# Cabibbo-Kobayashi-Maskawa Matrix and CP Violation in Standard Model

Shahram Rahatlou
University of Rome
SAPIENZA UNIVERSITÀ DI ROMA

Lecture 3

Introduction to CP Violation

Lezioni di Fisica delle Particelle Elementari

# Outline of Today's Lecture

- What is CP Violation and why do we care?

- CKM matrix revisited
  - CP Violation in the Standard Model

- Experimental method to measure CP Violating effects
  - Quantum interference

# Asymmetric Universe of Matter

- Universe is very empty but in a biased way

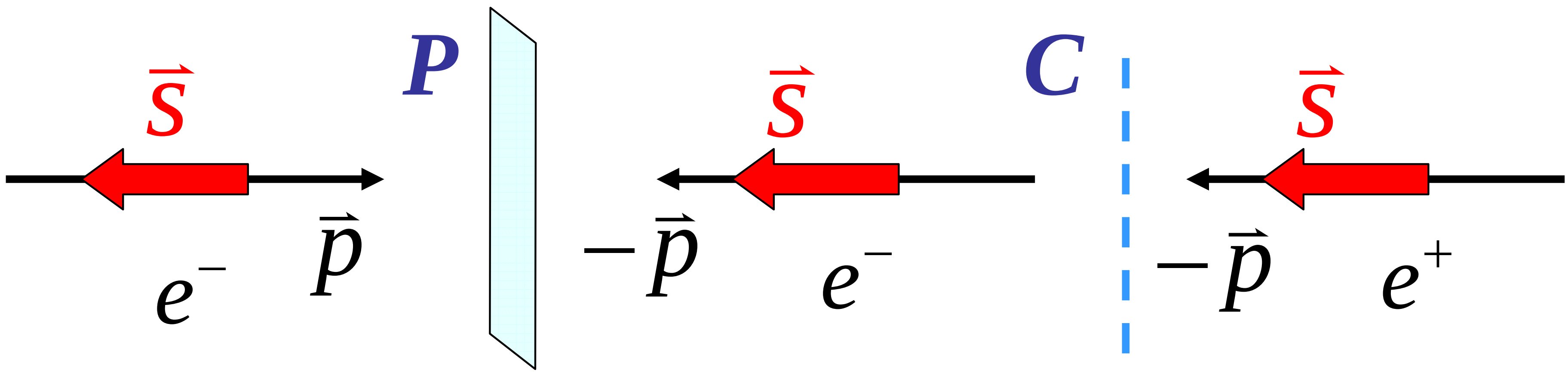
$$\frac{n_{baryon}}{n_{photons}} \approx 10^{-18} \qquad \frac{\text{N(anti-baryon)}}{\text{N(baryon)}} \leq 10^{-4}\text{-}10^{-6}$$

  - Absence of anti-nuclei amongst cosmic rays in our galaxy
  - Absence of intense $\gamma$–ray emission due to annihilation of distant galaxies in collision with antimatter galaxies

- The early universe believed to have equal amount of matter and anti-matter
  - What happened to the anti-matter?

- CP Violation is one of the three ingredients required to generate such an asymmetry after the Big Bang (A. Sakharov, 1967)
  - Baryon-number violating processes
  - Non-equilibrium state during expansion
  - C and CP Violation

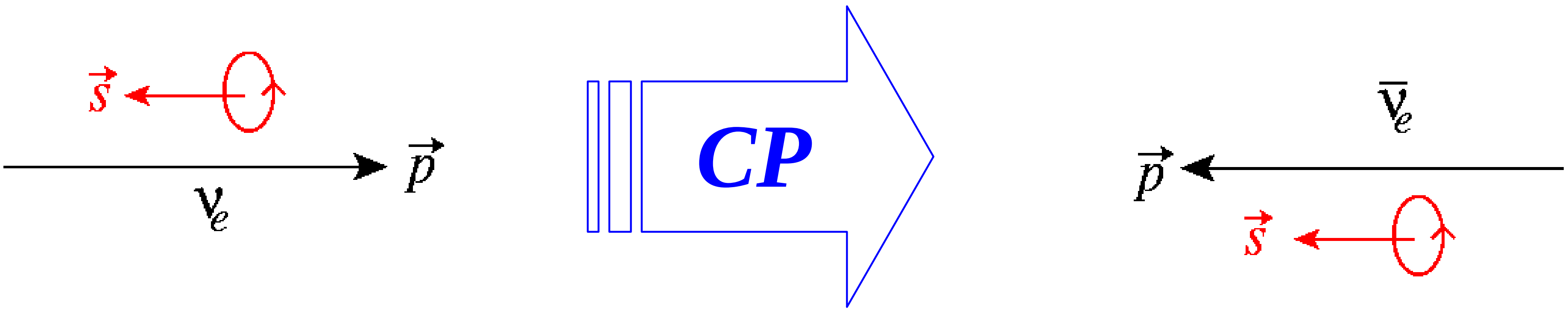
# C and P Symmetries and Fundamental Interactions




- Parity, *P*
  - Parity reflects a system through the origin. Converts right-handed coordinate systems to left-handed ones.
  - Vectors change sign but axial vectors remain unchanged
    - $\boldsymbol{x} \to -\boldsymbol{x}$ , $\boldsymbol{L} \to \boldsymbol{L}$
- Charge Conjugation, *C*
  - Charge conjugation turns a particle into its anti-particle
    - $\boldsymbol{e}^+ \to \boldsymbol{e}^-$, $\boldsymbol{K}^- \to \boldsymbol{K}^+$ , $\gamma \to \gamma$

# CP Symmetry, particles and anti-particles

- CP symmetry transforms a particle in its anti-particle




- CP is violated IF particles and anti-particles behave differently!

# Weak Interactions and Symmetry Violation

- P and C are good symmetries of the strong and electromagnetic interactions

- Parity violation observed in 1957
  - Asymmetry in $\beta$ decays of $^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \nu$
  - Electrons produced mostly in one hemisphere

- Charge-conjugation violation 1958
  - Only left-handed neutrinos and right-handed anti-neutrinos

- CP believed to be a good symmetry, but ...

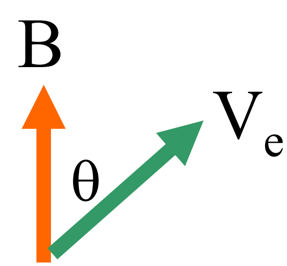
# A Shocker : Weak Interaction Violates Parity !

**1956**

Observation of a spatial asymmetry in the $\beta$-decay electrons from $^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \nu$


- Cold $^{60}$Co inside a Solenoidal B Field
- $^{60}$Co nuclei spin aligned with B field direction
- $^{60}$Co undergoes β decay .......electron emitted
- Measure electron intensity w.r.t B field dir.
- Result:Electrons preferentially emitted opposite spin dir.

$$I(\theta) = 1 - \frac{v_e}{c}\cos\theta$$



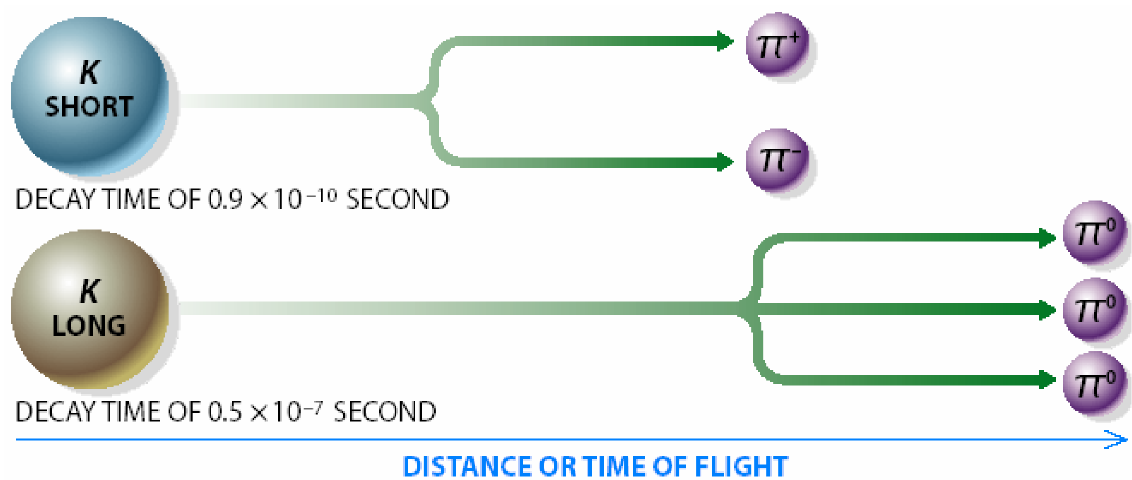

asymmetry of intensity → Weak interaction violated Parity

# CP Violation in Kaons

- CP conservation implies

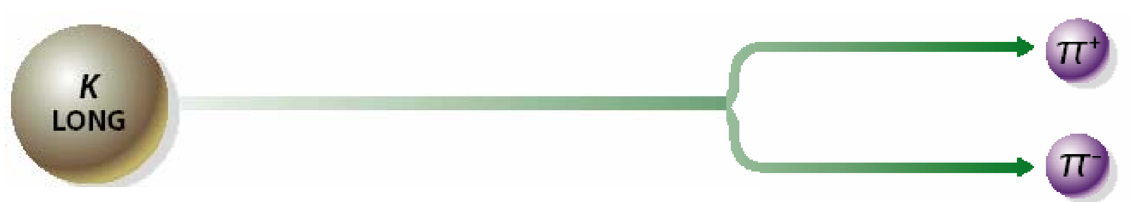
$CP = +1$

$CP = -1$




- CP violation in kaons observed in 1964

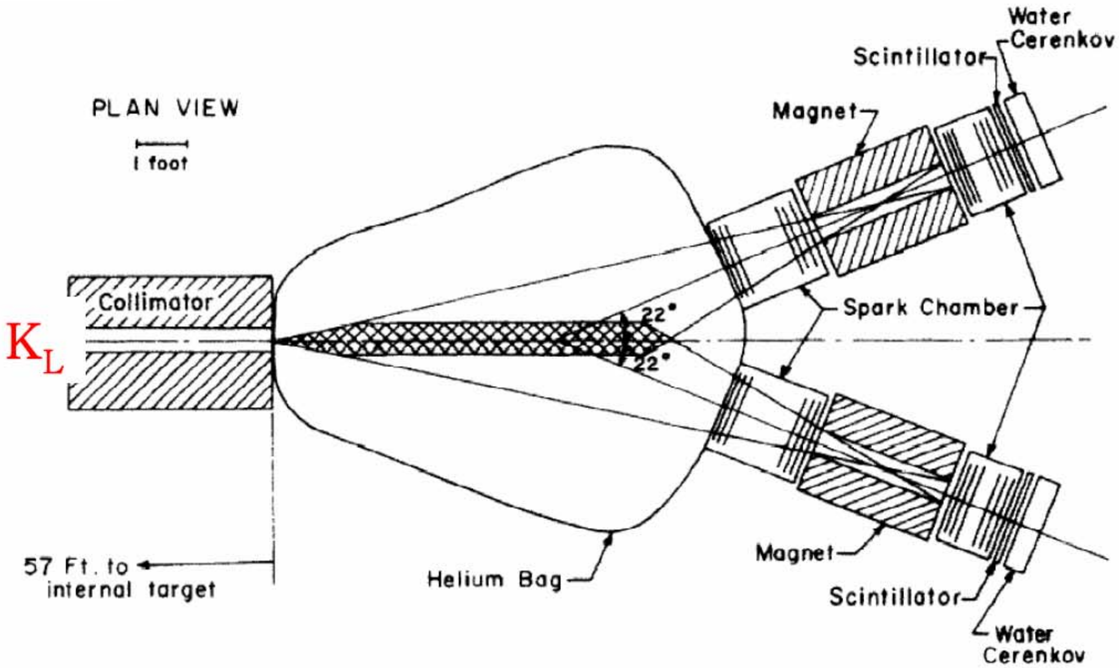
0.2% of the time!

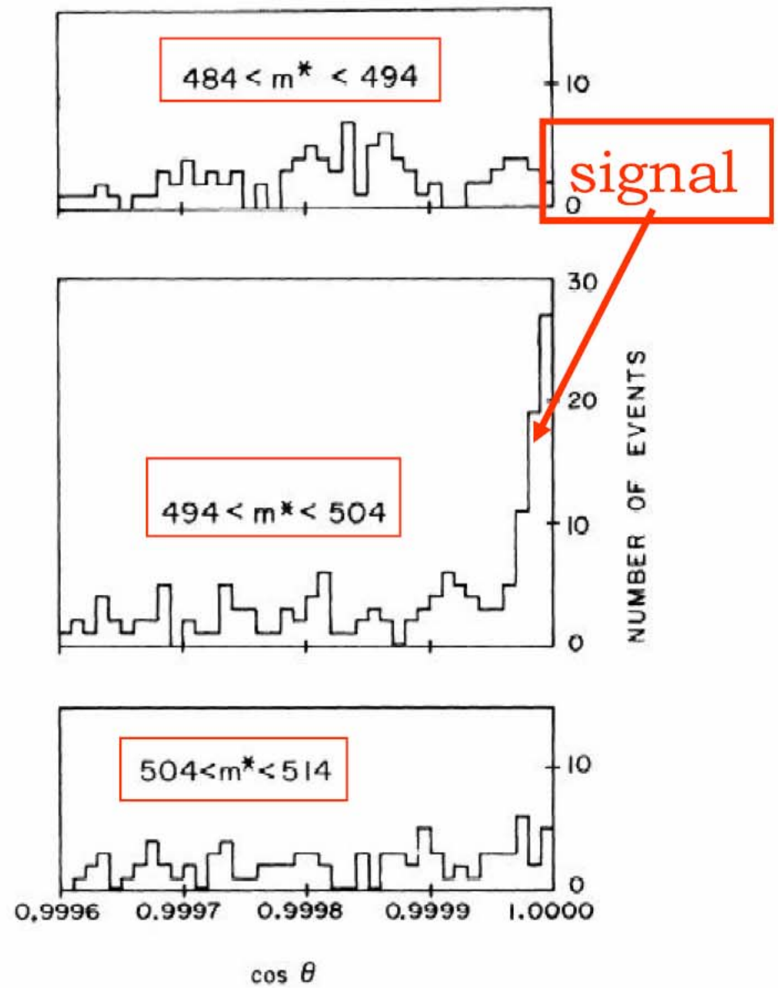



- No theoretical explanation!

# Observation of CP Violation in Kaons

$$\frac{A\left(\left|K_{\mathrm{L}}^{0}\right\rangle \to 2\pi\right)}{A\left(\left|K_{\mathrm{s}}^{0}\right\rangle \to 2\pi\right)} = (2.27 \pm 0.02)10^{-3}$$




2-body decay : the two π

are back-to-back: |cosθ|=1

# Complex Coupling Constants and CP Violation

| Fermion bilinear | Boson field $F$ | $\mathbf{P}\ F\ \mathbf{P}^\dagger$ | $\mathbf{C}\ F\ \mathbf{C}^\dagger$ | $\mathbf{CP}\ F\ \mathbf{CP}^\dagger$ |
|---|---|---|---|---|
| $\overline{\psi}\psi$ | Scalar $S^+(t,\vec{x})$ | $S^+(t,-\vec{x})$ | $S^-(t,\vec{x})$ | $S^-(t,-\vec{x})$ |
| $\overline{\psi}\gamma^5\psi$ | Pseudoscalar $P^+(t,\vec{x})$ | $-P^+(t,-\vec{x})$ | $P^-(t,\vec{x})$ | $-P^-(t,-\vec{x})$ |
| $\overline{\psi}\gamma_\mu\psi$ | Vector $V_\mu^+(t,\vec{x})$ | $V_\mu^+(t,-\vec{x})$ | $-V_\mu^-(t,\vec{x})$ | $-V_\mu^-(t,-\vec{x})$ |
| $\overline{\psi}\gamma_\mu\gamma^5\psi$ | Axial $A_\mu^+(t,\vec{x})$ | $-A_\mu^+(t,-\vec{x})$ | $A_\mu^-(t,\vec{x})$ | $-A_\mu^-(t,-\vec{x})$ |

Table 2.1: Properties of charged boson fields and corresponding fermion bilinear terms under **P**, **C**, and **CP**. $\gamma^5$ and $\gamma^\mu$ are the Dirac matrices.

Generic interaction lagrangian with vector and axial fields

$$\begin{aligned}\mathcal{L} &= a\, V_\mu^+(t,\vec{x})V^{\mu-}(t,\vec{x}) + b\, A_\mu^+(t,\vec{x})A^{\mu-}(t,\vec{x}) + \\ &\quad c\, V_\mu^+(t,\vec{x})A^{\mu-}(t,\vec{x}) + c^*\, A_\mu^+(t,\vec{x})V^{\mu-}(t,\vec{x})\end{aligned}$$

a, b: real constants
c: complex constant

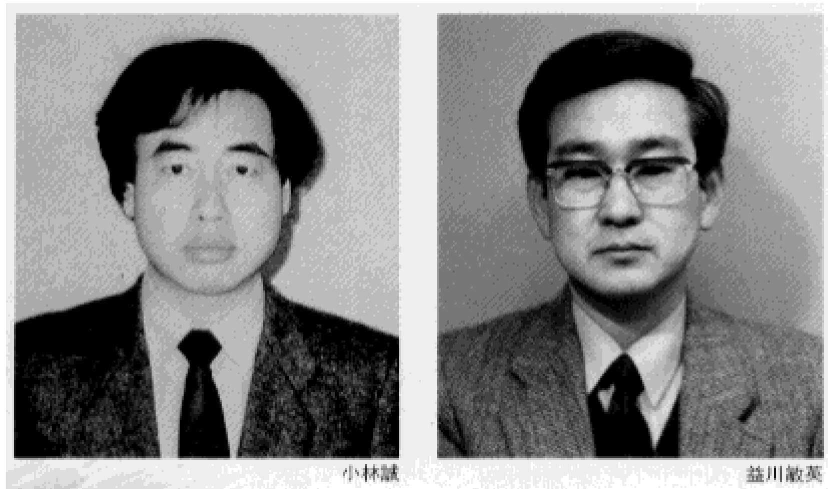
Lagrangian after CP transformation

$$\begin{aligned}\mathbf{CP}\mathcal{L}\mathbf{CP}^\dagger &= a\, V_\mu^-(t,-\vec{x})V^{\mu+}(t,-\vec{x}) + b\, A_\mu^-(t,-\vec{x})A^{\mu+}(t,-\vec{x}) + \\ &\quad c\, V_\mu^-(t,-\vec{x})A^{\mu+}(t,-\vec{x}) + c^*\, A_\mu^-(t,-\vec{x})V^{\mu+}(t,-\vec{x})\,.\end{aligned}$$

Lagrangian invariant under CP IF AND ONLY IF c=c*! c must be real

# Reminder Kobayashi-Maskawa Mechanism of CP Violation

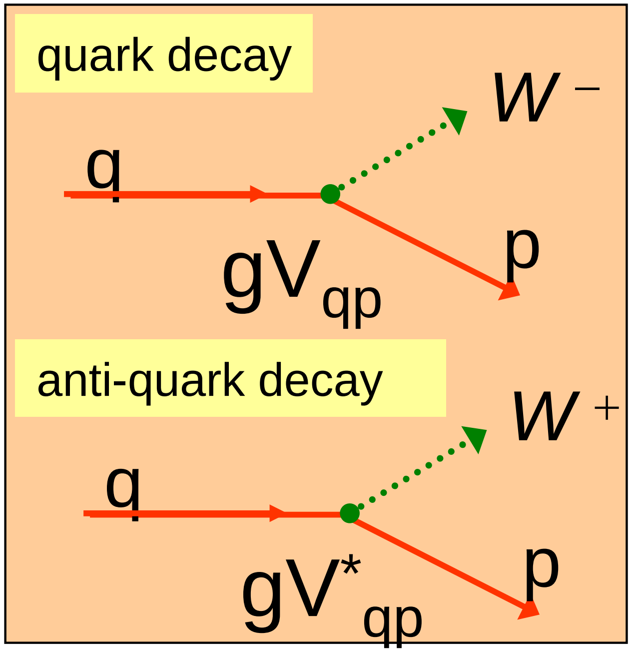
**1972**




Two Young Postdocs at that time !

- Proposed a daring explanation for CP violation in K decay:
- CP violation appears only in the charged current weak interaction of quarks
- There is a single source of CP Violation $\Rightarrow$ Complex Quantum Mechanical Phase $\delta_{KM}$ in inter-quark coupling matrix
- Need at least **3 Generation of Quarks** (then not known) to facilitate this
- **CP is NOT an approximate symmetry**, $\delta_{KM} \cong 1$, it is MAXIMALLY violated !

# CKM Matrix Revisited




$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{CKM} \equiv \begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

| Quark families | # Angles | # Phases | # Irreducible Phases |
|---|---|---|---|
| n | n(n-1)/2 | n(n+1)/2 | n(n-1)/2-(2n-1)=(n-1)(n-2)/2 |
| 2 | 1 | 3 | 0 |
| 3 | 3 | 6 | 1 |
| 4 | 6 | 10 | 3 |

**Only Source of CP Violation in SM**

CP Violation built in the Standard Model through Kobayashi-Maskawa Mechanism!

Only one complex phase! All CP violating effects in SM related to each other
B and K decays CP Violating phenomena are cause by the same complex phase

# Unitarity of CKM Matrix

$$V^\dagger V = VV^\dagger = 1$$

- All rows and columns must be orthonormal
  - 3 conditions for diagonal elements
  - 6 conditions for off-diagonal elements

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

Magnitude of each term

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0 \qquad \lambda \;\lambda\; \lambda^5$$
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \qquad \lambda^3 \;\lambda^3\; \lambda^3$$
$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0 \qquad \lambda^4 \;\lambda^2\; \lambda^2$$
$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0 \qquad \lambda^3 \;\lambda^3\; \lambda^3$$
$$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = 0 \qquad \lambda^4 \;\lambda^2\; \lambda^2$$
$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0 \qquad \lambda \;\lambda\; \lambda^5$$

Only condition with comparable size of all pieces and involving b decays

# Unitarity Triangles

Unitarity condition of CKM Matrix → orthonormality of rows & columns

$$\sum_{(i=u,c,t)} V_{ij} V^{*}_{ik} = \delta_{jk} \quad ; \quad \sum_{(i=d,s,b)} V_{ij} V^{*}_{kj} = \delta_{ik}$$

⇒ three conditions are interesting for understanding SM predictions for CP violation

$$V_{ud}V^{*}_{us} + V_{cd}V^{*}_{cs} + V_{td}V^{*}_{ts} = 0,$$

$$V_{us}V^{*}_{ub} + V_{cs}V^{*}_{cb} + V_{ts}V^{*}_{tb} = 0,$$

$$V_{ud}V^{*}_{ub} + V_{cd}V^{*}_{cb} + V_{td}V^{*}_{tb} = 0.$$

Each relation requires sum of three complex quantities to vanish

→ can be represented in the complex plane as a triangle

→ known as Unitarity Triangles

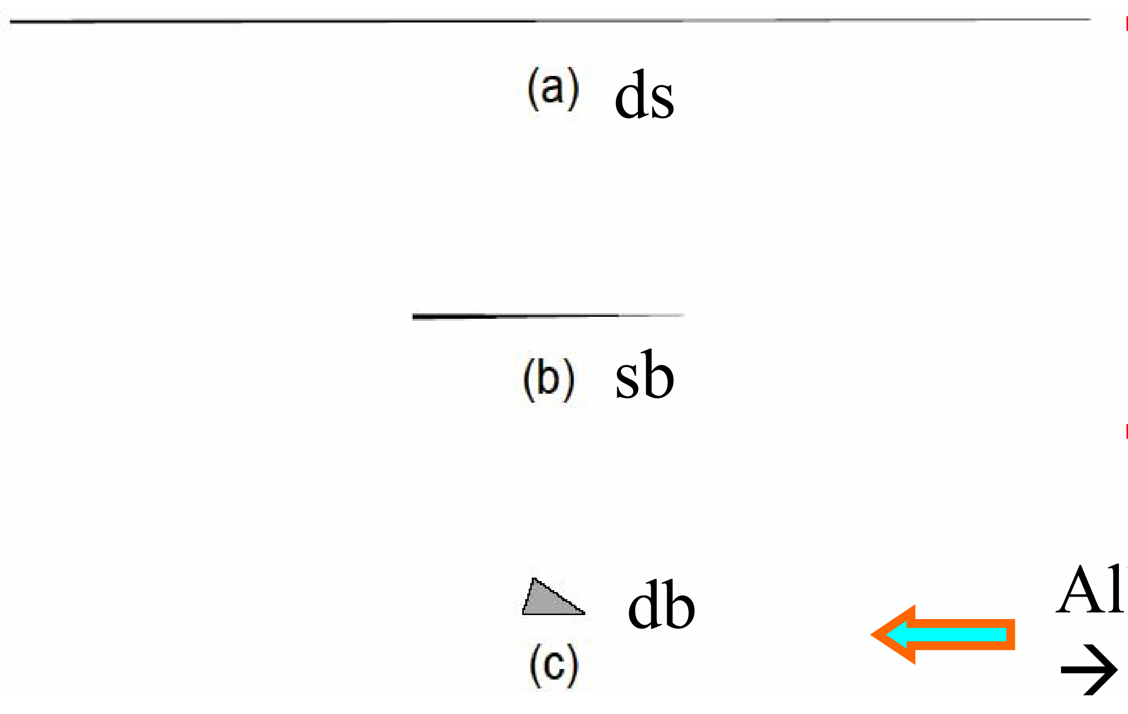
With the knowledge of $|V_{ij}|$ magnitudes, its instructive to draw the triangles

# Three Unitarity Triangles Drawn to Common Scale

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0,$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0,$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0.$$



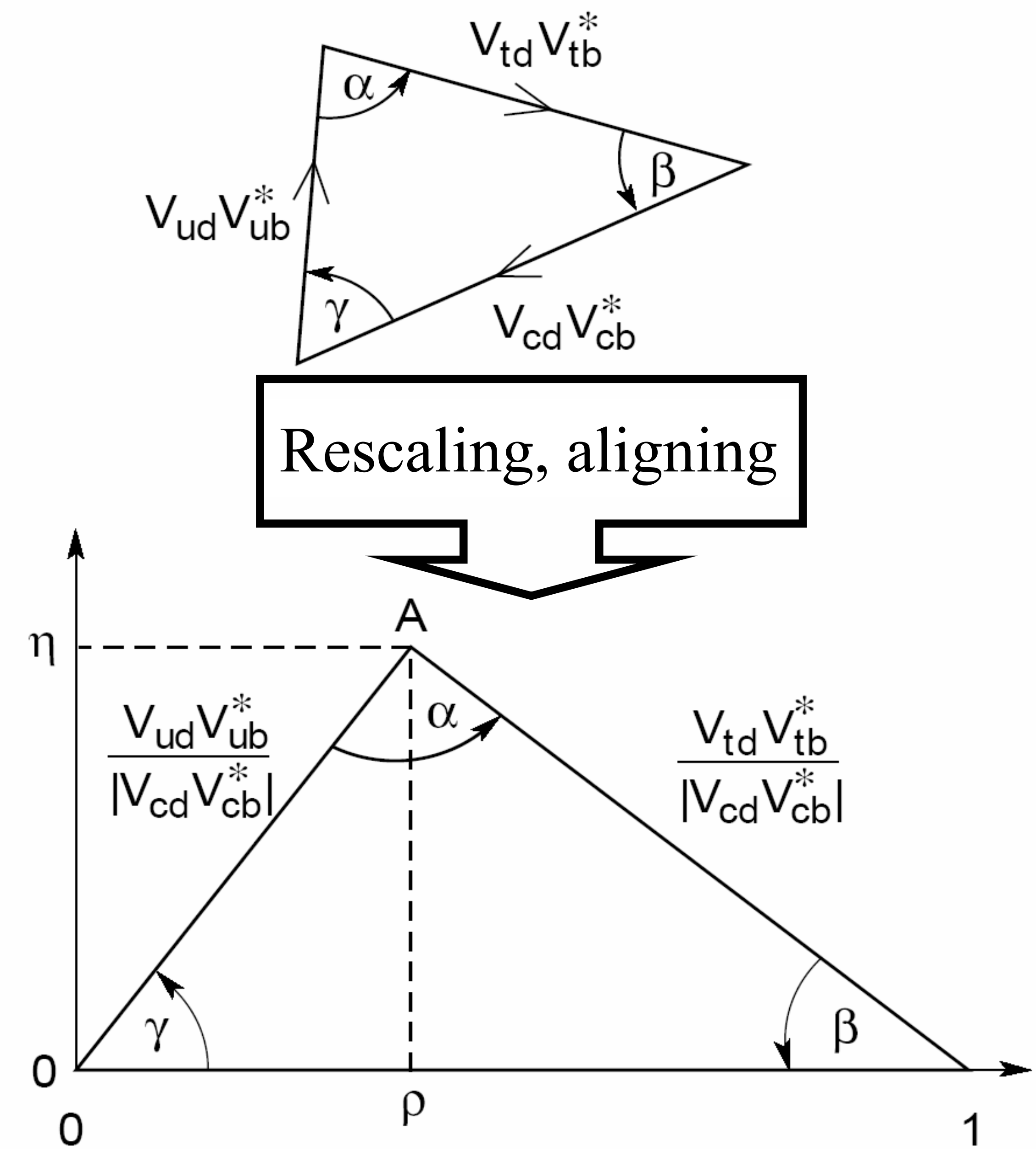

One side is much shorter than the other two→ triangle collapses on a line

All sides of comparable length ( $\lambda^3$)
→ All angles are large

Experimentally ⇒ hard to measure small numbers
easier to measure larger numbers as in (c)

# CKM Unitarity Triangle in B Decays

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$




Angles of Unitarity Triangle

$$\alpha = \phi_2 \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right],$$

$$\beta = \phi_1 \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right],$$

$$\gamma = \phi_3 \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

All lengths involve b decays
Large CP Asymmetries predicted , $\propto$ UT angles

# Measuring Complex Phase of CKM Matrix

- Branching fractions and lifetimes sensitive to magnitude of CKM elements
  - Decay probabilities usually include $|V_{ij}|^2$
  - We looked for decays involving only one CKM element to make interpretation of experimental result possible

- Complex phase of CKM is a relative phase between matrix elements

- We need processes with interference of two different CKM elements

$$A_1 = Ae^{i\alpha}$$

$$A_2 = Be^{i\beta}$$

$$A_{tot} = A_1 + A_2$$

Sensitive to phase difference!

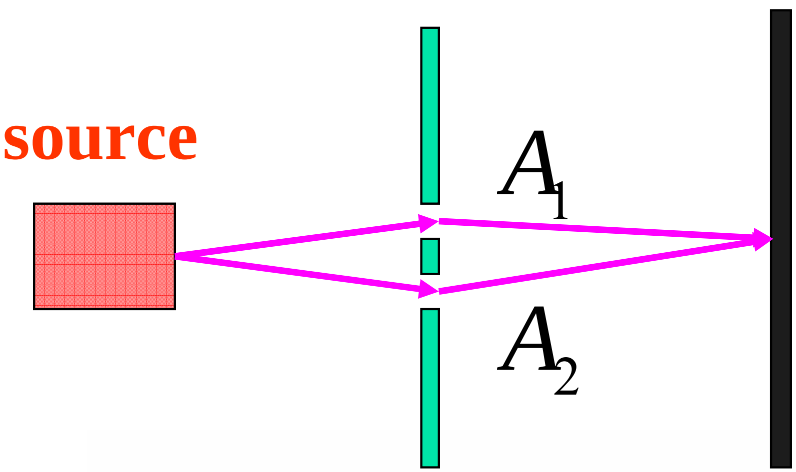
$$|A_{tot}|^2 = |A|^2 + |B|^2 + ABe^{i(\alpha-\beta)} + ABe^{-i(\alpha-\beta)}$$

# CP Violation

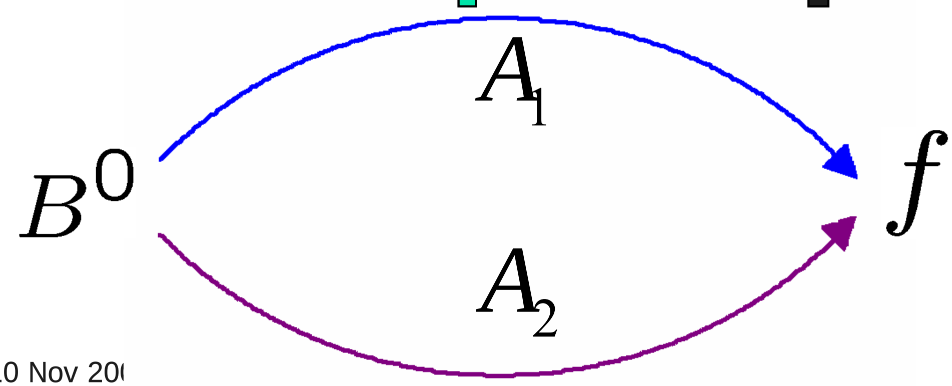
- CP violation can be observed by comparing decay rates of particles and antiparticles

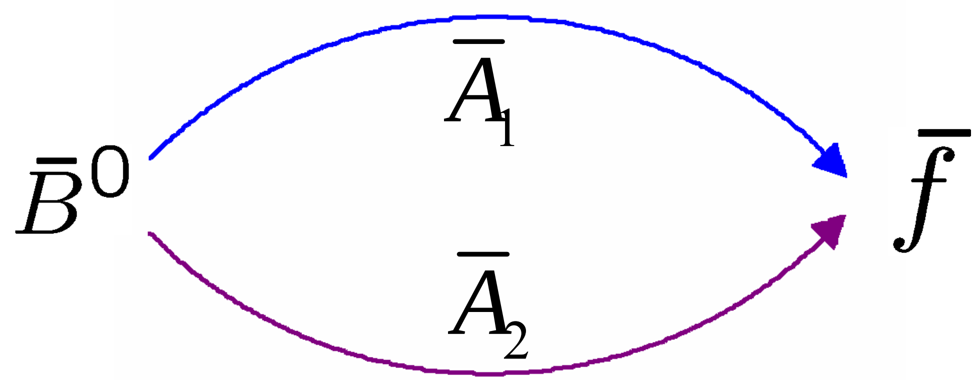
$$\Gamma(a \to f) \neq \Gamma(\bar{a} \to \bar{f}) \Rightarrow \text{CP Violation}$$

- The difference in decay rates arises from a different interference term for the matter vs. antimatter process. Analogy to double-slit experiment:



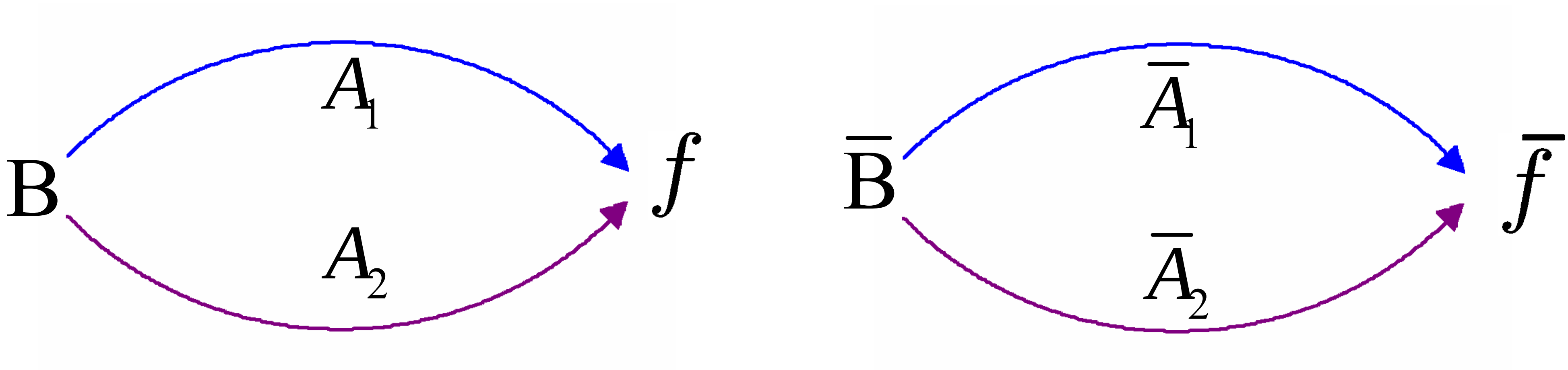

Classical double-slit experiment:
Relative phase variation due to different path lengths: interference pattern in space



# CP Violation in B Meson System

Identify B final states which are arrived at by two paths




In $B^0$ system, $B^0$ □ $\bar{B}^0$ oscillation provides one path with

the other path(s) come from weak decay of B hadron

In $B^{\pm}$system $\Rightarrow$ no oscillation possible,

2 (or more) amplitudes must come from different weak decay of B

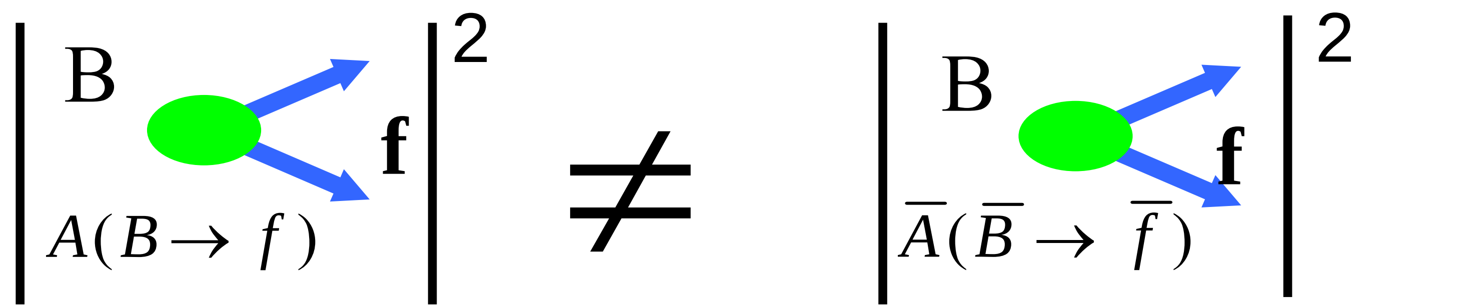
B Meson is heavy $\Rightarrow$ many final states, multiple "paths."

2 classes of B decays come into play: "Tree" $\Rightarrow$ spectator decay like

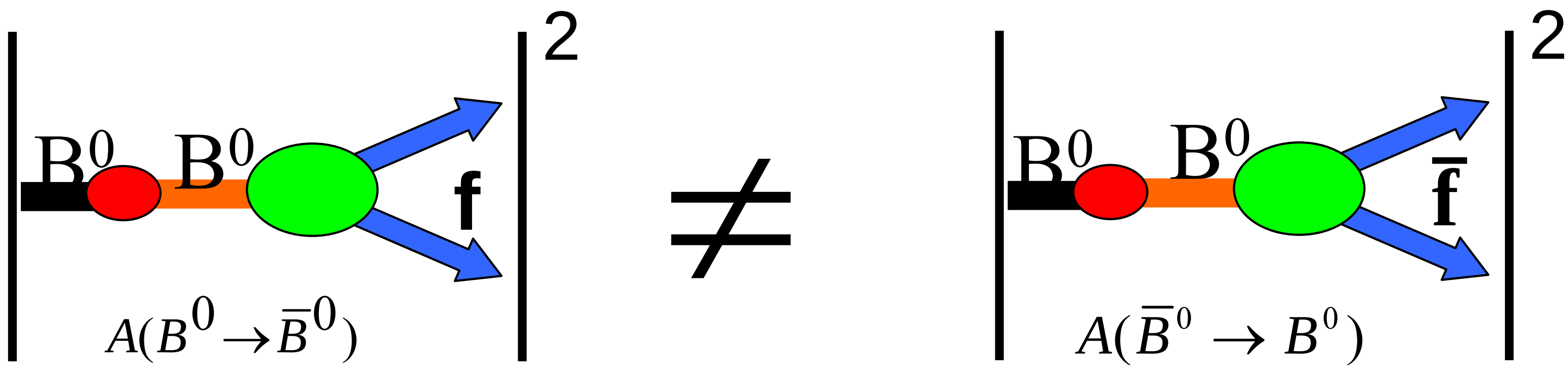
"Penguin" $\Rightarrow$ FCNC loop diagrams with u,c,t

# Overview of CP Violating Processes

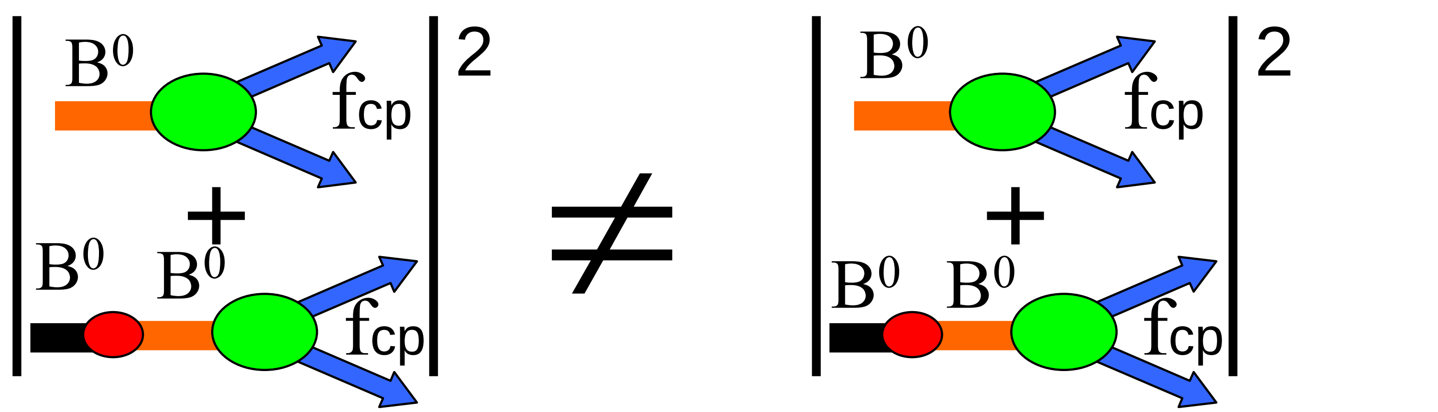
**CP Violation in Decay a.k.a. Direct CPV**




**CP Violation in Mixing**




**CP Violation in interference between Mixing and Decay**



# CP Violation Is a Quantum Phenomenon

- CPV is due to Quantum interference between two or more amplitudes

- Phase of QM amplitudes is the key

- Need to consider two types of phases
  - *CP-conserving phases*: <u>don't change sign under CP</u>
    - Sometimes called *strong phases* since they can arise from strong, final-state interactions
  - *CP-violating phases*: <u>these do change sign under CP transformation</u>
    - originate in the Weak interaction sector

$$A = Ae^{i\varphi}e^{i\delta}$$

$$\bar{A} = Ae^{-i\varphi}e^{i\delta}$$

# How can CP asymmetries arise ?

- Suppose a decay can occur through two different processes, with amplitudes $A_1$ and $A_2$

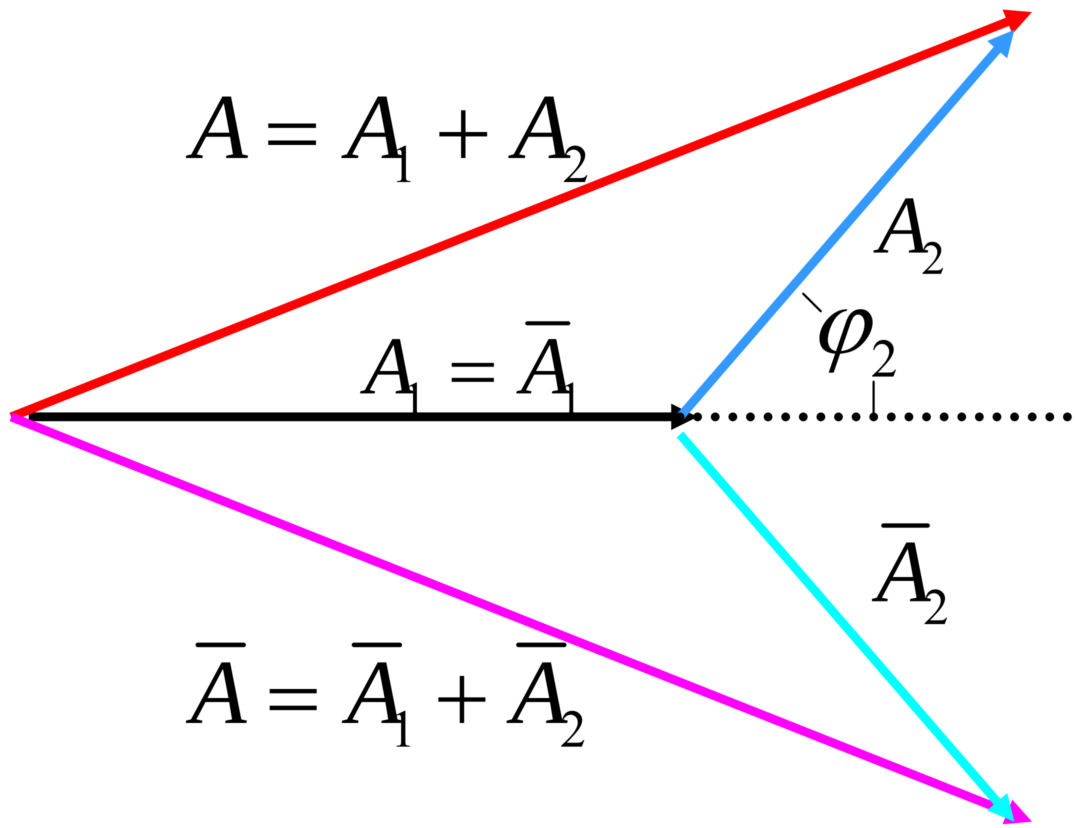
- First, consider the case in which there is a (relative) CP-violating phase between $A_1$ and $A_2$ only

$$A = A_1 + a_2 e^{i\varphi_2}$$

$$\bar{A} = A_1 + a_2 e^{-i\varphi_2}$$

➔ **No Direct CP asymmetry!**

(Decay rate is different from what it would be without the phase)



# How can CP asymmetries arise ?

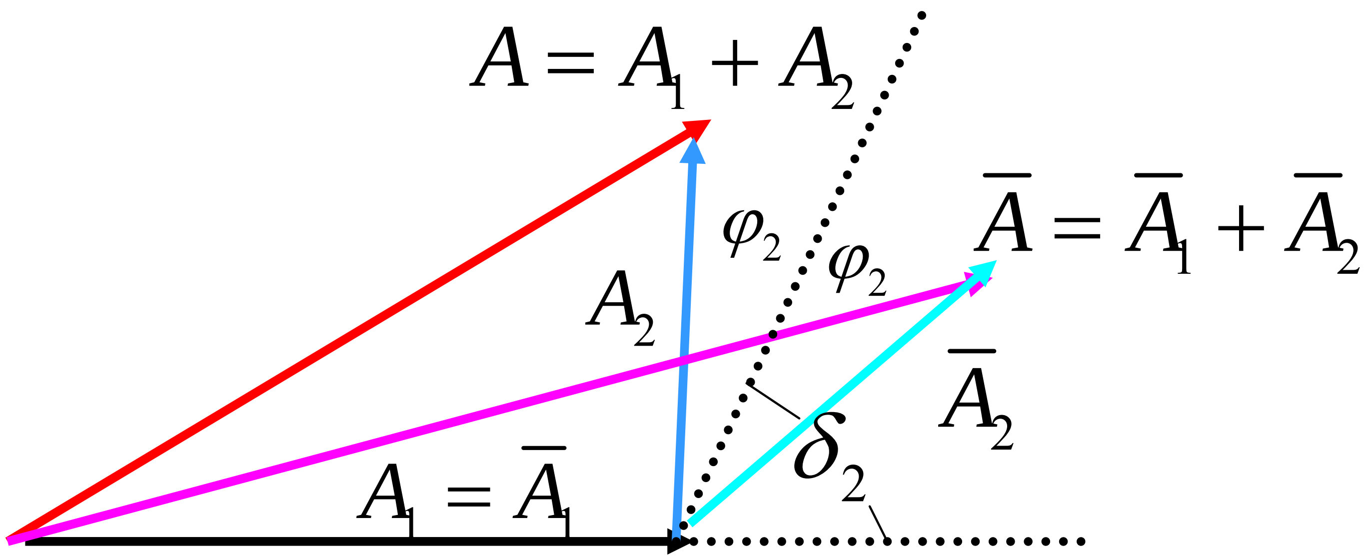
- Next, introduce a relative *CP-conserving* phase in addition to the relative *CP-violating* phase

$$A = A_1 + a_2 e^{i(\varphi_2 + \delta_2)}$$

$$\overline{A} = A_1 + a_2 e^{i(-\varphi_2 + \delta_2)}$$

- Now have a Direct CP Violation

$$|A| \neq |\overline{A}|$$



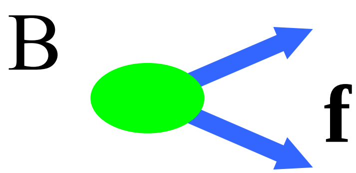
# Definition of CP Asymmetry

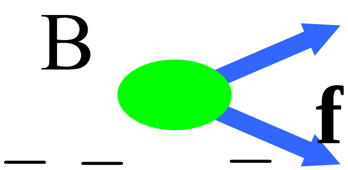
$$Asymmetry = \frac{\left|\bar{A}\right|^2 - \left|A\right|^2}{\left|\bar{A}\right|^2 + \left|A\right|^2} = \frac{2\left|A_1\right|\left|A_2\right|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{\left|A_1\right|^2 + \left|A_2\right|^2 + \left|A_1\right|\left|A_2\right|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)}$$

To extract the CP-violating phase from an observed CP asymmetry, we need to know the value of the CP-conserving phase difference

***B* system: extraordinary laboratory for quantum interference experiments: many final states, multiple "paths"→ Lots of channels for CP Violation**

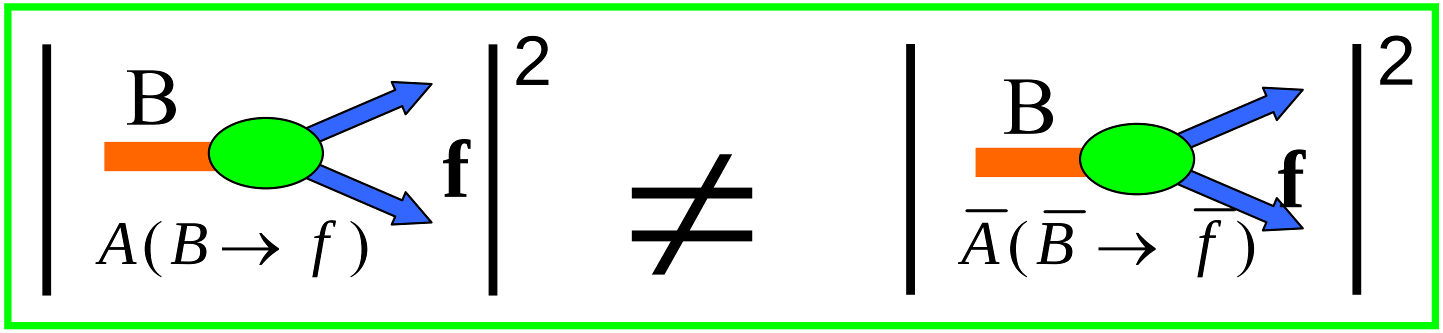
# Direct CP Violation

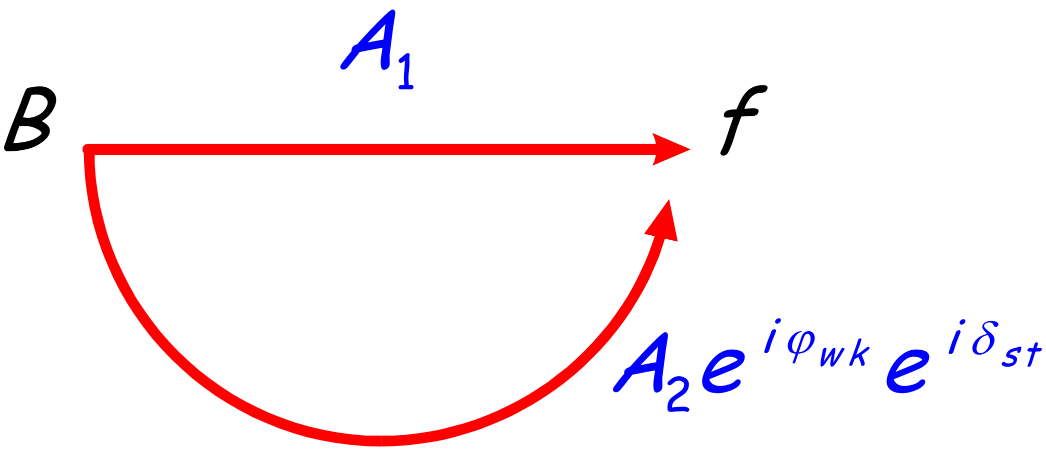


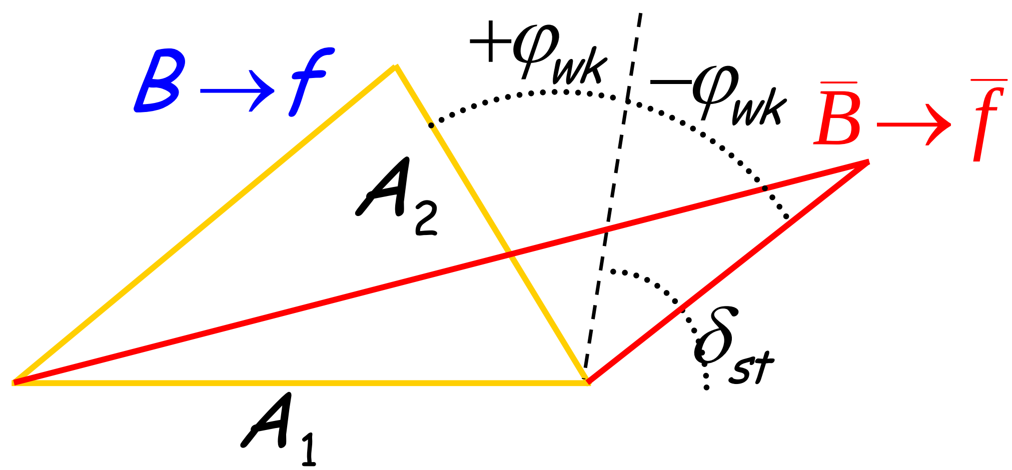




$$\left| A(B \to f) \right|^2 \neq \left| \overline{A}(\overline{B} \to \overline{f}) \right|^2$$

# CPV in Decay a.k.a. Direct CP Violation



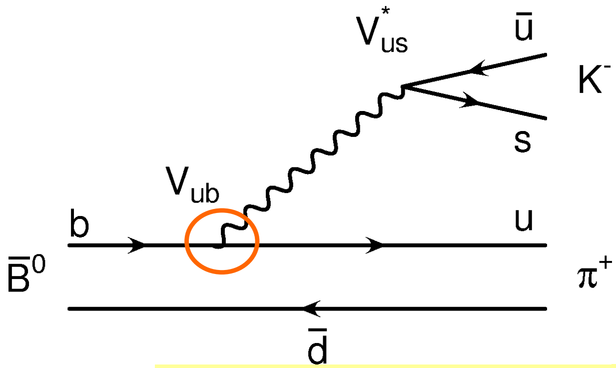



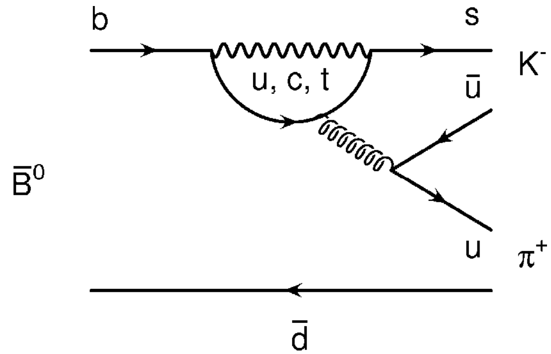

$$\Gamma(B \to f) = \left| A_1 + A_2 e^{i\varphi_{wk}} e^{i\delta_{st}} \right|^2, \quad \Gamma(\bar{B} \to \bar{f}) = \left| A_1 + A_2 e^{-i\varphi_{wk}} e^{i\delta_{st}} \right|^2$$

$$A_{CP} = \frac{Br\left(\bar{B} \to \bar{f}\right) - Br\left(B \to f\right)}{Br\left(\bar{B} \to \bar{f}\right) + Br\left(B \to f\right)} \equiv \frac{\left|\bar{A}_{\bar{f}}\right|^2 - \left|A_f\right|^2}{\left|\bar{A}_{\bar{f}}\right|^2 + \left|A_f\right|^2} \neq 0 \to \text{Direct } CPV$$

# Direct CP Violation in $B^0 \to K^- \pi^+$

T

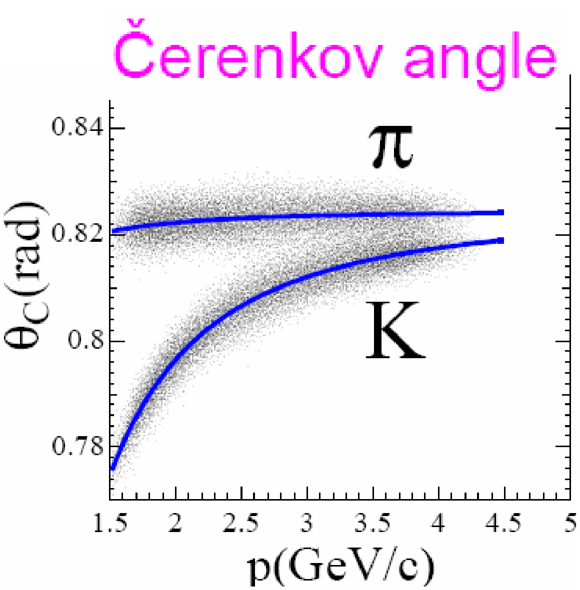


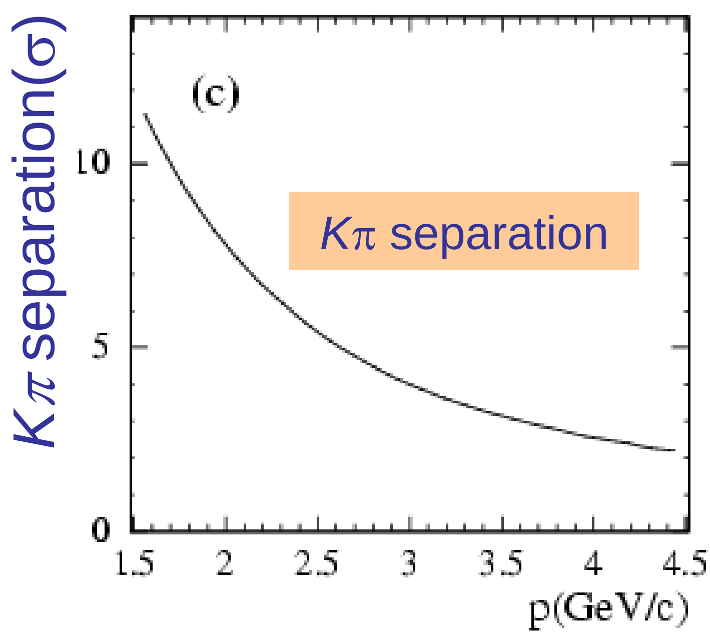



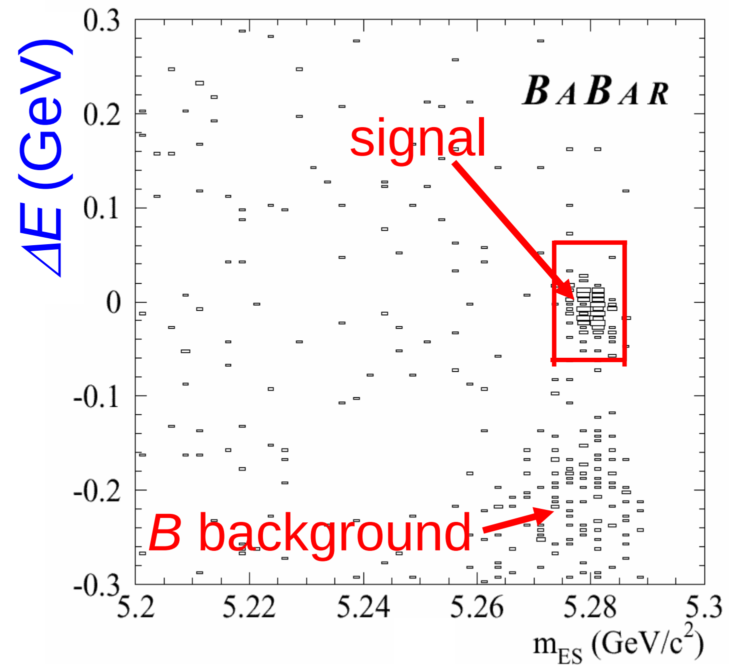

P

$SM\ amplitude = \lambda^2 e^{i\gamma} T + P$ $A_{K\pi} \propto \sin(\gamma)$

- Loop diagrams from New Physics (e.g. SUSY) can modify SM asymmetry via P
- Clean mode with “large” rate : $BF(B^0 \to K^+\pi^-) = (18.2 \pm 0.8)\times 10^{-6}$
- Measure *charge* asymmetry, reject large $B \to \pi\pi$ background with Particle ID



# Observation of Direct CPV in $B0 \to K^- \pi^+$

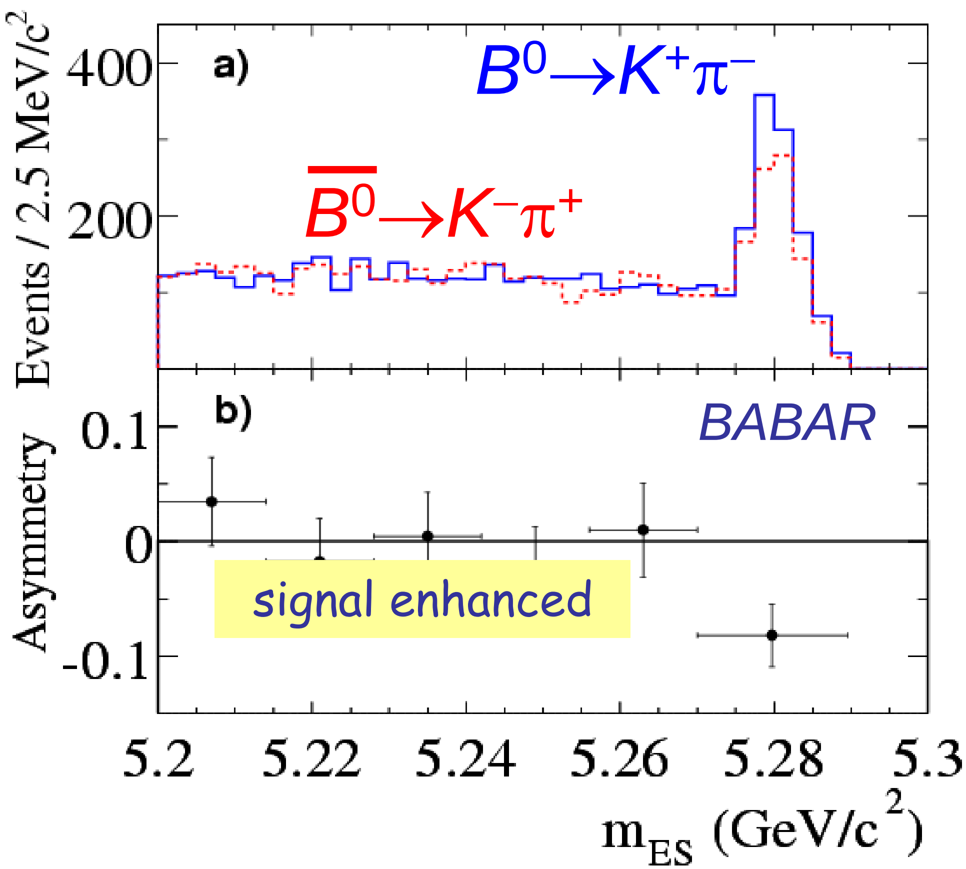
$$A_{K^-\pi^+} \equiv \frac{\Gamma(\overline{B} \to K^-\pi^+) - \Gamma(B \to K^+\pi^-)}{\Gamma(\overline{B} \to K^-\pi^+) + \Gamma(B \to K^+\pi^-)}$$

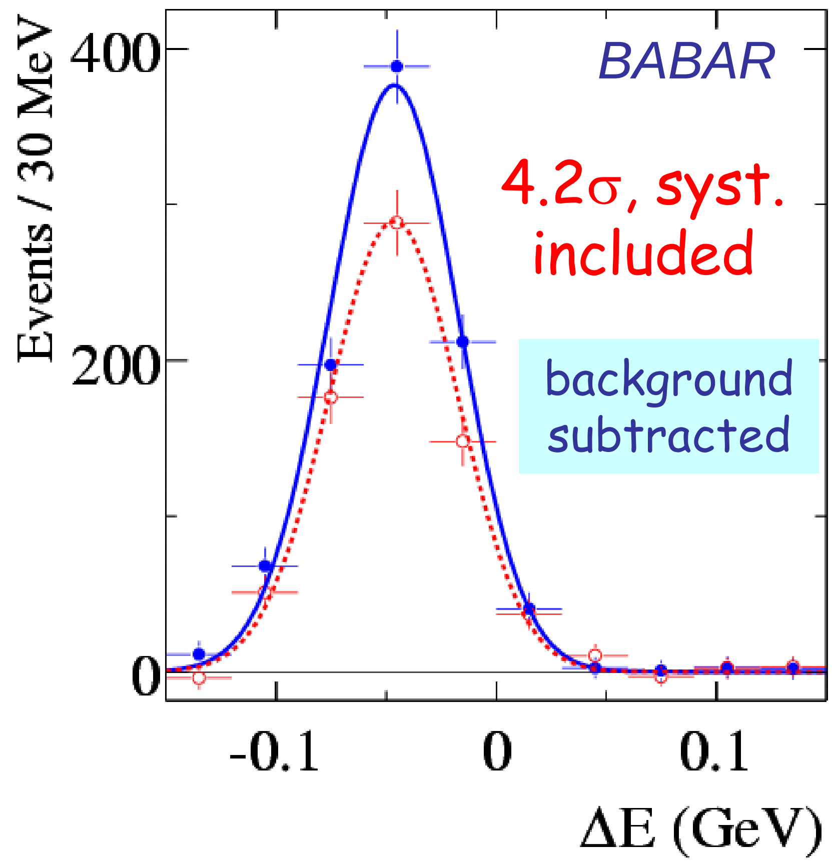
$$n_{K\pi} = 1606 \pm 51$$

$$A_{K\pi} = -0.133 \pm 0.030 \pm 0.009$$

$$n\left(B^0 \to K^+\pi^-\right) = 910$$

$$n\left(\overline{B}^0 \to K^-\pi^+\right) = 696$$

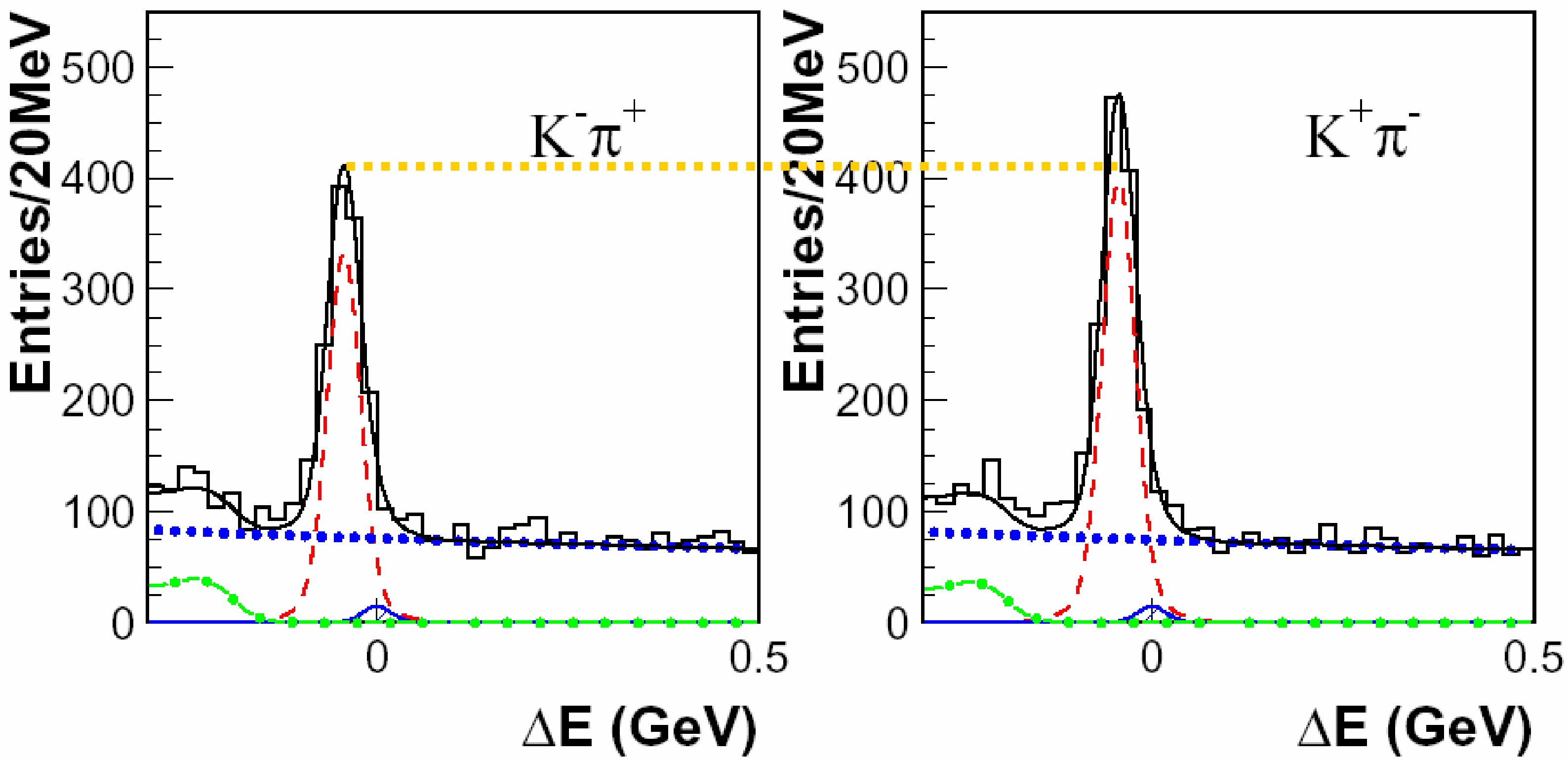


# Confirmation of Direct CPV by Belle at ICHEP04

$A_{CP} = -0.101 \pm 0.025 \pm 0.005$

3.9σ significance




274M $B\bar{B}$

Signal=2139 ±53

Non-Perturbative QCD uncertainties large,
Standard Model CP Violation not precisely predictable

⇒ insufficient to prove or rule out contribution from New Physics