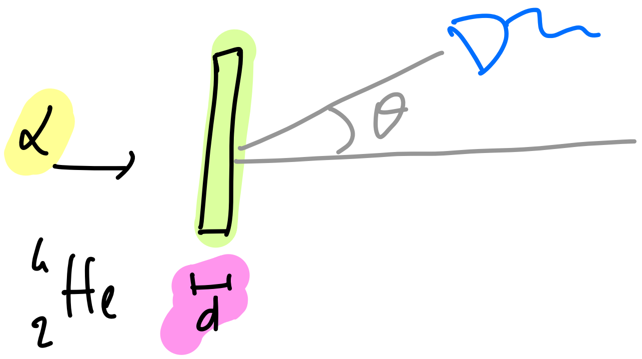
SCATTERING RUTHERFORD QUANTISTICO



$$T_\alpha = \frac{p_\alpha^2}{2m_\alpha} \equiv \frac{p^2}{2m} \simeq 5.5\,\text{MeV}$$

$$m_\alpha < 4 m_p$$

$$m_\alpha \sim 3.7\,\text{GeV}$$

$$\beta = \frac{p}{E} = \frac{\sqrt{2mT}}{T+m} \sim 0.05$$

il processo è

$$\alpha + N \to \alpha + N$$

misuro il numero di "reazioni" nell'unità di tempo

PARTICELLE INCIDENTI AL SEC.

DENSITA' DI BERSAGLI

$$\frac{dN_r}{dt} \propto \frac{dN_\alpha}{dt} \cdot n_b \cdot d$$

SPESSORE DEL BERSAGLIO

$$= \underbrace{\sigma}_{\text{DINAMICA}} \underbrace{\frac{dN_\alpha}{dt}\, n_b\, d}_{\text{COME FACCIO L'ESPERIMENTO}}$$

$$\text{Sezione d'urto} = \frac{dN_r/dt}{\frac{dN_\alpha}{dt}\, n_b\, d}$$

ipotesi

1ª $|i\rangle$ e $|f\rangle$: α LIBERA

$$\psi(\vec{r},t) \propto e^{i(\vec{p}\cdot\vec{r} - Et)}$$

2ª interazione EM

$$\tilde{V}(\vec{r}) = \frac{(Ze)(ze)}{4\pi\varepsilon_0}\cdot\frac{1}{r}$$

ENERGIA POTENZIALE

$$H = H_0 + \tilde{V}$$

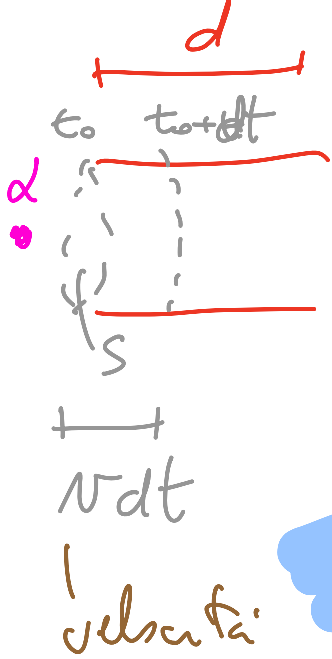
MQ posso scegliere la normalizzazione

di $\psi = A\, e^{\frac{i(\vec{p}\vec{r} - Et)}{\hbar}}$ $\qquad \hbar = c = 1$

$A \in \mathbb{C}$

P(particella α in un volume V) $= 1$

la quale risulta $u_A = \frac{1}{V}$



$$V = S \cdot d$$

$$n_b = \frac{1}{V}$$

$$\text{"}d\sigma\text{"} = \frac{dN_r/dt}{\frac{dN_\alpha}{dt} \cdot n_b \cdot d}$$

- $n_b = 1/V$
- $\dfrac{dN_\alpha}{dt} = \dfrac{dN_\alpha}{S v\, dt} \cdot S v$ — velocità delle α

  $= n_\alpha \cdot S \cdot v$
- $\dfrac{dN_\alpha}{dt} \cdot d = n_\alpha S v d$

  $= n_\alpha v V$

$$d\sigma = \frac{dN_r/dt}{n_b \cdot n_\alpha \cdot v \cdot V}$$

$$= \frac{dN_r/dt}{1/V \cdot 1/V \cdot v \cdot V} =$$

$$d\sigma = \frac{dN_r/dt}{\mathcal{N}}\, V$$

- $|i\rangle$ e $|f\rangle$ particelle libere

$$\psi_{i,f} = A\, e^{i(\vec p_{i,f}\vec r - E_{i,f}t)} = \frac{1}{\sqrt{V}}\, e^{i(\vec p_{i,f}\vec r - E_{i,f}t)}$$

$$P(\alpha \text{ in } V) = 1 = \int_V d^3r\, \psi^*\psi$$

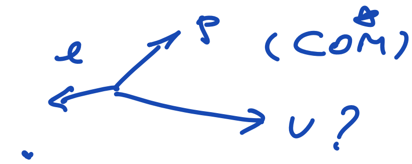
$$= V\cdot|A|^2 = 1 \Longrightarrow A = \frac{1}{\sqrt{V}}$$

$$\boxed{\frac{dN_r}{dt}} = d\Gamma_{i\to f} = \frac{2\pi}{\hbar}\underbrace{|\langle f|\tilde V|i\rangle|^2}_{\text{DINAMICA}}\ \underbrace{\rho(E_i)}_{\text{CINEMATICA}}$$

(4) $n \to p + e + \nu$

$\mu \to e + \nu + \nu$

(A)



$p_x^e = 15$ MeV

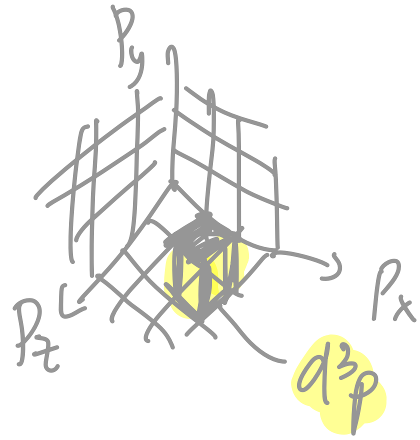
$p_x^p = 750$ MeV

$p_x^\nu = ? \to -765$ MeV

$$\rho(E_i) = \left.\frac{dn}{dE}\right|_{E_i=E_f}$$

$$= \left.\frac{dn}{dp}\frac{dp}{dE}\right|_{E_i=E_f}$$

$$E\ (\equiv T) = \frac{p^2}{2m} \to \frac{dE}{dp} = \frac{p}{m} \Longrightarrow \frac{dp}{dE} = \frac{m}{p}$$



$$\psi(x) \propto e^{ipx}$$

nello spazio 3D x, y, z:

$$V \equiv a^3$$

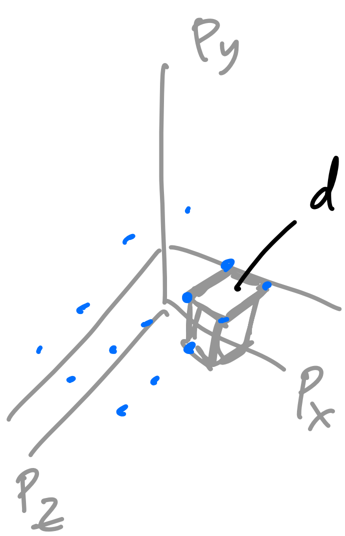
$$\psi(x+a) = \psi(x)$$

$$\psi(x+a) = e^{ip_x(x+a)}$$
$$= e^{ip_x x}$$

$$e^{ip_x a} = 1 \rightarrow p_x a = 2k\pi$$
$$k \in \mathbb{Z}$$

$$p_x = 2k\frac{\pi}{a}$$

10 APRILE



$$d^3p = p^2\,dp\,d\Omega$$

$$\frac{dn}{dE} = \frac{dn}{dp}\frac{dp}{dE}$$

$$\frac{dn}{dp} = ?$$

$V = a^3$

ψ è localizzata in V

$$\psi(x, y, z) = \psi(x \pm a, y, z)$$

$$\psi(x, y, z) = \psi(x, y \pm a, z)$$

$$\psi(x, y, z \pm a) = \psi(x, y, z)$$

$$e^{ip_x(x+a)} = e^{ip_x x}$$

$$e^{ip_x a} = 1$$

$$p_x a = 2k\pi \qquad k = 0, \pm 1 \ldots$$

$$p_x = k \cdot \left(\frac{2\pi}{a}\right)$$

$$\text{"} \; d^3p = k' \cdot \left(\frac{2\pi}{a}\right)^3 \text{"}$$

$$dn = \frac{d^3p}{(2\pi/a)^3} = \frac{p^2\,dp\,d\Omega}{(2\pi)^3 / V}$$

quanti stati ho nello spazio degli impulsi nel volume d^3p

$$\frac{dn}{dp} = \frac{p^2\,d\Omega}{(2\pi)^3} V$$

$$\rho(E_i) = \left.\frac{dn}{dE}\right|_{E_i=E_f} = \frac{dn}{dp}\frac{dp}{dE} = \frac{p^2\,d\Omega}{(2\pi)^3}V\cdot\frac{m}{p}$$

$$= \frac{p\,d\Omega\,m}{(2\pi)^3}V$$

$$\frac{dN_1}{dt} = \frac{2\pi}{\underbrace{\hbar}_{\hbar=1!}}\left|\langle f|\tilde{V}|i\rangle\right|^2\rho(E_i)$$

$$|i\rangle = \frac{1}{\sqrt{V}}e^{i(\vec{p}_i\cdot\vec{r}-E_i t)}$$

$E_i = E_f$

$$|f\rangle = \frac{1}{\sqrt{V}}e^{i(\vec{p}_f\cdot\vec{r}-E_f t)}$$

$|\vec{p}_i| = |\vec{p}_f| = p$

$$H = H_0 + \tilde{V}$$

$$\tilde{V} = \tilde{V}(\vec{r}) = \tilde{V}(r) = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(ze)}{r}$$

$$\langle f|\tilde{V}|i\rangle = \frac{1}{V}\frac{1}{4\pi\varepsilon_0}(Ze)(ze)\cdot\int d^3r\;\psi_f^*\cdot\frac{1}{r}\psi_i$$

$$= \frac{1}{V}\frac{1}{4\pi\varepsilon_0}(Ze)(ze)\int_0^\infty r^2 dr \int_0^{2\pi} d\phi \int_0^\pi \sin\tilde{\theta}\, d\tilde{\theta}\; e^{-i\vec{p}_f\cdot\vec{r}}\,\frac{1}{r}\cdot e^{i\vec{p}_i\cdot\vec{r}}$$

$$\vec{q} \equiv \vec{p}_f - \vec{p}_i \qquad e^{-i\vec{q}\cdot\vec{r}}$$

$$\vec{q}\cdot\vec{r} = q r \cos\tilde{\theta}$$

$$y = \cos\tilde{\theta}$$

$$dy = -\sin\tilde{\theta}\, d\tilde{\theta}$$

$$\theta = 0 \quad y = 1$$
$$\theta = \pi \quad y = -1$$

$$= \frac{1}{V}\frac{1}{4\pi\varepsilon_0}(Ze)(ze)\int_0^\infty r\,dr \overbrace{\int_0^{2\pi} d\phi}^{2\pi} \int (-dy)\, e^{-iqry}$$

$$= \frac{1}{V}\frac{-1}{4\pi\varepsilon_0}(Ze)(ze)\int_0^\infty r\,dr\,(2\pi)\int_1^{-1} dy\; e^{-iqry}$$

$$= \frac{1}{V}\frac{2\pi}{4\pi\varepsilon_0}(Ze)(ze)\int_0^\infty r\,dr\; \underbrace{\frac{e^{-iqr} - e^{+iqr}}{-iqr}}_{\propto\, 2\sin(qr)}$$

ipotes.

$$\tilde{V} = \frac{1}{4\pi\varepsilon_0}(ZeZe)\frac{1}{r} = \lim_{M_\gamma\to 0}\frac{1}{4\pi\varepsilon_0}(ZeZe)\cdot\frac{1}{r}\, e^{-M_\gamma r}$$

$\langle f|V|i\rangle =$

$$= \lim_{m_\gamma \to 0} \frac{-1}{4\pi\varepsilon_0} \frac{1}{V} (Ze)(ze) \int_0^\infty r\,dr\,(2\pi) \int_1^{-1} dy\, e^{-iqry} e^{-m_\gamma r}$$

$$= \lim_{m_\gamma \to 0} -\frac{2\pi (Ze)(ze)}{4\pi\varepsilon_0} \frac{1}{V} \int_0^\infty r\,dr \left[\frac{e^{-(iq+m_\gamma)r}}{-iqr} - \frac{e^{(iq-m_\gamma)r}}{iqr} \right]$$

$r=0$ sopadicu

$$= \lim_{m_\gamma \to 0} + \frac{1}{V} \frac{2\pi}{4\pi\varepsilon_0} (Ze)(Ze) \left(\frac{1}{iq}\right) \left[\frac{1}{-(iq+m_\gamma)} - \frac{1}{(iq-m_\gamma)} \right]$$

$$= \lim_{m_\gamma \to 0} -\frac{1}{V} \frac{2\pi}{4\pi\varepsilon_0} (Ze)(Ze) \left(\frac{1}{iq}\right) \left[\frac{iq - m_\gamma + iq + m_\gamma}{-q^2 - m_\gamma^2} \right]$$

$$= \lim_{m_\gamma \to 0} \frac{1}{V} \frac{2\pi}{4\pi\varepsilon_0} (Ze)(Ze) \frac{-2}{q^2 + m_\gamma^2}$$

$$= \lim_{m_\gamma \to 0} \frac{4\pi}{4\pi\varepsilon_0} \frac{1}{V} (Ze)(Ze) \frac{1}{q^2 + m_\gamma^2}$$

$$= \frac{1}{V} \frac{1}{4\pi\varepsilon_0} (Ze)(Ze) \cdot (4\pi) \cdot \frac{1}{q^2}$$

$$\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{\hbar c} \equiv \alpha \sim \frac{1}{137}$$

$$= \frac{1}{V}(4\pi\hbar c)(\alpha)\cdot\frac{1}{q^2}\cdot z Z$$

$$= \frac{1}{V}4\pi\hbar c\,(\sqrt{\alpha}\cdot z)(\sqrt{\alpha}\cdot Z)\cdot\frac{1}{q^2}$$

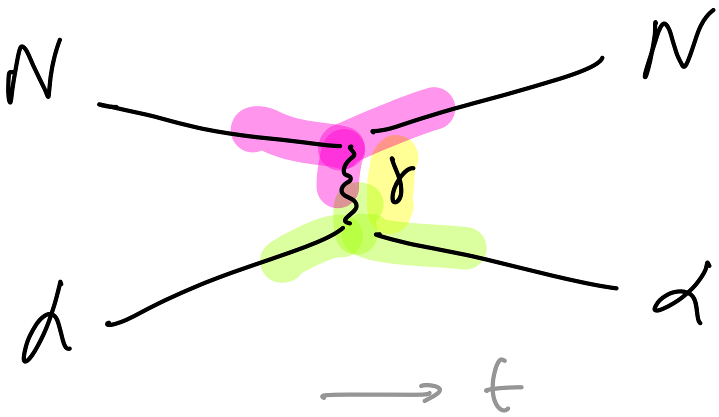


DIAGRAMMA DI FEYNMAN

$$\frac{dW_n}{dt} = \frac{2\pi}{\hbar}|\langle f|V_1|i\rangle|^2\rho(E_i)$$

$$= \frac{1}{V^2}(4\pi\hbar c)^2(\alpha z Z)^2\cdot\frac{1}{q^4}\,\frac{p\,d\Omega\, m}{(2\pi)^3 v}\cdot V$$

$$\vec{q} = \vec{p}_f - \vec{p}_i$$

$$q^2 = p_f^2 + p_i^2 - 2p_f p_i\cos\theta$$

$$= 2p^2(1 - 2\cos\theta)$$

$$= 4p^2\sin^2\theta/2$$

$$q^4 = 16p^4\sin^4\theta/2$$

$$d\sigma = \frac{dN_1/dk}{v} V$$

$$= \frac{(4\pi\hbar c)^2 (\alpha z Z)^2}{16 p^4 \sin^4 \theta/2} \, p \frac{m}{v} \frac{d\Omega}{(2\pi)^3}$$

$T = p^2/2m \implies$ $(P = mv)$

$$\implies \frac{d\sigma}{d\Omega} = \frac{(\alpha z Z)^2}{16 T^2} \frac{1}{\sin^4 \theta/2}$$