Exercise 1:

A charged pion π^+ moves with velocity $\beta = 0.73$ in the laboratory system of reference.

1. Determine the lifetime in the laboratory, knowing that the proper lifetime $\tau_0 = 2.6 \times 10^{-8}$.
2. What is the average distance pions make in the laboratory?
3. What it would have been without relativistic effects?
4. How many pions survive after 10 m?

Solution:

1. $\tau = \gamma \cdot \tau_0 = \frac{1}{sqrt 1-\beta^2} = 1.463\tau_0 = 38$ ns
2. $d = v \cdot \tau = \beta c \cdot \gamma\tau_0 = 8.3$ m
3. $d_{classical} = v \cdot \tau_0 = 5.7$ m
4. $\frac{N(t)}{N_0} = e^{-t/\gamma\tau_0} = e^{-L/\beta c\gamma\tau_0} = 30\%$

Exercise 2:

A cylinder rotates about its axis with angular velocity ω. Show that for an observer who moves along the axis of the cylinder with a speed V the cylinder will appear twisted and find the torsion magnitude per unit length of the cylinder.

Solution:

Let us imagine a set of points along the cylinder generatrix. Each cross-section of the cylinder is a clock where the corresponding point plays the role of the end of the clock's arrow. In the frame S, these clocks are synchronized, that is, all of them show, for example, 12:00. In the frame S', however, they are not synchronized, and when the clock on the far left end of the cylinder shows 12:00 other clocks along the cylinder show a different time. Consequently, our points no longer lie on one line in the frame S at any given instant of this frame. This indicates that the cylinder in the frame S' must be twisted.

The position of a point in the cylinder surface is defined, in the S frame, by an angle $\varphi = \omega t$. Since the transverse coordinates do not change, transverse angles do not change. But time changes, hence we can write:

$$\varphi' = \varphi = \omega t = \omega\gamma(t' + \frac{\beta}{c}x') \tag{1}$$

For two points x'_1 and x'_2 observed at the same time t', with $\Delta x' = x'_2 - x'_1$:

$$\Delta\varphi' = \omega\gamma\frac{V}{c^2}\Delta x' \tag{2}$$

and hence the twist per unit length is:

$$\frac{\Delta\varphi'}{\Delta x'} = \omega\gamma\frac{V}{c^2} \tag{3}$$