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2016 J. Phys.: Condens. Matter 28 134001

(http://iopscience.iop.org/0953-8984/28/13/134001)

IOP Publishing
Journal of Physics: Condensed Matter
J. Phys.: Condens. Matter **28** (2016) 134001 (6pp)
doi:10.1088/0953-8984/28/13/134001

Increasing 'ease of sliding' also increases friction: when is a lubricant effective?

M A Annunziata, A Baldassarri, F Dalton, A Petri and G Pontuale

Consiglio Nazionale delle Ricerche, Istituto dei Sistemi Complessi, Dipartimento di Fisica, Sapienza Università Piazzale Aldo Moro 2, 00185 Rome, Italy

E-mail: alberto.petri@isc.cnr.it

Received 20 July 2015, revised 18 October 2015
Accepted for publication 27 November 2015
Published 2 March 2016

Abstract
We investigate experimentally the effective Coulomb friction exerted by a granular medium on a shearing plate, varying the medium depth. The plate is driven by a spring connected to a motor turning at a constant speed and, depending on the system configuration, performs continuous sliding or stick and slip in different proportions. We introduce an order parameter which discriminates between the different regimes expressing the fraction of time spent in slipping. At low driving speed, starting from zero layers of interstitial granular material, the average friction coefficient decreases when a few layers are added, while the order parameter stays close to zero. By further increasing the granular depth, the friction undergoes a sudden increase but the order parameter does not change notably. At an intermediate driving speed, however, both the friction and the order parameter undergo a sudden increase, which for the order parameter amounts to several orders of magnitude, indicating that the plate is more braked but nevertheless keeps sliding more easily. For medium-high driving speeds, full sliding is obtained for only one layer of interstitial matter, where friction has a minimum, and is maintained for all increasing depths while friction increases. These observations show that the ease of slipping is not determined by friction alone, rather by the highly complex interplay between driving velocity, friction, and the depth of the medium.

Keywords: granular matter, confined shear, friction

(Some figures may appear in colour only in the online journal)

1. Introduction

The nature of granular flow has long attracted attention (Jaeger *et al* 1996) due to its role in tribology (Elrod 1987), lubrication (Lacombe et al2000), geophysics, including earthquakes (Mair and Marone 1999, Aharonov and Spark 2004), and landslides (Mohammed and Fritz 2012). In addition, granular systems constitute a rich instance of complexity, and therefore often supply useful models for investigating out-of-equilibrium and non-linear mechanisms in macro- and meso-scale systems, while whole properties can be usefully considered in addressing problems at the microscopic scale (Puglisi 2015), and vice versa (Levent Demirel and Granick 2002).

A granular material (GM) subjected to an externally applied force (either shear or normal) enters a highly inhomogeneous state in which the majority of the applied force is carried by *force chains* (Campbell 2006) comprising a minority of the grains. Even gravity is sufficient to bring about such a state in a static assembly of GM. (Duran 2000, Majmudar and Behringer 2005) performed correlation analyses on reconstructed experimental force chains in 2D systems, and found a characteristic chain length of about 3–5 grain diameters. More recently, Zhang *et al* (2014) measured the distribution of chain length in both sheared and compressed 2D systems, finding a slightly different value in the two cases but still around 3–4 grain diameters. It may then be expected that the nature of friction between two planes will change drastically with the introduction, and changing depth, of a granular bed. This has been investigated at low pressures and a slow constant shear rate by Marone *et al* (2008), up to a depth of about $n = 3$ grain diameters (i.e. $n = 3$ single layers of grains). In that study, the authors found strong evidence for a rolling transition; when the height of the bed is increased slightly above a monolayer, particles begin to frustrate one another and the macroscopic friction coefficient rises dramatically. In another work, Siavoshi *et al* (2006) investigated the effect of layer depth on the friction of a granular sample in the steady-sliding regime and in the presence of an index-matched interstitial fluid (so as to visualize bulk structure and dynamics). Even in that case the results indicated that an initially high friction, in the absence of interstitial grains, became lubricated in the presence of one or two layers of GM, following a gradual rise for increasing layers, until attaining a limit value independent of the number of layers.

In this paper we experimentally analyze the effect of varying the thickness of the granular bed on the Coulomb friction exerted by the GM on a shearing plate. At variance with cited previous works on the subject (Marone *et al* 2008, Siavoshi *et al* 2006) the plate is not driven at a fixed velocity, but rather it is connected to the driving engine through a spring. In this way the shear rate is not imposed but results from the dynamics of the system. Obviously, the average speed of the plate is equal to the motor speed. Nevertheless the instantaneous value of the velocity of the plate, as well as the instantaneous force exerted by the motor (directly related to the spring elongation) may vary. For large driving motor speeds the plate exhibits continuous sliding, with limited fluctuations of both velocity and force (Dalton *et al* 2005). On the other hand, for low enough engine speeds, and for a spring stiffness which is not too high, the plate performs a much more erratic motion, with an irregular stick–slip behavior. Interestingly, this regime has been described by means of a simple stochastic model (Baldassarri *et al* 2006), which allowed one to estimate the force chain length between 5 and 10 grain diameters. More specifically, the study recovered the statistical distributions of the size and duration of slips, which look wide and skewed, often similar to power laws, suggesting a scale invariance typical of critical phenomena. A similar behavior, addressed as 'crackling noise' (Sethna *et al* 2001), has been observed in several different physical phenomena. In this specific case, critical dynamics would emerge since the specific drive mechanism sets the granular system *at the edge of mobility*, close to the jamming point (Dalton and Corcoran 2000, de Arcangelis *et al* 2011). The coupled system spring-plate reaches a self-organized critical state, characterized by a very intermittent dynamics: the more time the plate sticks, the larger the spring force grows and, consequently, the larger the speed fluctuations become during the slips. Both the stick time and the friction force are dynamically determined by the system, depending on the drive features as well as the granular arrangement.

The aim of this work is to investigate the relation between sticking properties and friction force considering systems with different layers of grains, from which we expect different lubricating properties, driven by a spring pulled at different velocities. We will see that the system displays rather complex behavior. By increasing the velocity (with a fixed number of layers), in the presence of a granular bed the friction first decreases and then increases, exhibiting a minimum (this is not the case in the absence of interstitial grains, also called solid-on-solid (SOS)). A similar behavior is observed by increasing the number of layers (at a fixed driving velocity). On the other hand, the time that the system spends slipping increases monotonically with both the number of layers and the driving velocity, no matter what the friction behavior. This is quite surprising, since the system appears more 'slippy' even for increasing friction.

The structure of this paper includes a short description of the experimental set-up and protocol (section 2). In section 3 the order parameter discriminating stick–slip from continuous sliding phases is introduced. Section 4 contains an illustration of how this order parameter behaves as a function of the GM depth and the driving velocity. Section 5 shows the correspondent behavior of the friction force, and finally section 6 summarizes the results and suggests directions for future investigation.

2. The apparatus

Our experimental set-up consists of a GM confined to an annular cell (see figure 1). Shear is applied by an overhead rotating top plate (free to move vertically) which is driven through a torsion spring by a motor with driving velocity ω_D. The spring constant is chosen as $\kappa = 0.36$ Nm rad^{-1}; the GM is made of glass beads $2(\pm 10\%)$ mm in diameter and of mass $m \simeq 15\ \mu$g. The channel has inner and outer radii of $r = 12.5$ cm and $R = 19.2$ cm, respectively, and a height of $h = 14$ cm.

This set-up (Dalton *et al* 2005, Baldassarri *et al* 2006, Petri *et al* 2008) allows one to measure the instantaneous torque and the instantaneous velocity of the plate, from which the friction can be derived. We investigate the drive speed range $0.01 \leqslant \omega_D \leqslant 10$ rad s^{-1}, where the system displays a transition between a clearly observable non-periodic, irregular, stick–slip motion, and a more regular steady-sliding regime.

For the purposes of the present paper, GM beds of various heights were subjected to different constant motor driving velocities ω_D. The channel in which the GM resides is fitted with a false floor which can be lowered as necessary in order to accommodate the additional GM. Both the floor and the top-plate are covered with a monolayer of glued GM in order to prevent trivial sliding of the top plate over the GM and simple rolling of the GM on the channel floor; however, the GM covering the top plate does not extend closer than 1 cm from the channel walls—this gap serves to prevent individual particles jamming the system.

The GM is composed of 2 mm glass beads which were added to the channel in steps of a single layer of mass $M_L = \frac{\pi(R^2 - r^2)}{A_1} m\, \phi \simeq 175$ g, where A_1 is the single particle cross-sectional area $A_1 \simeq \pi$ mm^2, and we estimated for the (2D) packing fraction $\phi \approx 0.55$. With only a single layer of GM, much material invariably migrates to the gap near the channel walls where the top plate is not covered, resulting in predominantly SOS behavior over the central part. The thinner case therefore is constituted by 1.5 layers, while the thicker is constituted by 13 layers. Experiments were performed in order for the system to perform about, or above, a thousand radiants for each couple of values of ω_D and n.

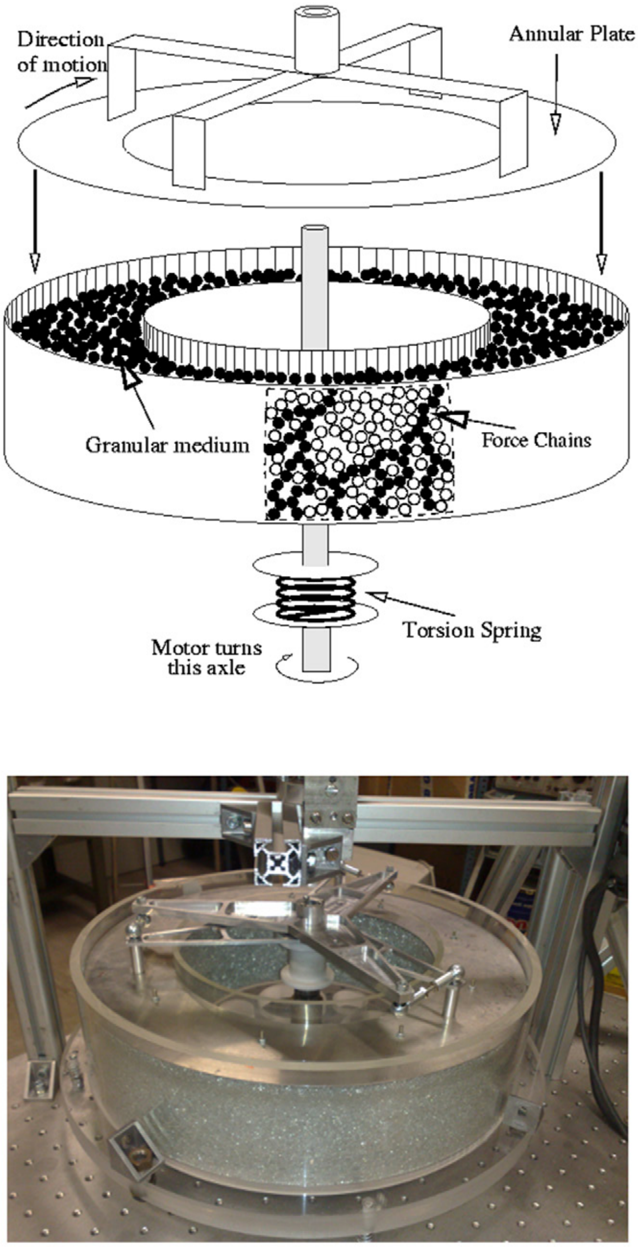

Figure 1. Experimental apparatus: a schematic sketch of the main components (top), and a picture of the real set-up (bottom).

3. Dynamical regimes and the order parameter

As the motor turns, the spring loads but the annular plate motion is initially retarded by the static friction with the GM. As soon as the dragging force is sufficient to destroy the granular structure (i.e. the force chains in the medium), the plate starts to slip, unloading the spring, and slows to a stop when the friction force overcomes the spring force. Due to the disordered nature of the GM, both the static and dynamic friction fluctuate around average values. By dragging the granular medium through the spring-driven plate, the system spontaneously self-organizes itself on the edge of a jamming–unjamming transition. As is well known, in this phase plate dynamics can display widely fluctuating and intermittent motion. This is particularly marked at low values of the driving velocity ω_D,

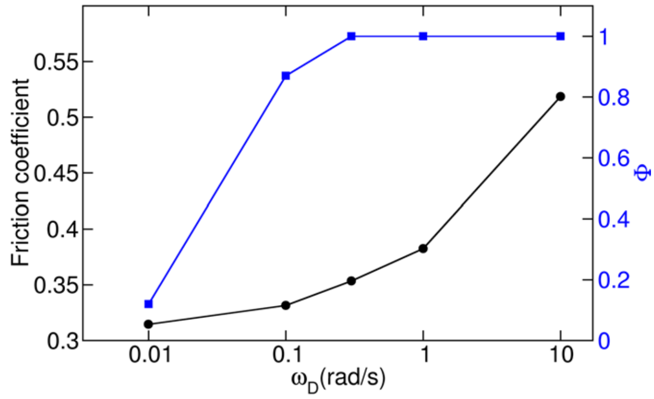

Figure 2. The variation of the order parameter Φ (right axis, blue squares) with driving velocity ω_D for 13 layers is shown along with the mean friction coefficient (left axis, black circles). Φ increases with driving velocity until steady sliding occurs, while the mean friction remains constant until $\omega_D \sim 1$, whereupon it increases rapidly.

where the system spends most of the time in a chaotic stick–slip dynamics. In this regime the statistical distributions of quantities characterizing the motion, such as slip extension and duration, and velocity distribution, are strongly skewed and close to power laws (Baldassarri *et al* 2006, Petri *et al* 2008), which is the form predicted from stochastic models (Baldassarri *et al* 2006, Colaiori 2008). Fluctuations decrease and distributions shrink for increasing driving (Dalton *et al* 2005) until continuous sliding is attained.

In order to characterize these observations in a quantitative way, we have defined an order parameter, Φ, as the fraction of time for which the top plate is moving. This was computed for each experiment by dividing the time that the plate spent slipping by the total duration of the experiment, and investigated as a function of the number n of layers and of the driving velocity ω_D for 36 different couples of values. An instance of the behavior described above is shown in figure 2, where the behavior of Φ as a function of the driving speed ω_D, in log scale, is reported together with that of the friction for a bed 13 layers deep. While the order parameter displays a rapid increase towards the sliding regime, the average friction exerted by the medium against the motion undergoes an increase exactly when the order parameter approaches unity. This is expected because approaching the steady-sliding regime ($\Phi \to 1$), friction becomes an increasing function of the velocity, proportional to ω_D^2 or to ω_D according to whether, respectively, grain inertia or viscosity dominate (Bagnold 1954), showing that Φ, i.e. the ease of slipping, is not directly related to the average friction, rather to ω_D.

4. The order parameter transition with increasing depth

The behavior of Φ observed in the instance of the previous section is qualitatively followed for any number n of layers. However, a dependence emerges on the number of layers for the critical driving velocity ω_c above which continuous sliding is displayed. Figure 3 plots the variation of Φ as function of ω_D

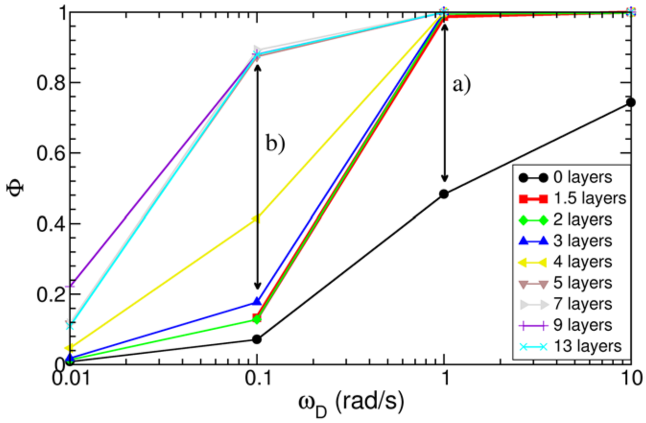

Figure 3. Two transitions are observed when Φ is plotted as a function of ω_D. (a) The lubrication transition causes even the minimum quantity of GM to trigger steady sliding at high ω_D. (b) The granular order parameter shows an abrupt transition at $\omega_D = 0.1$ rad s^{-1}.

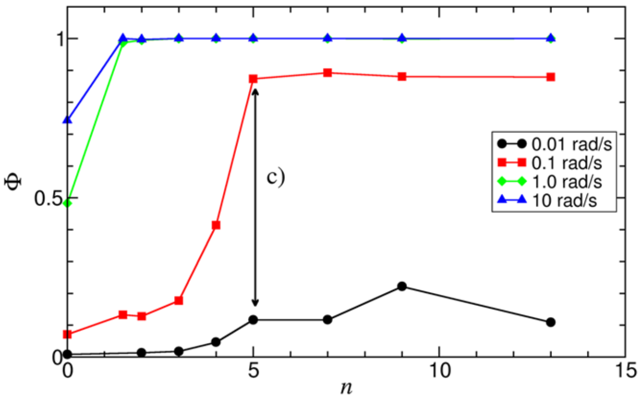

Figure 4. The order parameter Φ (defined as the fraction of time spent in motion) also shows a sharp transition at about four layers for $\omega_D \leqslant 0.1$ rad s^{-1}.

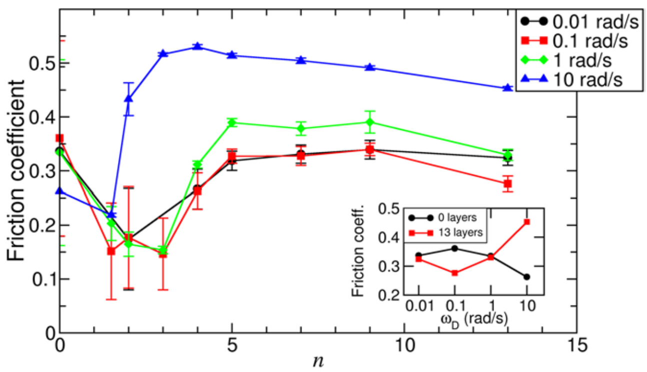

Figure 5. Plotting the mean friction coefficient applied to the GM as a function of granular bed depth reveals a transition to a stiffer system as the system transits approximately four to five layers. At higher driving speeds (10 rad s^{-1}), when the GM is highly energized, the transition is observed at about two layers. Additionally, there is a reduced friction at intermediate values between zero and four layers. The inset indicates the mean friction for the two extreme cases ϕ (squares) and 13 layers (bullets), as a function of the drive velocity ω_D.

for different n. As expected, the system is seen to engage more in stick–slip motion for low driving velocities. But it can also be seen that a *deeper GM bed favors sliding*. At high ω_D, there is always continuous sliding, but the value at which this is attained depends on the number of layers. For $1 < n < 3$, for ω_D between 0.01 and 0.1 rad s^{-1}, the order parameter shows a slow increase, following the same behavior as the SOS case. At intermediate speeds, for ω_D between 0.1 and 1 rad s^{-1}, however, the order parameter shows a faster increase up to almost $\Phi \simeq 1$, departing from the SOS value. On the other hand, for $n \geqslant 5$, the order parameter starts to grow sensibly already for $\omega_D \ll 1$, approaching steady sliding well before $\omega_D = 1$. These different behaviors displayed for different ranges of n give rise to an abrupt jump in the order parameter in correspondence to $n \approx 4$, as can be seen in figure 3(b). This sudden increase is maximum around the critical values of the driving where massive fluidization of the medium starts to occur, $\omega_D \approx 0.1$, and amounts to many orders of magnitude. However, it vanishes at high ω_D, when full fluidization is reached, figure 3(a) showing a lubrication transition due to the mere presence of the GM.

These kind of abrupt transitions are also clearly visible in figure 4, which highlights the influence of the number of layers. Specifically, how the relative proportion between time spent in stick and slip for a given driving velocity changes notably with n. We can see two main distinct behaviors. It is seen at first that at low driving speeds, Φ reaches an asymptotic value for a depth of roughly five layers. This happens soon after the transition (b) in figure 3. Second, for high driving speeds, full sliding is already reached at 1.5 layers, signaling that, at these drivings, the lubricated friction is the same for any $n \geqslant 1.5$ and is inherently different from the SOS (i.e. $n = 0$) case. These behaviors are the marks of two different mechanisms. The first is the saturation of the average shear band, which therefore appears to be typically around five layers. The second mechanism is the fluidization transition, causing steady-sliding at high ω_D, even with only one layer of GM. At these values of the plate speed, the medium is in the collisional phase and the number of layers becomes irrelevant. We can also see that for very low driving speeds ($\omega_D = 0.01$ rad s^{-1}) and few layers ($n \leqslant 3$) the system maintains the SOS values of the order parameter, indicating that very few particles are mobilized in this regime.

5. The transition in the average friction

In this section we report the results of our investigation on the frictional behavior when the number of granular layers is changed. Looking at the average friction coefficient in figure 5, it is seen that the greater ease for sliding when more layers are present, indicated by the increase of the order parameter Φ, is not without cost. In fact, the average friction, after a weakening with a minimum shown at about two to three layers for low driving speeds and 1.5 layers for high driving speeds, is seen to strengthen again with the number of layers. It is worth noting that the increase of the friction for increasing n cannot be attributed to the increase of Φ, and hence to the longer slipping time fraction. This because the difference between static and dynamic friction, and the relative fluctuations during motion, are limited to a few per cent (Dalton *et al* 2005). Rather it may be due to the increased depth of the shear band, as suggested by numerical simulations (Annunziata *et al* 2016).

Figure 5 also shows that the SOS friction is significantly higher than that of 1.5 layers[1]. The minimum in friction is found for a particle depth of roughly 1.5–3, suggesting a lubrication effect as explored by Marone *et al* (2008). For low driving velocities the friction remains low before reaching a steady value after approximately five layers have been added to the channel, as for the order parameter, confirming that this is the depth of the relevant shear band for stick–slip motion. For $\omega_D = 10$ rad s^{-1}, the friction increases to a constant value after only two to three layers have been added. This is also a confirmation that for high plate speeds the system fluidizes and there is not much change above two or three layers.

The inset to figure 5 illustrates the behavior of friction with drive velocity for a deep granular bed (squares) and for the SOS cases (bullets). As the driving velocity is increased, the granular average friction displays a weakening in correspondence to the fluidization region, where sliding contacts lose importance in favor of collisional interactions, before increasing in the steady-sliding state, as seen in the previous section. In contrast, we see that the SOS friction displays the opposite behavior.

6. Summary and some perspectives

We have investigated the dynamic response of a GM in the spring–slider configuration, producing both stick–slip and steady–sliding regimes, by varying the number of grain layers n and the nominal shear rate ω_D. In order to quantify the proneness to slip, we have introduced an order parameter expressing the fraction of time the plate spends slipping. In all cases it is observed to be a monotonic function of n and ω_D. For intermediate driving the increase is abrupt and concentrated around $n \simeq 4 \div 5$. At high driving speeds, however, the asymptotic value is already attained at $n \geqslant 2$.

In agreement with previous observations (where shear was imposed) (Marone *et al* 2008, Siavoshi *et al* 2006), Coulomb friction abruptly decreases when adding one layer to the bare SOS interface (rolling transition in Marone *et al* (2008)), and increases again when adding more layers until saturation occurs. At low and intermediate driving, as the depth of the granular bed is increased, the increase in friction is not smooth but undergoes a jump that takes place for a bed of about four to five particle diameters in depth, similarly to that observed in Siavoshi *et al* (2006). This latter circumstance may occur because long-lived grain–grain friction forces cease to dominate at high driving speeds, being replaced principally by short-lived collisional interactions between grains. Force chains are no longer the mechanism which sustains the applied torque and the transition depth does not depend on their length. This leads us to conclude that a non-trivial dependence on ω_D exists. The results also suggest that in the

[1] As noted above, there is reason to believe that 1.5 layers of GM may effectively behave as a monolayer between the top and bottom plates due to an accumulation of grains in the gap near the channel walls where no particles are glued to the top plate for any driving ω_D.

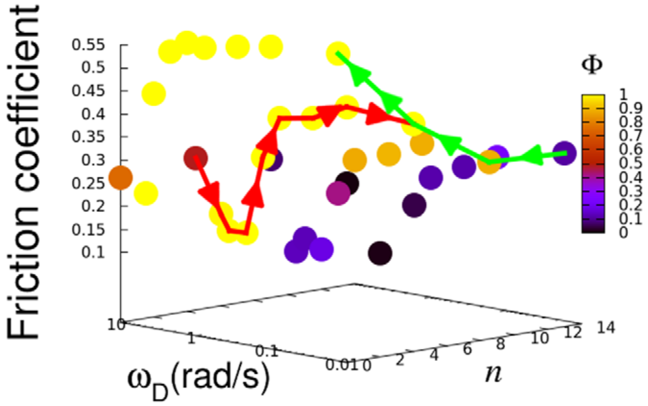

Figure 6. Phase diagram. The values assumed by the order parameter Φ are reported using a color-map as a function of the velocity of the plate ω_D and the number of layers. The z-axis shows the average friction coefficient. The diagram synthesizes the different dynamical regimes displayed by the system. Arrows are guides for the eye along constant drive and constant layer number trajectories.

low driving regime, shear bands form in the GM during stick–slip, such that for depths greater than five layers the channel floor is completely screened by the GM. The microscopic mechanisms which underly the flow and the friction change as the number of layers becomes comparable to the shear band. In fact, molecular dynamics simulations show that shear band formation is an intermittent phenomenon depending on the instantaneous velocity of the plate and possibly on its history (Annunziata *et al* 2016). Investigating which dynamics takes place within the GM will be subject of forthcoming simulational work.

In summary, increasing the grain depth always increases the time spent slipping, also at the cost of increasing the average friction, showing that in this self-organized dynamics it is not this quantity that determines the ease of slipping. These observations are summarized by the phase diagram displayed in figure 6. It shows the value assumed by the order parameter Φ and of the friction coefficient as functions of the driving velocity ω_D and of the number of layers, n. As anticipated, the sliding phase (brighter colors) is favored by both increasing the driving velocity or the number of layers, and does not always correspond to the minimum friction. So the answer to the question in the title depends on the definition of what an effective lubricant is. If, as usual, effectiveness means decreasing friction, or viscosity, then adding layers does not produce an effective lubrication. But if one looks at the probability of getting stuck, it is extremely effective in decreasing this, although at the cost of larger friction. The findings reported here concern systems with macroscopic particles. An intriguing point and very stimulating issue is whether similar phase diagrams can characterize lubrication of systems at smaller scales (Vanossi *et al* 2013b, 2013a), such as colloids, liquid crystals, and atoms, or could be relevant for lubrication of biological systems such as membranes, joints or surfaces. On the other hand, further investigation on granular systems can shed light on the main mechanisms at the base of the observed behavior, discriminating among, e.g. the melting–freezing and/or inertial effects of a lubricant, an issue which is also relevant at the nano-scale (Braun 2010, Rosenhek-Goldiana *et al* 2015).

Acknowledgments

This work benefited from the grant FIRB RBFR081IUK_003 of the Italian Ministry for Education and Research and is supported in part by COST Action MP1303. Thanks are due to A Morbidini of the Italian INAF, for realizing some parts of the set-up, and to S Zapperi for stimulating discussions.

References

Aharonov E and Spark D 2004 *J. Geophys. Res.: Solid Earth* **109**
Annunziata M, Baldassarri A, Dalton F and Petri A 2016 in preparation
Bagnold R A 1954 *Proc. R. Soc.* A **225** 49–63
Baldassarri A, Dalton F, Petri A, Zapperi S, Pontuale G and Pietronero L 2006 *Phys. Rev. Lett.* **96** 118002
Braun O 2010 *Tribol. Lett.* **39** 283–93
Campbell C S 2006 *J. Fluid Mech.* **539** 273–97
Colaiori F 2008 *Adv. Phys.* **57** 287–359
Dalton F and Corcoran D 2000 *Phys. Rev.* E **63** 61312–4
Dalton F, Farrelly F, Petri A, Pietronero L, Pitolli L and Pontuale G 2005 *Phys. Rev. Lett.* **95** 138001
de Arcangelis L, Ciamarra M P, Lippiello E and Godano C 2011 *J. Phys.: Conf. Ser.* **319** 012001
Duran J 2000 *Partially Ordered Systems* (New York: Springer)
Elrod H G 1987 *Tribology Series* vol 7 (Amsterdam: Elsevier) pp 75–88
Jaeger H M, Nagel S R and Behringer R P 1996 *Rev. Mod. Phys.* **68** 1259
Lacombe F, Zapperi S and Herrmann H J 2000 *Eur. Phys. J.* E **2** 181–9
Levent Demirel A and Granick S 2002 *J. Chem. Phys.* **117** 7745
Mair K and Marone C 1999 *J. Geophys. Res.* **104** 28899–914
Majmudar T S and Behringer R P 2005 *Nature* **435**
Marone C, Carpenter B M and Schiffer P 2008 *Phys. Rev. Lett.* **101** 248001
Mohammed F and Fritz H M 2012 *J. Geophys. Res. Oceans* **117** C11015
Petri A, Baldassarri A, Dalton F, Pontuale G, Pietronero L and Zapperi S 2008 *Eur. Phys. J.* B **64** 531–5
Puglisi A 2015 *Transport and Fluctuations in Granular Fluids* (Heidelberg: Springer)
Rosenhek-Goldiana I, Kampfa N, Yeredorb A and Kleina J 2015 *Proc. Natl Acad. Sci.* **112** 7117–22
Sethna J P, Dahmen K A and Myers C R 2001 *Nature* **410** 242–50
Siavoshi S, Orpe A V and Kudrolli A 2006 *Phys. Rev.* E **73** 010301
Vanossi A, Benassi A, Varini N and Tosatti E 2013a *Phys. Rev.* B **87** 045412
Vanossi A, Manini N, Urbakh M, Zapperi S and Tosatti E 2013b *Rev. Mod. Phys.* **85** 529–52
Zhang L, Wang Y and Zhang J 2014 *Phys. Rev.* E **89** 012203