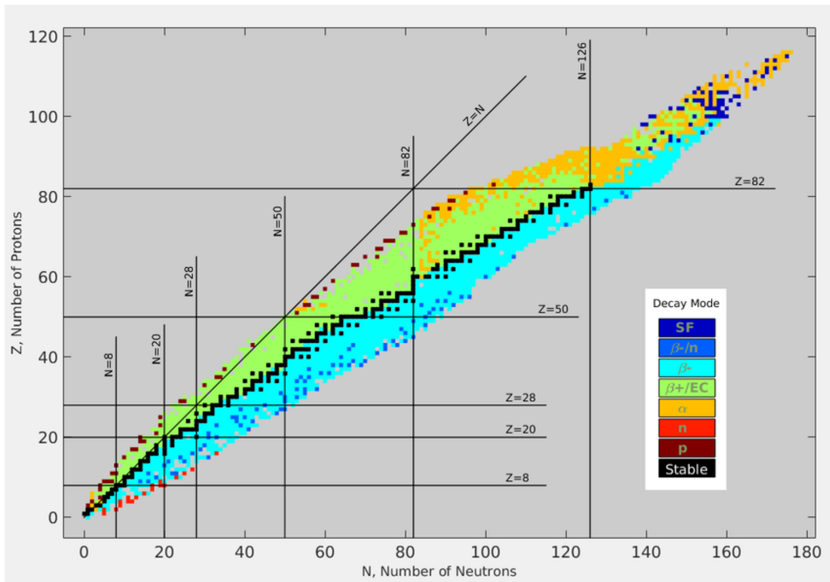
$e^{iKx} \to$ è indip. del tempo



DECADIMENTO β

A costante $\qquad n \Leftrightarrow p$

$$n \to p + e^-$$

Conserva carica?

$$0 \to +1 + (-1)$$

Conserva $\underline{P_\mu}$?

$$E_i = m_n c^2 = E_f \geq m_p c^2 + m_e c^2$$

Controesempio: $p \to n + e^+$

conserva $L+S$?

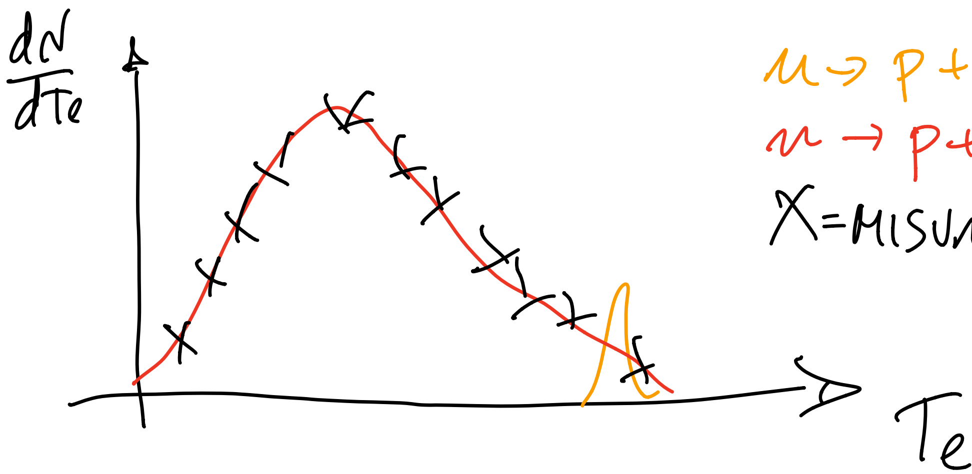
$\frac{1}{2} \to \underbrace{\frac{1}{2} + \frac{1}{2}}$

$J_{TOT} = \frac{1}{2} - \frac{1}{2} = 0$

$J_{TOT} = \frac{1}{2} + \frac{1}{2} = 1$

serve

$$n \to p + e^- + X$$



$n \to p + e^-$

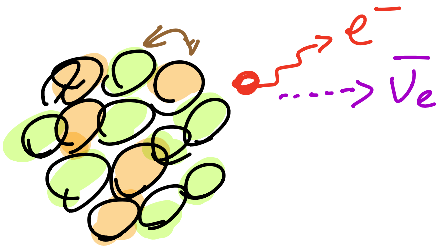
$n \to p + e^- + \bar{\nu}_e$

X = MISURA

introduco l'ANTINEUTRINO ELETTRONICO

$\bar{\nu}_e$

- carica 0
- assumo (per il momento) $m_\nu = 0$

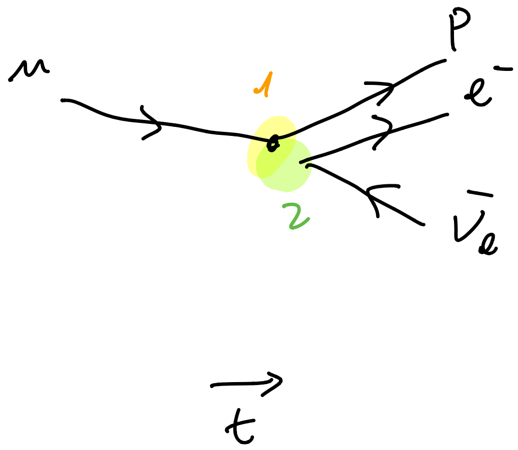


- β^+ $(A,Z) \to e^+ + \nu_e + (A,Z-1)$
- β^- $(A,Z) \to e^- + \bar{\nu}_e + (A,Z+1)$
- EC $(A,Z) + e^- \to (A,Z-1) + \nu_e$

- interazione a corto raggio
- $Q \simeq 1$ MeV
- ci dev'essere, nel calcolo di τ, una differenza fra e^- e e^+
- e^-, $\bar{\nu}_e$ non interagiscono FORTE col nucleo
- assumo che $m_\nu \simeq 0$

FERMI: interazione di contatto

prendiamo $n \to p + e^- + \bar{\nu}_e$ (con n fermo)



$$H = H_0 + H_I$$

$$H_I \equiv H(\vec{r}, \vec{r}_1, \vec{r}_2) \equiv g \cdot \delta^{(3)}(\vec{r} - \vec{r}_1) \times \delta^{(3)}(\vec{r} - \vec{r}_2)$$

Voglio

$$\tau = \frac{1}{\lambda}$$

/ RATE DI DECADIMENTO

$$\lambda = \frac{2\pi}{\hbar} \left| \langle f | H_I | i \rangle \right|^2 \cdot \rho(\underbrace{E_i}_{=m_n})$$

$$|i\rangle = |n\rangle$$
$$|f\rangle = |p\, e^- \bar{\nu}_e\rangle$$

Scelgo

- β^- : $n \to p\, e^- \bar{\nu}_e$
- V volume del nucleo

$$\langle f|H_I|i\rangle = \iiint d\vec{r}\, d\vec{r}_1\, d\vec{r}_2\, \psi_p^*(\vec{r}_1)\,\psi_e^*(\vec{r}_2)$$
$$\times\, \psi_{\bar{\nu}}^*(\vec{r}_2)\, H_I \cdot \psi_n(\vec{r}_1)$$

- $n \to p + e^- + \bar{\nu}_e$

$$H_I = g \cdot \delta^{(3)}(\vec{r}-\vec{r}_1)\,\delta^{(3)}(\vec{r}-\vec{r}_2)$$

- $(A,Z) \to (A,Z+1) + e^- + \bar{\nu}_e$

$$H_I = g \cdot O_x\, \delta^{(3)}(\vec{r}-\vec{r}_1)\,\delta^{(3)}(\vec{r}-\vec{r}_2)$$

$$= g\int d\vec{r}\, \psi_p^*(\vec{r})\,\psi_e^*(\vec{r})\,\psi_{\bar{\nu}}^*(\vec{r}) \cdot \psi_n(\vec{r})$$

$$\psi_e(\vec{r}) = \frac{1}{\sqrt{V}}\, e^{-\frac{i}{\hbar}\vec{p}_e\cdot\vec{r}}$$

$$\psi_{\bar{\nu}}(\vec{r}) = \frac{1}{\sqrt{V}}\, e^{-\frac{i}{\hbar}\vec{p}_\nu\cdot\vec{r}}$$

$$\psi_e\psi_{\bar{\nu}} = \frac{1}{V}\cdot e^{-\frac{i}{\hbar}(\vec{p}_e+\vec{p}_\nu)\cdot\vec{r}} = \frac{1}{V}\left(1 + \left(-\frac{i}{\hbar}(\vec{p}_e+\vec{p}_\nu)\cdot\vec{r}\right) + \dots\right)$$

abbiamo $Q \sim 1\ \mathrm{MeV}$

perciò $p_e \sim 1\,\mathrm{MeV}/c$

$r \sim R \sim 1\,\mathrm{fm}$

$$\left|\frac{\vec{p}_e}{\hbar}\cdot\vec{r}\right| \sim \frac{1\,\mathrm{MeV}/c \; 1\,\mathrm{fm}}{197\,\mathrm{MeV}\,\mathrm{fm}\,c}$$

$$\sim 10^{-2}$$

(vogliamo $\lambda \propto |\psi_e \psi_\nu|^2 \to$ sono termini $O(10^{-4})$)

sto ignorando $\frac{(\vec{p}_e + \vec{p}_\nu)}{\hbar}\cdot\vec{r} = \frac{\vec{L}}{\hbar}$

$$\langle f|H_I|i\rangle = \iiint d\vec{r}\, d\vec{r}_1\, d\vec{r}_2\; \psi_p^*(\vec{r}_1)\,\psi_e^*(\vec{r}_2) \times \psi_\nu^*(\vec{r}_2)\, H_I \cdot \psi_n(\vec{r}_1)$$

$$= \frac{1}{V}\, g \int d\vec{r}\, \psi_p^*(\vec{r})\, \psi_n(\vec{r})$$

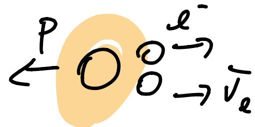
$$= \frac{1}{V}\, g\, M_{fi}$$ $(\vec{L} \equiv \vec{0})$

ELEMENTO DI MATRICE NUCLEARE

$$\lambda = \frac{2\pi}{\hbar}\, |\langle f|H_i|i\rangle|^2 \cdot \rho(E_i)$$

NON DIPENDE da $\vec{p}_\nu, \vec{p}_e$

posso scegliere qualunque $Q = T_e + T_\nu$



$$E = \frac{p^2}{2m_{nucleo}} = \frac{(\vec{p}_e + \vec{p}_\nu)^2}{2m_{nucleo}} \qquad p_e + p_\nu \sim Q$$

$$\vec{P}_{A,Z+1} + \vec{p}_e + \vec{p}_\nu = \vec{0} \qquad \leq \frac{1\,\text{MeV} \times Q}{2 \cdot 938\,\text{MeV}} = 10^{-4} \cdot Q$$

$\rho(E_i) = ?$ la voglio in funzione di T_e

$$dN_e = \frac{d^3p_e}{\underbrace{(2\pi\hbar)^3}_{V}} = V\,\frac{4\pi p_e^2\, dp_e}{(2\pi\hbar)^3}$$

$$dN_\nu = V \cdot \frac{4\pi\, p_\nu^2\, dp_\nu}{(2\pi\hbar)^3}$$

$$dN = dN_e \times dN_\nu$$

caratterizzare la cinematica di $X \to Y + e^- + \bar{\nu}_e$

$$Q = M_x - M_y - M_e - M_\nu$$

$$E_i = E_f$$

$$E_i = M_X = (T_Y + M_Y) + E_e + E_\nu$$

definisco

$$W \equiv M_X - M_Y = T_Y + E_e + E_\nu$$

$$\simeq E_e + E_\nu$$

$$\simeq \sqrt{p_e^2 c^2 + (m_e c^2)^2} + \sqrt{p_\nu^2 c^2 + (m_\nu c^2)^2}$$

$$p_\nu^2 c^2 = E_\nu^2 - (m_\nu c^2)^2$$

$$= (W - E_e)^2 - (m_\nu c^2)^2$$

faccio il differenziale assumendo E_e fissata

$$2 p_\nu \, dp_\nu c^2 = 2 (W - E_e) \, dW$$

$p_\nu c$ x sopra:

$$p_\nu^2 \, dp_\nu \, c^3 = (W - E_e) \, dW \, p_\nu c$$

$$= (W - E_e) \sqrt{(\underbrace{W - E_e}_{E_\nu})^2 - (m_\nu c^2)^2}$$

$$\times dW$$

$$dN = dN_e \cdot dN_\nu = \frac{(4\pi)^2 \cdot V^2}{(2\pi\hbar)^6} \cdot P_e^2 dP_e \cdot P_\nu^2 dP_\nu$$

$$= \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^3} \cdot P_e^2 dP_e \cdot (W - E_e)\sqrt{(W-E_e)^2 - (m_\nu c^2)^2}$$

$$\times dW$$

$W = E_e + E_\nu \simeq E_f$

$$\rho(E_i) = \left.\frac{dN}{dE}\right|_{E=E_f} = \frac{dN}{dW}$$

$$= \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^3} \cdot P_e^2 dP_e (W - E_e)\sqrt{(W-E_e)^2 - (m_\nu c^2)^2}$$

$$P_e^2 c^2 + m_e^2 c^4 = E_e^2$$

$$2 P_e c^2 dP_e = 2 E_e dE_e$$

$E_e = m_e + T_e$

$dE_e = dT_e$

$$\rho(E_i) = \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^5} \cdot P_e \cdot (m_e + T_e) dE_e (W - E_e)$$

$$\times \sqrt{(W - E_e)^2 - (m_\nu c^2)^2}$$

$$= \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^5} \cdot P_e (m_e c^2 + T_e)\, dE_e \cdot E_\nu P_\nu c$$

$$= \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^5} \cdot p_e (m_e c^2 + T_e)\, dT_e\, E_\nu p_\nu c$$

è la densità degli stati con energia cinetica dell'elettrone fra T_e e $T_e + dT_e$

$$d\vec{\lambda} = \frac{2\pi}{\hbar} |\langle f | H_I | i \rangle|^2 \rho(E_i)$$

$$= \frac{2\pi}{\hbar} \cdot \frac{g^2}{V^2} |M_{fi}|^2 \frac{(4\pi)^2 V^2}{(2\pi\hbar)^6 c^4} \cdot p_e (m_e c^2 + T_e) E_\nu p_\nu dT_e$$

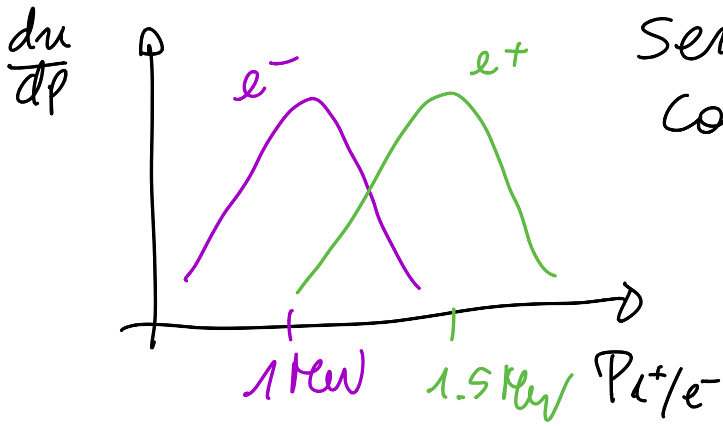
$$G_F \equiv \frac{g}{(\hbar c)^3}$$ ha le dimensioni di $[E]^{-2}$

$$= \frac{2\pi}{\hbar} G_F^2 (\hbar c)^6 \cdot |M_{fi}|^2 \cdot \frac{(4\pi)^2}{(2\pi\hbar)^6 c^4} \cdot$$

$$\times\, p_e (m_e c^2 + T_e) E_\nu p_\nu dT_e \times F(\pm Z, T_e)$$

$$n \rightarrow p + e^- + \bar{\nu}_e$$

$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$



Serve una correzione coulombiana (che tiene conto di e^- vs e^+)

correzione

$$F = F(\pm Z, T_e)$$

$$d\lambda = \frac{G_F^2 c^2}{2\pi^3 \hbar} |M_{fi}|^2 F(\pm Z, T_e) \cdot$$

$$\times E_\nu p_\nu p_e (m_e c^2 + T_e) dT_e$$

posso
- calcolare lo spettro di T_e

$$\frac{d\lambda}{dT_e} \propto E_\nu p_\nu p_e (m_e c^2 + T_e) \cdot F(\pm Z, T_e)$$

- calcolare il rate $\quad$ T_e va da 0 a Q

$$\frac{1}{\tau} = \lambda = \int d\lambda = \frac{G_F^2 c^2}{2\pi^3 \hbar} |M_{fi}|^2 \int_0^Q dT_e \, F(\pm Z, T_e)$$

$$\times\, E_\nu\, P_\nu\, P_e\, (m_e c^2 + T_e)$$

$$= \frac{G_F^2}{2\pi^3 \hbar} |M_{fi}|^2 f(\pm Z, Q)(m_e c^2)^5$$

con $f(\pm Z, Q) = \frac{1}{(m_e c^2)^5} \int_0^Q E_\nu P_\nu c P_e c (m_e c^2 + T) \times F(\pm Z, T_e)\, dT_e$

voglio verificare $m_\nu = 0$ sia vero

$$(A, Z) \rightarrow (A, Z+1) + e^- + \bar{\nu}_e$$

$$m_{ex}(T_e) = M(Z, A) - M(Z+1, A) - m_\nu = Q - m_\nu$$

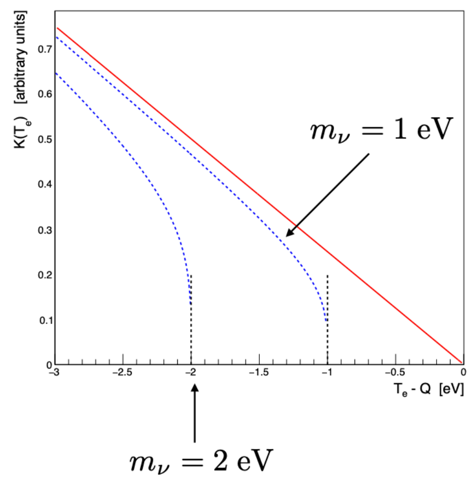
lo spettro

$$\frac{d\lambda}{dT_e} = \frac{G_F^2 (m_e c^2)^5}{2\pi^3 \hbar} |M_{fi}|^2 F(\pm Z, T_e) \times P_e (T_e + mc^2) \cdot \underbrace{(Q - T_e)}_{E_\nu} \underbrace{\sqrt{(Q-T_e)^2 - (m_\nu c^2)^2}}_{P_\nu}$$

se genero il plot di KURIE

$$K(T_e) = \sqrt{\frac{d\lambda/dT_e}{F \times p_e (T_e + mc^2)}} \propto \sqrt{}$$

$K(T_e)$



$T_e - Q$

potete dire

$$m_\nu < 2\,eV$$