Periodically time-modulated bistable systems: Stochastic resonance

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We characterize the notion of stochastic resonance for a wide class of bistable systems driven by a periodic modulation. On developing an adiabatic picture of the underlying relaxation mechanism, we show that the intensity of the effect under study is proportional to the escape rate in the absence of perturbation. The adiabatic model of stochastic resonance accounts for the role of finite damping and finite noise correlation time as well. Our predictions compare well with the results of analogue simulation.

I. INTRODUCTION

A typical feature of stochastic relaxation in a periodically modulated bistable system is the so-called stochastic resonance (SR). The interplay of intrinsic noise and periodic driving mechanism produces a sharp enhancement of the signal power spectrum corresponding to the forcing frequency. Such an effect is apparent even when the perturbation is weak enough not to appreciably affect the rate of the noise-induced switch process. The nonstationary properties of one-dimensional bistable systems driven by a sinusoidal forcing term have been illustrated in Ref. 1 by means of a perturbation approach. The SR mechanism is related to the oscillating behavior of the signal autocorrelation function (ACF) for times much larger than the relevant decay time (the reciprocal of the Kramers rate) in the unperturbed system. Recently a characteristic SR behavior has been detected in complex systems[2,3] which resists the perturbation approach of I. SR turns out to be of potential application in the modeling of a variety of physical phenomena.

In the present paper we report and discuss the results of analogue simulation for the process described by the differential equation (prime and overdot denote x and t derivation, respectively)

$$\ddot{x}+\gamma\dot{x}=-V'(x,t)+\xi(t)\ , \tag{1.1}$$

where γ is the damping constant, $\xi(t)$ is a Gaussian zero-mean-valued noise with correlation function

$$\langle\xi(t)\xi(0)\rangle=\frac{\gamma D}{\tau}e^{-|t|/\tau}\ , \tag{1.2}$$

and the potential $V(x,t)$ comprises a deterministic term $V(x)$ and a periodic time dependent perturbation $P(x,t)$, i.e.,

$$V(x,t)=V(x)+P(x,t)\ , \tag{1.3}$$

with

$$V(x)=-\frac{a}{2}x^2+\frac{b}{4}x^4 \tag{1.4}$$

and

$$P(x,t)=-Ah(x)\cos(\Omega t+\theta)\ . \tag{1.5}$$

The overdamped limit of the process (1.1) and (1.2),

$$\dot{x}=-V'(x)+\xi(t)\ , \tag{1.6}$$

with noise correlation function

$$\langle\xi(t)\xi(0)\rangle=\frac{D}{\tau}e^{-|t|/\tau}\ , \tag{1.7}$$

has been investigated in I in the limit of vanishing correlation time, $\tau\to 0$. The findings of our analogue simulation allow us to appreciate the general nature of SR and its most remarkable features (Sec. II). The interpretation of our experimental data relies on an adiabatic picture of the relevant relaxation process, the prediction of which is comparable with the theory of I for the overdamped limit (Sec. III). The role of damping and finite noise correlation time is well explained within our approximations (Sec. IV).

II. STOCHASTIC RESONANCE

The notation of SR is based on the remark[4] that the output signal $x(t)$ from a stochastic bistable system may be modulated in time by applying an external periodic perturbation. A way of quantifying this effect is to look at a discrete stochastic process associated with $x(t)$. Let $T(n)$ denote the first-crossing time[1] of the nth sampling record of the output signal $x(t)$ (with fixed length much larger than the forcing period $T_\Omega=2\pi/\Omega$). The first-crossing time thus determined corresponds to measuring the switch time of $x(t)$ between its stable values. In Fig. 1 we display the distribution of $T(n)$, $N(T)$, for the process (1.1)–(1.5) with $h(x)=x$, as previously reported in Ref. 3. At $A=0$ we recover the usual distribution of the first-passage times in an unperturbed bistable potential[5]

$$N_0(T)=(2T_k)^{-1}e^{-T/2T_k}\ , \tag{2.1}$$

where T_k is the reciprocal of the Kramers rate out of a single metastable well. In the presence of deterministic forcing term, $A\neq 0$, instead, the number of crossings peaks at $T=\pi/\Omega$. This implies that the process $x(t)$ switches almost periodically between its stable minima with frequency $\nu_\Omega=\Omega/2\pi$. The peak of $N(T)$ reaches a maximum for a certain value of ν_Ω [Fig. 1(b)]. For lower

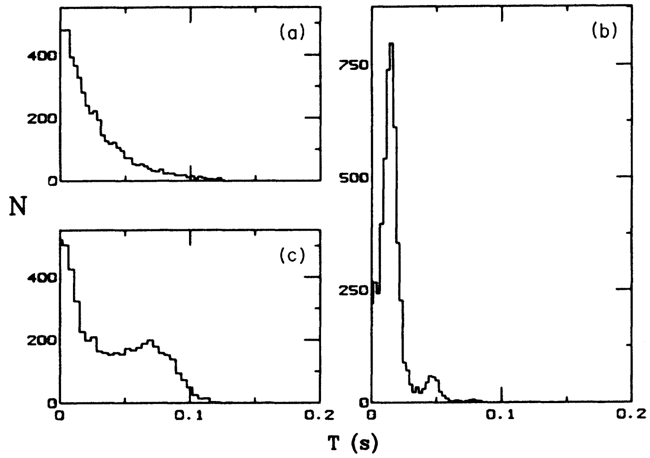

FIG. 1. First crossing-time distribution $N(T)$ for the system (1.1)–(1.5) with (a) $A=0$, (b) $Ax_m=0.5\Delta V$ and $\nu_\Omega=30$ Hz, and (c) $Ax_m=0.5\Delta V$ and $\nu_\Omega=6$ Hz. Other parameter values are $x_m=7.3$ V, $\sqrt{2a}=10.2$ kHz, $\Delta V/D=3$, $\gamma=0.64\sqrt{2a}$, and $\tau=50\ \mu$s. The estimated error is less than 5%.

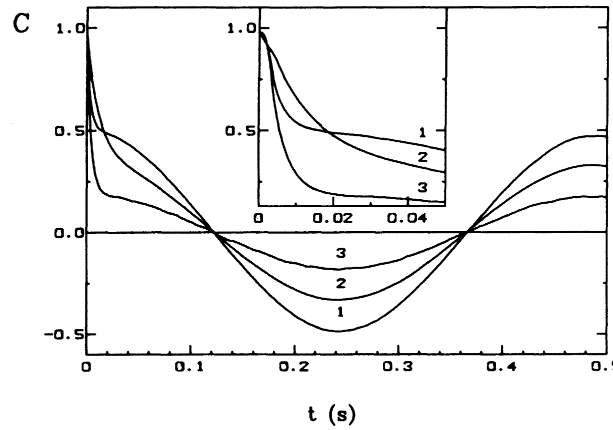

FIG. 2. Output-signal autocorrelation function $C(t)$ for $\nu_\Omega=2$ Hz and (1) $\Delta V/D=2$, $Ax_m=0.5\Delta V$, (2) $\Delta V/D=3$, $Ax_m=0.22\Delta V$, (3) $\Delta V/D=2$, $Ax_m=0.22\Delta V$. The other parameter values are as in Fig. 1. No appreciable statistical error is expected.

forcing frequencies the SR peak smooths out, whereas in the opposite limit it merges into the exponentially decaying branch of $N(T)$ about $T=0$. Furthermore, at high frequencies side peaks located at about the odd multiples of π/Ω become detectable. The explanation of such behavior reveals an important feature of SR. For low forcing frequencies $\nu_\Omega<T_k^{-1}$, the relaxation process in a bistable symmetric potential can be envisaged as the statistical superposition of two relaxation processes in the asymmetric potentials $V_\pm(x)\equiv V(x)\mp Ax$ driven by the noise $\xi(t)$. If the perturbation $P(x,t)$ is taken small, $V_\pm(x)$ are bistable asymmetric potentials with absolute minimum at $x_+>0$ and $x_-=-x_+<0$, respectively. Under this approximation we can define two Kramers rates, μ_K^+ out of the deeper well ($\mu_K^+<\mu_K$) and μ_K^- out of the shallow one ($\mu_K^->\mu_K$) for both $V_\pm(x)$. The forcing mechanism alternatively tilts $V(x)$ in the configurations V_+ and V_- for half a forcing period so that, if the condition $\mu_K^+<2\nu_\Omega<\mu_K^-$ is fulfilled, the output signal switches between the relevant stable values with frequency ν_Ω, giving $x(t)$ little chance to leave the absolute minimum $x_\pm$ of $V_\pm(x)$ during one half-period π/Ω. The periodicity of the switching signal is blurred by too fast a hopping dynamics (out of the deeper well), $2\nu_\Omega<\mu_K^+$, or too slow a forcing mechanism (compared to the escape rate from the shallow well), $2\nu_\Omega>\mu_K^-$. The side peaks of Fig. 1(b) can be explained by noticing that there is a finite probability that $x(t)$ sojourns about the relative minimum of $V_\pm(x)$ longer than half a forcing period, so that $x(t)$ may be trapped in a semiaxis during an odd π/Ω multiple time interval. As a consequence, the intensity of the kth side peak decreases relatively to the main peak with an exponential law, $\exp(-k\mu_K\nu_\Omega)$. From the above discussion we learn that SR is an adiabatic process occurring at low forcing frequencies.

A second characterization of SR is provided by the study of the autocorrelation function of the signal $x(t)$, $C(t)$, and its Fourier transform $C(\nu)$. In Figs. 2 and 3 we display $C(t)$ and $C(\nu)$ for the process (1.1)–(1.5) with $h(x)=x$ at different values of the parameters A and D. $C(t)$ exhibits an oscillating behavior for large values of t, $t\gg T_k$, with frequency ν_Ω. Accordingly, the spectrum $C(\nu)$ exhibits a δ-function-like spike at $\nu_\Omega=\Omega/2\pi$. Such an important property of $C(t)$ has been determined analytically in I for the overdamped process (1.6)–(1.7) with $\tau=0$. The generalization of this property to more general cases can be realized as follows. The external perturbation tilts $V(x)$ from V_+ to V_- and vice versa every half forcing period. $V_\pm$ are the configurations which maximize the absolute mean value of x and x_T. Provided that the relaxation process within the deeper well is fast compared to both the escape mechanism and the forcing dynamics, the asymptotic behavior of $\langle x(t)x(0)\rangle$ is then represented by an oscillating function with amplitude x_T^2 and frequency ν_Ω. Furthermore, the dependence of x_T on D admits of two interesting limits:

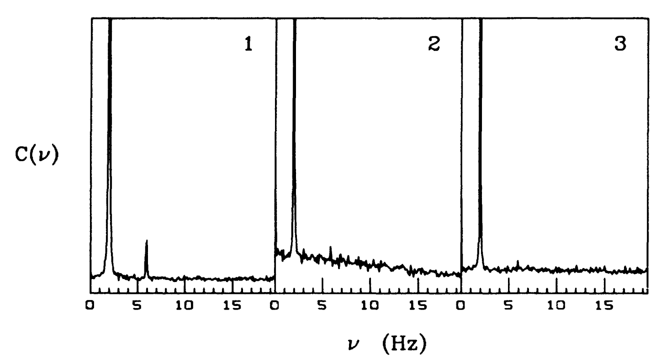

FIG. 3. Fourier transform of the correlation functions in Fig. 2 (arbitrary units).

x_T vanishes for both $D \to 0$ and $D \to \infty$. In fact, on increasing D the potential barrier and the time-dependent modulation $P(x,t)$ become negligible, the symmetry of the problem is thus gradually restored. On decreasing D, at fixed Ω, instead, μ_K grows eventually much smaller than ν_Ω, so that the hopping mechanism is only marginally affected by the periodic forcing.[6] Following the authors of Ref. 4, we agree to term this phenomenon *stochastic resonance*. Finally, it is clear from this line of reasoning (see also I) that no oscillating behavior for $C(t)$, and therefore no SR can be observed in bistable potentials, the symmetry of which is not altered by the external perturbation $P(x,t)$. For the process (1.1)–(1.5) this amounts to requiring that $h(x)$ has to be an odd function of x.

We conclude this section by introducing a characterization of SR introduced by the authors of Ref. 2: the signal-to-noise ratio (SNR). One determines the signal power spectrum $|x(\nu)|^2$ and measures the ratio of the strength of the δ-function-like spike at the forcing frequency to the background spectrum at the same frequency.[2,3] This quantity accounts for the energy transfer from the (unperturbed) background spectrum, determined by the system response to the noise, to the ordered mode driven by the periodic modulation. The SNR thus relates the asymptotic oscillatory behavior of the perturbed system with its stochastic dynamics at the time scale of the hopping mechanism. SNR vanishes for both $D \to 0$ and $D \to \infty$, and it has been shown to peak at about $D/\Delta V \simeq 0.5$ under different experimental circumstances.[2,3] Contrary to x_T, the curves for SNR versus D approach a limiting curve for vanishingly small values of the forcing frequency (Fig. 4). Most importantly, the dependence of SNR on the relevant system parameters can be discussed in some detail within the adiabatic approximation of Sec. III.

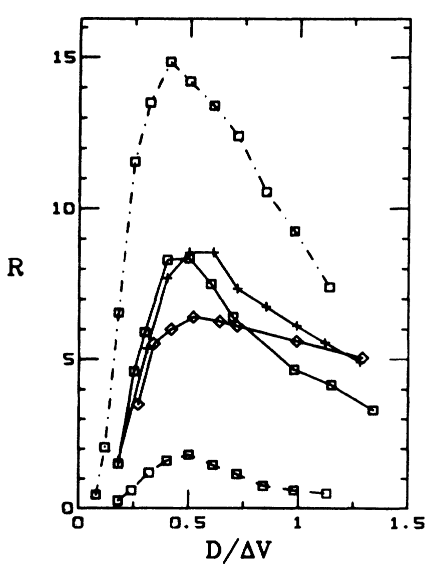

FIG. 4. SNR vs $D/\Delta V$ at several values of $Ax_m/\Delta V$ (dashed line, 0.22; solid line, 0.5; dot-dashed line, 0.66) and ν_Ω (squares, 15 Hz; crosses, 30 Hz; lozenges, 500 Hz). The other parameter values are as in Fig. 1. The relevant averages have been taken over 5000 digitized spectra.

III. ADIABATIC APPROACH

In order to describe the oscillatory behavior of the signal ACF we develop here in some detail the adiabatic argument introduced in Sec. II. Let us assume that the forcing frequency ν_Ω is much smaller than any other characteristic frequency of the system under investigation and, in particular, that $\nu_\Omega \ll \mu_K$. The adiabatic approximation consists of determining the statistical quantities of interest at a fixed value of the perturbation and, then, letting the perturbation vary in time. For the sake of simplicity we confine ourselves to the case of an additive modulation, $h(x)=x$, i.e.,

$$V(x,t)=V(x)-Ax\cos(\Omega t+\theta) . \tag{3.1}$$

Our approach, however, applies for any choice of $h(x)$ (odd) provided that the perturbation can be considered small. The mean value of $x(t)$ in the presence of perturbation (3.1) oscillates between $\pm x_T$ with

$$x_T=\frac{\int dx\, x e^{-V_+(x)/D}}{\int dx\, e^{-V_+(x)/D}} . \tag{3.2}$$

The second moment of $x(t)$, instead, can be cast in the form

$$\langle x^2(\theta)\rangle=\frac{\int dx\, x^2 e^{-V(x,\theta)/D}}{\int dx\, e^{-V(x,\theta)/D}} \tag{3.3}$$

where $V(x,\theta)=V(x)-Ax\cos\theta$. To account for the slow variation of $V(x,t)$ with time, one must average $\langle x^2(\theta)\rangle$ over the phase θ (see paper I), i.e.,

$$\langle\langle x^2\rangle\rangle=\frac{1}{2\pi}\int_0^{2\pi} d\theta \langle x^2(\theta)\rangle . \tag{3.4}$$

On combining Eqs. (3.2) and (3.3) we obtain our estimate for the amplitude C_T (see Figs. 2 and 3):

$$C_T(D)=\frac{x_T^2}{\langle\langle x^2\rangle\rangle} . \tag{3.5}$$

We note immediately that $C_T(D)$ of Eq. (3.5) is independent of Ω, consistently with the adiabatic approximations introduced above. Equation (3.5) can be approximated analytically in the limit of low-noise intensity $D \ll \Delta V$ and small perturbation $Ax_m \ll D$. $\pm x_m$ denote here the bistable minima of the unperturbed potential $V(x)$. Under such circumstances the stationary probability distribution corresponding to the tilted potential V_+ can be substituted by two normalized δ functions centered at about the two local minima of the function $V_+(x)$. The relative and absolute minima of $V_+(x)$ are located in $x_1 \simeq A/V''(x_m)-x_m$ and $x_2 \simeq A/V''(x_m)+x_m$, respectively. Making use of the approximations $V(x_1)\simeq V(x_m)-Ax_m$ and $V(x_2)\simeq V(x_m)+Ax_m$, we obtain for x_T

$$x_T = \frac{x_1 e^{-Ax_m/D} + x_2 e^{Ax_m/D}}{e^{-Ax_m/D} + e^{Ax_m/D}}$$
$$\simeq \frac{A}{V''(x_m)} + x_m \tanh\left[\frac{Ax_m}{D}\right] . \tag{3.6}$$

The θ-averaged variance of $x(t)$, $\langle\langle x^2 \rangle\rangle$, instead, coincides at the leading order with the second moment of the stationary distribution of the unperturbed system. Without a gross inaccuracy[6] we can approximate $\langle\langle x^2 \rangle\rangle$ with x_m^2. Finally, in the limit considered here, $\Delta V \gg D \gg Ax_m$, Eq. (3.5) reduces to

$$C_T(D) \simeq \tanh^2\left[\frac{Ax_m}{D}\right] . \tag{3.7}$$

The major limitation of the adiabatic approximation is due to the fact that on decreasing D at a fixed value of the forcing frequency, μ_K becomes smaller than ν_Ω, thus breaking the starting assumption $\nu_\Omega \ll \mu_K$. Moreover, our estimate for $C_T(D)$ has to be taken with some caution even in the limit of vanishingly small ν_Ω: as a matter of fact the argument of the hyperbolic tangent has to be very small, so that

$$C_T(D) \simeq \left[\frac{Ax_m}{D}\right]^2 . \tag{3.8}$$

In Fig. 5 we compare our adiabatic prediction for C_T (3.5) with the corresponding prediction of the perturbation approach,[1] for the process (1.6)–(1.7) in the limit of white noise ($\tau \rightarrow 0$). As expected, the adiabatic approach turns unsatisfactory on decreasing the noise intensity, when it fails to reproduce the sharp drop of $C_T(D)$ to zero. The adiabatic result, instead, looks reliable in the intermediate region of D values, which also includes the peak of the curve $C_T(D)$, the agreement improving by lowering the forcing frequency. For a comparison with analogue simulation data, the reader is referred to Sec. III of I.

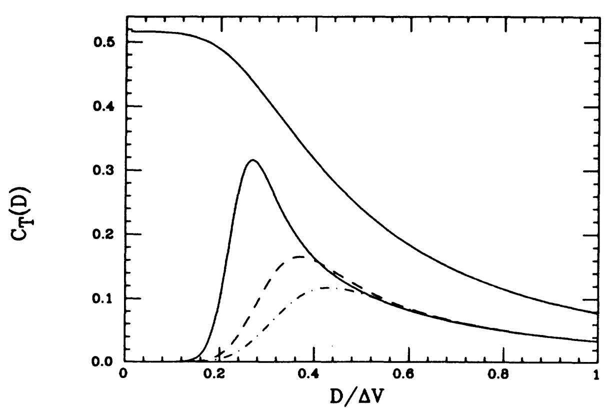

FIG. 5. $C_T(D)$ vs $D/\Delta V$ for different values of the forcing frequency. The upper curve represents the adiabatic prediction (3.5), the lower ones are the results of the perturbative approach presented in I for $\nu_\Omega = 10$ Hz (solid), $\nu_\Omega = 30$ Hz (dashed), and $\nu_\Omega = 50$ Hz (dot-dashed). Other parameter values are $Ax_m = 0.5\Delta V$, $x_m = 7.3$ V, $\tau = 0$, and $a = 6850$ Hz.

IV. SIGNAL-TO-NOISE RATIO

The adiabatic approach of Sec. III fails to reproduce the SR behavior of $C_T(D)$. However, an interesting characterization of SR has been introduced recently,[2] which can be studied by having recourse to our adiabatic picture of the process under investigation, namely, the signal-to-noise ratio (SNR). The intensity S of the δ-function-like spike showing up in the Fourier transform of the signal ACF in correspondence with its oscillating tail can be easily determined within the adiabatic approach, i.e.,

$$S(\nu_\Omega) = \frac{C_T}{\Delta \nu} , \tag{4.1}$$

where $\Delta\nu$ is the finite bandwidth actually employed in the procedure of frequency analysis. On the other hand, the background of the $x(t)$ spectrum, $B(\nu)$, is closely reproduced by the power spectrum of the unperturbed process. Under the condition that at long times the unperturbed relaxation dynamics in the potential $V(x)$ is dominated by the hopping mechanism with rate μ_K, the background $B(\nu)$ for the relevant stationary process can be determined through standard techniques[5]

$$B(\nu) = \frac{2\mu_K}{(\pi\nu)^2 + (\mu_K)^2} . \tag{4.2}$$

Such an assumption is certainly tenable in the limit of low forcing frequency $\nu_\Omega \ll \mu_K$ and small perturbation, $Ax_m \ll D$. On taking the limit $\nu_\Omega \rightarrow 0$ of (4.2), our estimate for SNR, $R(D)$, follows immediately:

$$R(D) = \frac{C_T(D)}{\Delta\nu} \mu_K(D) . \tag{4.3}$$

It is clear, now, why our adiabatic approximation (4.3) for $R(D)$ reproduces, indeed, the typical SR behavior for a wide class of physical systems. For large values of D, $R(D)$ tends to zero due to the fact that, in the cases we consider here, $C_T(D)$ vanishes with increasing D faster than $\mu_K(D)$ increases. In the limit of small D values, instead, $\mu_K(D)$ vanishes, whereas $C_T(D)$ approaches a constant value (see Fig. 5). We know that the adiabatic determination of $C_T(D)$, (3.5), is not tenable for small noise intensity where $\mu_K < \nu_\Omega$; however, the drop of $R(D)$ for D tending to zero is apparently dominated by the Arrhenius factor in $\mu_K(D)$. For instance, on comparing Fig. 5 with Fig. 1 in I, we learn that the adiabatic prediction for $C_T(D)$ reproduces fairly closely the decreasing branch of the relevant simulation curve. Lowering the forcing-frequency shifts the peak of $C_T(D)$ towards smaller D values. On the other hand, every experimental evidence (Sec. II) locates the SR peak at $D/\Delta V \simeq 0.5$, irrespective of the value of the forcing frequency and the intensity of the perturbation. This argument comforts our attempt at interpreting the features of SR under diverse physical conditions by means of the adiabatic prediction (4.3).

Let us consider the response of the system (1.1)–(1.5) with $h(x)=x$ and $\tau\rightarrow 0$ at varying the damping constant.[1] In Fig. 6 we display the dependence of R on $D/\Delta V$ for three values of γ, measured by means of our analogue simulator. The curves peak at about the same noise intensity, whereas the magnitude of the SNR strongly depends on γ. It is immediately clear that such a dependence is consistent with our prediction (4.3). On increasing γ, the maximum value of $R(D)$, $R_{\max}$, increases at small damping and decreases at large damping. The very same behavior is exhibited by the Kramers rate μ_K, which is known to be proportional to γ for $\gamma \ll \sqrt{2a}$ and to the reciprocal of γ for $\gamma \gg \sqrt{2a}$.[7] We checked that $R_{\max}$ reaches its maximum value for the same γ value where μ_K is maximum, too.

Another check of the adiabatic approach is provided by the dependence of R on the noise correlation time τ, (1.2), at fixed damping constant. In Fig. 7 we show that for τ small the profile of the curve $R(D)$ does not change, while $R_{\max}$ decreases with increasing τ. For τ large, instead, the peak of $R(D)$ shifts toward larger D values. A similar behavior has been observed in the overdamped version of the same process, (1.6)–(1.7). In both cases the results of analogue simulation are in good agreement with our estimate of R, (4.3). Let us consider, for instance, the overdamped case of Fig. 8. We know that for $a\tau \ll D/\Delta V$, μ_K is proportional to τ, i.e., $\mu_K(\tau) \simeq \mu_K(0)(1-\frac{3}{2}a\tau)$. For $a\tau > 1$, instead, μ_K decreases exponentially with τ according to a phenomenological law $\mu_K(\tau) \simeq \mu_K(0)\exp(-\Delta V/D\frac{8}{27}a\tau)$.[8] The extra D dependence of μ_K implied by the strong color regime is responsible for the shift of the SR peak to larger values of the noise intensity. Finally, we checked that (4.3) describes the τ dependence of $R(D)$, quantitatively, as well, in both regimes of small and larger τ.

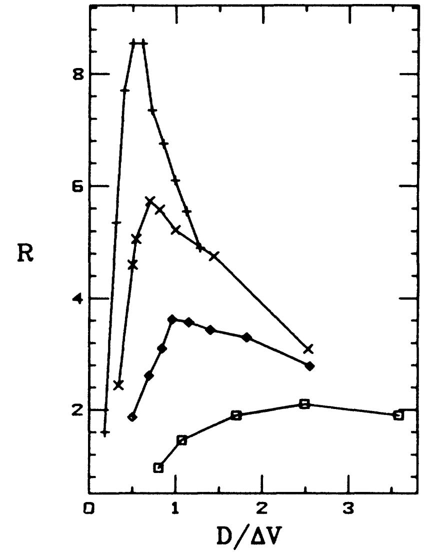

FIG. 7. SNR vs $D/\Delta V$ for different values of the noise correlation-time τ: $\tau=50$ μs (pluses), $\tau=200$ μs (crosses), $\tau=440$ μs (lozenges), $\tau=1$ ms (squares). Other parameter values are $\nu_\Omega=30$ Hz, $Ax_m=0.5\Delta V$, $x_m=7.3$ V, $\gamma=0.6\sqrt{2a}$, and $\sqrt{2a}=10.2$ kHz.

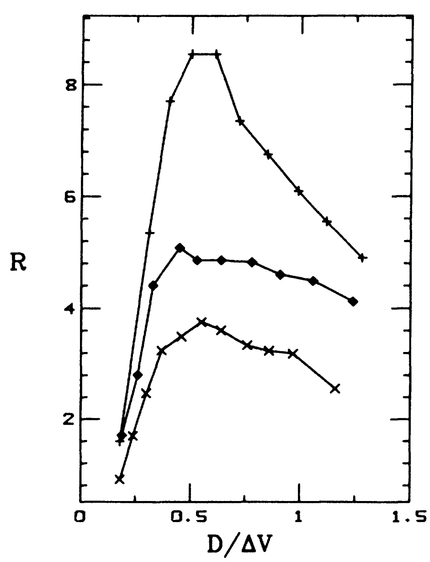

FIG. 6. SNR vs $D/\Delta V$ for three different values of the damping constant: $\gamma=0.6\sqrt{a}$ (pluses), $\gamma=0.06\sqrt{2a}$ (lozenges), $\gamma=2.05\sqrt{2a}$ (crosses). Other parameter values are $\nu_\Omega=30$ Hz, $Ax_m=0.5\Delta V$, $x_m=7.3$ V, $\tau=50$ μs, and $\sqrt{2a}=10.2$ kHz.

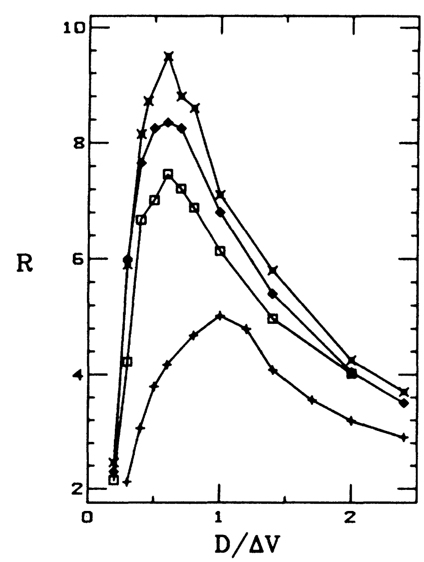

FIG. 8. SNR vs $D/\Delta V$ for different values of the noise correlation-time τ in the overdamped case (1.6)–(1.7): $\tau=30$ μs (crosses), $\tau=50$ μs (lozenges), $\tau=100$ μs (squares), $\tau=200$ μs (pluses). Other parameter values are $\nu_\Omega=30$ Hz, $Ax_m=0.5\Delta V$, $x_m=7.3$ V, and $a=6850$ Hz.

V. CONCLUSIONS

We have characterized the SR as an adiabatic mechanism which takes place in a bistable system modulated by a periodic perturbation with forcing frequency comparable with the escape rate in the unperturbed process. SR is well described in terms of the amplitude of the oscillating tail of the signal ACF for a variety of bistable systems. Unfortunately, the adiabatic approach does not allow a correct determination of this observable. An experimentally well motivated, but somewhat more involved, signature of SR is provided by the signal-to-noise ratio. The scarce dependence of this quantity on the forcing frequency is intrinsic to its definition. For the same reason its dependence on the various parameters may be explained within the adiabatic picture. In fact, the signal-to-noise ratio relates the diffusive properties of the unperturbed system (noise) with its modulation in the presence of periodic forcing (signal). At the present stage of our analysis we conclude that a complete account of the SR properties of a bistable system is provided by the asymptotic behavior of the signal ACF.

ACKNOWLEDGMENTS

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