Exercise:

Determine the tracking efficiency for charged pions as a function of momentum in the KLOE detector exploiting the decay:

$\phi \rightarrow \pi^+\pi^-\pi^0$

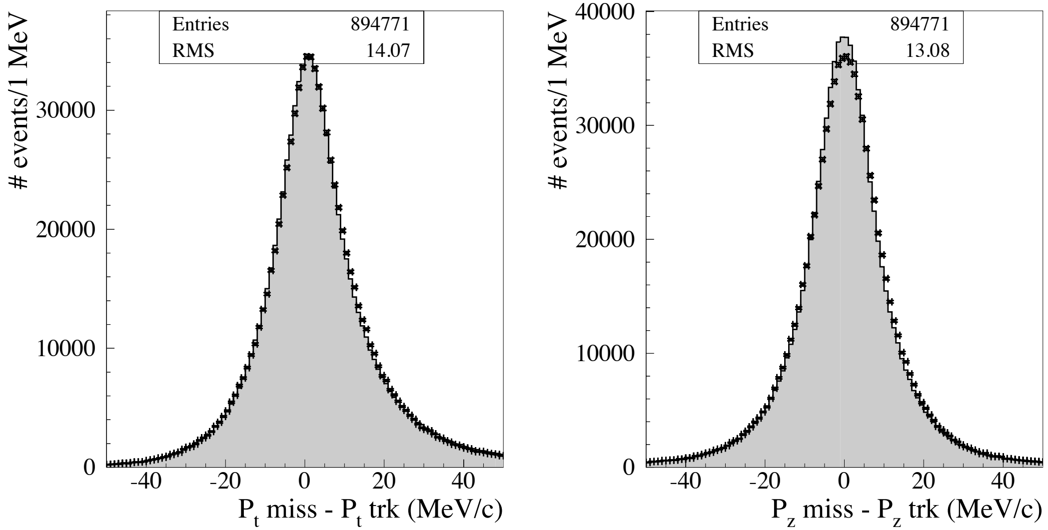
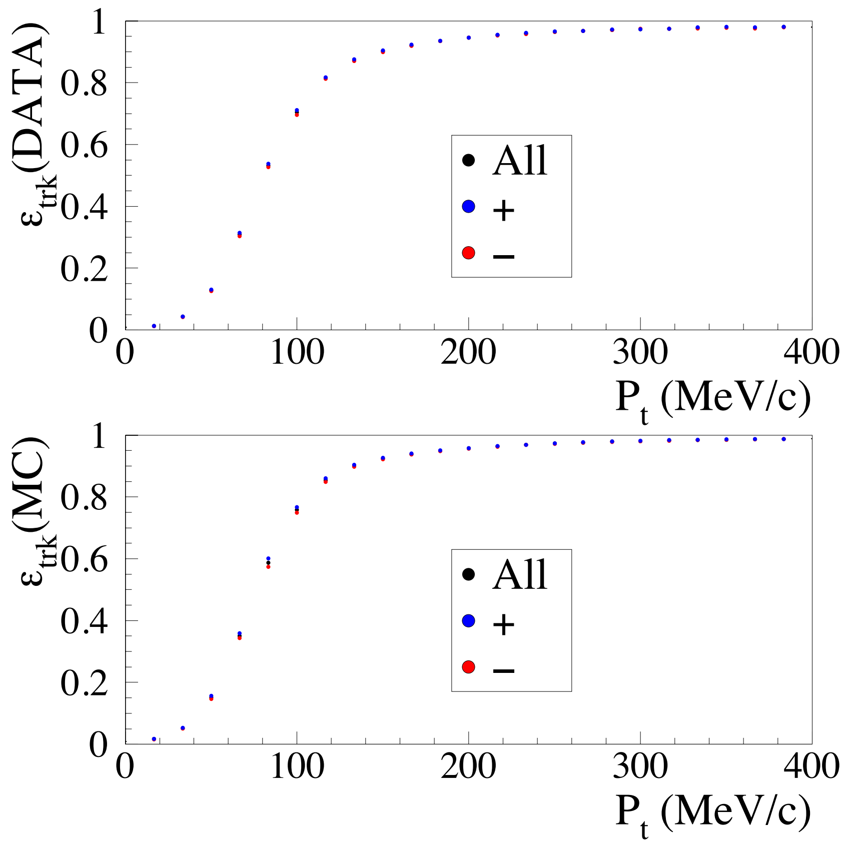


Fig. 3. Data (dots) and MC (filled histogram) comparison. Difference between the missing momentum and the momentum of the second track (when found), P_t (left) and P_z (right).



1
0.8
0.6
0.4
0.2
0
ε_{trk}(DATA)
0
100
200
300
400
P_t (MeV/c)
All
+
−
1
0.8
0.6
0.4
0.2
0
ε_{trk}(MC)
0
100
200
300
400
P_t (MeV/c)
All
+
−

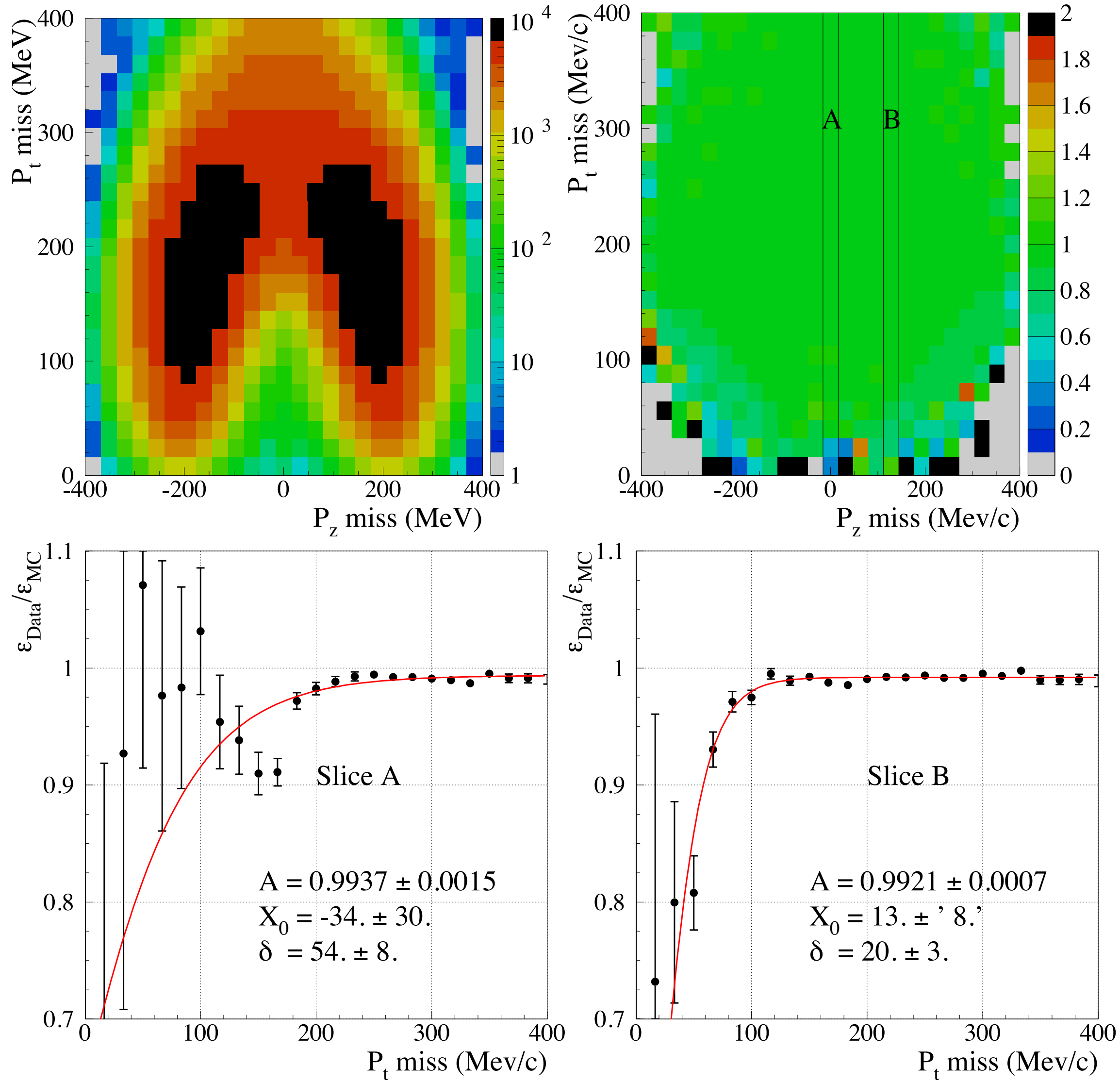


Fig. 7. Top left: *Pointer* spectrum. Top right: ratio of Data/MC tracking efficiency as a function of P_t and P_z. Two slices have been highlighted as examples (slice A corresponds to $-20 < P_z < 20$ MeV, slice B to $120 < P_z < 160$ MeV). Bottom: ratio of Data/MC tracking efficiency as a function of P_t for slice A (left) and B (right). The ratio has been fitted using the function defined in equation (5); the fit functions and the parameters are shown.

$$C_\varepsilon(X) = A\left(1 - \frac{1}{1 + e^{\frac{X - X_0}{\delta}}}\right)$$

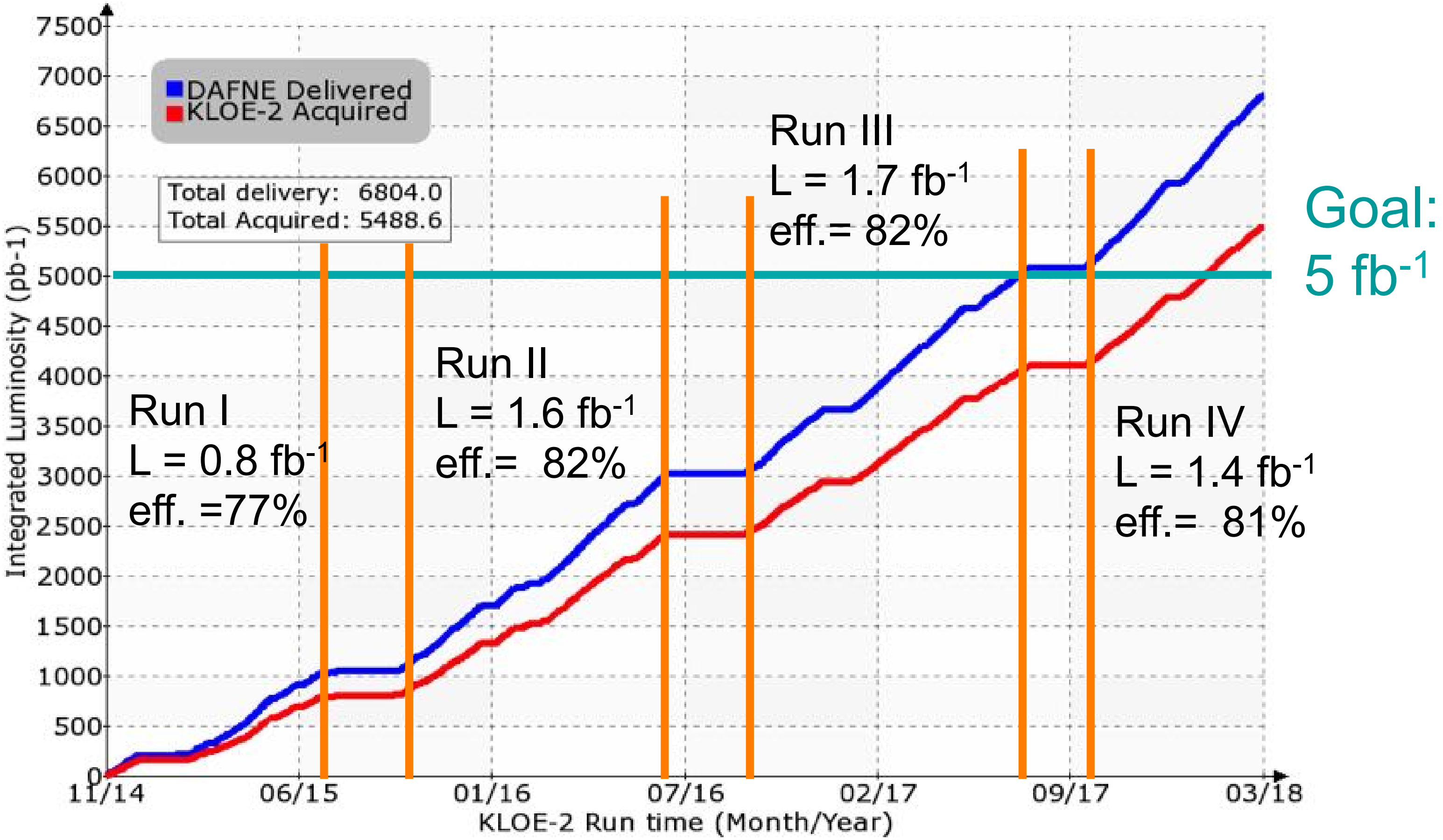
Proposed exercises

In DAFNE operations for KLOE-2 experiment:

Top-up injection
2 mA injections at a rate of 2 Hz with 60% duty cycle
Veto of KLOE-2 DAQ for 50ms at each single injection
Dead time DAQ 4 μs
Trigger rate ~ 8 kHz

Determine DAQ inefficiency

Proposed exercises



Proposed exercise

We want to set-up a trigger to detect $Z \to \mu^+\mu^-$ decays in pp collisions at LHC. We have a low threshold (LT, $p_T >$4 GeV) and a high threshold (HT, $p_T >$ 20 GeV) single muon triggers. The efficiencies of the two triggers for the muons coming from Z decays are ϵ(LT)=89.2%, ϵ(HT)=62.1%. Determine the efficiencies for triggering on Z decays in the two configurations: (1) LT1 AND LT2, (2) HT1 OR HT2 .

Proposed exercise

The values of the parameter $\mu=\sigma/\sigma_{SM}$ for the Higgs boson for the three main decay channels measured in 2014 by ATLAS were:

$$\mu_{\gamma\gamma} = 1.55 \pm 0.30$$

$$\mu_{ZZ} = 1.43 \pm 0.37$$

$$\mu_{WW} = 0.99 \pm 0.29$$

Evaluate the compatibility among the three independent ATLAS results and calculate the best overall estimate of μ from ATLAS. Then evaluate the compatibility with the SM expectation (μ=1).

Consider the Higgs production (M_H =125 GeV) at a pp collider at $\sqrt{s} = 14$ TeV. Evaluate the interval in rapidity y and the minimum value of x for direct Higgs production.

Proposed exercise

Consider the Higgs production (M_H =125 GeV) at a pp collider at $\sqrt{s} = 14$ TeV. Evaluate the interval in rapidity y and the minimum value of x for direct Higgs production.

Bayesian vs frequentist intervals (revisited)

Bayesian intervals

posterior → prior

$$p(\theta_{true}/x_0) = \frac{L(x_0/\theta_{true})\pi(\theta_{true})}{\int d\theta_{true} L(x_0/\theta_{true})\pi(\theta_{true})}$$

Bayesian interval $$\int_{\theta_1}^{\theta_2} p(\theta_{true}/x_0)d\theta_{true} = \beta$$

The interval $[\theta_1, \theta_2]$ is called **credible interval**.

The edges θ_1, θ_2 of the Bayesian intervals are not uniquely defined

$$\int_{\theta_1}^{\theta_2} p(\theta_{true}/x_0) d\theta_{true} = \beta$$

Central intervals: the pdf integral is the same above and below the interval:

$$\int_{-\infty}^{\theta_1} p(\theta_{true}/x_0) d\theta_{true} = \frac{1-\beta}{2}$$

$$\int_{\theta_2}^{+\infty} p(\theta_{true}/x_0) d\theta_{true} = \frac{1-\beta}{2}$$

Upper limits: θ_{true} is below a certain value. In this case the interval is between 0 (if θ is a non-negative quantity) and θ_{up}:

$$\int_{0}^{\theta_{up}} p(\theta_{true}/x_0) d\theta_{true} = \beta$$

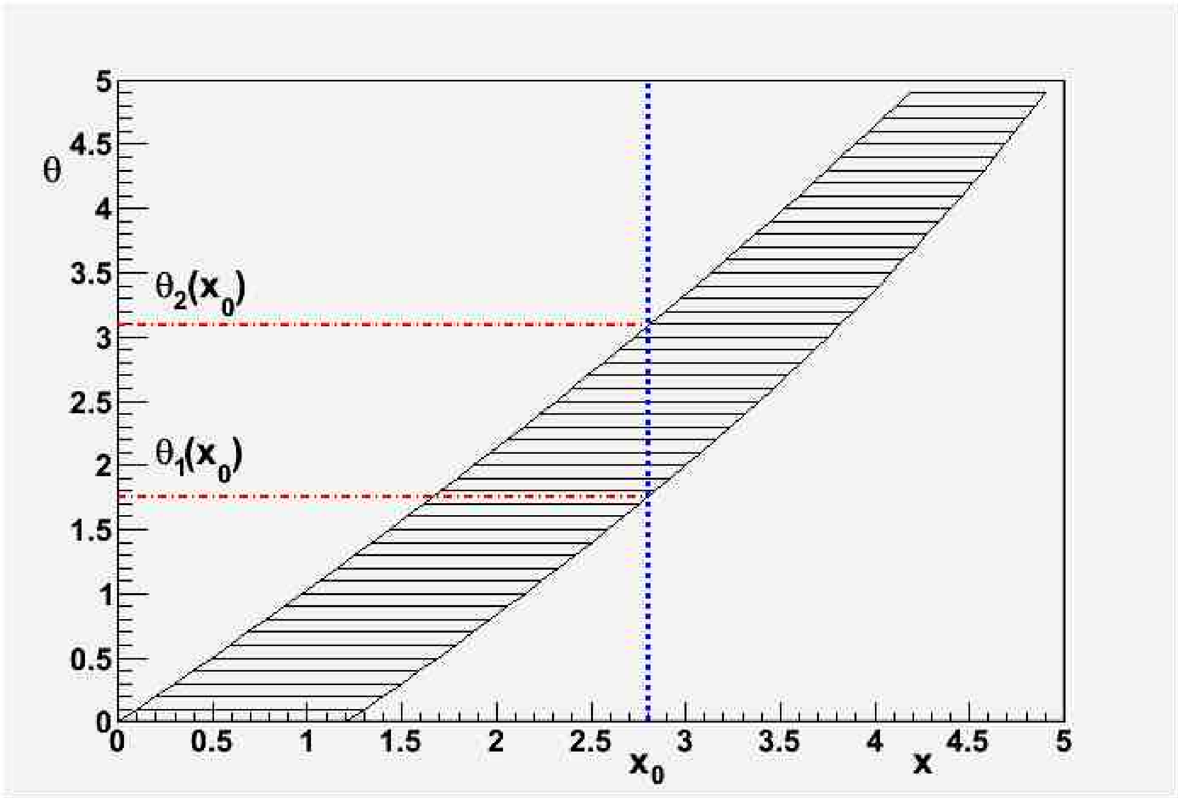
Lower limits: θ_{true} is above a certain value θ_{low}:

$$\int_{\theta_{low}}^{+\infty} p(\theta_{true}/x_0) d\theta_{true} = \beta$$

Frequentist intervals

Neynman construction of the confidence intervals

$$\int_{x_1(\theta)}^{x_2(\theta)} L(x/\theta)dx = \beta$$



Coverage: $$p(\theta_1(x_0) < \theta_{true} < \theta_2(x_0)) = \beta$$

Comments:

Bayes:

- Non informative prior (does it exist?)
- Recursive Bayes estimation => Bayes filter

$$\text{posterior} \propto \text{prior} \times \text{likelihood}$$

$$\textit{revised} \propto \textit{current} \times \textit{new likelihood}$$

$$\pi_{n+1}(\theta) \propto \pi_n(\theta) \times L_{n+1}(\theta) = \pi_n(\theta) f(x_{n+1} \,|\, \mathbf{x_n}, \theta).$$

In this dynamic perspective we notice that at time n we only need to keep a representation of π_n and otherwise can ignore the past.

The current π_n contains all information needed to revise knowledge when confronted with new information $L_{n+1}(\theta)$.

We sometimes refer to this way of updating as *recursive*.

Confidence Interval & Coverage

- You claim, $CI_{\mu}=[\mu_1,\mu_2]$ at the 95% C.L
  i.e. In an ensemble of experiments C.L (95%) of the obtained confidence intervals will contain the true value of μ.

  - If your statement is accurate, you have full coverage
  - If the true C.L is>95%, your interval has an over coverage
  - If the true C.L is <95%, your interval has an undercoverage

Signal searches: upper and lower limits

(consider the simple example of counting experiment)

- **Discovery**: the Null Hypothesis H_0, based on the Standard Model is falsified by a goodness-of-fit test. This means that new physics should be included to account for the data. This is an important discovery.
- **Exclusion**: the Alternative Hypothesis H_1, based on an extension of the Standard Model (or on a new theory at all), doesn't pass the goodness-of-fit test. H_1 is excluded by data.

Exclusion means that the search has given a negative result. However a negative result is not a complete failure of the experiment, but it gives important informations that have to be expressed in a quantitative way so that theorists or other experimentalists can use them for further searches. These quantitative statements about negative results of a search for new phenomena are normally the "upper limits" or the "lower limits".

By **upper limit** we mean a statement like the following: such a particle, if it exists, is produced with a rate (or cross-section) below this quantity, with a certain probability. On the other hand, by **lower limit** statements like: this decay, if exists, takes place with a lifetime larger than this quantity, with a certain probability. Both statements concern an exclusion.

Bayes limits

$$L(n_0/s) = \frac{e^{-s} s^{n_0}}{n_0!}$$

Assume background b=0

If we count n_0=0

$$L(0/s) = e^{-s}$$

Let's consider Bayes theorem and assume uniform prior (π=cost for s>0 and π=0 for s<0)

$$p(s/0) = \frac{L(0/s)\pi(s)}{\int L(0/s)\pi(s)ds} = L(0/s) = e^{-s}$$

Given a probability content α (e.g. α=95%) the upper limit s_{up} will be such that:

$$\int_{s_{up}}^{\infty} p(s/0)ds = 1 - \alpha$$

$$\int_{s_{up}}^{\infty} e^{-s} ds = e^{-s_{up}} = 1 - \alpha$$

We easily find s_{up}=2.3 for α=90% and s_{up}=3 for α=95%.

Bayes limits

Assume background $b \neq 0$ with negligible uncertainty and same prior as before

If we count $n_0 \geq 0$

$$p(s/n_0) = \frac{e^{-(s+b)}(s+b)^{n_0}}{n_0!}$$

$$\int_{s_{up}}^{\infty} \frac{e^{-(s+b)}(s+b)^{n_0}}{n_0!} ds = 1 - \alpha$$

Bayes limits

$$\int_{s_{up}}^{\infty} \frac{e^{-(s+b)}(s+b)^{n_0}}{n_0!} ds = 1 - \alpha$$

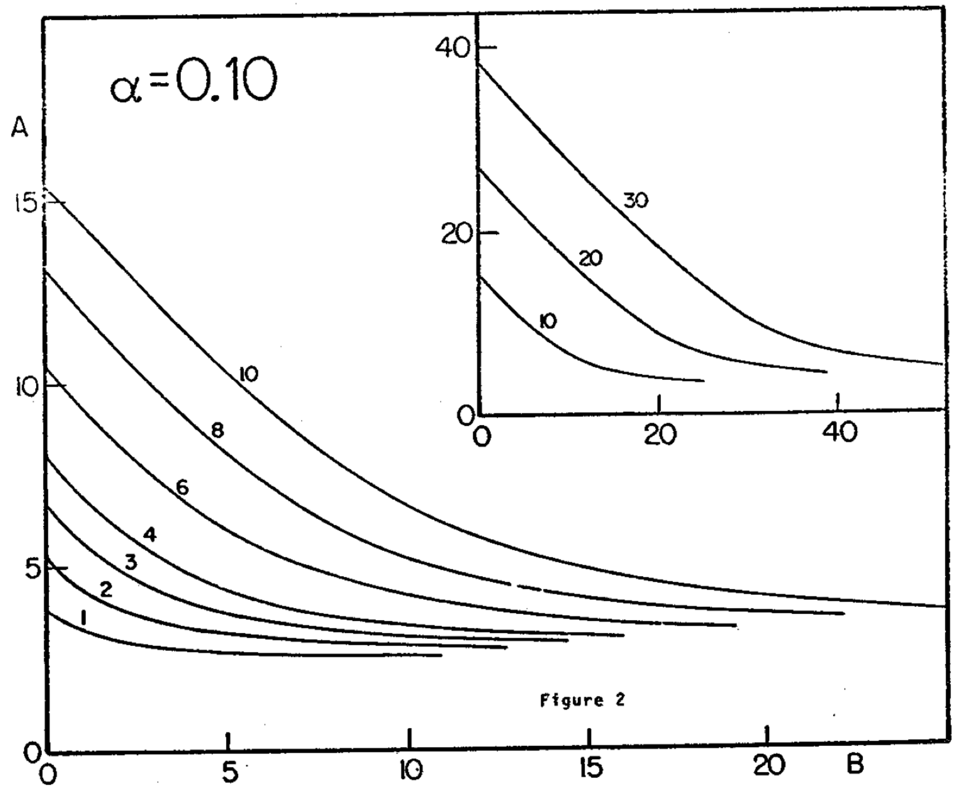


FIGURE 18. 90% limit s_{up} (A in the figure) vs. b (B in the figure) for different values of n_0. These are the upper limits resulting from a bayesian treatment with uniform prior. (taken from O.Helene, Nucl.Instr. and Meth. 212 (1983) 319)

Poisson

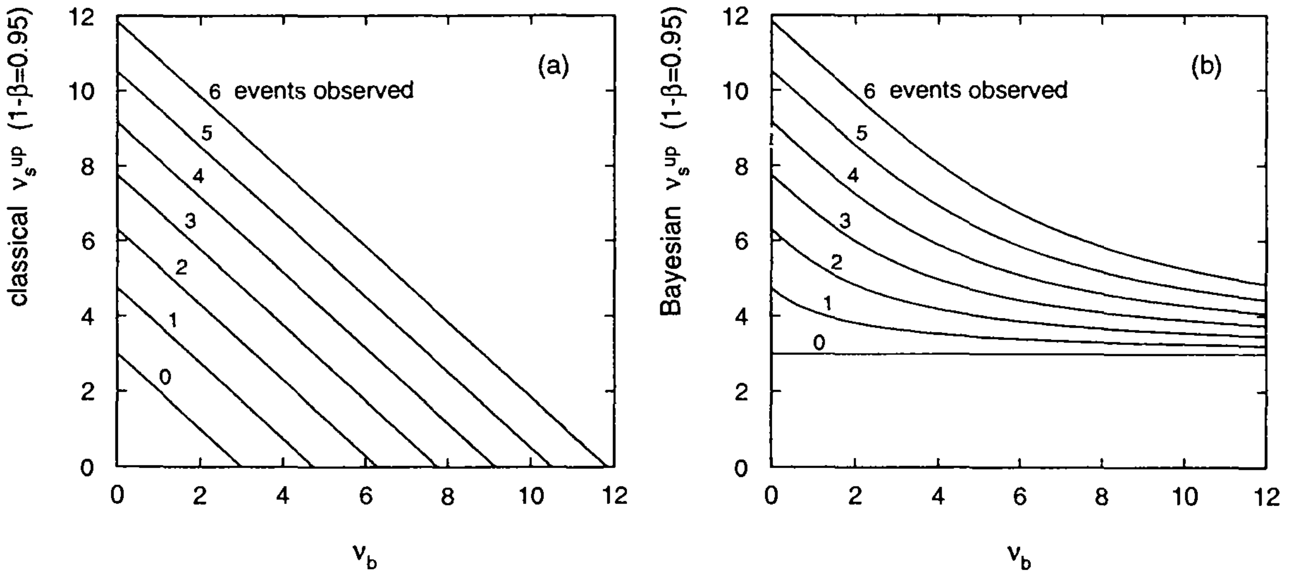


Fig. 9.9 Upper limits ν_s^{up} at a confidence level of $1 - \beta = 0.95$ for different numbers of events observed n_{obs} and as a function of the expected number of background events ν_{b}. (a) The classical limit. (b) The Bayesian limit based on a uniform prior density for ν_s.

Bayes limits

Assume background b ≠ 0 with uncertainty described by a pdf f(b) within interval bmin, bmax

$$p(s/n_0) = \frac{e^{-(s+b)}(s+b)^{n_0}}{n_0!}$$

↓

$$p(s/n_0) = \int_{b_{min}}^{b_{max}} \frac{e^{-(s+b')}(s+b')^{n_0}}{n_0!} f(b-b')db'$$

In general the width of f(b) affects the limit, large uncertainty on b => increase of S_{up}
The result in general depends on the prior (π(s)= cost, 1/s, 1/√s) (not in the case n_0=b=0)
General result for any n_0, transition from upper limit to central interval:

$$\hat{s} = n_0 - b \pm \sqrt{n_0 + \sigma^2(b)}$$

flip-flop problem (see next)