## Summarizing

- $N_{\text {cand }}$ : poissonian process $\boldsymbol{\rightarrow}$ the higher the better
- $\varepsilon$ : binomial process $\rightarrow$ high $N_{\text {gen }}$ and high $\boldsymbol{\varepsilon}$
- $N_{b}$ : normalized $\approx$ poissonian process $\rightarrow$ high $R$ and high $N_{g e n}$, low $N_{\text {exp }}$
- Moreover: unfortunately efficiency and background cannot be both improved simultaneously...


## Efficiency vs. background

What happens if I move the cut ?


## Efficiency-background relation

Example: selection of b-jets in ATLAS.
"b-jet" is the signal;
"light jet" is the background.
MC samples of $\boldsymbol{b}$-jets and light-jets Application of 5 different selection recipes each with a "free-parameter".
For each point I evaluate

- b-jet efficiency

$$
=\mathrm{N}_{\text {sel }} / \mathrm{N}_{\text {gen }}(\mathrm{b} \text {-jet sample })
$$

- light-jet rejection

$$
=\mathrm{N}_{\text {gen }} / \mathrm{N}_{\text {sel }}(\text { light-jet sample })
$$

Choice of a working point, "compromise".


Unlucky situation: if you gain in efficiency you increase your bckg and viceversa...

## Combining uncertainties

- Given the uncertainties on $\boldsymbol{N}_{\text {cand }}, \boldsymbol{\varepsilon}$ and $\boldsymbol{N}_{b}$, how can we estimate the uncertainty on $\boldsymbol{N}_{X}$ ?
$-\rightarrow$ Uncertainty Propagation. General formulation

$$
\left(\frac{\sigma\left(N_{X}\right)}{N_{X}}\right)^{2}=\left(\frac{\sigma(\varepsilon)}{\varepsilon}\right)^{2}+\frac{\sigma^{2}\left(N_{\text {cand }}\right)+\sigma^{2}\left(N_{b}\right)}{\left(N_{\text {cand }}-N_{b}\right)^{2}}
$$

Assumption: three indipendent contributions NB: if $\boldsymbol{N}_{\text {cand }} \approx \boldsymbol{N}_{\boldsymbol{b}}$ the relative uncertainty becomes very large (the Formula cannot be applied anymore...)
Can we say we have really observed a signal ???
Or we are simply observing some fluctuation of the background ?
3.2. Cut-based selection. The most natural way to proceed is to apply cuts. We find among the physical quantities of each event those that are more "discriminant" and we apply cuts on these variables or on combinations of these variables. The selection procedure is a sequence of cuts, and is typically well described by tables or plots that are called "Cut-Flows". An example of cut-flow is shown in Table 1. The choice of each single cut is motivated by the shape of the MC signal and background distributions in the different variables. From the cut-flow shown in Table 1 we get: $\epsilon=2240 / 11763=$

Table 1. Example of cut-flow. The selection of $\eta \pi^{0} \gamma$ final state with $\eta \rightarrow \pi^{+} \pi^{-} \pi^{0}$ from $e^{+} e^{-}$collisions at the $\phi$ peak $(\sqrt{s}=1019 \mathrm{MeV}$, is based on the list of cuts given in the first column. The number of surviving events after each cut is shown in the different columns for the MC signal (column 2) and for the main MC backgrounds (other columns). (taken from D. Leone, thesis, Sapienza University A.A. 2000-2001).

| Cut | $\eta \pi^{0} \gamma$ | $\omega \pi^{0}$ | $\eta \gamma$ | $K_{S} \rightarrow$ neutrals | $K_{S} \rightarrow$ charged |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Generated Events | 11763 | 33000 | 95000 | 96921 | 112335 |
| Event Classification | 6482 | 17602 | 55813 | 18815 | 14711 |
| 2 tracks +5 photons | 3112 | 724 | 110 | 371 | 3100 |
| $E_{\text {tot }}-\left\\|\vec{P}_{\text {tot }}\right\\|$ | 2976 | 539 | 39 | 118 | 1171 |
| Kinematic fit I | 2714 | 236 | 5 | 24 | 66 |
| Combinations | 2649 | 129 | 1 | 19 | 0 |
| Kinematic fit II | 2247 | 2 | 0 | 1 | 0 |
| $E_{\text {rad }}>20 \mathrm{MeV}$ | 2240 | 1 | 0 | 0 | 0 |

$(19.04 \pm 0.36) \%^{3}$ and $R=33000$ for $\omega \pi^{0}$. For the other background channels only a lower limit on $R$ can be given, since in the end no events pass the selection.

1. A charged kaon $\left(K^{+}\right)$beam is produced with a rate of $1.2 \times 10^{2} \mathrm{~Hz}$. Our detector takes data for $\Delta \mathrm{t}=24$ hours and aims to count the total number of decays $K^{+} \rightarrow e^{+} \nu_{e}$. The efficiency of our detector for this final state is $\epsilon=63.2 \%$ with negligible uncertainty. Evaluate the minimum value of the rejection power needed for the $K^{+} \rightarrow \mu^{+} \nu_{\mu}$ decay if we want to maintain the uncertainty on $\mathrm{N}\left(K^{+} \rightarrow e^{+} \nu_{e}\right)$ below $15 \%$ (neglect other possible backgrounds and the uncertainties on background).
2. In an $e^{+} e^{-}$experiment at a center of mass energy $\sqrt{s}=1.5 \mathrm{GeV}$, we aim to count the number of $e^{+} e^{-} \rightarrow K^{+} K^{-}$final states. At the end of the experiment, after the selection, we get $N_{\text {cand }}=136$. We estimate the background to be $N_{b}=13.2 \pm 0.9$. The selection efficiency is obtained by selecting 5922 events from a sample of $10^{4}$ Montecarlo simulated $e^{+} e^{-} \rightarrow K^{+} K^{-}$final states. Calculate $\mathrm{N}\left(e^{+} e^{-} \rightarrow K^{+} K^{-}\right)$ with its uncertainty. What is the dominant contribution to the uncertainty? How many st.dev. is the signal from 0 ?

Data l'espressione:

$$
\frac{\left|\eta_{+-}\right|^{2}}{\left|\eta_{00}\right|^{2}}=\frac{\left\lceil\frac{\left.B R\left(K_{L} \rightarrow \pi^{+} \pi^{-}\right)\right\rceil}{\left[B R\left(K_{S} \rightarrow \pi^{+} \pi^{-}\right)\right.}\right\rceil}{\left\lfloor\frac{\left.B R\left(K_{L} \rightarrow \pi^{0} \pi^{0}\right)\right\rceil}{\left[B R\left(K_{S} \rightarrow \pi^{0} \pi^{0}\right)\right.}\right\rfloor 1+6 \Re e\left(\frac{\varepsilon^{\prime}}{\varepsilon}\right)}
$$

Dimostrare che :

$$
\delta \Re e\left(\frac{\varepsilon^{\prime}}{\varepsilon}\right)_{s t a t}=\frac{1}{6} \frac{1}{\sqrt{(2 / 3) N_{L}^{0}}}
$$

con $\mathrm{N}_{\mathrm{L}}^{0}$ numero di conteggi $\mathrm{K}_{\mathrm{L}}->\pi^{0} \pi^{0}$.
In quale approssimazione vale la formula?

La relazione fra $B R_{S, L}^{ \pm 0}$ e $N_{S, L}^{ \pm, 0}$ è data da:

$$
N_{S, L}^{ \pm 0}=N_{S, L}^{ \pm 0}(o b s)-B c k_{s, L}^{ \pm, 0}=N_{K K} \cdot \rho_{S, L}(t a g) \cdot B R_{S, L}^{ \pm, 0} \cdot\left\langle\rho_{S, L}^{ \pm, 0}\right\rangle \cdot \iint_{F V} g\left(l-l^{\prime}\right) I(l) d l d l^{\prime}
$$

dove:

- $N_{S, L}^{ \pm, 0}(o b s)$ e' il numero effettivamente osservato di decadimenti $\pi^{+} \pi^{-}, \pi^{0} \pi^{0}$;
- $N_{K K}$ e' il numero totale di coppie $\mathrm{K}_{\mathrm{S}}, \mathrm{K}_{\mathrm{L}}$ prodotte;
- $\rho_{S, L}(t a g)$ e' l'efficienza di identificazione;
- $B R_{s, L}^{ \pm 0} \mathrm{e}^{\prime}$ il "branching ratio" corrispondente al decadimento $K_{S, L} \rightarrow \pi^{+} \pi^{-}, \pi^{0} \pi^{0}$;
$-\left\langle\rho_{S, L}^{ \pm .0}\right\rangle \mathrm{e}^{\prime}$ l'efficienza media di rivelazione dei decadimenti $K_{S, L} \rightarrow \pi^{+} \pi^{-}, \pi^{0} \pi^{0}$;
$-\iint_{F V} g\left(l-l^{\prime}\right) I(l) d l d l^{\prime}$ rappresenta la convoluzione dell'intensita' $I(l)=e^{-l_{s L}}$ dei
decadimenti con la risoluzione sperimentale $\mathrm{g}\left(1-\mathrm{l}^{\prime}\right)$ sul cammino di decadimento 1 , integrata sul volume fiduciale del rivelatore.
- $B c k_{s, L}^{ \pm 0}$ e' il contributo degli eventi di fondo.

$$
N_{L}^{0}=\underbrace{3 \mu \mathcal{L}}_{\sigma_{e^{+} e^{-\rightarrow \phi}}^{3 \mu b}} \cdot \underbrace{0.66}_{\int L d t} \cdot \underbrace{0.6 R\left(\phi \rightarrow K_{S} K_{L}\right)}_{\rho_{L}(t a g)} \underbrace{0.34}_{B R_{L}^{0}} \cdot \underbrace{10^{-3}}_{\text {fiducial volume }} \cdot\left(e^{\left.-3 q_{350}-e^{-15 q_{350}}\right)}\right.
$$

Data una luminosita' integrata di $\mathcal{L}=10^{4} \mathrm{pb}^{-1}$ qual'e' il fattore di reiezione del fondo $\mathrm{K}_{\mathrm{L}}{ }^{->} 3 \pi^{0}$ (sul segnale $\mathrm{K}_{\mathrm{L}^{-}}>2 \pi^{0}$ ) necessario per avere un errore su $\operatorname{Re}\left(\varepsilon^{\prime} / \varepsilon\right)<3 \times 10^{-4}$ assumendo di conoscere il fondo con una precisione del $20 \%$ ?

