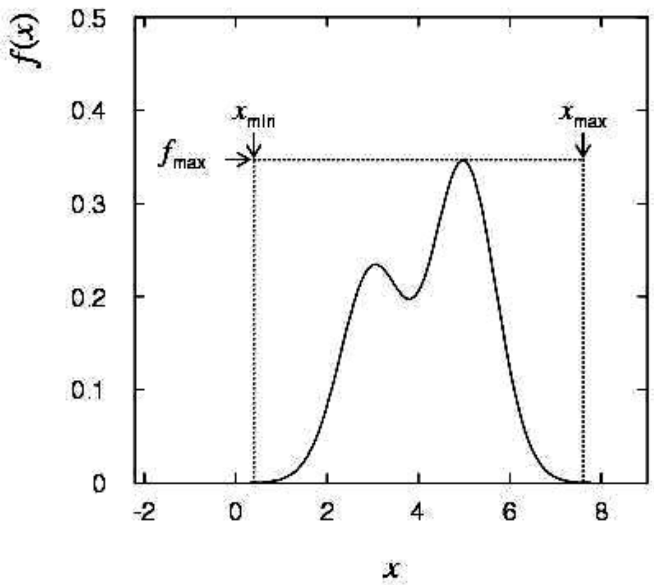
The acceptance-rejection method

Enclose the pdf in a box:



(1) Generate a random number x, uniform in $[x_{min}, x_{max}]$, i.e.

$x = x_{\text{min}} + r_1(x_{\text{max}} - x_{\text{min}})$, r_1 is uniform in [0,1].

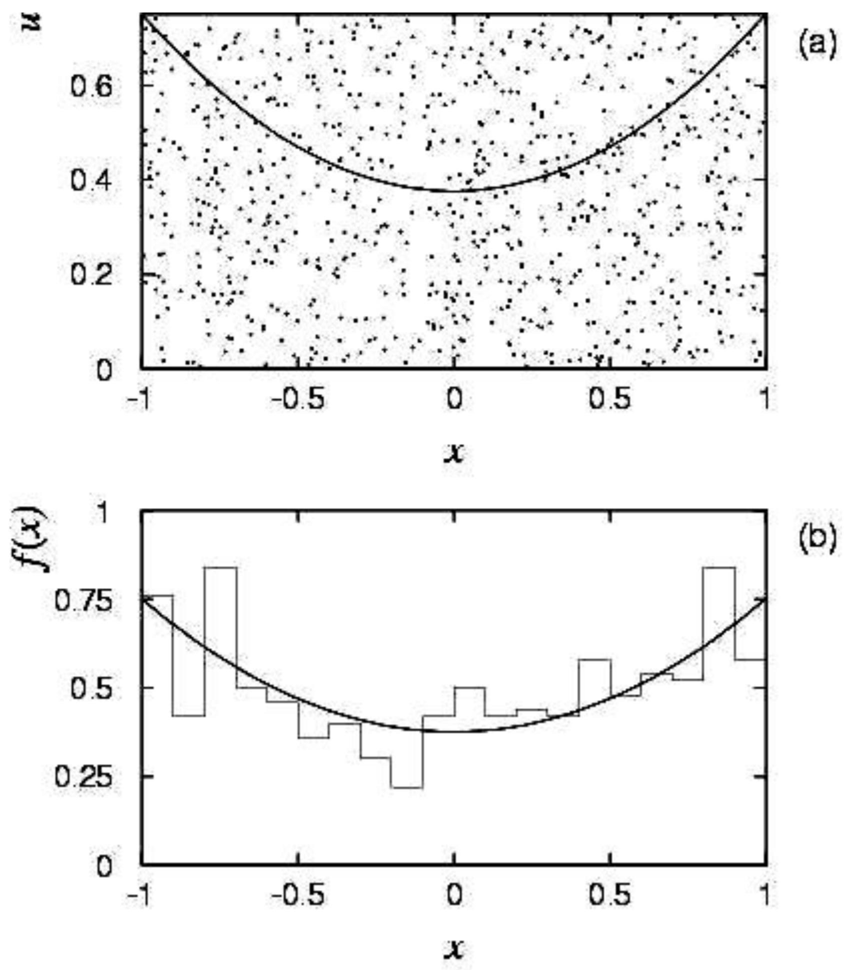
(2) Generate a 2nd independent random number u uniformly distributed between 0 and f_{max}, i.e. $u = r_2 f_{\text{max}}$.

(3) If $u < f(x)$, then accept x. If not, reject x and repeat.

Example with acceptance-rejection method

$$f(x) = \frac{3}{8}(1 + x^2)$$

$$(-1 \leq x \leq 1)$$



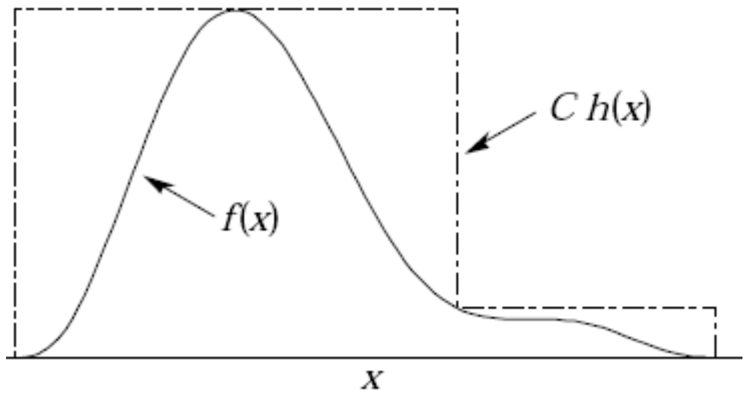
If dot below curve, use *x* value in histogram.

Improving efficiency of the acceptance-rejection method

The fraction of accepted points is equal to the fraction of the box's area under the curve.

For very peaked distributions, this may be very low and thus the algorithm may be slow.

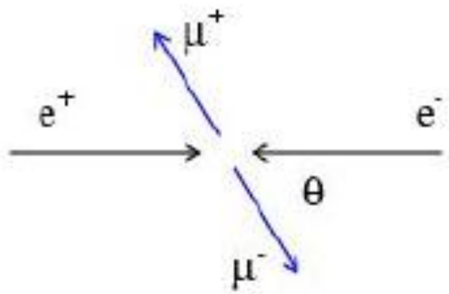
Improve by enclosing the pdf $f(x)$ in a curve $C\,h(x)$ that conforms to $f(x)$ more closely, where $h(x)$ is a pdf from which we can generate random values and C is a constant.



Generate points uniformly over $C\,h(x)$.

If point is below $f(x)$, accept x.

Monte Carlo event generators



Simple example: $e^+e^- \to \mu^+\mu^-$

Generate $\cos\theta$ and ϕ:

$$f(\cos\theta; A_{\mathrm{FB}}) \propto (1 + \frac{8}{3} A_{\mathrm{FB}} \cos\theta + \cos^2\theta) \, ,$$

$$g(\phi) = \frac{1}{2\pi} \quad (0 \le \phi \le 2\pi)$$

Less simple: ‘event generators’ for a variety of reactions:

$e^+e^- \to \mu^+\mu^-$, hadrons, ...

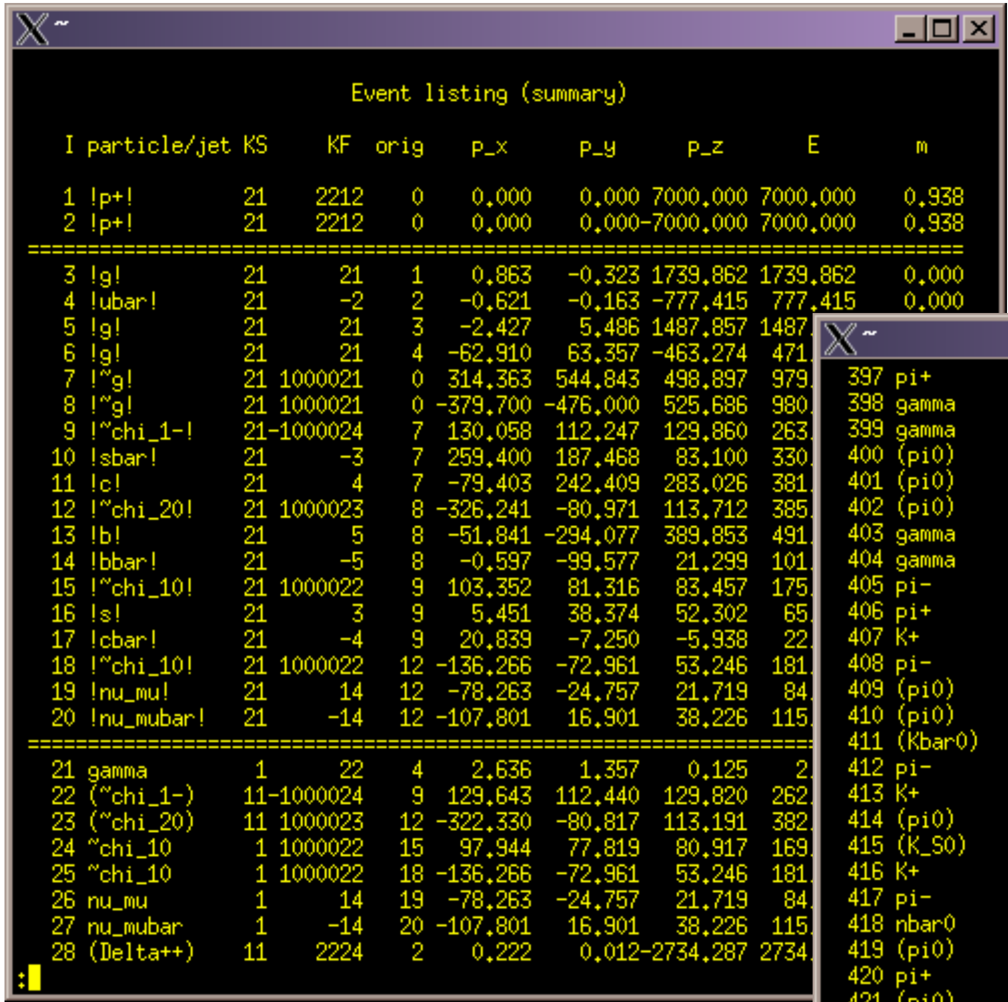
$pp \to$ hadrons, D-Y, SUSY,...

e.g. PYTHIA, HERWIG, ISAJET...

Output = ‘events’, i.e., for each event we get a list of generated particles and their momentum vectors, types, etc.

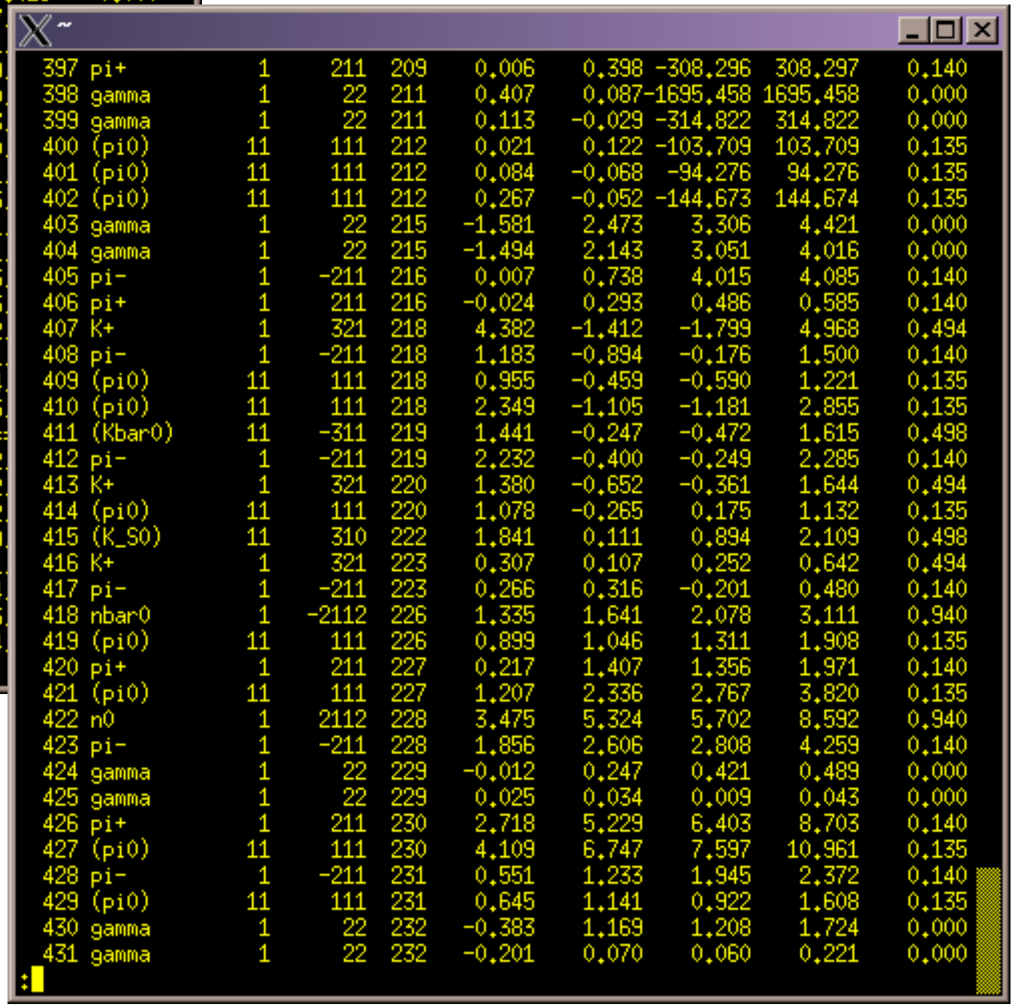
A simulated event

⋮



Event listing (summary)

I	particle/jet	KS	KF	orig	p_x	p_y	p_z	E	m
1	!p+!	21	2212	0	0.000	0.000	7000.000	7000.000	0.938
2	!p+!	21	2212	0	0.000	0.000	-7000.000	7000.000	0.938
3	!g!	21	21	1	0.863	-0.323	1739.862	1739.862	0.000
4	!ubar!	21	-2	2	-0.621	-0.163	-777.415	777.415	0.000
5	!g!	21	21	3	-2.427	5.486	1487.857	1487.	
6	!g!	21	21	4	-62.910	63.357	-463.274	471.	
7	!~g!	21	1000021	0	314.363	544.843	498.897	979.	
8	!~g!	21	1000021	0	-379.700	-476.000	525.686	980.	
9	!~chi_1-!	21	-1000024	7	130.058	112.247	129.860	263.	
10	!sbar!	21	-3	7	259.400	187.468	83.100	330.	
11	!c!	21	4	7	-79.403	242.409	283.026	381.	
12	!~chi_20!	21	1000023	8	-326.241	-80.971	113.712	385.	
13	!b!	21	5	8	-51.841	-294.077	389.853	491.	
14	!bbar!	21	-5	8	-0.597	-99.577	21.299	101.	
15	!~chi_10!	21	1000022	9	103.352	81.316	83.457	175.	
16	!s!	21	3	9	5.451	38.374	52.302	65.	
17	!cbar!	21	-4	9	20.839	-7.250	-5.938	22.	
18	!~chi_10!	21	1000022	12	-136.266	-72.961	53.246	181.	
19	!nu_mu!	21	14	12	-78.263	-24.757	21.719	84.	
20	!nu_mubar!	21	-14	12	-107.801	16.901	38.226	115.	
21	gamma	1	22	4	2.636	1.357	0.125	2.	
22	(~chi_1-)	11	-1000024	9	129.643	112.440	129.820	262.	
23	(~chi_20)	11	1000023	12	-322.330	-80.817	113.191	382.	
24	~chi_10	1	1000022	15	97.944	77.819	80.917	169.	
25	~chi_10	1	1000022	18	-136.266	-72.961	53.246	181.	
26	nu_mu	1	14	19	-78.263	-24.757	21.719	84.	
27	nu_mubar	1	-14	20	-107.801	16.901	38.226	115.	
28	(Delta++)	11	2224	2	0.222	0.012	-2734.287	2734.	



397	pi+	1	211	209	0.006	0.398	-308.296	308.297	0.140
398	gamma	1	22	211	0.407	0.087	-1695.458	1695.458	0.000
399	gamma	1	22	211	0.113	-0.029	-314.822	314.822	0.000
400	(pi0)	11	111	212	0.021	0.122	-103.709	103.709	0.135
401	(pi0)	11	111	212	0.084	-0.068	-94.276	94.276	0.135
402	(pi0)	11	111	212	0.267	-0.052	-144.673	144.674	0.135
403	gamma	1	22	215	-1.581	2.473	3.306	4.421	0.000
404	gamma	1	22	215	-1.494	2.143	3.051	4.016	0.000
405	pi-	1	-211	216	0.007	0.738	4.015	4.085	0.140
406	pi+	1	211	216	-0.024	0.293	0.486	0.585	0.140
407	K+	1	321	218	4.382	-1.412	-1.799	4.968	0.494
408	pi-	1	-211	218	1.183	-0.894	-0.176	1.500	0.140
409	(pi0)	11	111	218	0.955	-0.459	-0.590	1.221	0.135
410	(pi0)	11	111	218	2.349	-1.105	-1.181	2.855	0.135
411	(Kbar0)	11	-311	219	1.441	-0.247	-0.472	1.615	0.498
412	pi-	1	-211	219	2.232	-0.400	-0.249	2.285	0.140
413	K+	1	321	220	1.380	-0.652	-0.361	1.644	0.494
414	(pi0)	11	111	220	1.078	-0.265	0.175	1.132	0.135
415	(K_S0)	11	310	222	1.841	0.111	0.894	2.109	0.498
416	K+	1	321	223	0.307	0.107	0.252	0.642	0.494
417	pi-	1	-211	223	0.266	0.316	-0.201	0.480	0.140
418	nbar0	1	-2112	226	1.335	1.641	2.078	3.111	0.940
419	(pi0)	11	111	226	0.899	1.046	1.311	1.908	0.135
420	pi+	1	211	227	0.217	1.407	1.356	1.971	0.140
421	(pi0)	11	111	227	1.207	2.336	2.767	3.820	0.135
422	n0	1	2112	228	3.475	5.324	5.702	8.592	0.940
423	pi-	1	-211	228	1.856	2.606	2.808	4.259	0.140
424	gamma	1	22	229	-0.012	0.247	0.421	0.489	0.000
425	gamma	1	22	229	0.025	0.034	0.009	0.043	0.000
426	pi+	1	211	230	2.718	5.229	6.403	8.703	0.140
427	(pi0)	11	111	230	4.109	6.747	7.597	10.961	0.135
428	pi-	1	-211	231	0.551	1.233	1.945	2.372	0.140
429	(pi0)	11	111	231	0.645	1.141	0.922	1.608	0.135
430	gamma	1	22	232	-0.383	1.169	1.208	1.724	0.000
431	gamma	1	22	232	-0.201	0.070	0.060	0.221	0.000

⋮

PYTHIA Monte Carlo

pp → gluino-gluino

Monte Carlo detector simulation

Takes as input the particle list and momenta from generator.

Simulates detector response:

multiple Coulomb scattering (generate scattering angle),
particle decays (generate lifetime),
ionization energy loss (generate Δ),
electromagnetic, hadronic showers,
production of signals, electronics response, ...

Output = simulated raw data $\rightarrow$ input to reconstruction software:

track finding, fitting, etc.

Predict what you should see at 'detector level' given a certain hypothesis for 'generator level'. Compare with the real data.

Estimate 'efficiencies' = #events found / # events generated.

Programming package: **`GEANT`**

Monte Carlo integration method

- x uniform random variable in [a,b]:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n} f\left(x_i\right)=\frac{1}{b-a}\int_a^b f\left(x\right)dx$$

- Hit or miss method, x,y u.r.v., x in [a.b], y in [0,c]:

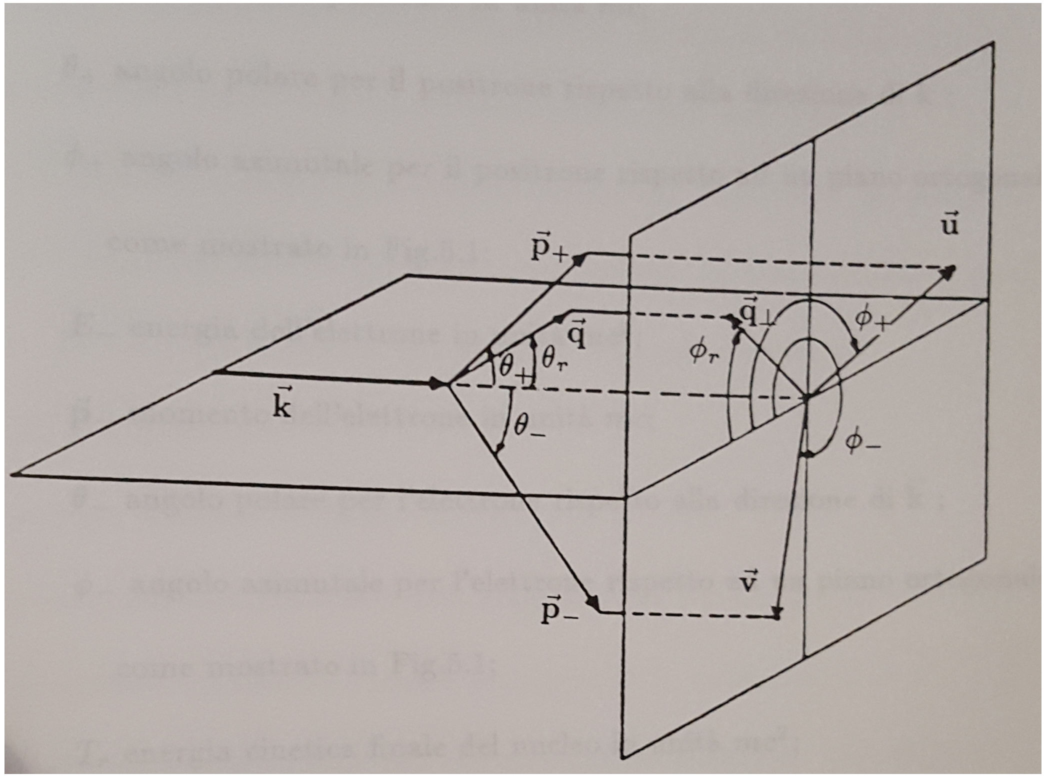
$$\int_a^b f\left(x\right)dx=\frac{N_{hit}}{N_{TOT}}c(b-a)$$

Differential pair production cross section from circularly polarized photons

$$d^5\sigma = (a + b\vec{\mathbf{P}}_\gamma \cdot \vec{\mathbf{n}})\mathrm{dE}_+\mathrm{d}\Omega_+\mathrm{d}\Omega_-$$

$$\vec{\mathbf{P}}_\gamma = \pm P_\gamma \widehat{\mathbf{k}}$$

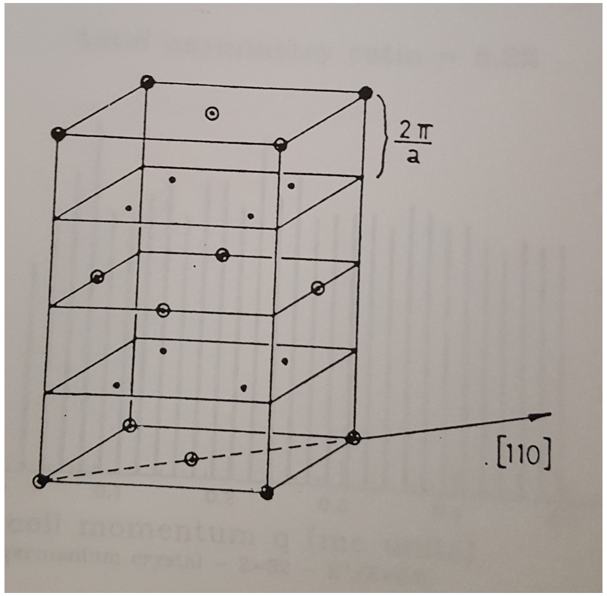
$$\vec{\mathbf{n}} = \vec{\mathbf{u}} \wedge \vec{\mathbf{v}};$$

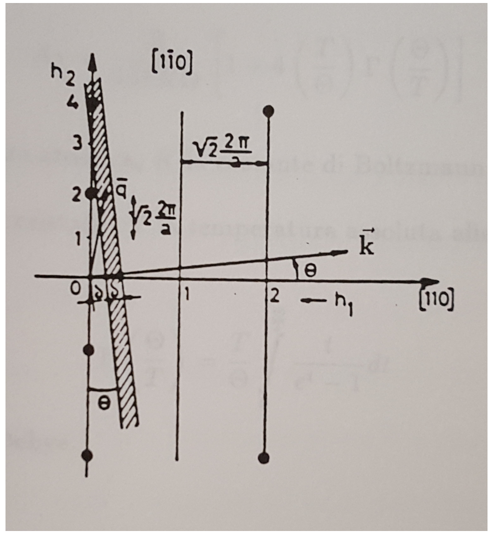


$$d\sigma = \left| \sum_{\vec{\mathbf{L}}} e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{L}}} \right|^2 d\sigma(\vec{\mathbf{q}})$$

$$\left| \sum_{\vec{\mathbf{L}}} e^{i\vec{\mathbf{q}}\cdot\vec{\mathbf{L}}} \right|^2 \propto \sum_{\vec{\mathbf{g}}} \delta(\vec{\mathbf{q}} - \vec{\mathbf{g}})$$

$$\vec{\mathbf{q}} = \vec{\mathbf{g}}$$

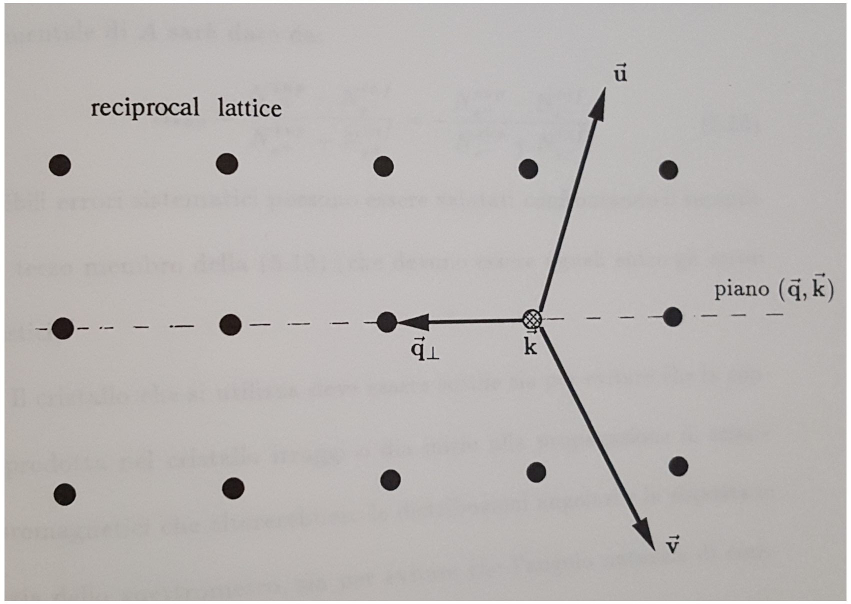


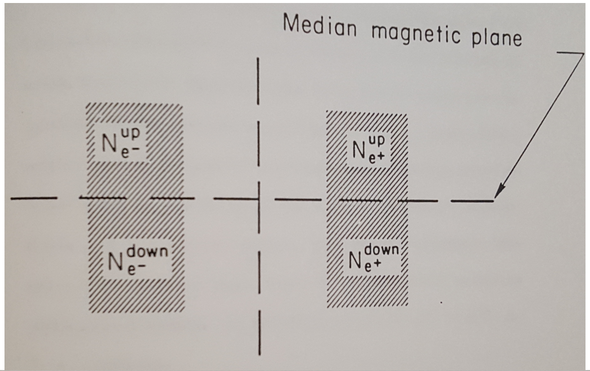


$$d^5\sigma(\vec{\mathbf{q}}) = \frac{d^5\sigma}{dE_+ d\Omega_+ d\Omega_-} |J| dq d\Omega_r dE_- d\phi_-$$

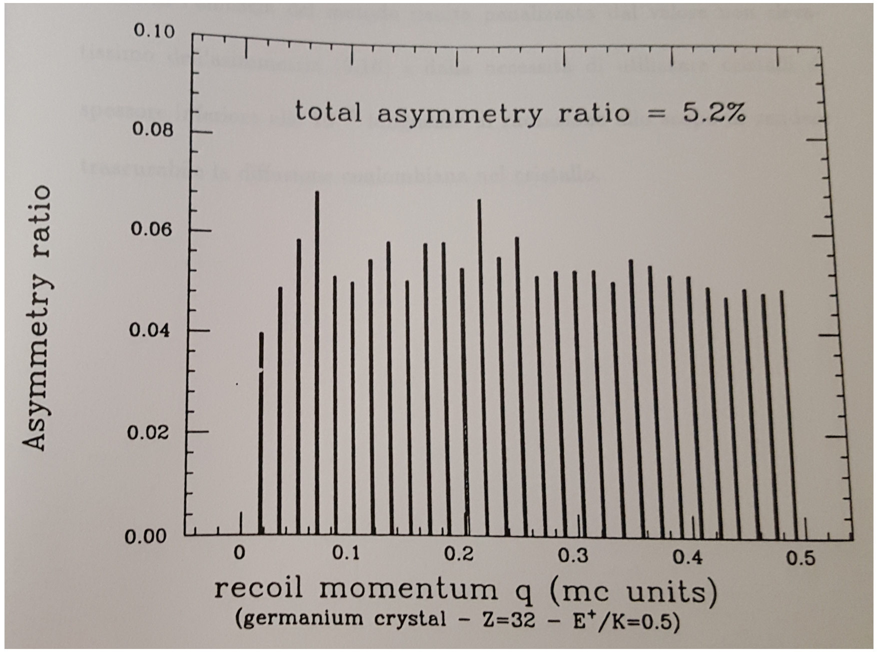
$$d\sigma_c^5 = \left| \sum_{\vec{\mathbf{L}}} \exp(i\vec{\mathbf{q}} \cdot \vec{\mathbf{L}}) \right|^2 \exp(-A_T q^2) d\sigma^5(\vec{\mathbf{q}})$$

$$\left| \sum_{diam} \exp(i\vec{\mathbf{q}} \cdot \vec{\mathbf{L}}) \right|^2 = \frac{1}{8} N \frac{(2\pi)^3}{\Delta} \sum_{\vec{\mathbf{g}}} D(\vec{\mathbf{g}}) \delta(\vec{\mathbf{q}} - \vec{\mathbf{g}})$$





$$A_{exp} = \frac{N_{e^+}^{sup} - N_{e^+}^{inf}}{N_{e^+}^{sup} + N_{e^+}^{inf}} = -\frac{N_{e^-}^{sup} - N_{e^-}^{inf}}{N_{e^-}^{sup} + N_{e^-}^{inf}}$$



Recap

- Let's remind at this point that our aim is to learn how to design an experiment.
- We have seen:
  - Definition of the process we want to study
  - Selection of the events correponding to this process
  - Measurement of the quantities related to the process
  - Other measurements related to the physics objects we are studying.
- Now, in order to really design an experiment we need:
  - To see how projectiles and targets can be set-up
  - To see how to put together different detectors to mesure what we need to measure

How to design an EPP experiment

How to design an EPP experiment

- Define which process I want to study:
  - ➔ initial state (particles, energy, required intensities,…)
  - ➔ final state(s) (which particles to detect, which energies, which are the main possible backgrounds etc.): exclusive vs. inclusive.
  - ➔➔ Overall Montecarlo simulation of the process, to understand the main parameters in the game (kinematics, rates, number of particles, backgrounds)
- Overall design parameters:
  - Center of mass energy $\sqrt{s}$
  - Luminosity L / flux ϕ to obtain the requires statistical accuracy. For this I need to know (or at least to estimate) the cross-section of the process.
  - Detector general structure: depends on what we want to measure:
    - charged particles momenta ➔ magnetic field
    - neutral particles detection and particles energy ➔ calorimetry
    - special particles: neutrinos, muons, neutrons,…

Collider experiments

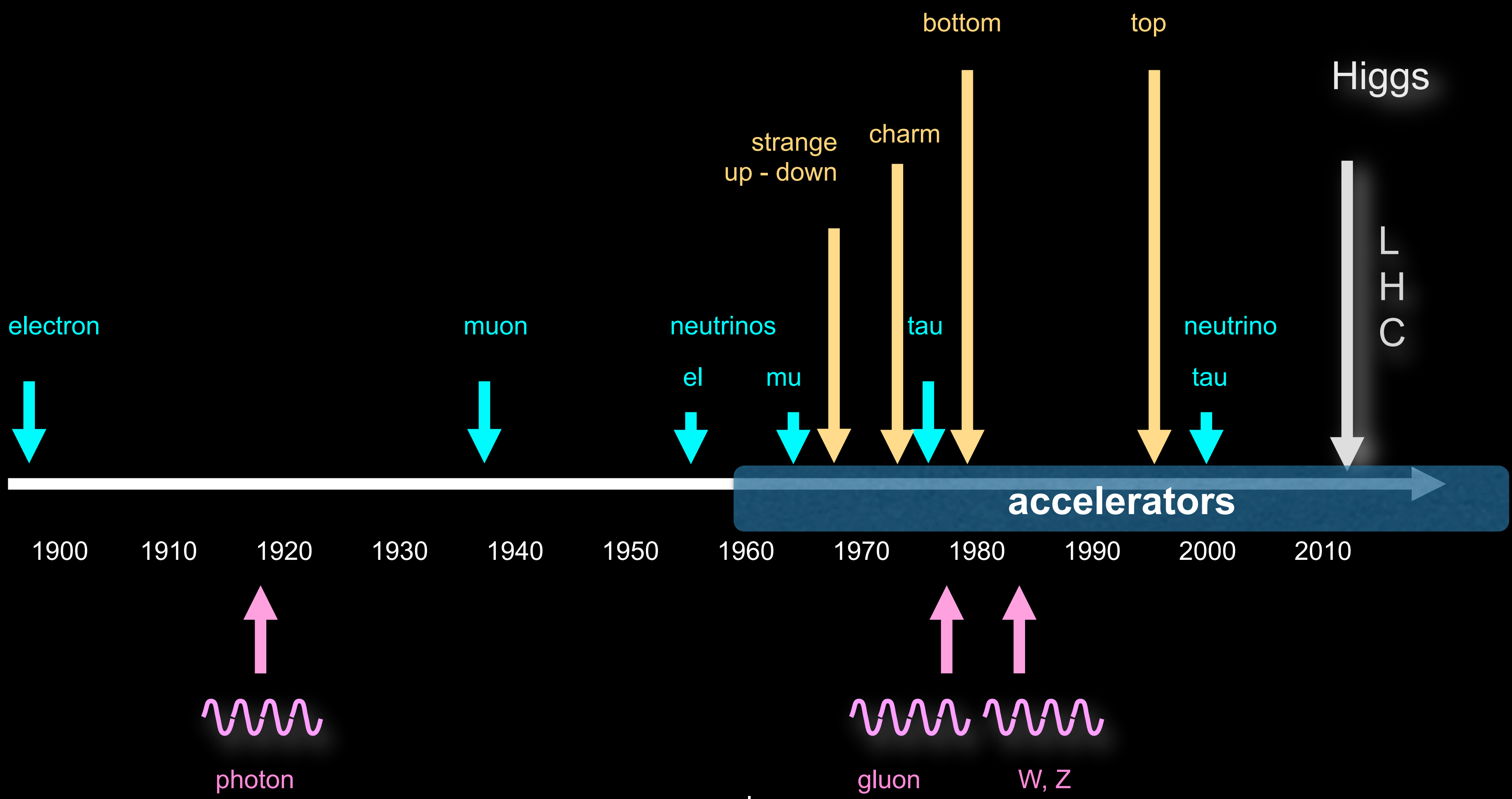
The main parameters of the colliders

LHC: ATLAS+CMS parameters

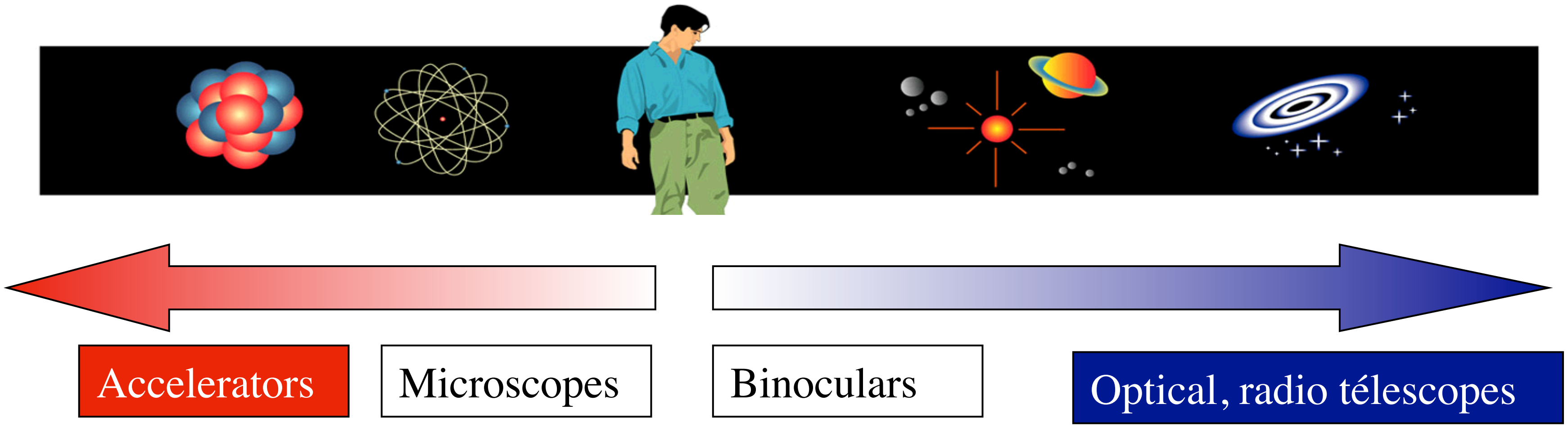
Particle Accelerator Physics

- A new discipline, separation of the communities;
- Many byproducts:
  - Beams for medicine
  - Beams for archeology and determination of age
- Two main quantities define an accelerator: the **center of mass energy** and the **beam intensity** (normally called luminosity)
- Few general aspects to be considered (we consider colliders here):
  - The **center of mass energy** is a "design" quantity: it depends on the machine dimensions, magnets and optics.
  - The **luminosity** is a quantity that has to be reached: it depends on several parameters. In many cases it doesn't reach the "design" value. It is the key quantity for the INTENSITY frontier projects.

60 years of experiments at accelerators have discovered the set of fundamental particles



Accelerators gain us one frontier of the physics spectrum

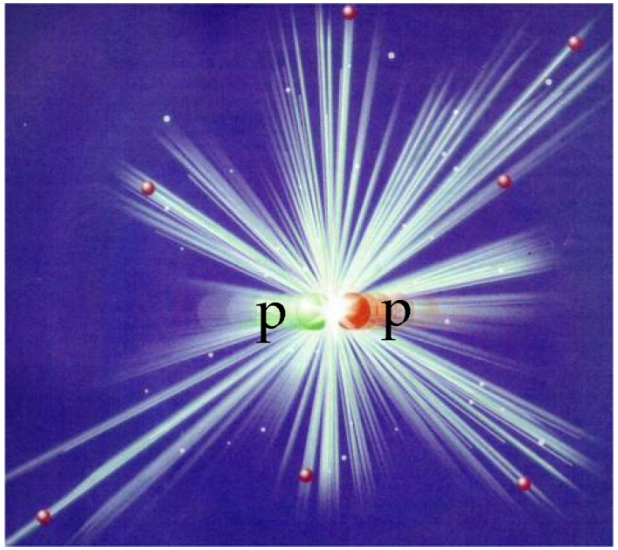


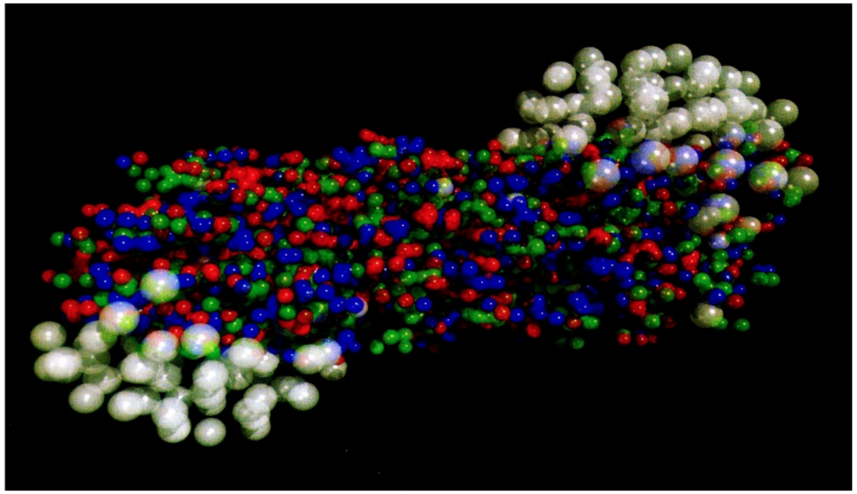
Particle physics looks at matter in its smallest dimensions and accelerators are very fine microscope or, better, *atto-scope!*

$\lambda = h/p$; @LHC: $T = 1$ TeV $\Rightarrow \lambda \cong 10^{-18}$ m = 1 *am* (actually 30 *zm*)

…back to Big Bang

- Trip back toward the Big Bang: $t_{\mu s} \cong 1/E^2_{Gev}$
- $T \cong 100$ fs after Big Bang for single particle creation (3 TeV)
- $T \cong 1$ μs for collective phenomena QGS (Quark-Gluon Soup)





But we are left with the task of explaining how the rich complexity that developed in the ensuing 13.7 billion years came about…

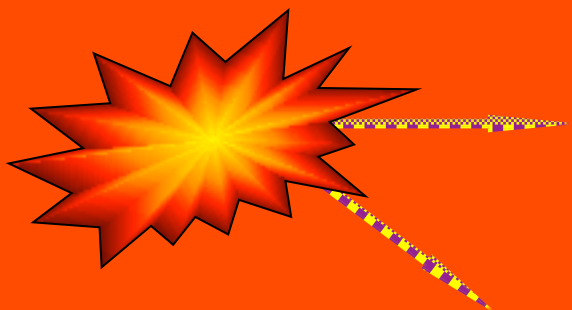
Which is a much more complex task!

Accelerators: the two frontiers

2 routes to new knowledge about the fundamental structure of the matter

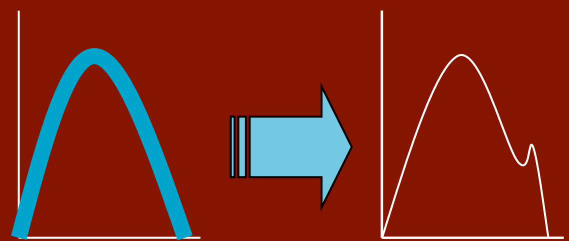
High Energy Frontier

New phenomena
(new particles)
created when the
“usable” energy > mc^2 [×2]

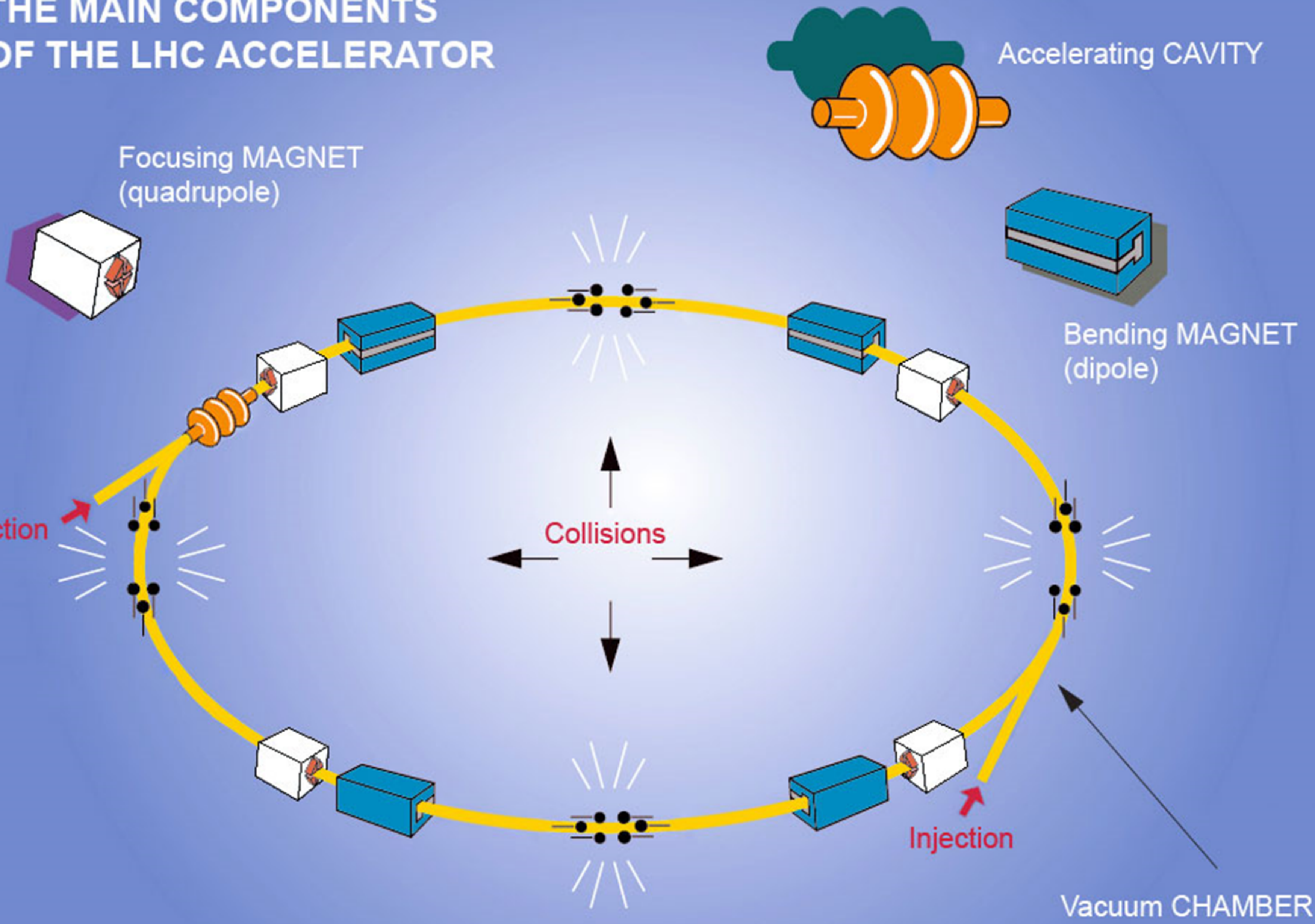


High Precision Frontier

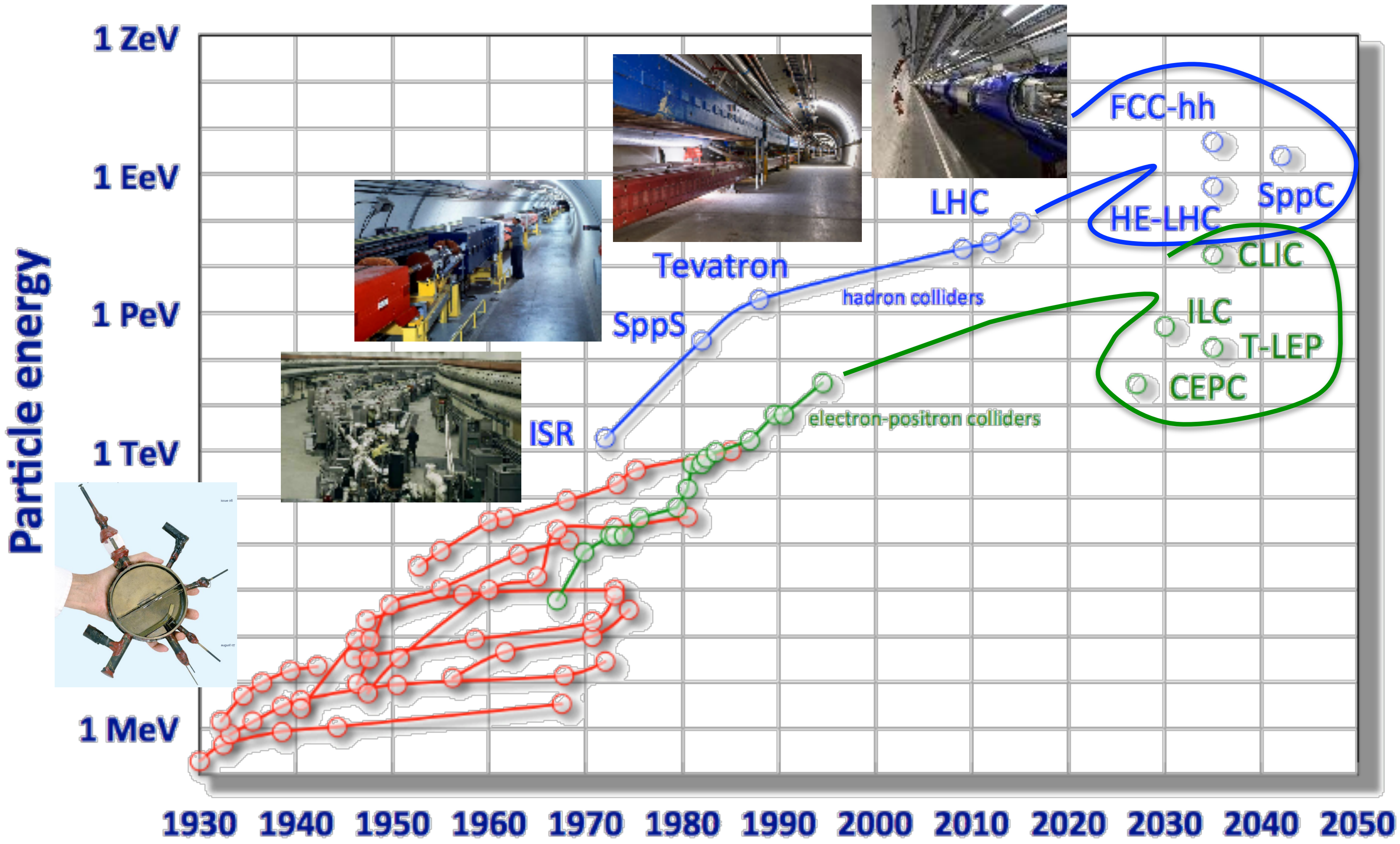
Known phenomena studied with high precision *may* show inconsistencies with theory



THE MAIN COMPONENTS OF THE LHC ACCELERATOR

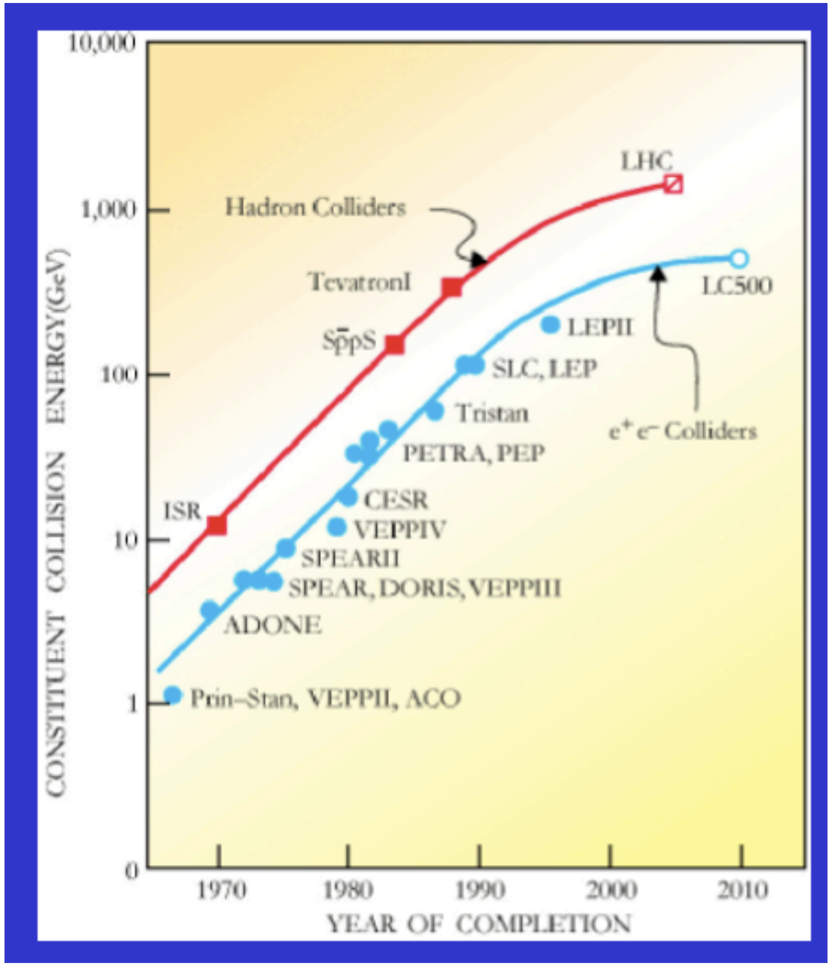


Livingston plot

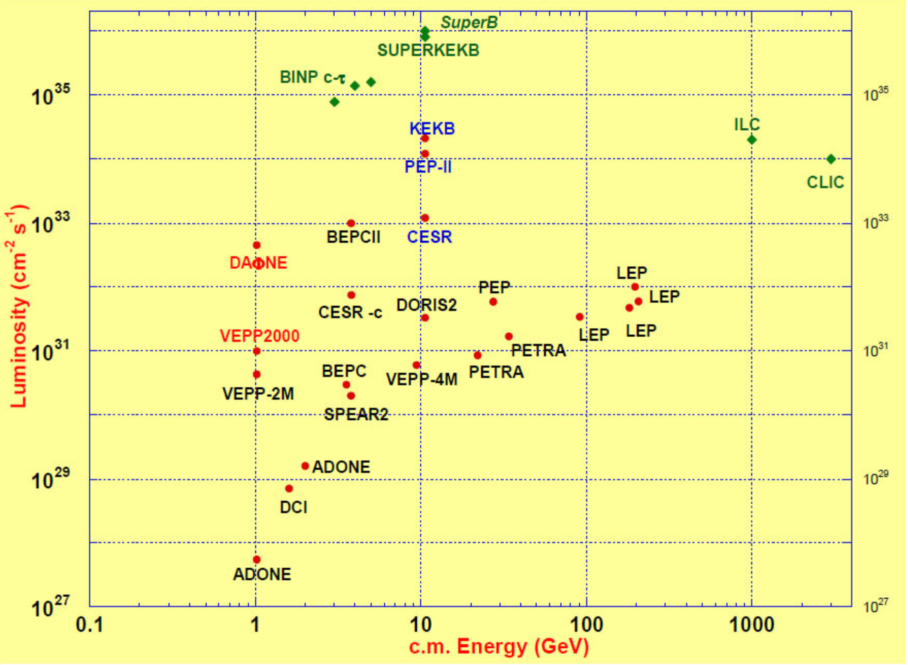


From Luca.Bottura-CERN

Colliders: “Livingston” plots



Here it can be seen the separation
Between *Energy* and *Intensity* frontiers !



Colliders: general aspects - I

- **Storage rings**:

  beams are accumulated in circular orbits and are put in collisions.

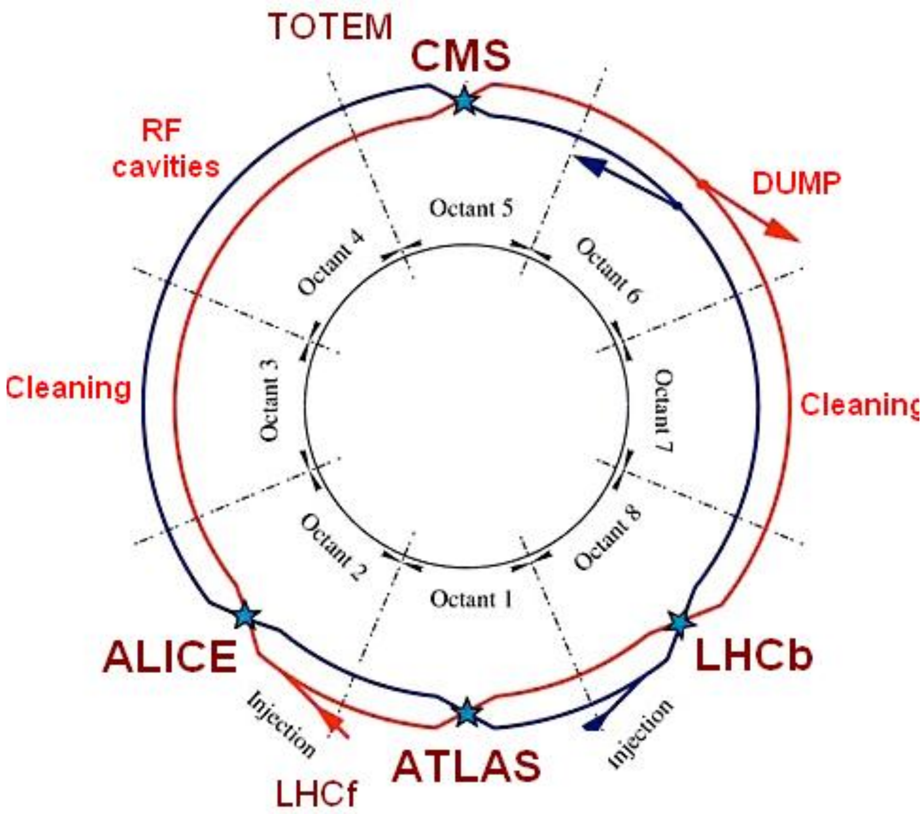
  - "bunches" of particles (typically $N \approx 10^{10}$-10^{12} / bunch) in small transverse dimensions (σ_X, σ_Y down to < mm level) and higher longitudinal dimensions (σ_Z at cm level) like *needles* or *ribbons.*
  - the bunches travel along a ≈ circular trajectory (curvilinear coordinate s)
    - magnetic fields to bend them (dipoles) and to focalize them (quadrupoles or higher order)
    - electric fields to increase their energies (RadioFrequency cavities)
  - Multi-bunch operation n_b (increase of luminosity BUT reduction of inter-bunch time)
  - One or more interaction regions (with experiments or not..)
  - History:
    - *e^+e^-: Ada, Adone, Spear,… Lep, flavour-factories*
    - *pp: ISR, LHC*
    - *ppbar: SpS, Tevatron*
    - *ep: HERA*
    - *muon colliders are considered today (never built)*

- **Linear colliders**:

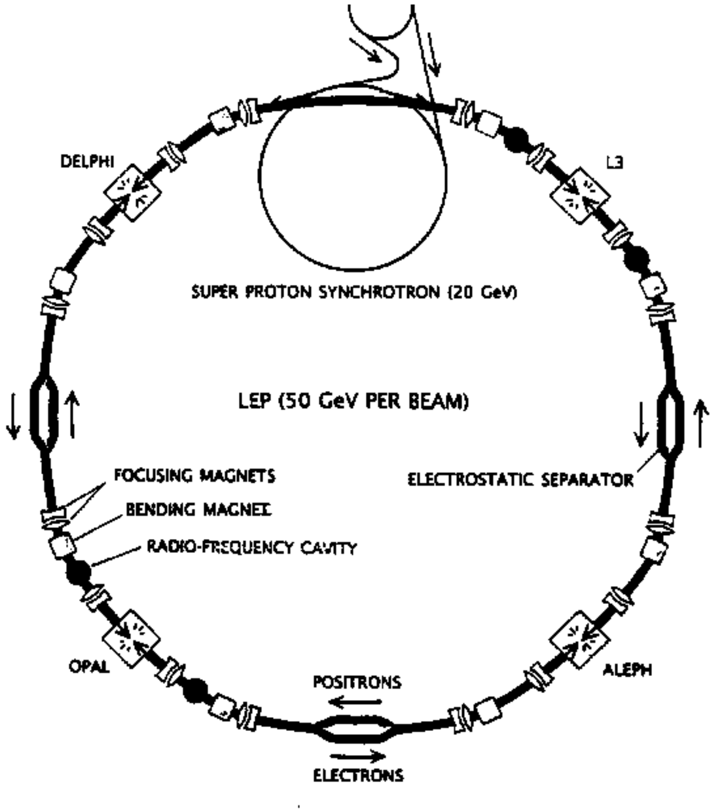
  ambituous projects aiming to reach higher electron energies without the large energy loss due to synchrotron radiation.

Colliders: general aspects - II

LHC scheme: up to 7 TeV per beam

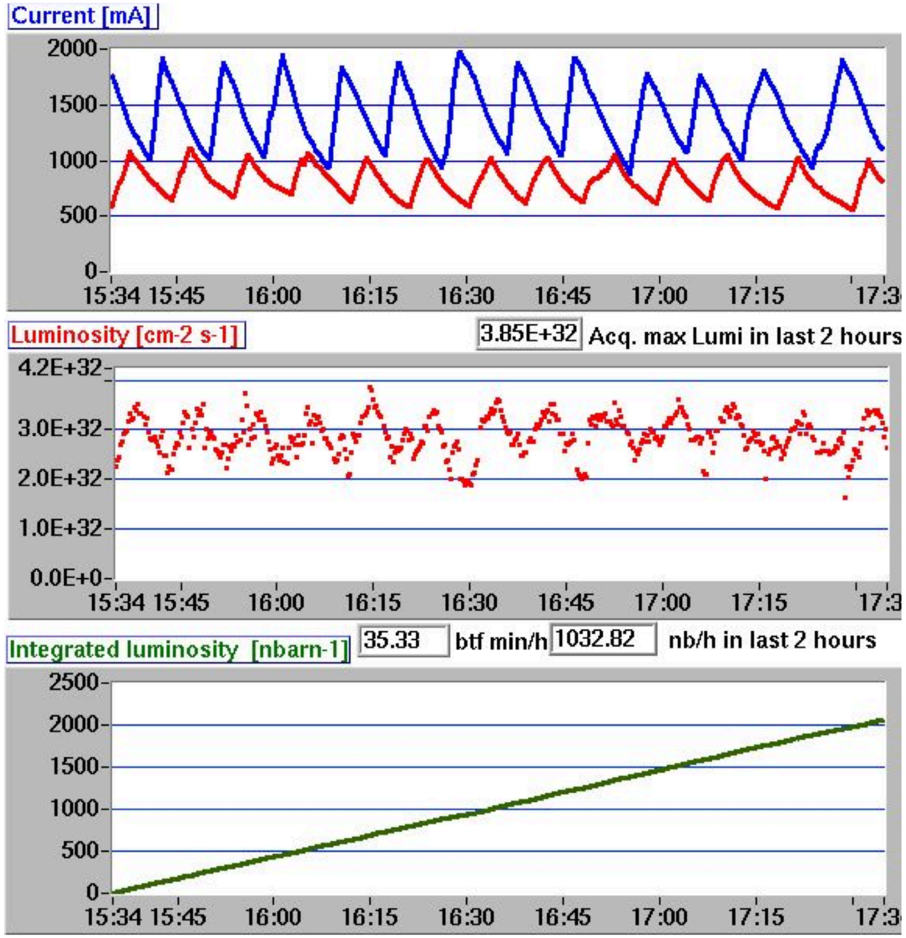


LEP scheme: up to 100 GeV per beam



Colliders: general aspects - III

- Two different operation modes:
  - Single injection (LHC)
  - “top-up” injection, continuos mode.
- Important quantities for the experiment operation are:
  - Integrated luminosity
  - Machine background



LifeTime: 50% reduction in 10 minutes

Colliders: general aspects - IV

"Typical" LHC operation mode: single- injection

LifeTime: 25% reduction in 9 h

Collider parameters - I

		CESR (Cornell)	CESR-C (Cornell)	LEP (CERN)	ILC (TBD)	CLIC (TBD)
	Physics start date	1979	2002	1989	TBD	TBD
	Physics end date	2002	2008	2000	—	—
Main parameters	Maximum beam energy (GeV)	6	6	100 - 104.6	250 (upgradeable to 500)	1500 (first phase: 250)
	Delivered integrated luminosity per exp. (fb^{-1})	41.5	2.0	0.221 at Z peak 0.501 at 65 – 100 GeV 0.275 at >100 GeV	—	—
	Luminosity (10^{30} $cm^{-2}s^{-1}$)	1280 at 5.3 GeV	76 at 2.08 GeV	24 at Z peak 100 at > 90 GeV	1.5×10^4	6×10^4
Impact on detector operation	Time between collisions (μs)	0.014 to 0.22	0.014 to 0.22	22	$0.55^\dagger$	$0.0005^\ddagger$
	Full crossing angle (μ rad)	±2000	±3300	0	14000	20000
	Energy spread (units 10^{-3})	0.6 at 5.3 GeV	0.82 at 2.08 GeV	0.7→1.5	1	3.4
	Bunch length (cm)	1.8	1.2	1.0	0.03	0.0044
	Beam radius (μm)	H: 460 V: 4	H: 340 V: 6.5	H: 200 → 300 V: 2.5 → 8	H: 0.474 V: 0.0059	H: 0.045 * V: 0.0009
	Free space at interaction point (m)	±2.2 (±0.6 to REC quads)	±2.2 (±0.3 to PM quads)	±3.5	±3.5	±3.5
	Luminosity lifetime (hr)	2–3	2–3	20 at Z peak 10 at > 90 GeV	n/a	n/a
	Turn-around time (min)	5 (topping up)	1.5 (topping up)	50	n/a	n/a
	Injection energy (GeV)	1.8–6	1.5–6	22	n/a	n/a
Techincal parameters	Transverse emittance ($10^{-9}\pi$ rad-m)	H: 210 V: 1	H: 120 V: 3.5	H: 20–45 V: 0.25 → 1	H: 0.02 V: 7×10^{-5}	H: 2.2×10^{-4} V: 6.8×10^{-6}
	β^*, amplitude function at interaction point (m)	H: 1.0 V: 0.018	H: 0.94 V: 0.012	H: 1.5 V: 0.05	H: 0.01 V: 5×10^{-4}	H: 0.0069 V: 6.8×10^{-5}

Collider parameters - II

Main parameters

Impact on detector operation

Techincal parameters

	KEKB (KEK)	PEP-II (SLAC)	SuperKEKB (KEK)
Physics start date	1999	1999	2015
Physics end date	2010	2008	—
Maximum beam energy (GeV)	e^-: 8.33 (8.0 nominal) e^+: 3.64 (3.5 nominal)	e^-: 7–12 (9.0 nominal) e^+: 2.5–4 (3.1 nominal)	e^-: 7 e^+: 4
Delivered integrated luminosity per exp. (fb^{-1})	1040	557	—
Luminosity (10^{30} $cm^{-2}s^{-1}$)	21083	12069 (design: 3000)	8×10^5
Time between collisions (μs)	0.00590 or 0.00786	0.0042	0.004
Full crossing angle (μ rad)	$\pm 11000^\dagger$	0	± 41500
Energy spread (units 10^{-3})	0.7	e^-/e^+: 0.61/0.77	e^-/e^+: 0.64/0.81
Bunch length (cm)	0.65	e^-/e^+: 1.1/1.0	e^-/e^+: 0.5/0.6
Beam radius (μm)	H: 124 (e^-), 117 (e^+) V: 1.9	H: 157 V: 4.7	e^-: 11 (H), 0.062 (V) e^+: 10 (H), 0.048 (V)
Free space at interaction point (m)	$+0.75/-0.58$ ($+300/-500$) mrad cone	± 0.2, ± 300 mrad cone	$e^-: +1.20/-1.28, e^+: +0.78/-0.73$ ($+300/-500$) mrad cone
Luminosity lifetime (hr)	continuous	continuous	continuous
Turn-around time (min)	continuous	continuous	continuous
Injection energy (GeV)	e^-/e^+: 8.0/3.5 (nominal)	e^-/e^+: 9.0/3.1 (nominal)	e^-/e^+: 7/4
Transverse emittance ($10^{-9}\pi$ rad-m)	e^-: 24 (57*) (H), 0.61 (V) e^+: 18 (55*) (H), 0.56 (V)	e^-: 48 (H), 1.8 (V) e^+: 24 (H), 1.8 (V)	e^-: 4.6 (H), 0.013 (V) e^+: 3.2 (H), 0.0086 (V)
β^*, amplitude function at interaction point (m)	e^-: 1.2 (0.27*) (H), 0.0059 (V) e^+: 1.2 (0.23*) (H), 0.0059 (V)	e^-: 0.50 (H), 0.012 (V) e^+: 0.50 (H), 0.012 (V)	e^-: 0.025 (H), 3×10^{-4} (V) e^+: 0.032 (H), 2.7×10^{-4} (V)

Collider parameters - III

Main parameters

Impact on detector operation

Techical parameters

	HERA (DESY)	TEVATRON* (Fermilab)	RHIC (Brookhaven)	LHC (CERN)		
Physics start date	1992	1987	2001	2009	2012 (expected)	nominal
Physics end date	2007	2011	—	—		
Particles collided	ep	$p\bar{p}$	pp (polarized)	pp		
Maximum beam energy (TeV)	e: 0.030 p: 0.92	0.980	0.25 48% polarization	3.5	4.0	7.0
Delivered integrated luminosity per exp. (fb^{-1})	0.8	12	up to 0.14 at 100 GeV/n up to 0.15 at 200 GeV/n	up to 5.6	—	—
Luminosity (10^{30} $\text{cm}^{-2}\text{s}^{-1}$)	75	431	145 (pk) 90 (avg)	3.7×10^3	5×10^3	1.0×10^4
Time between collisions (ns)	96	396	107	49.90	49.90	24.95
Full crossing angle (μ rad)	0	0	0	240	≈ 300	≈ 300
Energy spread (units 10^{-3})	e: 0.91 p: 0.2	0.14	0.15	0.116	0.116	0.113
Bunch length (cm)	e: 0.83 p: 8.5	p: 50 $\bar{p}$: 45	70	9	9	7.5
Beam radius (10^{-6} m)	e: $110(H), 30(V)$ p: $111(H), 30(V)$	p: 28 $\bar{p}$: 16	90	26	20	16.6
Free space at interaction point (m)	± 2	± 6.5	16	38	38	38
Initial luminosity decay time, $-L/(dL/dt)$ (hr)	10	6 (avg)	5.5	8	8	14.9
Turn-around time (min)	e: 75, p: 135	90	200	≈ 180	≈ 180	≈ 180
Injection energy (TeV)	e: 0.012 p: 0.040	0.15	0.023	0.450	0.450	0.450
Transverse emittance ($10^{-9}\pi$ rad-m)	e: $20(H), 3.5(V)$ p: $5(H), 5(V)$	p: 3 $\bar{p}$: 1	15	0.7	0.6	0.5
β^*, ampl. function at interaction point (m)	e: $0.6(H), 0.26(V)$ p: $2.45(H), 0.18(V)$	0.28	0.6	1.0	0.6	0.55