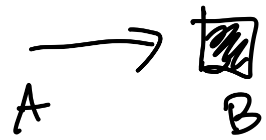
ENERGIA DI SOGLIA

(A)



$$A + B \rightarrow C_1 + C_2 \dots C_N$$

$$E_A = E_{THR} \Rightarrow (*) : C_i \text{ FERME}$$

$$(LAB) : C_i$$

(B) $A \rightarrow B + C + D$ non c'è E_{THR}

$$p \rightarrow n + e^- + \bar{\nu}$$

CAMBIO DI RIFERIMENTO

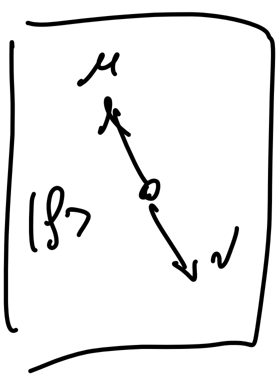
Esercizio 9 **Come cambia l'energia tra centro di massa e laboratorio**

Consideriamo il decadimento del pione in un muone e un antineutrino muonico,

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu,$$

dove $m_\pi = 140\,\mathrm{MeV/c^2}$, $m_\mu = 105.6\,\mathrm{MeV/c^2}$ e $m_\nu = 0$. Il pione si muove con una velocità $v = 0.27c$. Quanto vale l'energia del muone nei vari sistemi di riferimento?

(*) $$E^*_\pi = m_\pi = E^{*\,\text{fin}}_{TOT} = E^*_\mu + E^*_\nu$$



$$\sqrt{s} = E^*_\mu + E^*_\nu$$

$$|\vec{p}^{\,*}_\nu| = |\vec{p}^{\,*}_\mu| \equiv p^*$$

$$\sqrt{s} = \sqrt{m_\mu^2 + p^{*2}} + \sqrt{0^2 + p^{*2}}$$

$$\sqrt{s} - p^* = \sqrt{m_\mu^2 + p^{*2}}$$

$$s + p^{*2} - 2\sqrt{s}\,p^* = m_\mu^2 + p^{*2}$$

$A + B \to \mu + \nu$
se conosco $\sqrt{s}$

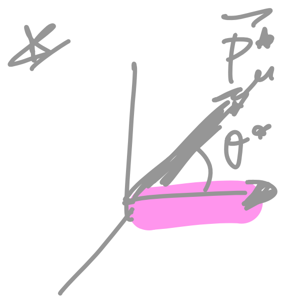
$$p^* = \frac{s - m_\mu^2}{2\sqrt{s}} = \frac{m_\pi^2 - m_\mu^2}{2 m_\pi}$$

$$\simeq 30 \text{ MeV}$$

$$E_\mu^* = \sqrt{p^{*2} + m_\mu^2} \simeq 110 \text{ MeV}$$

$\boxed{LAB}$

$$E_\mu = \gamma \left(E_\mu^* + \beta \, p_{\parallel}^* \right)$$



$$\left\{ ct = \gamma (ct') + \beta\gamma (x) \right.$$

$$= \gamma \left(E_\mu^* + \beta \, p^* \cos\theta^* \right)$$

$$\in \left[\gamma (E_\mu^* - \beta p^*) , \gamma (E_\mu^* + \beta p^*) \right]$$

$\pi \to \mu \nu$

$$\gamma \equiv \gamma_\pi = \frac{E_\pi}{m_\pi} = \frac{1}{\sqrt{1-\beta^2}} \simeq 1.04$$

$$\beta = \frac{p_\pi}{E_\pi} \simeq 0.27 c$$

γ_{CM}, β_{CM} in generale

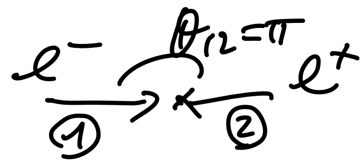
boost lungo x

$$\underline{P_{TOT}^*} = \begin{pmatrix} \sqrt{s} \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_{TOT} \\ P_x^{TOT} \\ P_y^{TOT} \\ P_z^{TOT} \end{pmatrix}$$

$$\sqrt{s} = \gamma E_{TOT} - \beta\gamma p_x^{TOT} \qquad (p_x \equiv p_{\parallel})$$
$$0 = -\beta\gamma E_{TOT} + \gamma p_x^{TOT} \implies \beta_{CM} = \frac{p_{TOT}^{\parallel}}{E_{TOT}}$$
$$\gamma_{CM} = \frac{E_{TOT}}{\sqrt{s}}$$

Esercizio 10 Collider asimmetrico

1. SuperKEKB[13] è un acceleratore di particelle che si trova in Giappone, e fa scontrare fasci di elettroni di energia $E_1 = 7\,\mathrm{GeV}$ con fasci di positroni (particelle della stessa massa degli elettroni, $m_e = 511\,\mathrm{keV/c^2}$, e carica opposta) di energia $E_2 = 4\,\mathrm{GeV}$.
   a) Si calcoli l'energia del centro di massa della collisione.



(1) $E_1 = 7\,\mathrm{GeV}$

$E_2 = 4\,\mathrm{GeV}$

$|\vec{p}_1| \simeq E_1$

$|\vec{p}_2| \simeq E_2$

$m_e \simeq 511\,\mathrm{keV} \ll \mathrm{GeV}$

$$\sqrt{s} = \sqrt{(E_1+E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2}$$
$$= \sqrt{E_1^2 + E_2^2 + 2E_1E_2 - p_1^2 - p_2^2 - 2\vec{p}_1\vec{p}_2}$$
$$= \sqrt{m_e^2 + m_e^2 + 2E_1E_2(1-\cos\theta_{12})}$$
$$= 2\sqrt{E_1E_2} = 2\sqrt{28} \simeq 10.58\,\mathrm{GeV}$$

$$\beta_{CM} = \frac{p_{TOT}^{\parallel}}{E_{TOT}} = \frac{7\,\mathrm{GeV} - 4\,\mathrm{GeV}}{7\,\mathrm{GeV} + 4\,\mathrm{GeV}} \simeq 0.27$$

$$\gamma_{CM} = \frac{E_{TOT}}{\sqrt{s}} = \frac{11\,\mathrm{GeV}}{10.58\,\mathrm{GeV}} \simeq 1.04$$

$$\beta\gamma \simeq 0.28$$

② $e^- + e^+ \to \Upsilon(4S)$

la Υ ha $c\tau = 9.6\ fm = 9.6\cdot 10^{-15}\ m$

$L_{LAB} = \beta_{CM}\gamma_{CM}\, c\tau_{\Upsilon} = 0.28 \times 9.6\cdot 10^{-15}\ m = 2.7\ fm$

2. La $\Upsilon(4S)$ decade in coppie di particelle $B^{\pm}$, dette *mesoni* B, di massa $m_B = 5.279\,\mathrm{GeV/c^2}$, per cui si ha nel complesso

$$e^+ + e^- \to \Upsilon(4S) \to B^+ + B^-.$$

a) Calcolare, nel riferimento del centro di massa, energia e impulso dei mesoni B.
b) I mesoni B sono a loro volta particelle instabili, e hanno una vita media di 1.638 ps. Quanto spazio percorreranno nel laboratorio lungo l'asse di collisione dei fasci?

ⓐ $\sqrt{s} = 10.58\ \mathrm{GeV} \equiv m_{\Upsilon}$

$m_B \equiv m_{B^+} = m_{B^-} = 5.3\ \mathrm{GeV}$

Conosco $\sqrt{s} \to E^*_{B^+}$? $E^*_{B^-} = ?$

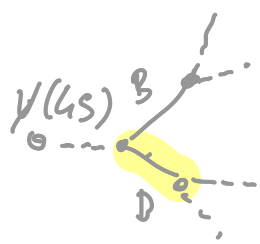
$$\sqrt{s} = E^*_{B^+} + E^*_{B^-} = \sqrt{m_B^2 + p^{*2}_{B^+}} + \sqrt{m_B^2 + p^{*2}_{B^-}}$$

$$= 2E^*_{B^+} \to E^*_{B^+} = E^*_{B^-} = \frac{\sqrt{s}}{2} = \frac{10.58}{2}\ \mathrm{GeV}$$

$$p^*_B = \sqrt{E^{*2}_B - m_B^2} \simeq 360\ \mathrm{MeV}$$

ⓑ $\tau_B = 1.638\ \mathrm{ps} = 1.638\cdot 10^{-12}\ s$

$$L_{//} = (\beta\gamma)^B_{//}\, c\tau_B$$

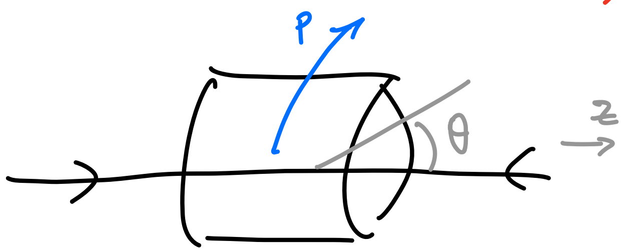


$$= \left(\frac{P}{E}\,\frac{E}{m}\right)^{B}_{\parallel} c\tau_B = \frac{P_\parallel^B}{m_B}\, c\tau_B$$

per cosa: calcolare $P_\parallel^B$ (che sarà fra $[P^B_{\parallel\,MIN}, P^B_{\parallel\,MAX}]$

$\simeq [0.10\ mm, 0,17\ mm]$

TRASFORMAZIONE DEGLI ANGOLI

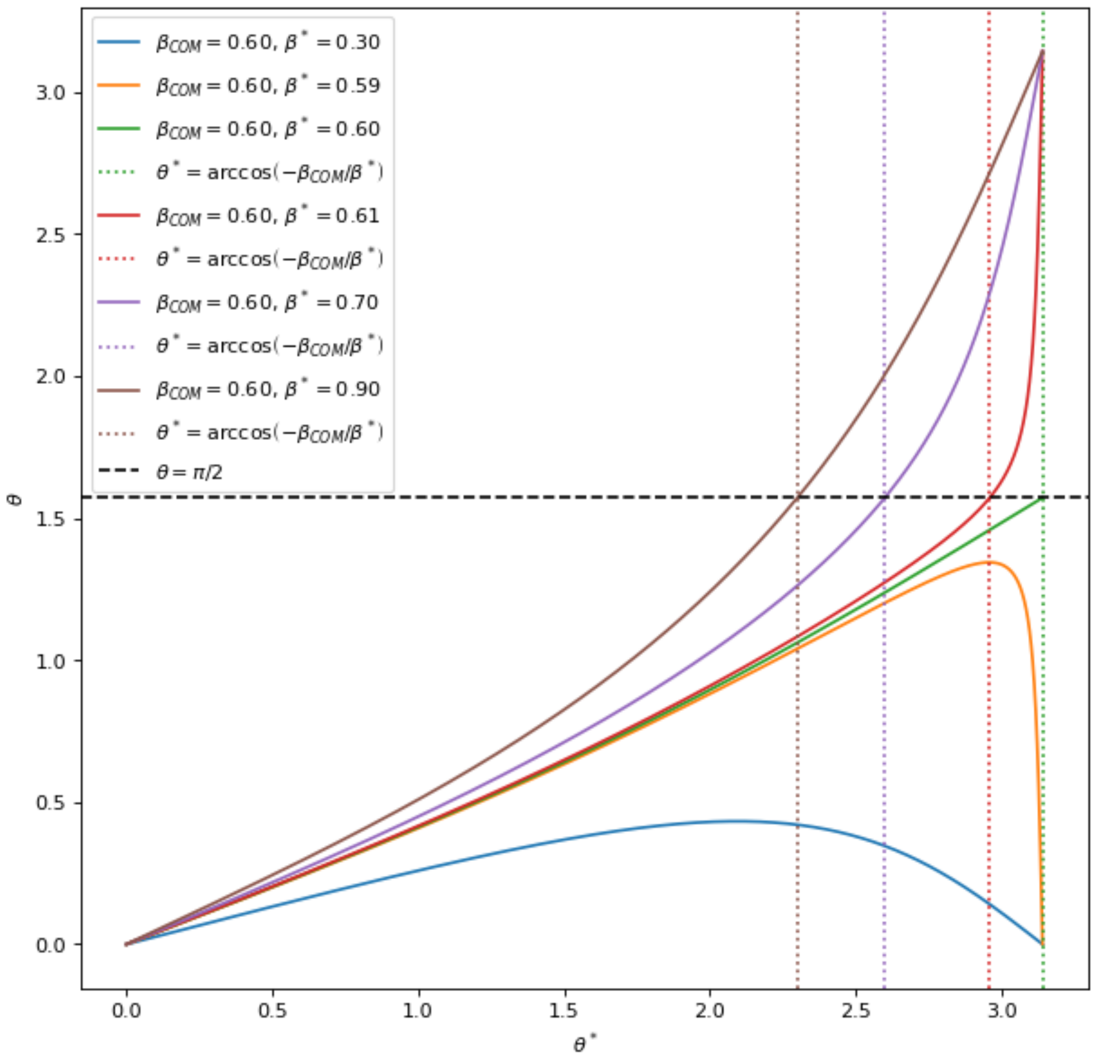


$$\underset{(LAB)}{P_p} = \begin{pmatrix} E \\ P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} E \\ p\sin\theta\cos\varphi \\ p\sin\theta\sin\varphi \\ p\cos\theta \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E^* \\ p^*\sin\theta^*\cos\varphi \\ p^*\sin\theta^*\sin\varphi \\ p^*\cos\theta^* \end{pmatrix}$$

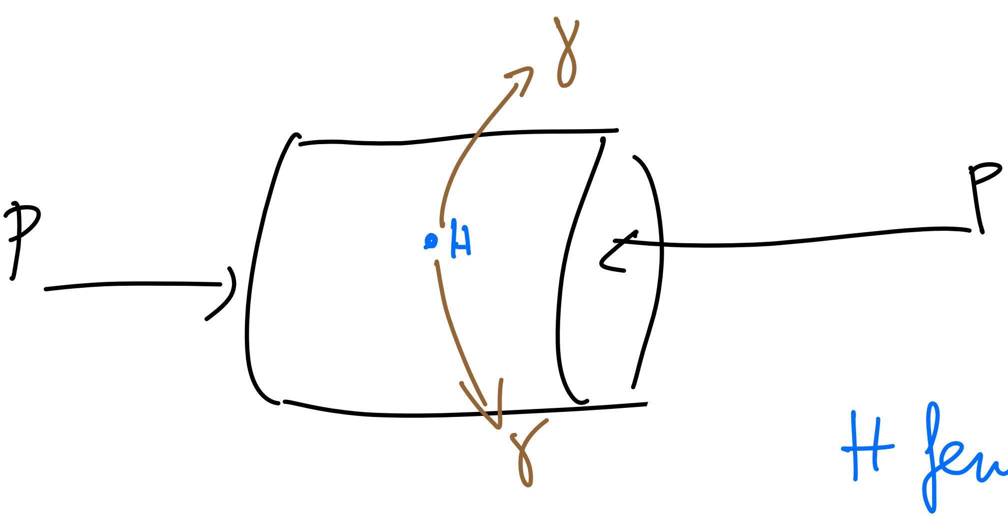
$$\tan\theta = \frac{\sin\theta^*}{\gamma\left(\frac{\beta}{\beta^*} + \cos\theta^*\right)}$$

PARTICELLA

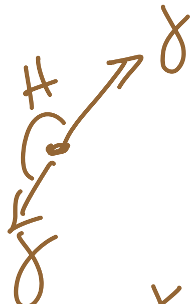
centro di massa



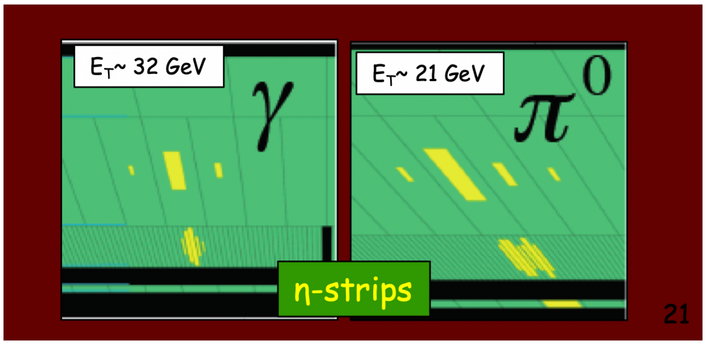
DECADIMENTO $H \to \gamma\gamma$



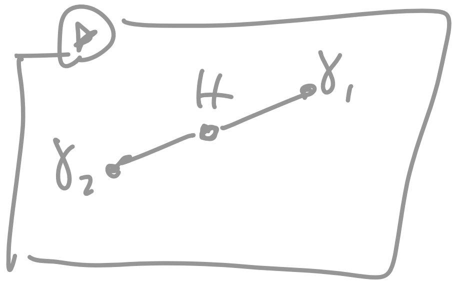
H fermo



H veloce



H velocissima



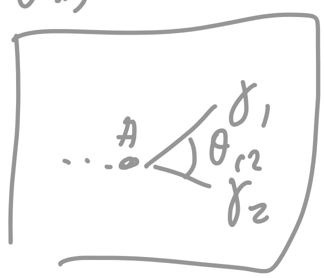
$$E^*_{\gamma_1} = E^*_{\gamma_2} = \frac{m_H}{2}$$

$$m_{\gamma_1} = m_{\gamma_2} = 0$$

$\boxed{LAB}$ MASSA INVARIANTE DI $\gamma\gamma$?

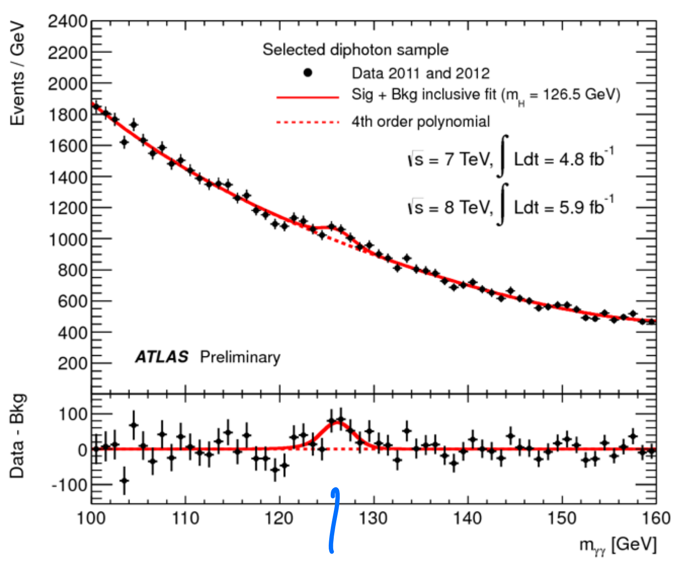
$$\sqrt{s} = \sqrt{2E_1E_2(1-\cos\theta_{12})} \overset{?}{=} m_H$$

LAB



A ☹ $p+p \to \gamma+\gamma$

B ☺ $p+p \to H \to \gamma+\gamma$ (+ altri)



m_H

↳ $\sqrt{s}$ dei 2 fotoni

$$m_H = \sqrt{2E_1E_2\left(1-\cos\theta_{12}\right)} = \sqrt{s}$$

$()^2$

$$s = 2E_1E_2\left(1-\cos\theta_{12}\right)$$

$$= 4E_1E_2\left(\sin^2\frac{\theta_{12}}{2}\right)$$

$$\sin^2\frac{\theta_{12}}{2} = \frac{s}{4E_1E_2} = \frac{m_H^2}{4E_1(E_H-E_1)}$$

Cons. energ.
$E_H = E_1 + E_2$

$$\sin\frac{\theta_{12}}{2} = \frac{m_H}{2\sqrt{E_1(E_H-E_1)}}$$

θ_{12} MIN $\longleftrightarrow$ denominatore MAX

$$\frac{d\left(E_1(E_H-E_1)\right)}{dE_1} = 0 = E_H - E_1 - E_1$$

UNICA INCOGNITA

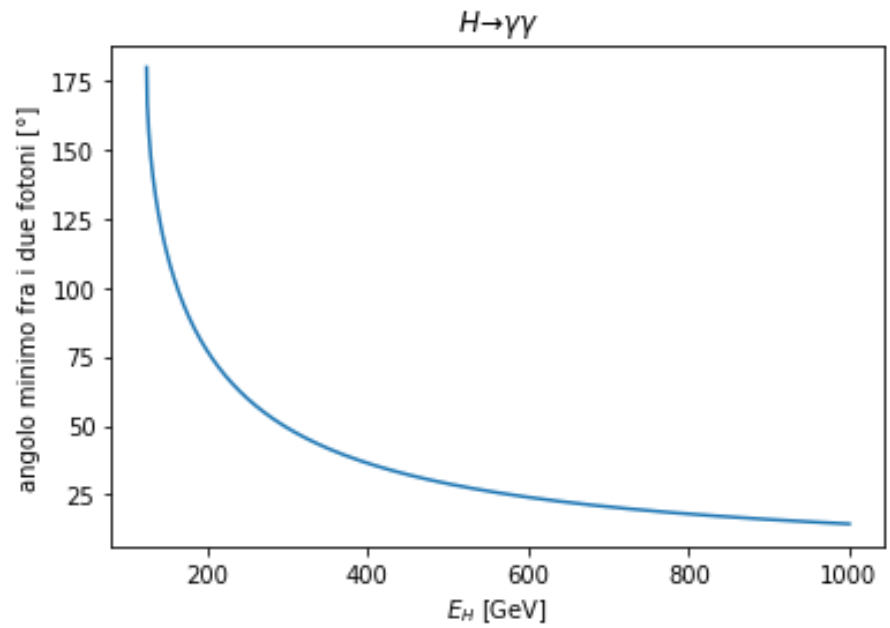
$$= E_H - 2E_1$$

$E_1 = \frac{E_H}{2}$ è il valore di E_1 per cui θ_{12} è MINIMO

$$\sin\frac{\theta_{12}}{2} = \frac{m_H}{2\sqrt{E_1(E_H-E_1)}} = \frac{m_H}{2\sqrt{\frac{E_H}{2}\left(E_H-\frac{E_H}{2}\right)}}$$

$$= \frac{m_H}{E_H} = \gamma_H$$

θ_{12} Piccolo $\rightarrow$ $\theta_{12}\big|_{MIN} \simeq \dfrac{2m_H}{E_H}$



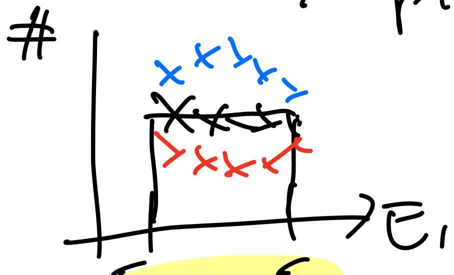
Come si distribuisce E_1 nel LAB?

$$E_1 = \gamma_H\left(E^* + \beta_H p^* \cos\theta^*\right)$$

$$E^* = \frac{m_H}{2}$$

$$= \frac{E_H + P_H\cos\theta^*}{2}$$

distribuzione del # fotoni in funzione di E_1



$$\frac{df}{dE_1} = \frac{df}{d\cos\theta^*}\,\frac{d\cos\theta^*}{dE_1}$$

SPIN DELLA PARTICELLA

$$\frac{dE_1}{d\cos\theta^*} = \frac{p_{1+}}{2} \quad \text{è un NUMERO}$$

spin 0: ISOTROPIA

$$\Omega^* = (\theta^*, \varphi^*)$$

$$\int \rho(\Omega^*)\, d\Omega^* = 1$$

$$d\Omega^* = d\cos\theta^*\, d\varphi^*$$

$$\rho(\Omega^*) \equiv \text{costante}$$