alpha\hbar c(Z-2)\left(\frac{1}{r}\right) - Q}\, dr$$

$$= \sqrt{\frac{2m}{\hbar^2}} \cdot \sqrt{Q} \int_a^b \sqrt{2\alpha\hbar c(Z-2)\cdot\frac{1}{r}\cdot\frac{1}{Q} - 1}\, dr$$

$$= \sqrt{\frac{2mQ}{\hbar^2}} \int_a^b \sqrt{\frac{b}{r} - 1}\, dr$$

$$x = r/b \qquad \sqrt{\frac{1}{x} - 1} = \sqrt{\frac{1-x}{x}}$$

$$dx = \frac{1}{b}\, dr \qquad \int \sqrt{\frac{1-x}{x}}\, dx = \sqrt{x - x^2} + \operatorname{asin}\sqrt{x}$$

$$= \sqrt{\frac{2mQ}{\hbar^2}} \cdot b \left[\sqrt{x - x^2} + \operatorname{asin}\sqrt{x}\right]_{a/b}^{1}$$

$$= \sqrt{\frac{2mQ}{\hbar^2}}\, b \left[\left(0 + \pi/2\right) - \left(\sqrt{\frac{a}{b} - \left(\frac{a}{b}\right)^2} + \operatorname{asin}\sqrt{\frac{a}{b}}\right)\right]$$

$$\frac{\pi}{2} - \operatorname{asin} y = \operatorname{acos} y$$

$$= \sqrt{\frac{2mQ}{\hbar}}\, b \left[\operatorname{acos}\sqrt{\frac{a}{b}} - \sqrt{\frac{a}{b} - \left(\frac{a}{b}\right)^2}\right]$$

$$= \sqrt{\frac{2mQ}{\hbar^2}} \cdot \frac{2\alpha \hbar c (Z-2)}{Q} \left[f\left(\frac{a}{b}\right)\right]$$

$$= G$$

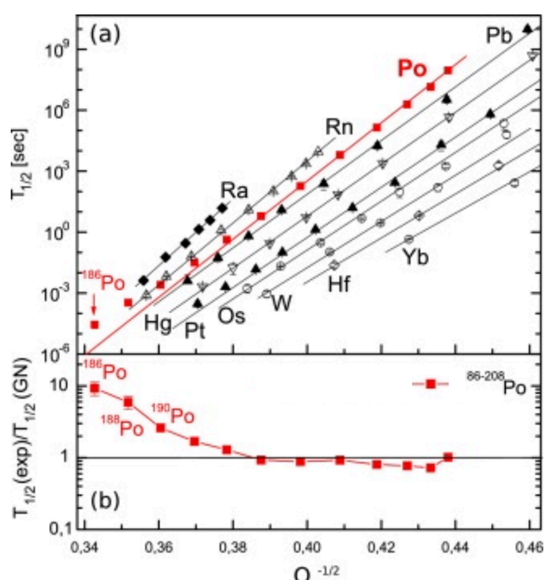
$$\lambda = \frac{1}{\tau} = f \cdot P = f \cdot 4 \cdot e^{-2G}$$

$$= \frac{4}{a} \sqrt{\frac{(Q + V_0)}{2m}}\, e^{-4\alpha \frac{\hbar c (Z-2)}{Q} \sqrt{\frac{2mQ}{\hbar^2}} f\left(\frac{a}{b}\right)}$$

$$\ln(\tau) = \ln\left[\left(\frac{4}{a}\sqrt{\frac{(Q+V_0)}{2m}}\right)^{-1}\right] + 4\alpha\,\frac{\hbar c\,(Z-2)}{\sqrt{Q}}\sqrt{\frac{2m}{\hbar^2}}\,f\!\left(\frac{a}{b}\right)$$

$$\ln(\tau) = A + \frac{1}{\sqrt{Q}}\cdot B$$

cioè la stessa forma della
legge di Geiger-Nuttall



9 MeV **5 MeV**

$$[Q] = [MeV]^{-\frac{1}{2}}$$