Other Proposed exercises

We perform a cross-section measurement and obtain the following values: N_{cand} =128, $N_b = 14 \pm 2$, $\epsilon = 0.523 \pm 0.002$, $L_{int} = 2.43$ pb$^{-1} \pm 1.8$ %: calculate the resulting cross-section with its uncertainty. In case this is a measurement of $e^+e^- \to \pi^+\pi^-$ at $\sqrt{s} = 1$ GeV, determine the value of the pion time-like form factor with its uncertainty. The formula relating the cross-section to the form factor $F_\pi(s)$ is the following:

$$\sigma(s) = \frac{\pi\alpha^2}{3s}\beta_\pi^3|F_\pi(s)|^2$$

Consider the reaction $e^+e^- \to K^+K^-$ at a Φ-factory. Which fraction of events have at least one kaon decaying within a sphere of R = 20 cm ? In which fraction of events both kaons decay within the same sphere ?

The SM expected semi-leptonic K_S charge asymmetry is 3×10^{-3}. At Dafne we expect to produce a sample of 1.2×10^9 tagged K_Ss. If the BR($K_S \to \pi e\nu$)=BR($K_S \to \pi^+e^-\overline{\nu}$)+BR($K_S \to \pi^-e^+\nu$)=$6.95\times10^{-4}$ which error can we reach on the asymmetry ?

Which average instantaneous luminosity is required to improve by a factor 3 such an uncertainty in one year of data taking (assuming a duty cicle of 50% and a tagging efficiency of 30%) ? [$\sigma(e^+e^- \to \phi)$= 3 μb at the ϕ peak].

Particle properties

- Once a particle has been identified (either directly or through its decay products), it is interesting to measure its properties:
  - Mass $\boldsymbol{M}$
  - Total Decay Width $\boldsymbol{\Gamma}$
  - LifeTime $\boldsymbol{\tau}$
  - Couplings $\boldsymbol{g}$
- If the particle is identified through its decay, all these parameters can be obtained through a dedicated analysis of the kinematics of its decay products.

Invariant Mass - I

- Suppose that a particle *X* decays to a number of particles (***N***), and assume we can measure the quadri-momenta of all them. We can evaluate the Invariant Mass of X for all the candidate events of our final sample:

$$M_{inv}^2 = \left(\sum_{k=1}^{N} \tilde{p}_k \right)^2$$

- It is a relativistically invariant quantity. In case of N = 2

$$M_{inv}^2 = m_1^2 + m_2^2 + 2\left(E_1E_2 - \vec{p}_1 \cdot \vec{p}_2\right)$$

- If N=2 and the masses are 0 or very small compared to p

$$M_{inv}^2 = 2E_1E_2\left(1 - \cos\theta\right) = E_1E_2 \sin^2 \theta/2$$

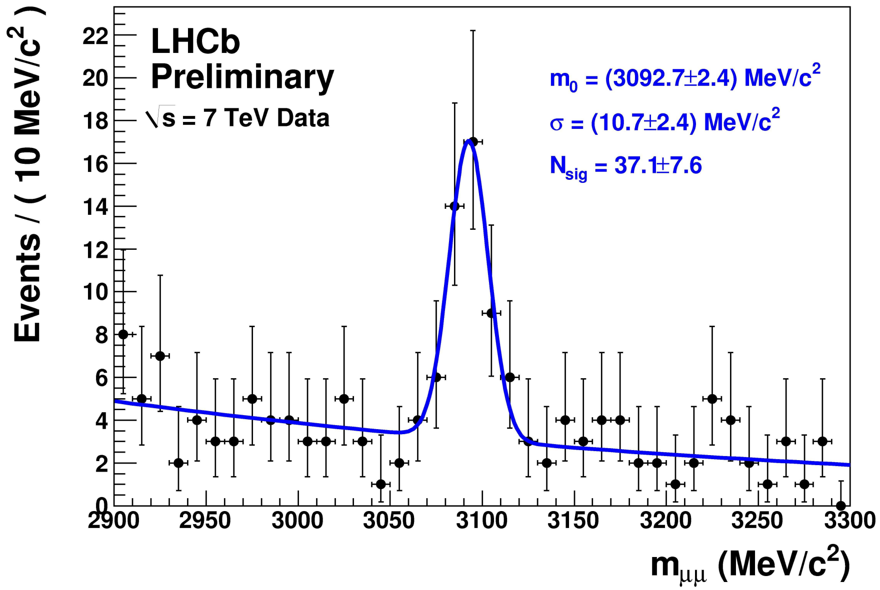
- Where $\boldsymbol{\theta}$ is the opening angle between the two daughter particles.

Invariant Mass - II

- Given the sample of candidates, we do the invariant mass distribution and we typically get a plot like that:

  - A peak (the signature of the particle)
  - A background (almost flat in this case) ➔ **unreducible** background.

$B^+ \Rightarrow J/\psi\ K^+$



- What information can we get from this plot (by fitting it) ?

(1) Mass of particle;

(2) Width of the particle (BUT not in this case…);

(3) Number of particles produced (related to $\boldsymbol{\sigma}$ or ***BR***)

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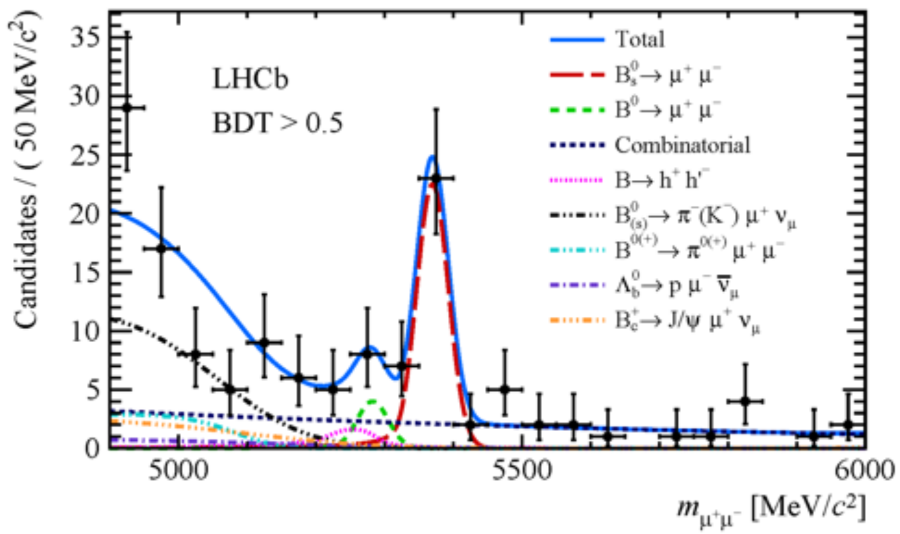

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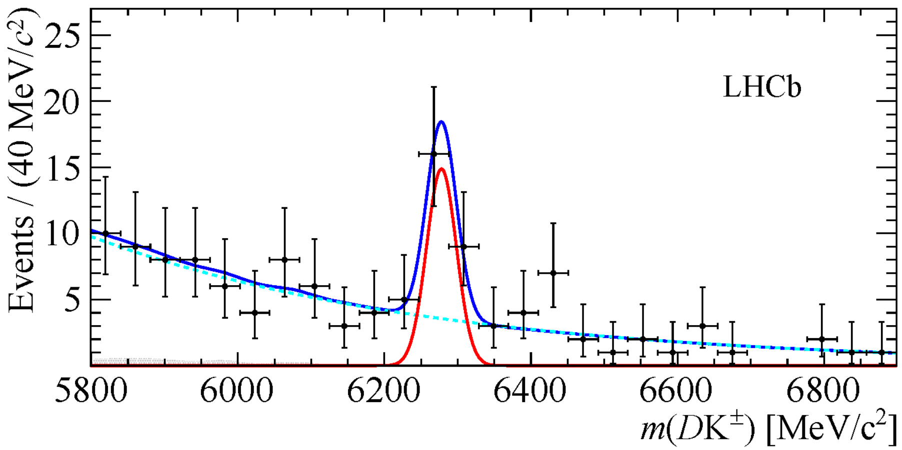

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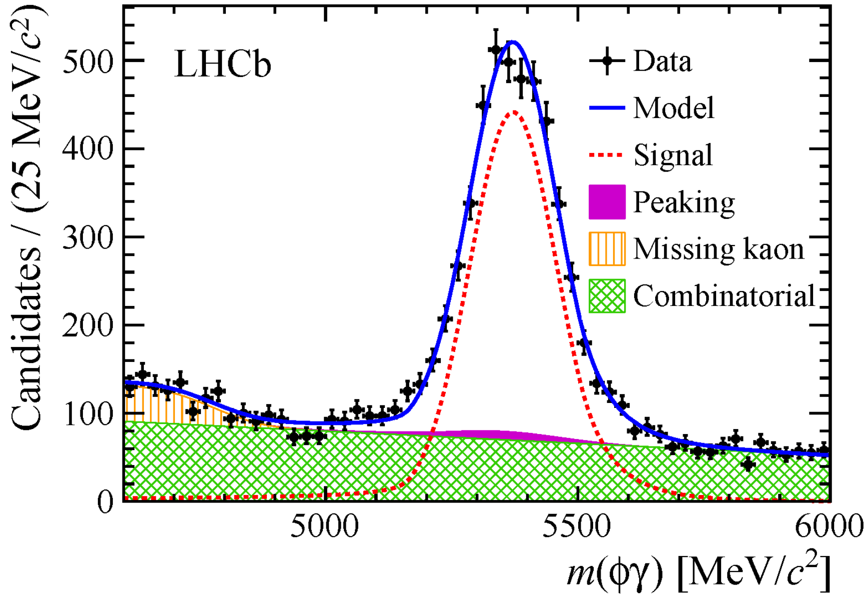

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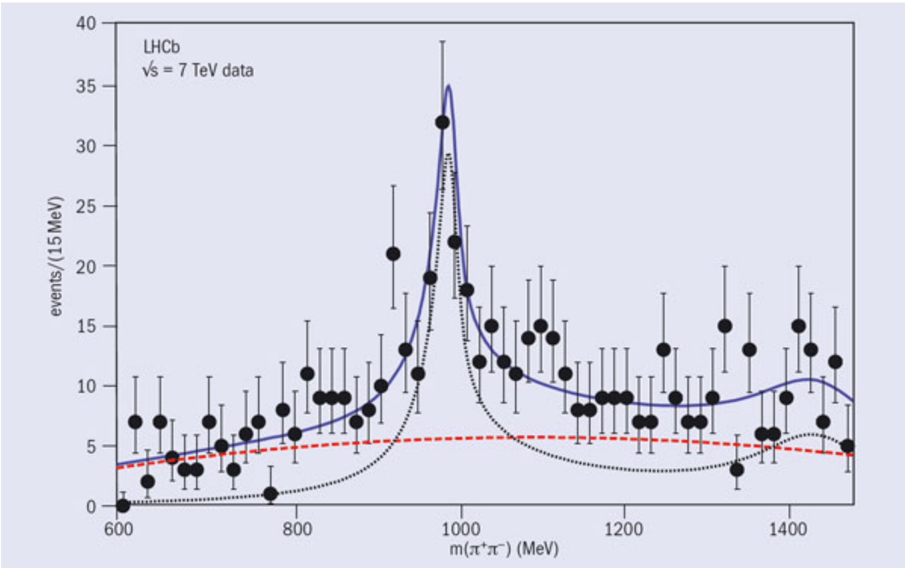
$B^0_s \to J/\psi\ f_0(980)$



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Suppose Poisson variable and n=0 is measured (no background) => 0±0 (freq) or 1±1 (Beyes) ?

Upper limit (lower limit =0)

By construction the probability to measure $x_0' < x_0$ if the true value $\mu=\mu_1(x_0)$ is $(1-\alpha)$ (only one limit) or the probability to measure $x_0' > x_0$ if the true value $\mu=\mu_1(x_0)$ is α

$$P(n>0/\lambda)=\sum_{n=1}^{\infty}\frac{\lambda^n e^{-\lambda}}{n!}=1-e^{-\lambda}=\alpha$$

frequentist

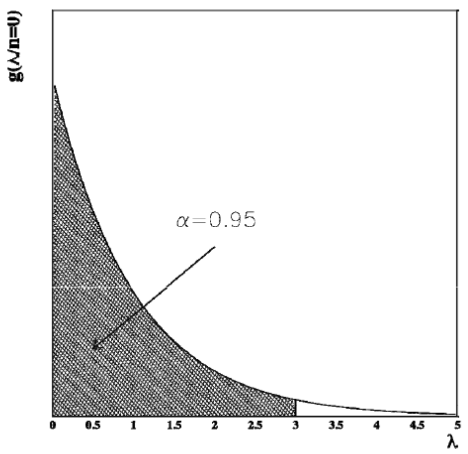
$$\bar{\lambda}=-\ln(1-\alpha)$$

$$g(\lambda/n=0)=\frac{p(n=0/\lambda)f_0(\lambda)}{\int_0^{\infty}p(n=0/\lambda)f_0(\lambda)d\lambda}=\frac{e^{-\lambda}}{\int_0^{\infty}e^{-\lambda}d\lambda}=e^{-\lambda}$$

Bayesian (uniform prior)

$$p(\lambda<\bar{\lambda})=\int_0^{\bar{\lambda}}e^{-\lambda}d\lambda=1-e^{-\bar{\lambda}}=\alpha$$

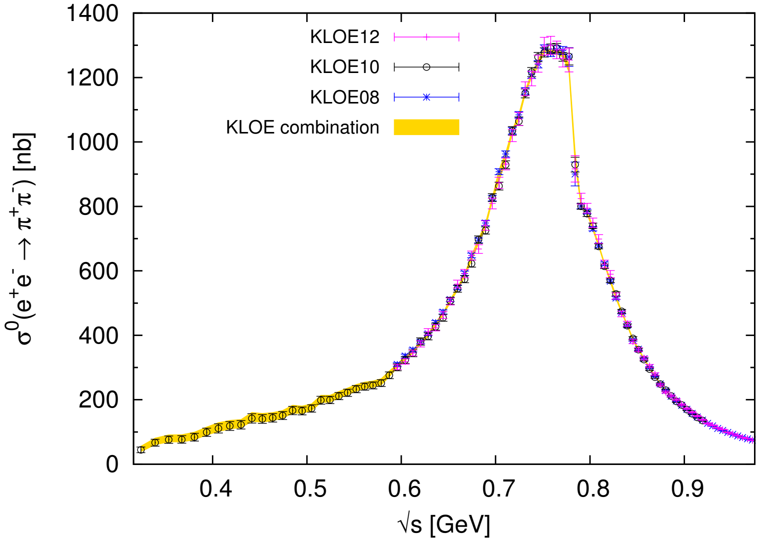
	90%	95%	99%
$\bar{\lambda}$	2.3	3.0	4.6

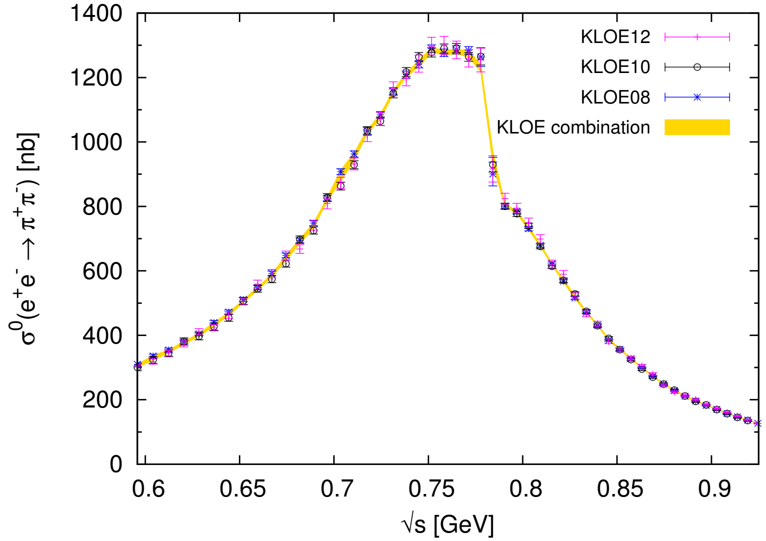


$\underline{\lambda}$ (68.3%)=1.15

Parenthesys: 2 kinds of background

- ***Unreducible background***: same final state as the signal, no way to disentangle. The only way to separate signal from unreducible background is to fit the inv.mass spectrum
- ***Reducible background***: a different final state that mimic the signal (e.g. because you are losing one or more particles, or because you are confusing the nature of one or more particles)
- Example:
  - Signal: $pp \rightarrow H \rightarrow ZZ^* \rightarrow 4l$
  - Unreducible background: $pp \rightarrow ZZ^* \rightarrow 4l$
  - Reducible backgrounds: $pp \rightarrow Zbb$ with $Z \rightarrow 2l$ and two leptons, one from each b-quark jet; $pp \rightarrow tt$ with each $t \rightarrow Wb \rightarrow l\nu$"l"j


1400
1200
1000
800
600
400
200
0
0.4
0.5
0.6
0.7
0.8
0.9
√s [GeV]
$\sigma^0(e^+e^- \to \pi^+\pi^-)$ [nb]
KLOE12
KLOE10
KLOE08
KLOE combination


1400
1200
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200
0
0.6
0.65
0.7
0.75
0.8
0.85
0.9
√s [GeV]
$\sigma^0(e^+e^- \to \pi^+\pi^-)$ [nb]
KLOE12
KLOE10
KLOE08
KLOE combination

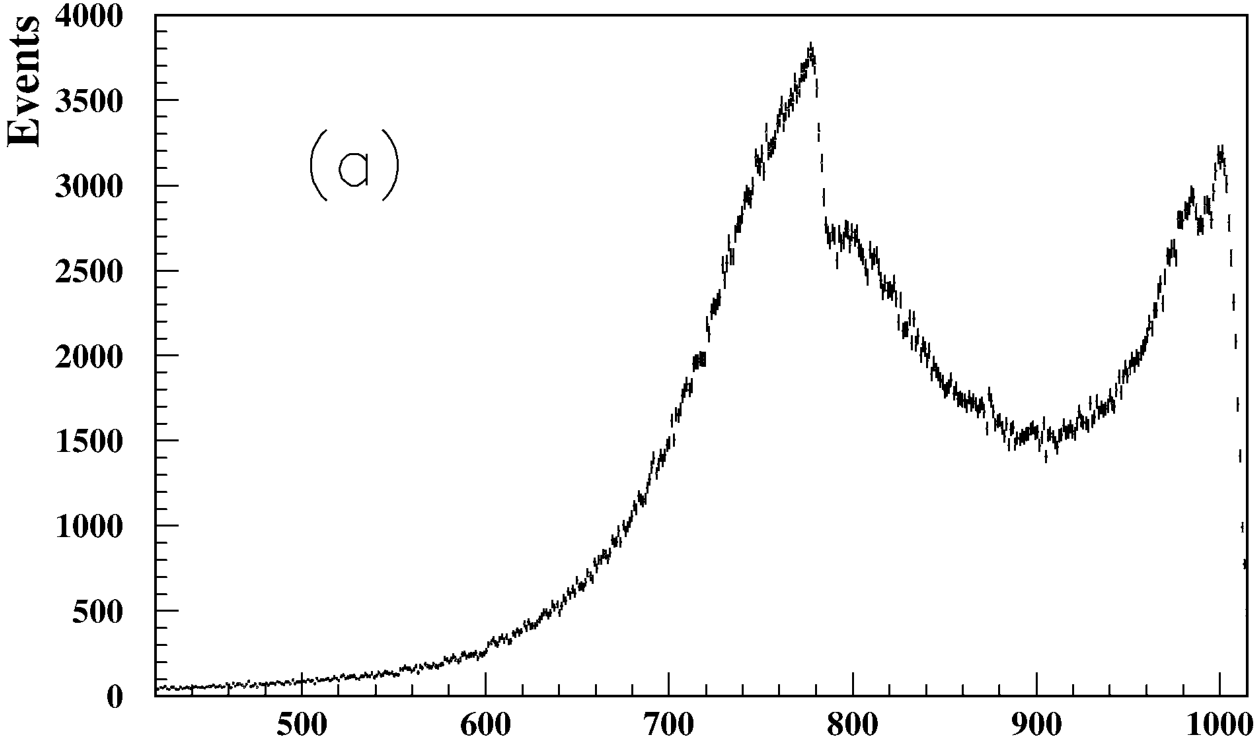
$f_0(500)$ mass 400-550 MeV width 400-700 MeV

$\rho(770)$ mass 775.26 MeV width 149.1 MeV

$\omega(782)$ mass 782.65 MeV width 8.49 MeV

$f_0(980)$ mass 990 MeV width 10-100 MeV

$e^+e^- \Rightarrow \pi^+\pi^- (\gamma)$



$M_{\pi\pi}$ (MeV)

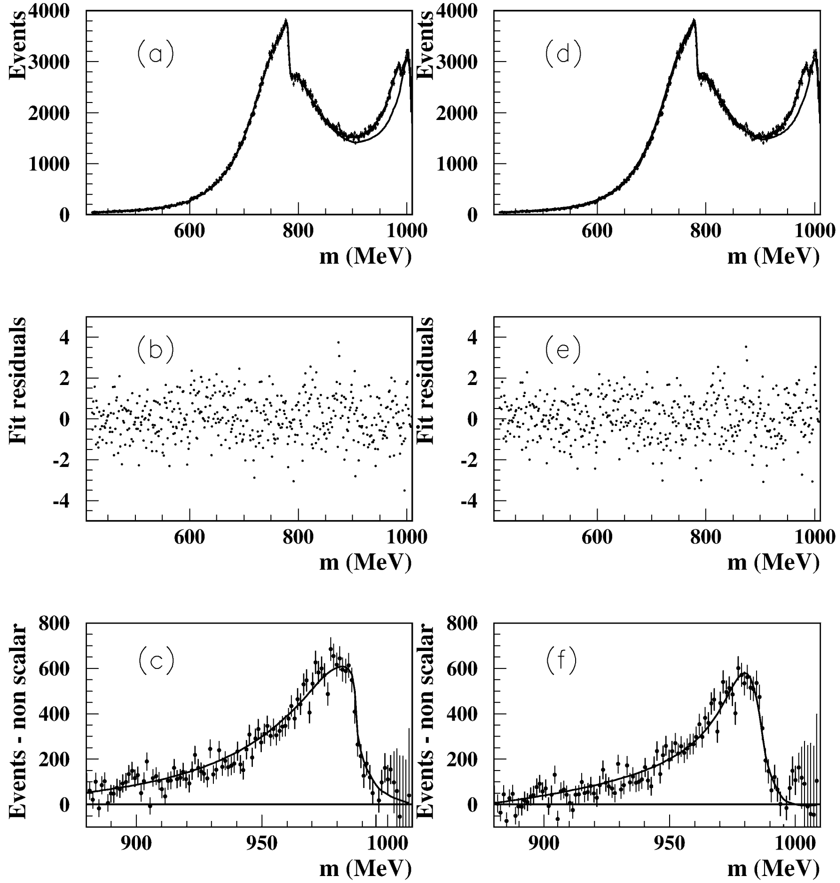


Fig. 3. Result of the KL fit (a)–(b)–(c) and of the NS fit (d)–(e)–(f). (a)–(d) Data spectrum compared with the fitting function (upper curve following the data points) and with the estimated non-scalar part of the function (lower curve); (b)–(e) fit residuals as a function of *m*; (c)–(f) the fitting function is compared to the spectrum obtained subtracting to the measured data the non-scalar part of the function in the f_0 region.

Mass and Width measurement

- Fit of the $\boldsymbol{M_{inv}}$ spectrum with a Breit-Wigner + a continuos background: BUT careful with mass resolution. It can be neglected only if $\sigma(M_{inv}) << \Gamma$
- If $\sigma(M_{inv}) \approx \Gamma$ or $\sigma(M_{inv}) > \Gamma$ there are two approaches (as we already know):
  - Folding: correct the theoretical distribution to be used in the fit:

$$\sigma_{fit}(E) = \int G_{res}(E - E_0)\,\sigma_{BW}(E_0)\,dE_0$$

  - Unfolding: correct the experimental data and fit with the theoretical function.
  - Use a gaussian (or a "Crystal Ball" function) neglecting completely the width.
- In many cases only the mass is accessible: the uncertainty on the mass is the one given by the fit (taking into account the statistics) + possible scale systematics.

Gaussian vs. Crystal Ball

- Gaussian: 3-parameters, A, μ, σ. Integral $=A\sigma\sqrt{2\pi}$

$$f(m / A,\mu,\sigma) = A\exp(-\frac{(m-\mu)^2}{2\sigma^2})$$

- Crystal-Ball: 5-parameters, $\underline{m}$, σ, α, n, N

$$f_{CB}(m,\bar{m},\sigma,\alpha,n) = N \cdot \begin{cases} e^{\frac{-(m-\bar{m})^2}{2\sigma^2}} & \text{per } \frac{m-\bar{m}}{\sigma} > -\alpha \\ A \cdot (B - \frac{m-\bar{m}}{\sigma})^{-n} & \text{per } \frac{m-\bar{m}}{\sigma} \leq -\alpha \end{cases}$$

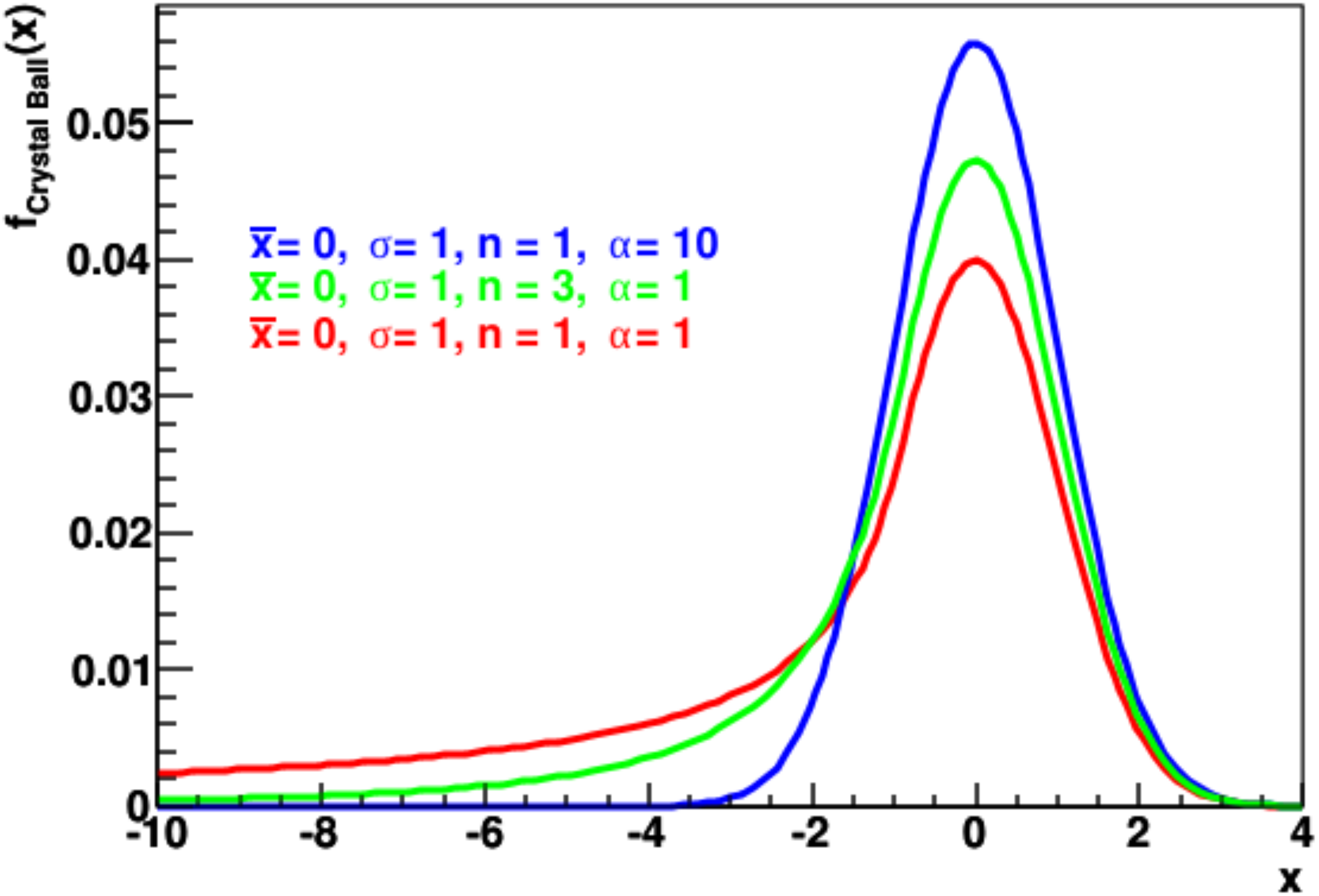
$$A = (\frac{n}{|\alpha|})^n e^{-\frac{\alpha^2}{2}},\ B = \frac{n}{|\alpha|} - |\alpha|$$

Essentially takes into account energy losses, useful in many cases.

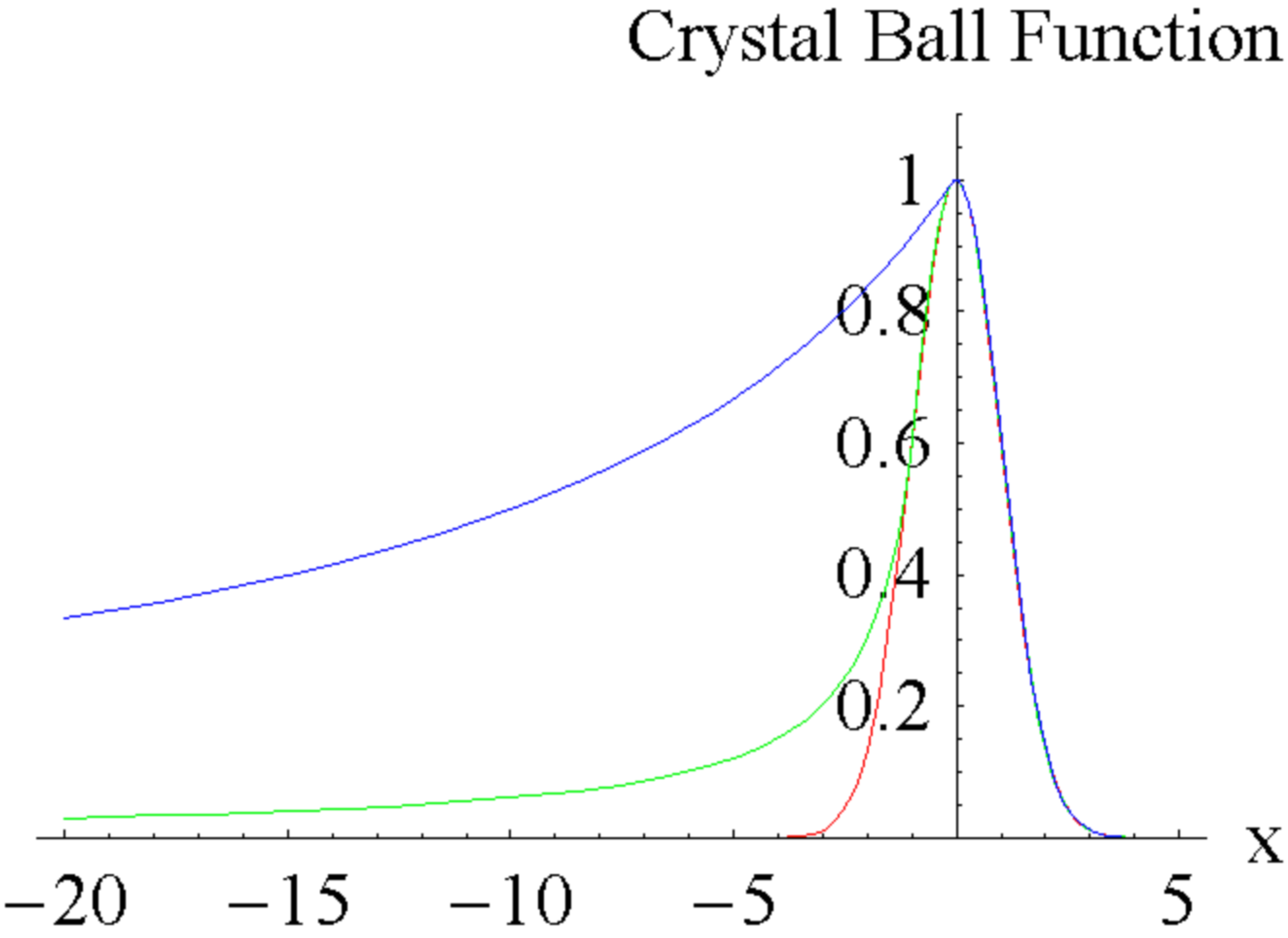
Crystal-Ball function and its first derivative are both continuos.

After Crystal-Ball collaboration, Crystal Ball hermetic NaI detector at SPEAR Stanford 1979 (then DESY, AGS-BNL, A2-Mainz Microtron…)

Crystal Ball function



Examples of the Crystal Ball function.



$\bar{x} = 0, \sigma = 1, N = 1$ Red: $\alpha = 10$, Green: $\alpha = 1$, Blue: $\alpha = 0.1$.

Template fits: not functions but histograms

In this case the fit is not done with a function with parameters BUT it is a "template" fit:

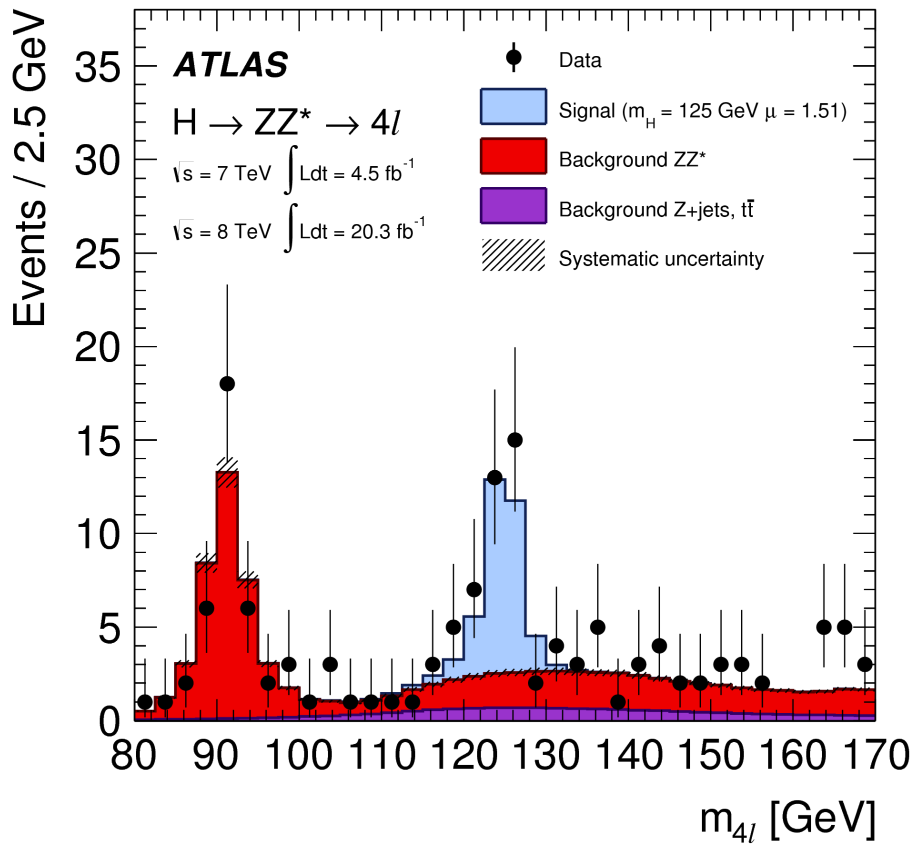
$$F = a\text{HIST1}(\boldsymbol{m_H}, \ldots) + \boldsymbol{b}\text{HIST2}$$

$\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{m_H}$ are free parameters

The method requires the knowledge (from MC) of the expected distributions. Such a knowledge improves our uncertainties.

NB: HIST1 and HIST2 take into account experimental resolution: so it is directly the folding method

An example: Higgs mass in the 4l channel.



Effect of the mass resolution on the significativity of a signal

- Let's consider now the case in which we look for a process and we expect a peak in a distribution at a definite mass: when may we say that we have observed that process ?
- Method of assessment: simple fit S+B (e.g. template fit).
  $S \pm \sigma(S)$ away from 0 at least 3 (5) standard deviations.
- Ingredients:
  - Mass resolution;
  - Background

$$\sigma^2(S) = \sigma^2(N) + \sigma^2(B) = N + \sigma^2(B)$$

neglecting $\sigma(B)$ $\approx N = S + B = S + 6\sigma_M b$

- Effect of mass resolution negligible on the uncertainty on S if:

$$S >> 6b\sigma_M \quad \Rightarrow \quad \sigma_M << \frac{S}{6b}$$

Background b= 50 /MeV in an interval of 60 MeV (+/- 3*10 MeV) B=b*60 MeV = 3000 (broad)
in an interval of 12 MeV (+/- 3*2 MeV) B=b*12 MeV = 600 (narrow)

Signal S=200

Significance = 3.5 (broad) and 7.1 (narrow)

S/6b = 0.67 MeV => in both cases σ_M <<S/6b not satisfied => resolution effect important and observation of the signal can be improved reducing the resolution

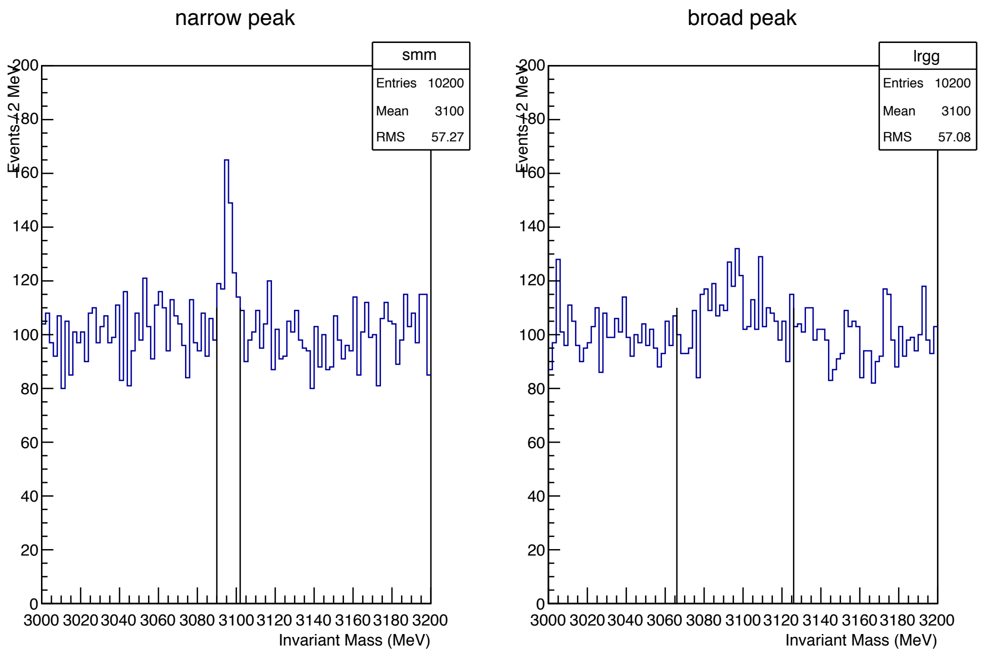


FIGURE 8. Simulation of $S = 200$ J/ψ events superimposed to a flat background of 10000 distributed on a range of 200 MeV (b=50 MeV−1). $\sigma_M = 2$ MeV (left) and $\sigma_M = 10$ MeV (right). The limits of $\pm 3\sigma_M$ intervals around the expected position of the peak are shown. Outside these limits are the sidebands.

H→γγ ATLAS: is the resolution negligible ?

Numbers directly from the plot:

$S \approx 1000$

$b \approx 5000/2$ GeV

$= 2500/\text{GeV}$

$\sigma_M \approx 10$ GeV$/6$

$= 1.7$ GeV

→ $S/6b$

$= 0.07$ GeV $<< \sigma_M$