

SCRIVIAMO LE COORDINATE DEL PUNTO

MATERIALE  
DI  
MASSA  
 $m$ .

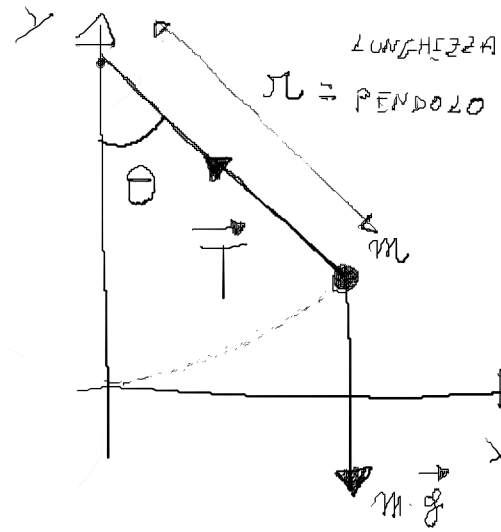
# PENDOLO SEMPLICE

$$\begin{cases} x = r \cdot \sin \theta \\ y = r - r \cos \theta \end{cases}$$

$\Downarrow$

$$\begin{cases} \frac{dx}{dt} = r \cos \theta \cdot \frac{d\theta}{dt} \\ \frac{dy}{dt} = r \sin \theta \cdot \frac{d\theta}{dt} \end{cases} \Rightarrow \begin{cases} \frac{d^2x}{dt^2} = -r \sin \theta \left( \frac{d\theta}{dt} \right)^2 + r \cos \theta \frac{d^2\theta}{dt^2} = a_x \\ \frac{d^2y}{dt^2} = +r \cos \theta \left( \frac{d\theta}{dt} \right)^2 + r \sin \theta \frac{d^2\theta}{dt^2} = a_y \end{cases}$$

$$\begin{cases} F_x = m a_x = -T \sin \theta \quad [1] \\ F_y = m a_y = -mg + T \cos \theta \quad [2] \end{cases}$$



$$[1]: m \cdot \left( -r \sin \theta \left( \frac{d\theta}{dt} \right)^2 + r \cos \theta \frac{d^2\theta}{dt^2} \right) = -T \sin \theta$$

$$\Rightarrow T = \frac{m}{\sin \theta} \cdot \left( r \sin \theta \left( \frac{d\theta}{dt} \right)^2 - r \cos \theta \frac{d^2\theta}{dt^2} \right)$$

$$[2]: m \cdot \left( r \cos \theta \left( \frac{d\theta}{dt} \right)^2 + r \sin \theta \frac{d^2\theta}{dt^2} \right) = -mg + \frac{m}{\sin \theta} \cdot \left( r \sin \theta \left( \frac{d\theta}{dt} \right)^2 - r \cos \theta \frac{d^2\theta}{dt^2} \right) \cos \theta$$

$$\Rightarrow r \cos \theta \left( \frac{d\theta}{dt} \right)^2 + r \sin \theta \frac{d^2\theta}{dt^2} = -g + r \cos \theta \left( \frac{d\theta}{dt} \right)^2 - r \frac{\cos^2 \theta}{\sin \theta} \frac{d^2\theta}{dt^2}$$

Moltiplichiamo per  $\sin \theta$ :  $r \sin^2 \theta \frac{d^2\theta}{dt^2} = -g \sin \theta - r \cos^2 \theta \frac{d^2\theta}{dt^2}$

$$\Rightarrow -g \sin \theta = r \sin^2 \theta \frac{d^2\theta}{dt^2} + r (1 - \sin^2 \theta) \frac{d^2\theta}{dt^2} \Rightarrow \boxed{-g \sin \theta = r \cdot \frac{d^2\theta}{dt^2}}$$

Per angoli "piccoli" ( $\sin \theta \approx \theta$ ):

$$\boxed{-g \theta = r \cdot \frac{d^2\theta}{dt^2}}$$

MOTO ARMONICO con PULSAZIONE:

$$\Rightarrow \omega = \sqrt{\frac{g}{r}}$$