Cabibbo-Kobayashi-Maskawa Matrix
and
CP Violation in Standard Model

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Lecture 5

Measurements of α, β, γ

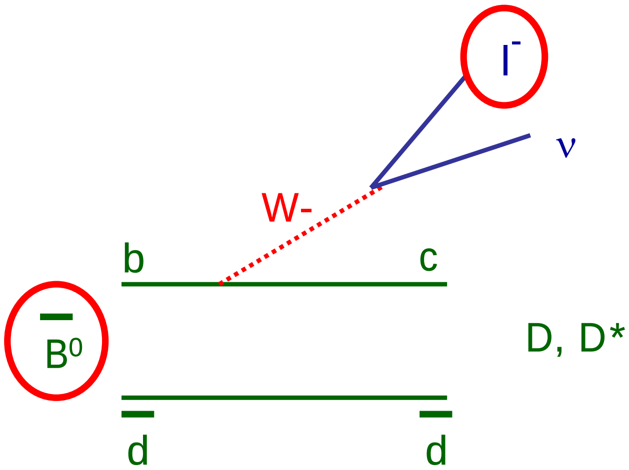
Constraints on Unitarity Triangle

Lezioni di Fisica delle Particelle Elementari

Outline of Today's Lecture

- Measurements of Angle β with rare B decays
  - Probing New Physics
- Measurements of Angle α
- Measurements of Angle γ
- Constraints on Unitarity Triangle from CKM Measurements

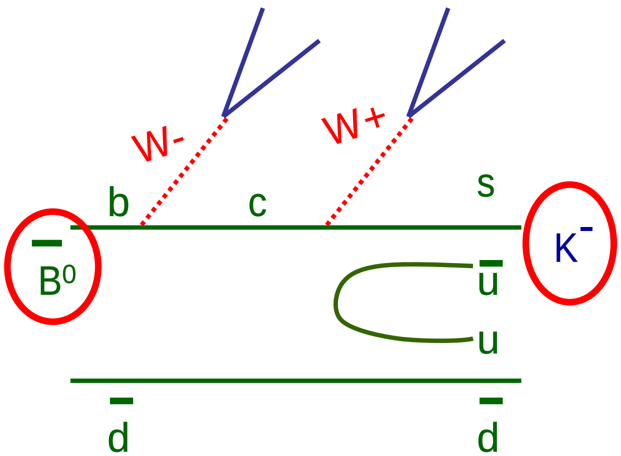
Separating B^0 and $\overline{B}^0$ mesons



$l^- \rightarrow \overline{B}^0$

$l^+ \rightarrow B^0$

Lepton Tag

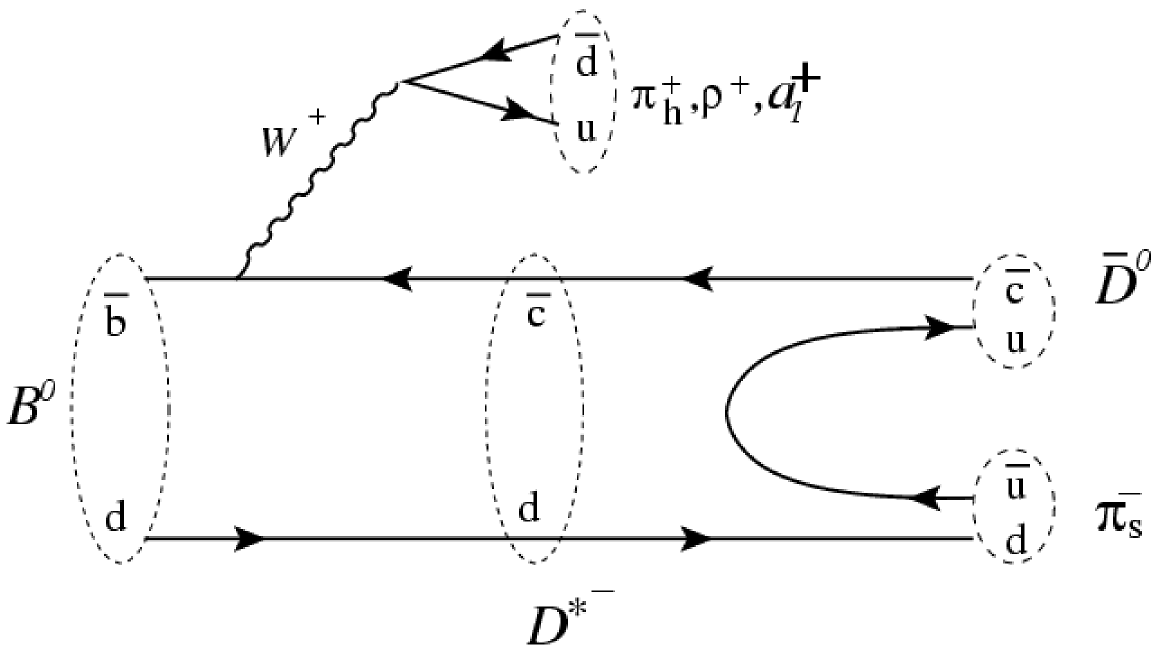


$\sum_{kaons} Q < 0 \Rightarrow \overline{B}^0$

$\sum_{kaons} Q > 0 \Rightarrow B^0$

Kaon Tag

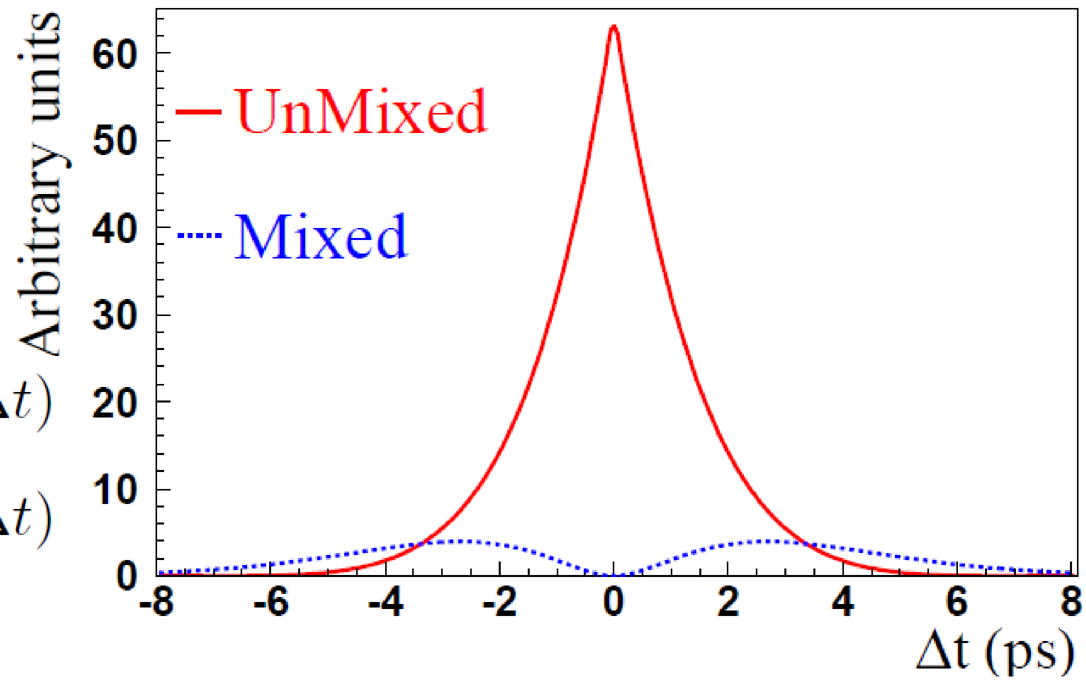
Kinematic Tag



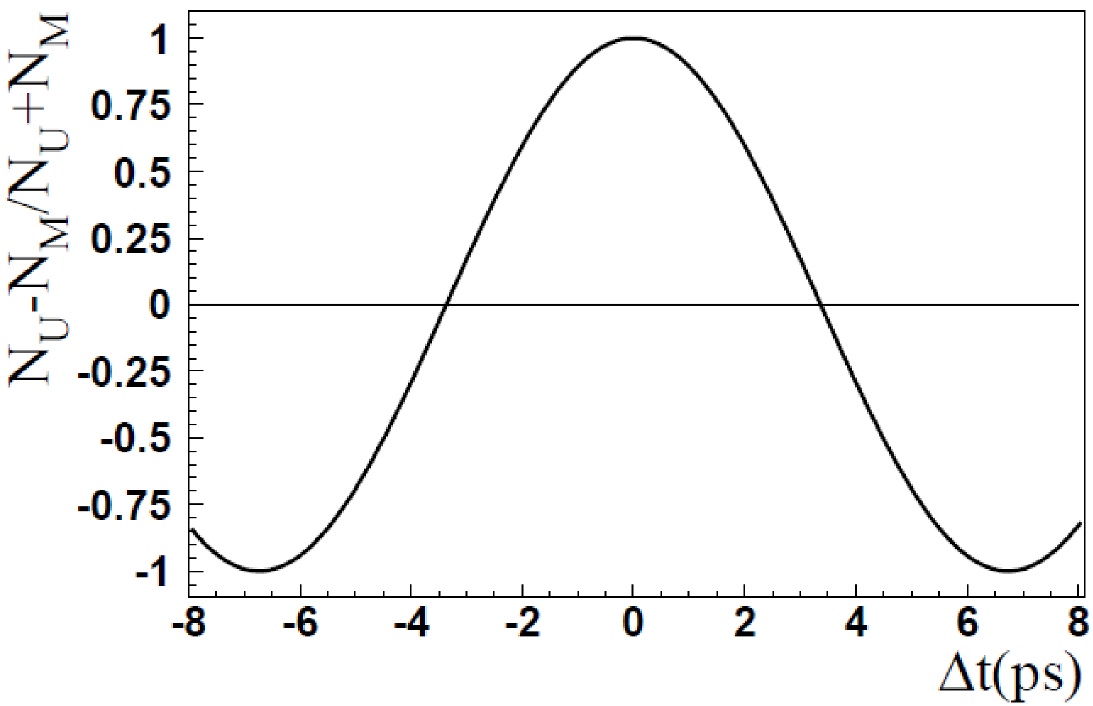
Time Dependent B Oscillation (Or Mixing) at $\Upsilon(4S)$

$$f_{\text{unmix}}(\Delta t) \propto e^{-\Gamma|\Delta t|}(1+\cos\Delta m_d \Delta t)$$

$$f_{\text{mix}}(\Delta t) \propto e^{-\Gamma|\Delta t|}(1-\cos\Delta m_d \Delta t)$$



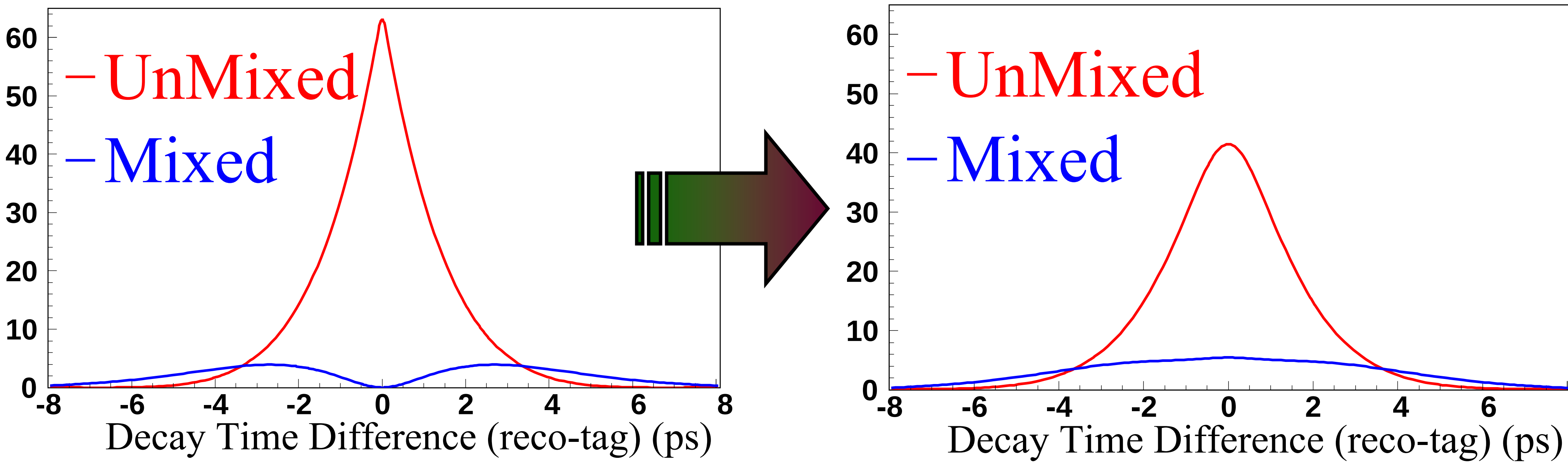
$$\mathcal{A}_{\text{mix}}(\Delta t) = \frac{f_{\text{unmix}} - f_{\text{mix}}}{f_{\text{unmix}} + f_{\text{mix}}}$$



Δt Spectrum of Mixed and Unmixed Events

perfect
flavor tagging & time resolution

realistic
mis-tagging & finite time resolution



$$f_{\substack{Unmix\\Mix}}(\Delta t) = \left\{ \; f_{\substack{Unmix\\Mix}}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B_d}}}{4\tau_{B_d}} \times \left(1 \pm \cos(\Delta m_d \Delta t) \right) \mathrm{ionFunction} \right.$$

Unmixed: $B^0_{flav}\bar{B}^0_{tag}$ *or* $\bar{B}^0_{flav}B^0_{tag}$

Mixed: $B^0_{flav}B^0_{tag}$ *or* $\bar{B}^0_{flav}\bar{B}^0_{tag}$

w: the fraction of wrongly tagged events

Δm_d: oscillation frequency

Flavor Tagging Performance

The large sample of fully reconstructed events provides the precise determination of the tagging parameters required in the CP fit

Tagging category	Fraction of tagged events ε (%)	Wrong tag fraction w (%)	$Q = \varepsilon (1-2w)^2$ (%)
Lepton	11.1 ± 0.2	**8.6 ± 0.9**	7.6 ± 0.4
Kaon	**34.7 ± 0.4**	18.1 ± 0.7	14.1 ± 0.6
NT1	7.7 ± 0.2	22.0 ± 1.5	2.4 ± 0.3
NT2	14.0 ± 0.3	37.3 ± 1.3	0.9 ± 0.2
ALL	67.5 ± 0.5		**25.1 ± 0.8**

Highest "efficiency"

Smallest mistag fraction

Δt Resolution Function

$$R(\delta\Delta t) = (1 - f_{tail} - f_{outl}) \cdot G_{core}(\delta\Delta t, S_{core}, \delta_{core,i})$$ ← Core

$$+ f_{tail} \cdot G_{tail}(\delta\Delta t, S_{tail}, \delta_{tail})$$ ← Tail

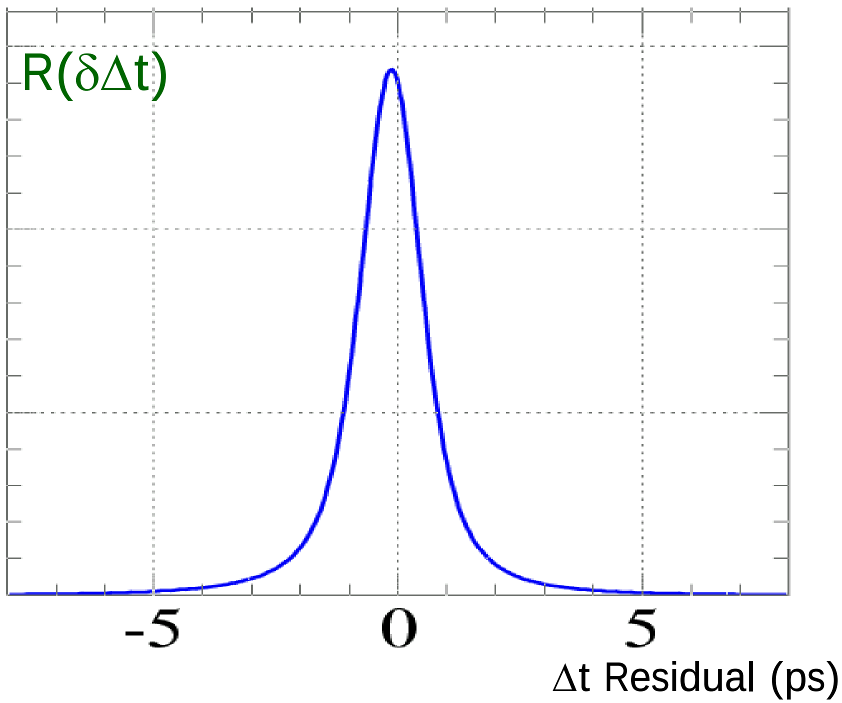
Outlier → $$+ f_{outl} \cdot G_{outl}(\delta\Delta t, \sigma_{outl} = 0\,ps, \delta_{outl} = 8\,ps)$$

$$\sigma_{core} = S_{core} \cdot \sigma_{\Delta t}^{evt}$$

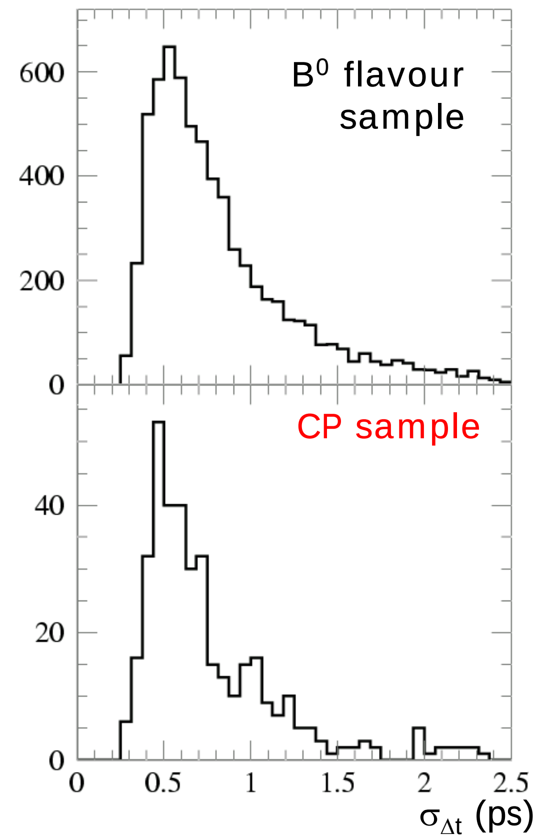
$$\sigma_{tail} = S_{tail} \cdot \sigma_{\Delta t}^{evt}$$

Use the event-by-event uncertainty on Δt

Parameter
S_{Core}
S_{Tail}
f_{Tail} (%)
f_{Outlier} (%)
$\delta_{\mathrm{Core,Lepton}}$
$\delta_{\mathrm{Core,Kaon}}$
$\delta_{\mathrm{Core,NT1}}$
$\delta_{\mathrm{Core,NT2}}$
δ_{Tail}

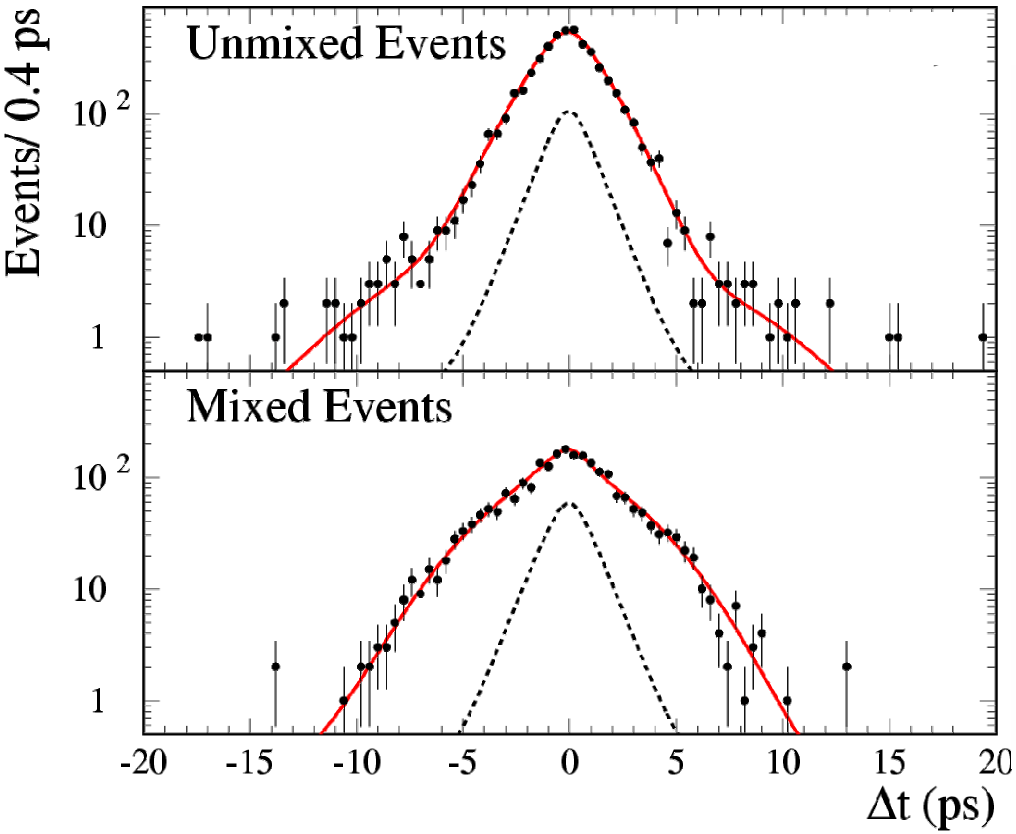


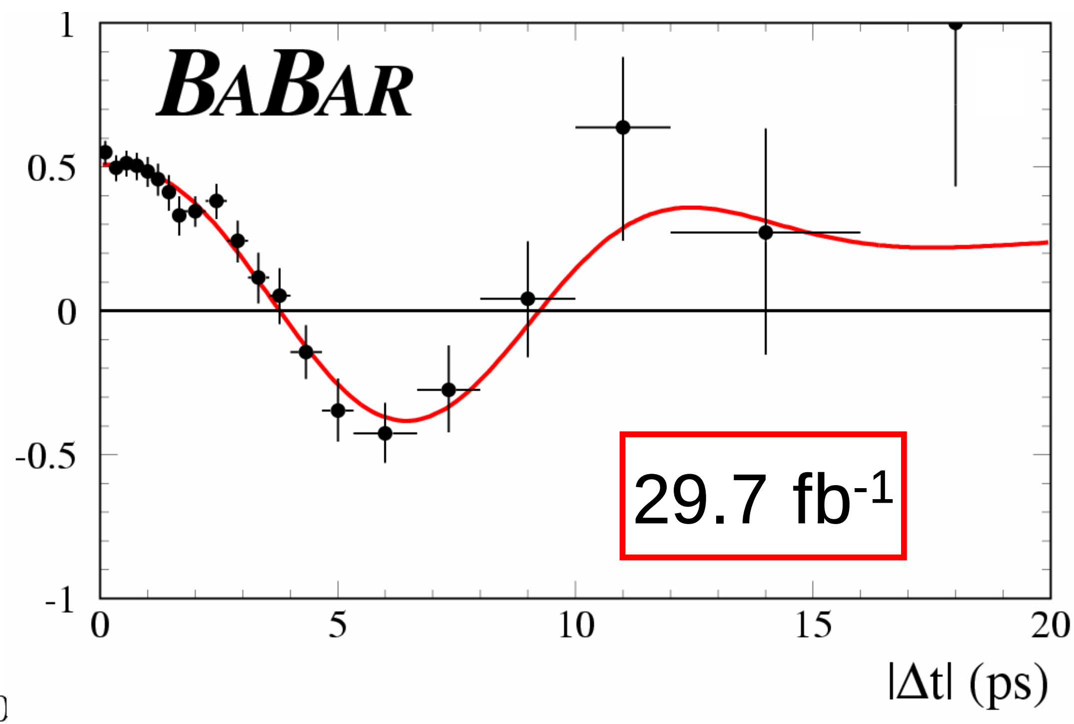
Different bias scale factor
For each tagging category



$B^0\bar{B}^0$ Mixing Fit Result

$$\text{Asymmetry}(\Delta t) = \frac{N(\text{unmixed}) - N(\text{mixed})}{N(\text{unmixed}) + N(\text{mixed})} \approx \left(1 - 2\langle w \rangle\right) \times \cos(\Delta m_d \Delta t)$$





Δm_d = 0.516 ± 0.016 (stat) ± 0.010 (syst) ps^{-1}

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Probing New Physics with β

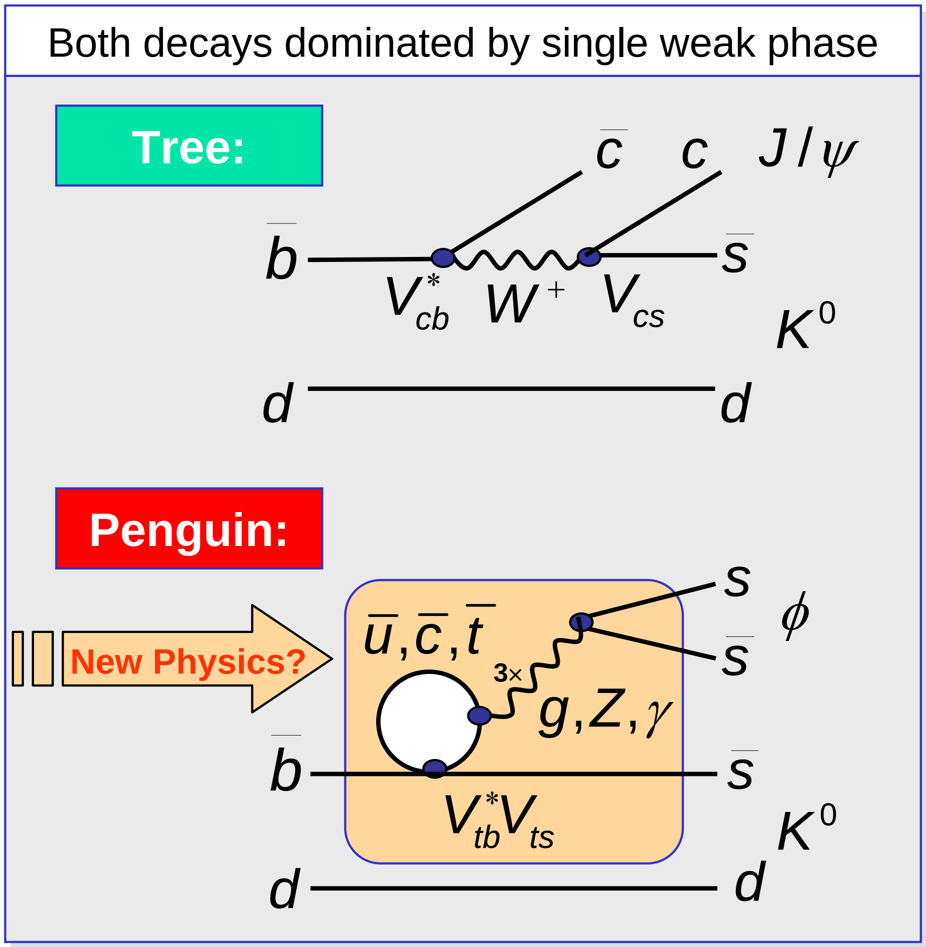
Compare sin2β *with* "sin2β" from CPV in Penguin decays of B^0

In SM, interference between B^0 mixing and dominant $b \to s s \bar{s}$ ($b \to s u \bar{u}$)

[penguin amplitudes have no CKM phase] gives the same CPV (due to $e^{-i2\beta}$) as in $b \to c \bar{c} s$

Loop diagrams sensitive to high virtual mass scales $\Rightarrow$ sensitive to new physics

NP coupling can bring in new phases that may cause deviations from expected "sin2β"



$b \to c\bar{c}s$

$$\lambda_{J/\psi K^0_{S,L}} = \eta_{J/\psi K^0_{S,L}} \left(\frac{q}{p}\right)_B \cdot \left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right) \cdot \left(\frac{q}{p}\right)_K = \eta_{J/\psi K^0_{S,L}} e^{-2i\beta}$$

$b \to s\bar{s}s$

$$\lambda_{\phi K^0_{S,L}} = \eta_{\phi K^0_{S,L}} \left(\frac{q}{p}\right)_B \cdot \left(\frac{V_{tb}V_{ts}^*}{V_{tb}^*V_{ts}}\right) \cdot \left(\frac{q}{p}\right)_K \sim \eta_{\phi K^0_{S,L}} e^{-2i\beta}$$

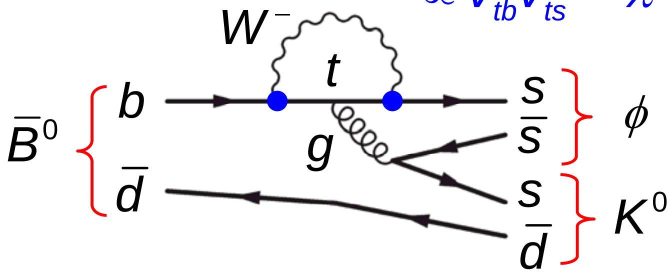
$\sin 2\beta$ [charmonium] $\stackrel{?}{=}$ $\sin 2\beta$ [s-penguin]

Must be if one amplitude dominates

Standard Model Polution in Penguin Mode

Decay amplitude of interest

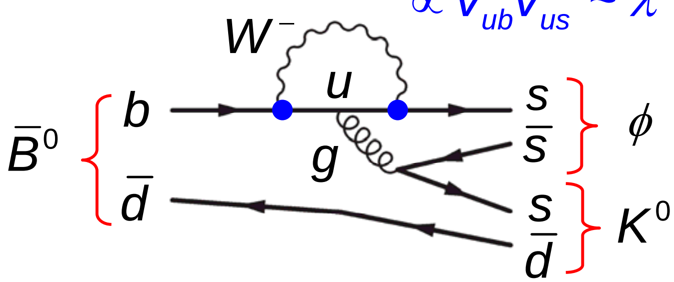
$\propto V_{tb}V_{ts}^{*} \sim \lambda^{2}$



SM Pollution

Naive (dimensional) uncertainties on sin2β

$\propto V_{ub}V_{us}^{*} \sim \lambda^{4}R_{u}e^{-i\gamma}$



$O(\lambda^{2})$

~ 5%

The « Golden » Penguin mode $B^0 \to \phi K^0$

BaBar

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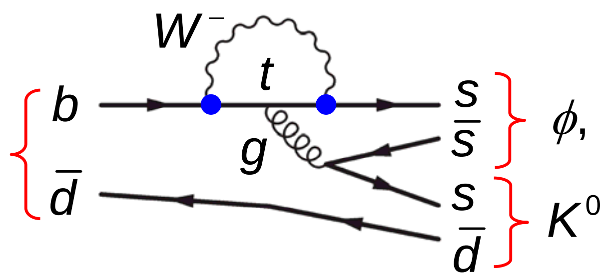
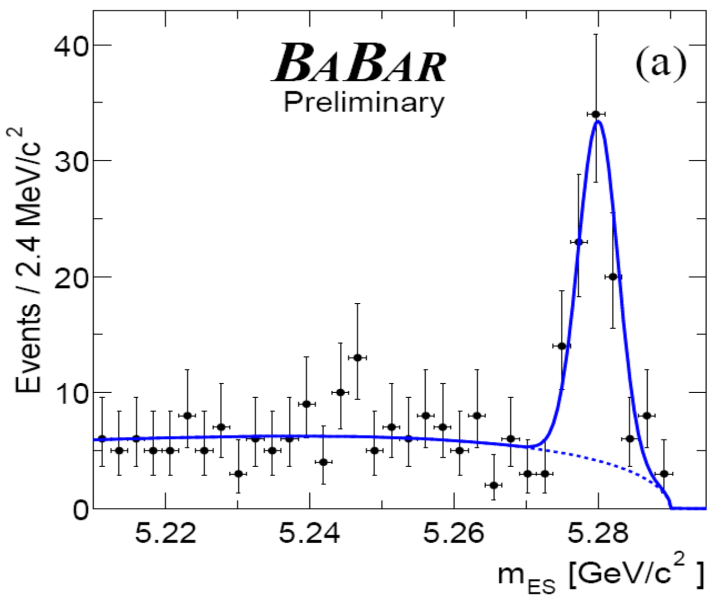
- Modes with K_S and K_L are both reconstructed

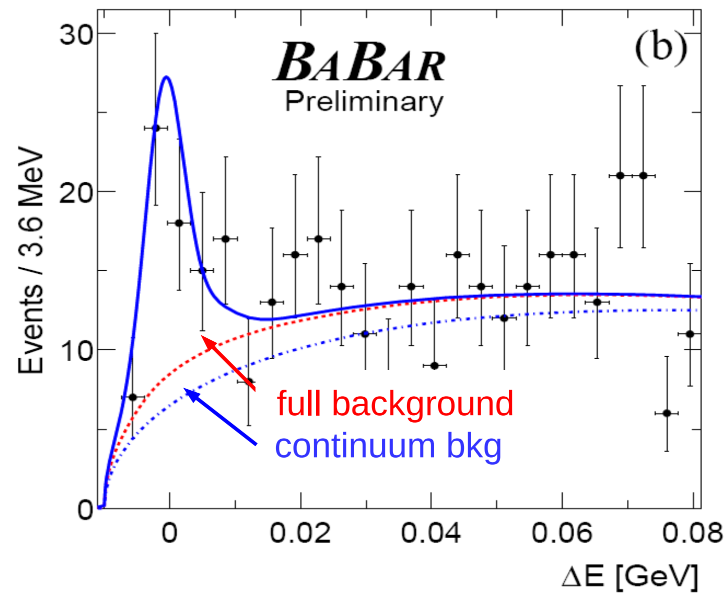


$B^0 \to \phi K^0_S \to K^+K^-\pi^+\pi^-$



114 ± 12 signal events

$B^0 \to \phi K^0_L$ *(Opposite CP)*



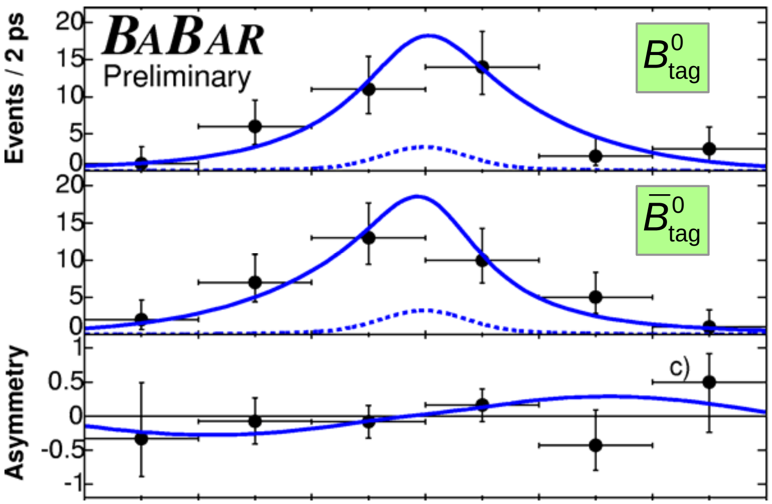
98 ± 18 signal events

Plots shown are 'signal enhanced' through a cut on the likelihood on the dimensions that are not shown, and have a lower signal event count

CP analysis of ‘golden penguin mode’ $B^0 \rightarrow \phi K^0$

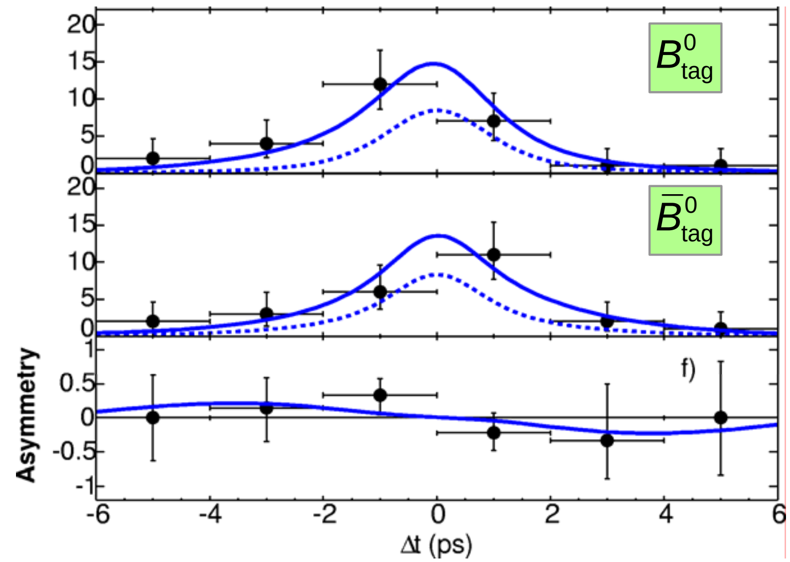
BaBar

$B^0 \rightarrow \phi K^0_S \rightarrow K^+K^-\pi^+\pi^-$



S(ϕK$_S$) = +0.29 ± 0.31(stat)

$B^0 \rightarrow \phi K^0_L$ *(Opposite CP)*



S(ϕK$_L$) = -1.05 ± 0.51(stat)

Combined fit result

(assuming ϕK$_L$ and ϕK$_S$ have opposite CP)

$$\eta_{\phi K0} \times S_{\phi K^0} = +0.50 \pm 0.25 \; {}^{+0.07}_{-0.04}$$

$$C_{\phi K^0} = +0.00 \pm 0.23 \pm 0.05$$

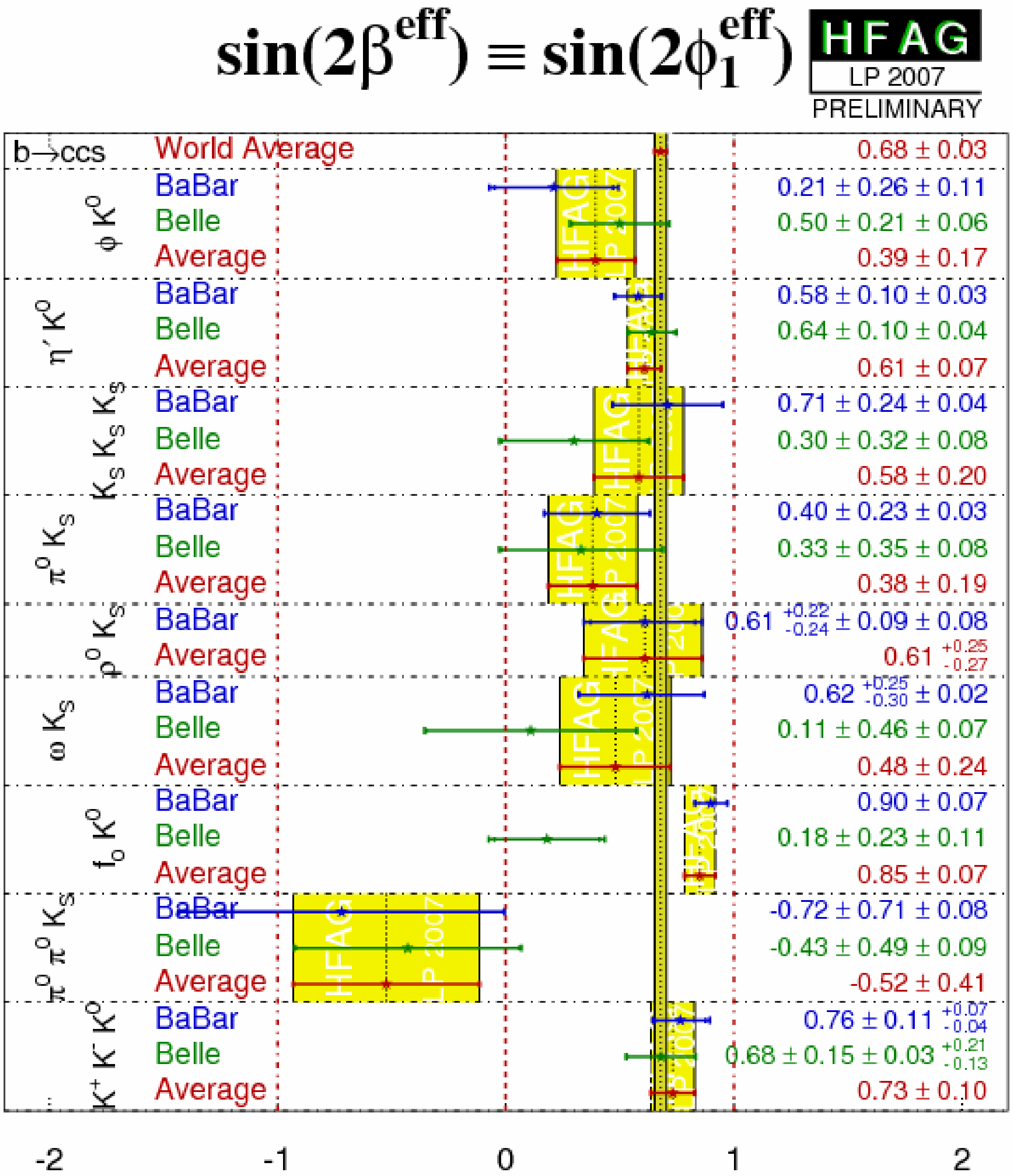
0.9σ

Standard Model Prediction

S(ϕK^0) = sin2β = 0.72 ± 0.05

C(ϕK^0) = 1-|λ| = 0

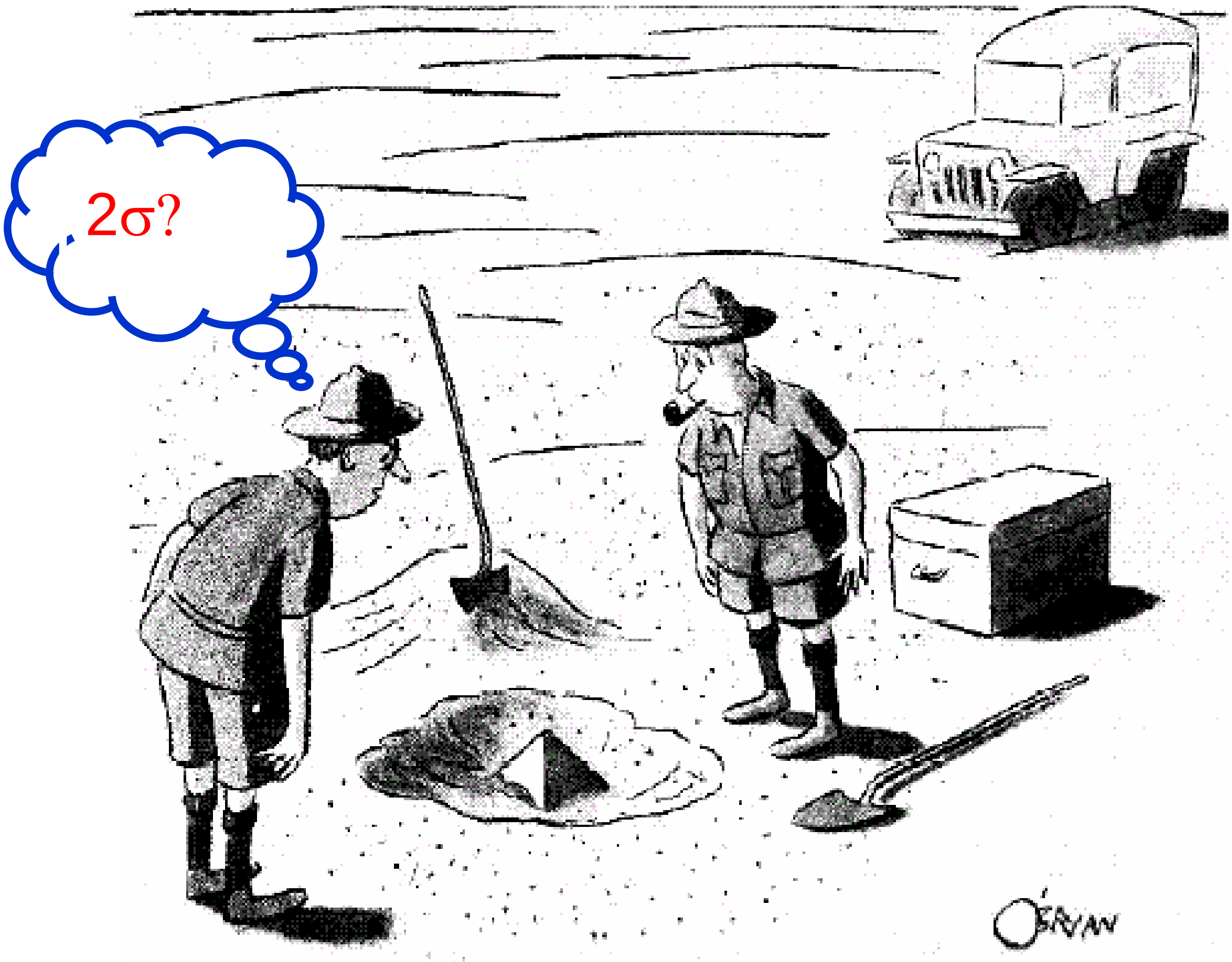
Time-Dependent CPV Results from Penguin Modes



All s-Penguin modes exhibit "sin2β" consistent with but slightly below $\sin 2\beta_{Tree}$ while Theory (Beneke et al) predicts same or higher

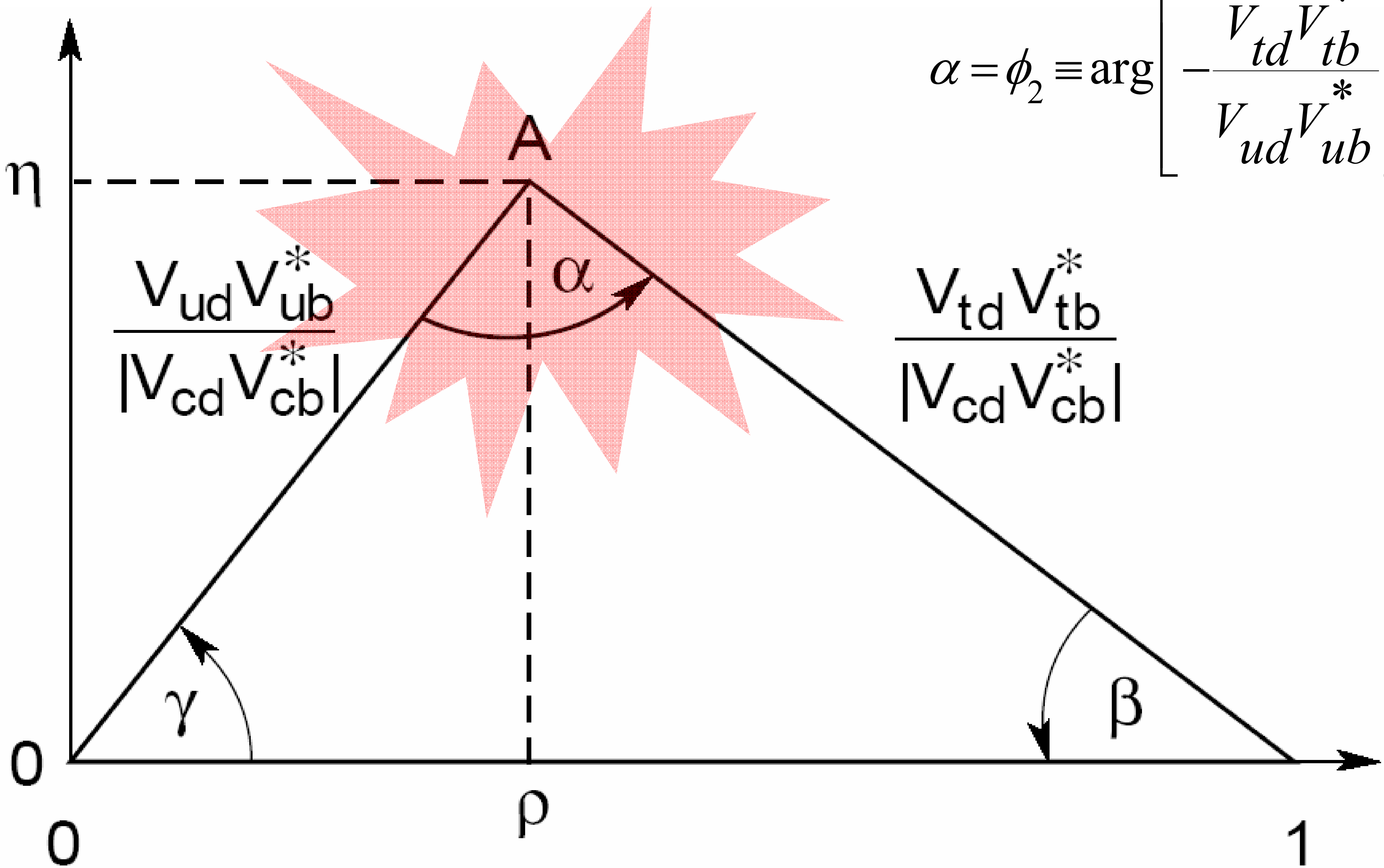
Recall: Don't get over excited for 2σ discrepancies!

What Are Penguins Telling Us ?

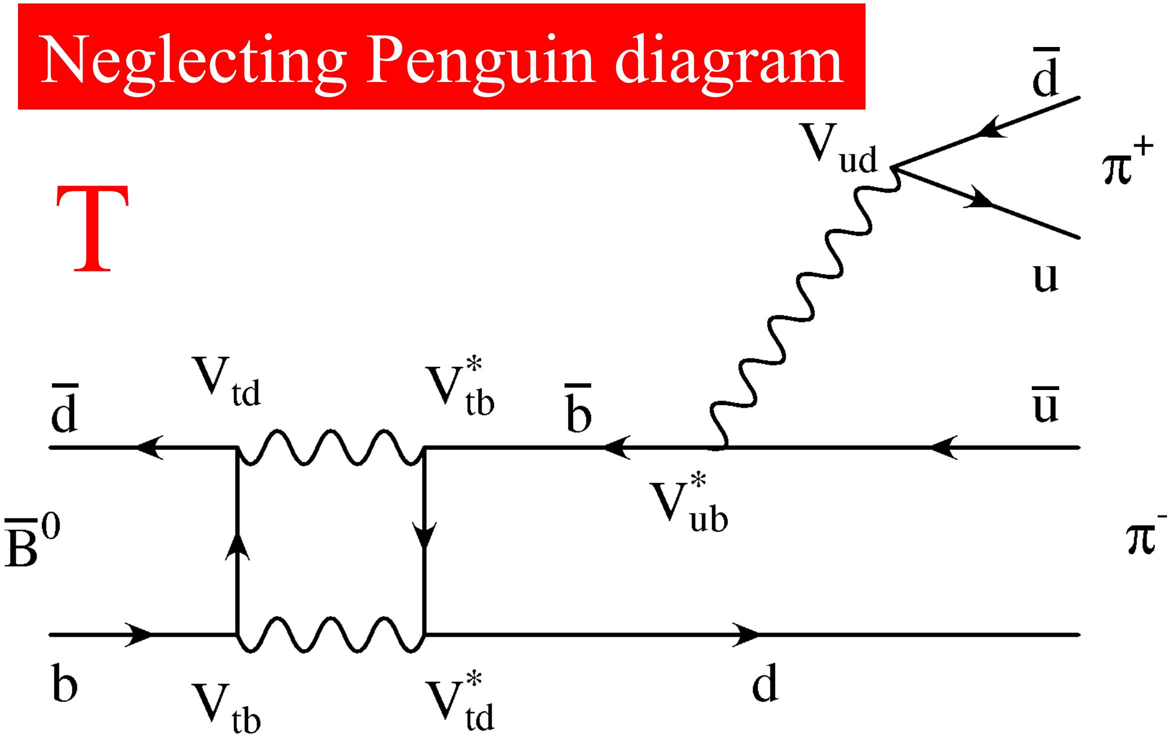


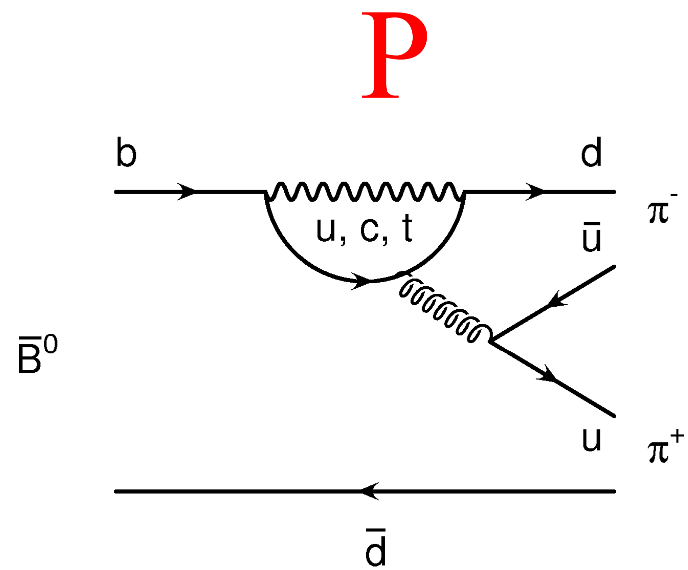
Discrepancy?

$$\alpha = \phi_2 \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$



CKM Angle α From *(b→ u u d) Process:* $B^0 \to \pi^+\pi^-$





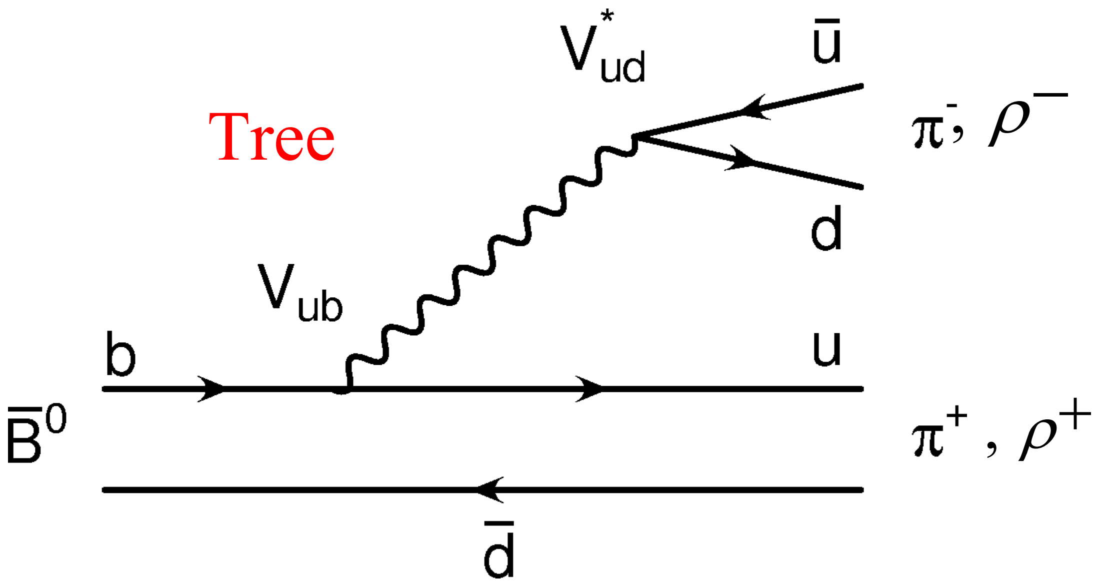
$$\lambda\left(B \to \pi^+\pi^-\right) = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right) \Rightarrow Im\lambda_{\pi\pi} = \sin(2\pi - 2\beta - 2\gamma) = \sin 2\alpha$$

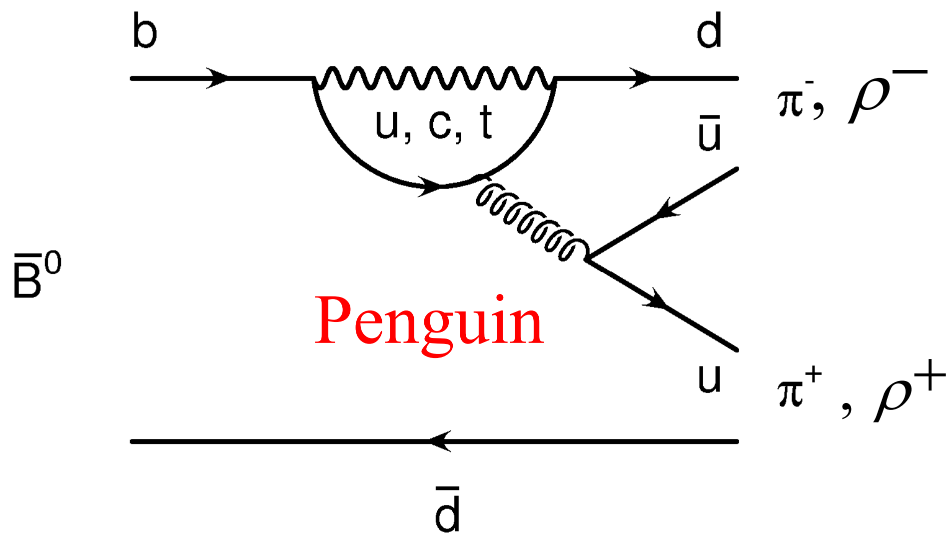
$e^{-2i\beta}$ $e^{-2i\gamma}$

Weak Phase in Penguin term is $\arg(V_{td}^* V_{tb})$ different from Tree, so it will modify

$Im\lambda_{\pi\pi}$ and $|\lambda_{\pi\pi}|$ depending on its relative strength w.r.t Tree. (Penguins are large!)

Decay Amplitudes In $B^0 \to \pi^+\pi^-, \rho^+ \rho^-$





$$\lambda_{\pi^+\pi^-} = e^{i2\alpha} \frac{T + P\, e^{+i\gamma}\, e^{i\delta}}{T + P\, e^{-i\gamma}\, e^{i\delta}}$$

$$A_{CP} = S \sin \Delta mt - C \cos \Delta mt$$

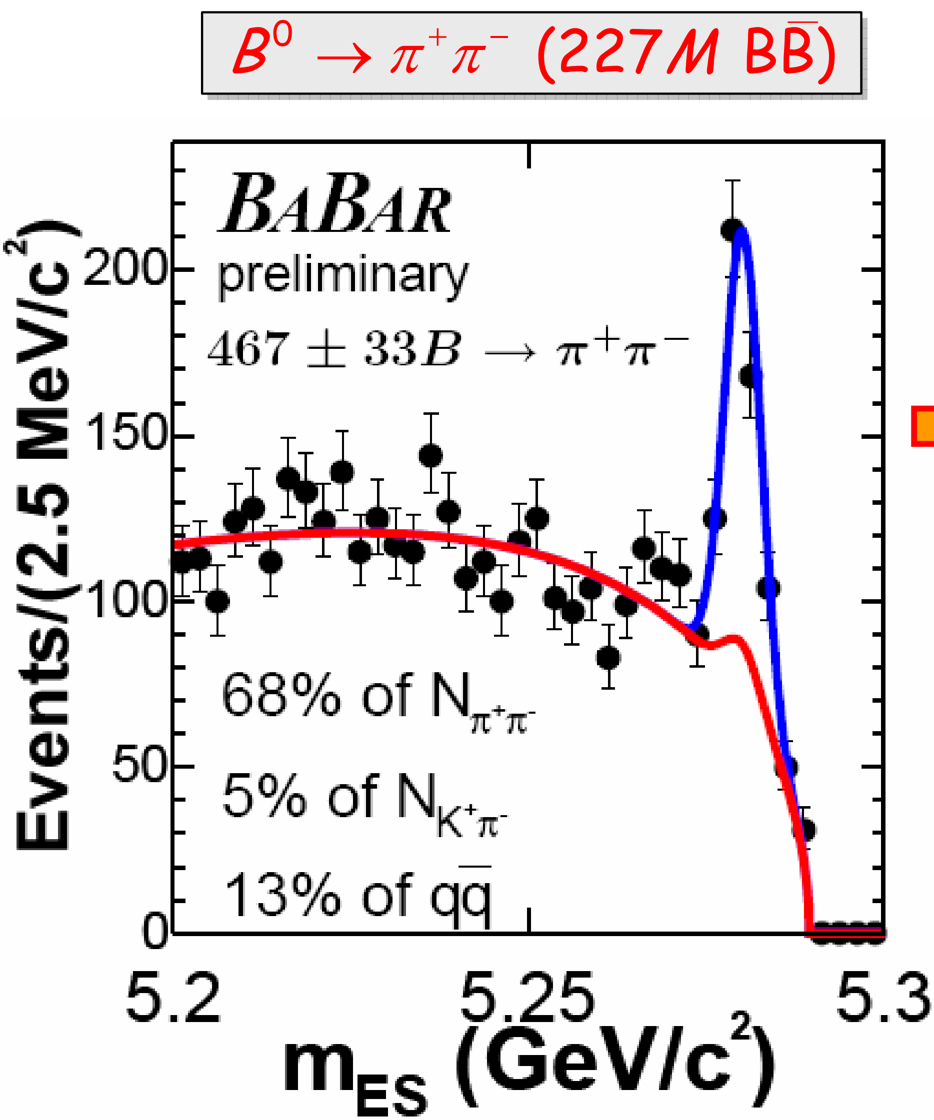
Ratio of amplitudes **|P/T|** and strong phase difference δ can *not* be reliably calculated!

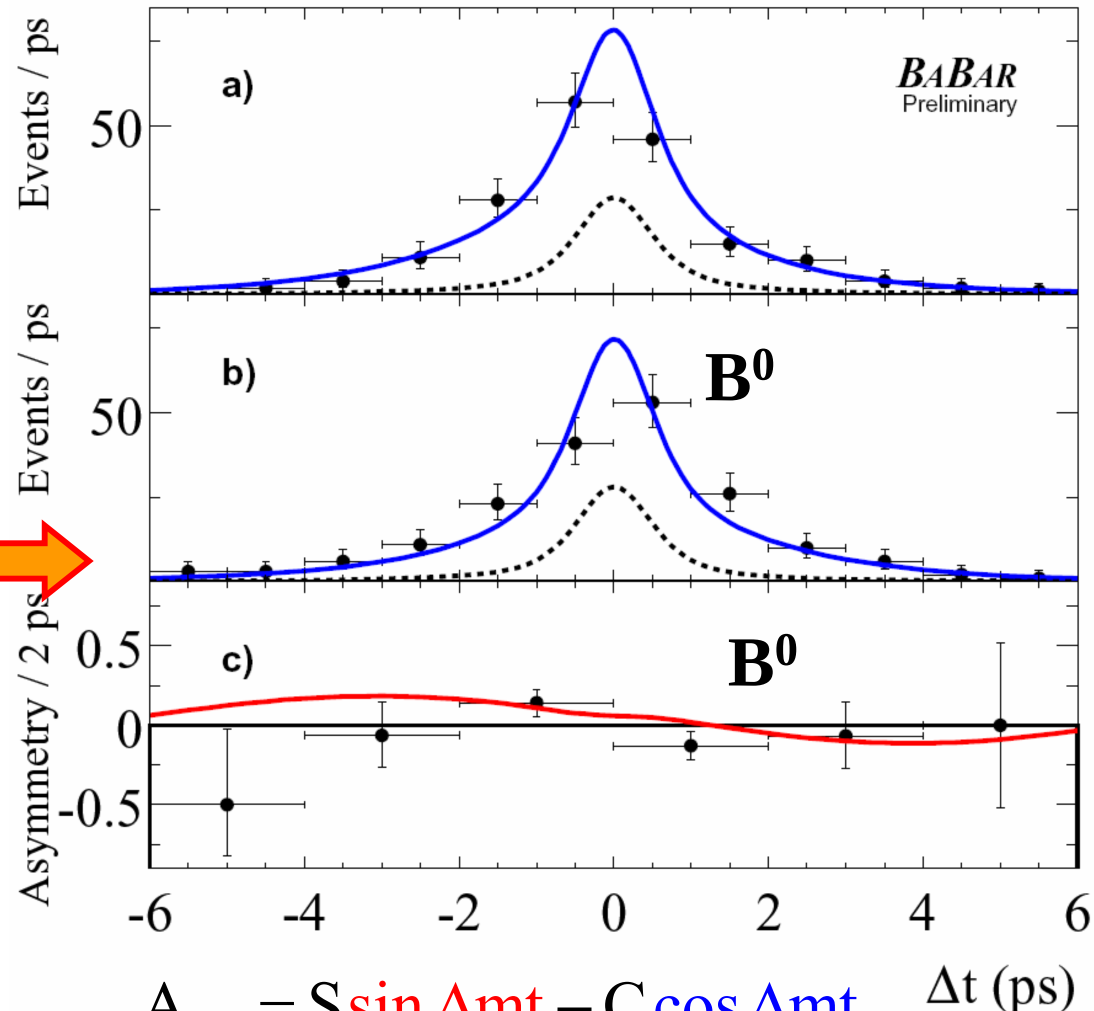
$$C_{\pi^+\pi^-} \propto \sin \delta$$

$$S_{\pi^+\pi^-} = \sqrt{1 - C^2_{\pi^+\pi^-}}\ \sin\, 2\alpha_{eff}$$

One can measure $\delta\alpha_{peng}$ using isospin relation and bounds to get α

Time Dependent Asymmetry Measurement: $B^0 \to \pi^+\pi^-$





$$A_{CP} = S \sin \Delta mt - C \cos \Delta mt$$

$$S_{\pi\pi} = \sin 2\alpha_{eff} = -0.30 \pm 0.17 \pm 0.03$$

$$C_{\pi\pi} = -0.09 \pm 0.15 \pm 0.04$$

$\sin 2\alpha = ??$

Estimating Penguin Pollution in $B^0 \to \pi^+\pi^-$, $\rho^+ \rho^-$

$B \to \pi^+\pi^-, \pi^+\pi^0$ and $\pi^0\pi^0$ related by SU(2) $\Rightarrow$ Isospin relation between amplitudes A^{+-}, A^{+0} and A^{00}

$B \to \pi\pi$ states can have I=0 or 2; Gluonic Penguins contribute only to I=0 (ΔI=1/2 rule)

$B \to \pi^+\pi^0$ has only tree amplitude $\Rightarrow$ $|A^{+0}| = |A^{-0}|$

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}$$

$$\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = A^{-0}$$

$A^{+-} = A(B^0 \to \pi^+\pi^-)$

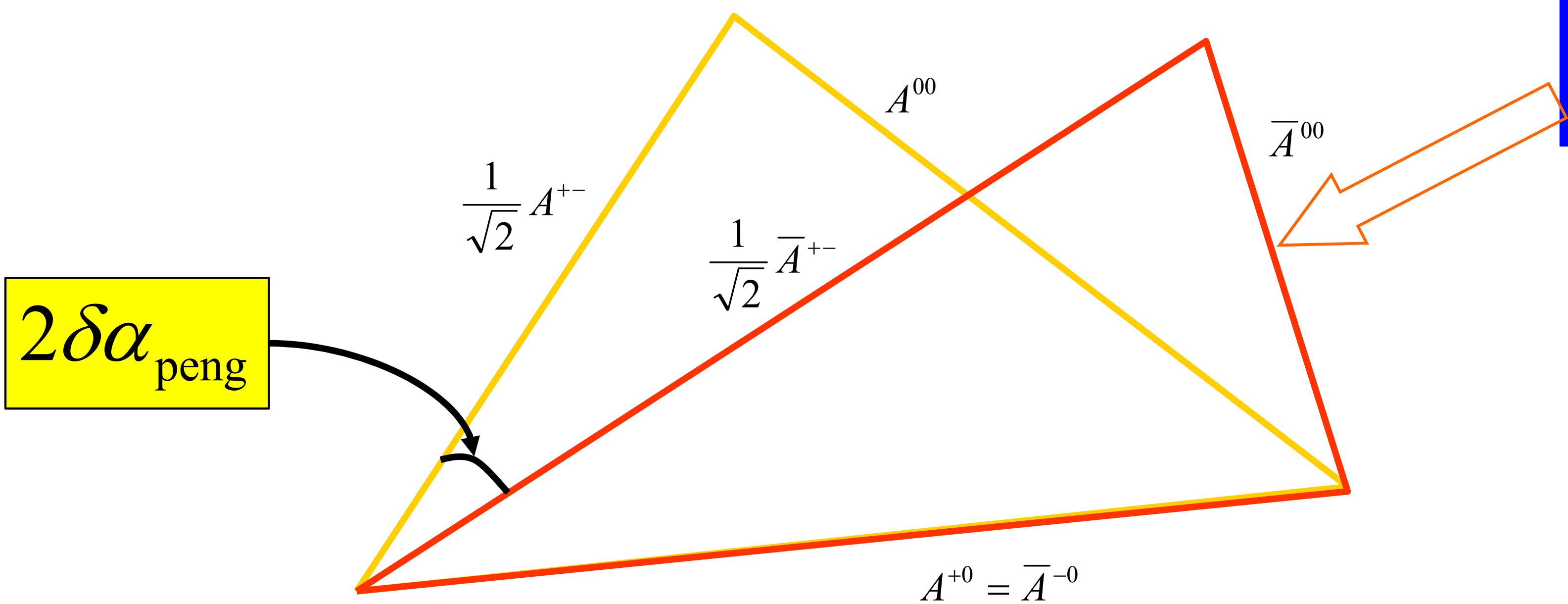
$\bar{A}^{+-} = A(\bar{B}^0 \to \pi^+\pi^-)$

$A^{00} = A(B^0 \to \pi^0\pi^0)$

$\bar{A}^{00} = A(\bar{B}^0 \to \pi^0\pi^0)$

$A^{+0} = A(B^+ \to \pi^+\pi^0)$

$\bar{A}^{-0} = A(B^- \to \pi^-\pi^0)$

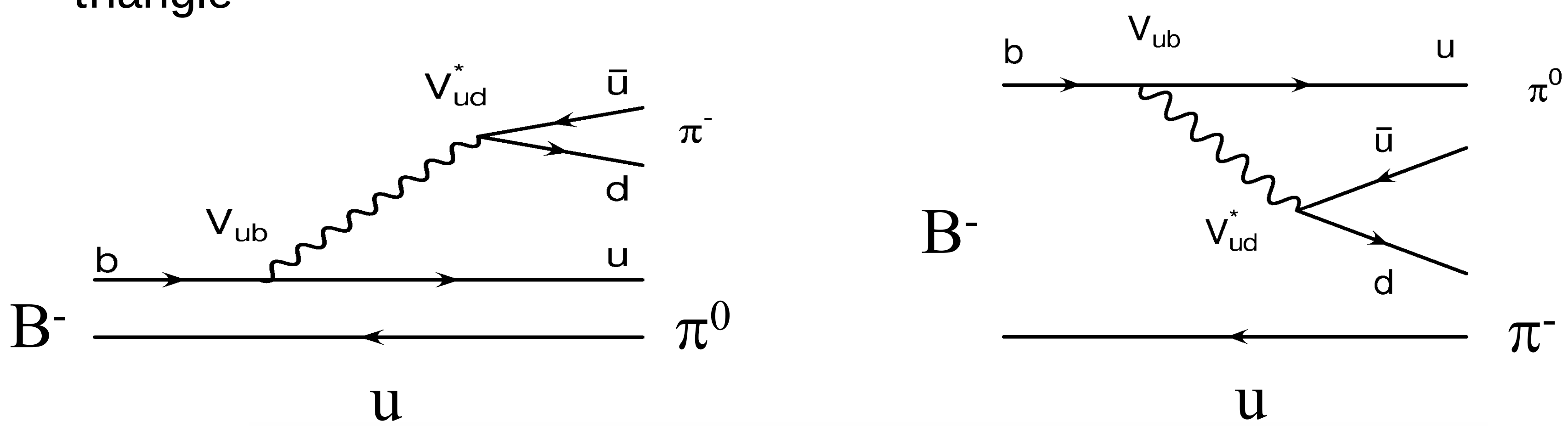


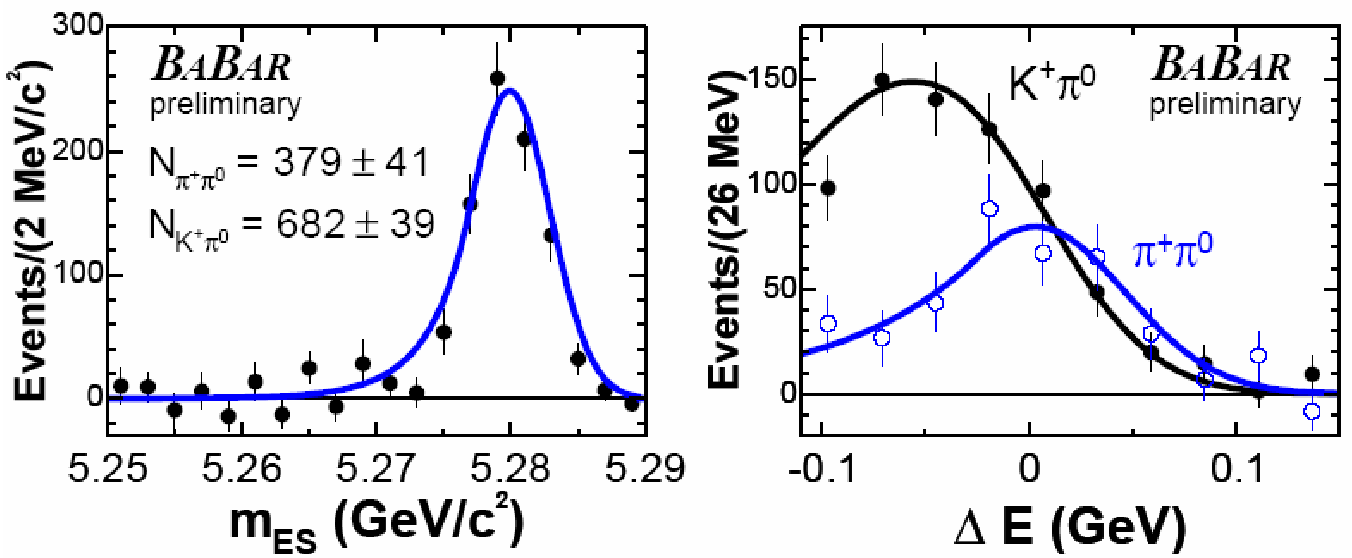
To constrain $\delta\alpha_{penguin}$ by isospin analysis requires A^{00} and $\bar{A}^{00}$ to be very small or very large ! $\Rightarrow$ Measure and constrain C_{+-}, C_{00}, A^{+0}, A^{00} by rate and asymmetry measurements

Needs lots of statistics for $B \to \pi^0\pi^0$ and $B \to \pi^0\pi^0$ rate measurements

Constraining α: $B^- \to \pi^- \pi^0$ Rate Measurement

- $B^- \to \pi^- \pi^0$ ($I=2$, $\Delta I=1/2$) has only tree amplitude, no penguin $\Rightarrow$ Base of Isospin triangle

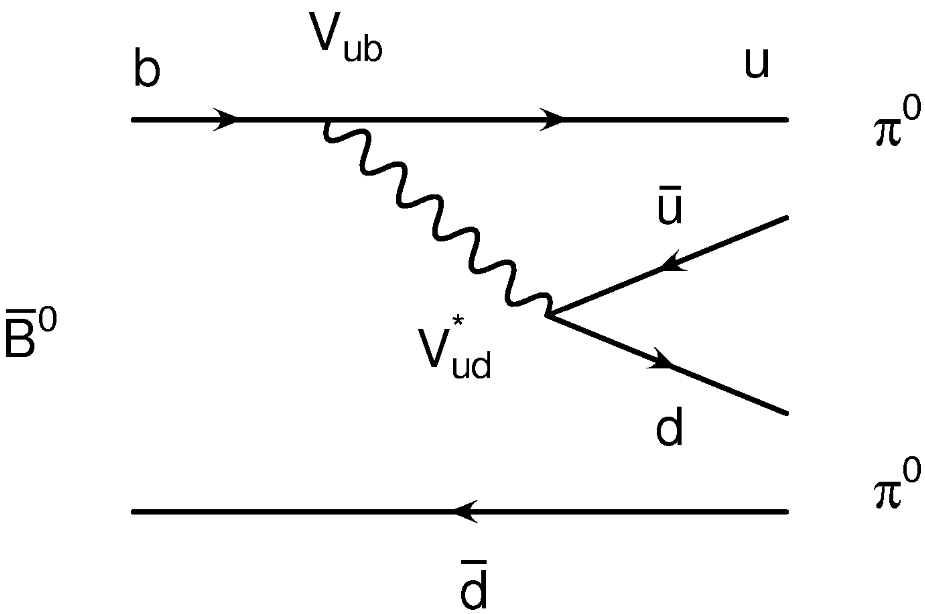


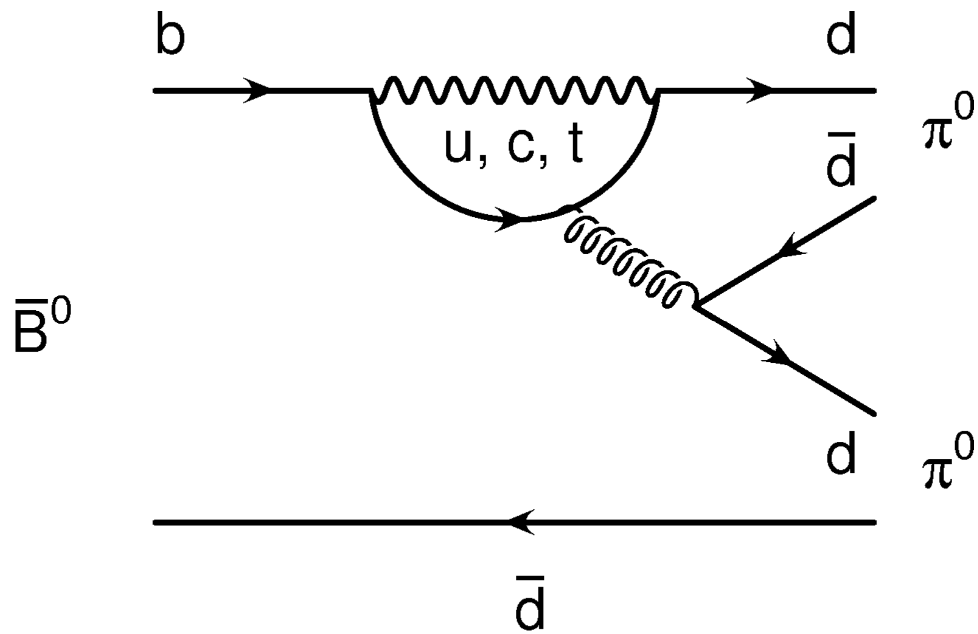


BaBar

$$\mathcal{B}(B^+ \to \pi^+\pi^0) = (5.8 \pm 0.6 \pm 0.4) \times 10^{-6}$$

Constraining α : $B \to \pi^0\pi^0$ Decay Diagrams





Difficult to calculate rates for such processes

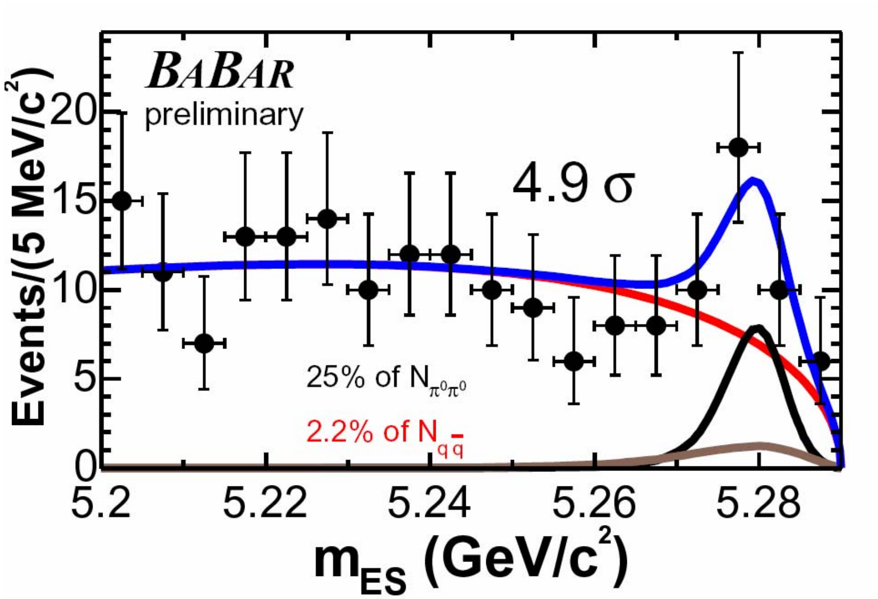
Smaller the better for constraining α

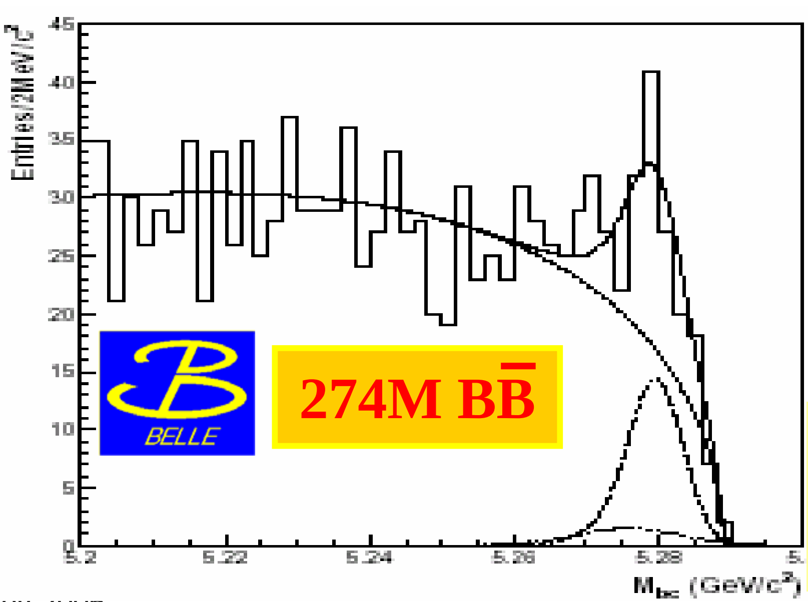
Grossman-Quinn bound:

$$\sin^2(\alpha - \alpha_{\rm eff}) \leq \frac{\mathcal{B}(B^0 \to \pi^0\pi^0) + \mathcal{B}(\overline{B}^0 \to \pi^0\pi^0)}{\mathcal{B}(B^+ \to \pi^+\pi^0) + \mathcal{B}(B^- \to \pi^-\pi^0)}$$

$B \to \pi^0\pi^0$: Rate and Flavor Tagged Rate Asymmetry

$B \to \pi^0\pi^0$ is large !





First measurements

$$BF_{\pi^0\pi^0} = (1.17 \pm 0.32 \pm 0.10) \times 10^{-6}$$
$$C_{\pi^0\pi^0} = -0.12 \pm 0.56 \pm 0.06$$

4.9σ BaBar

Measured by flavor of the other B → **B**$_{\mathbf{tag}}$

$$BF_{\pi^0\pi^0} = (2.32^{+0.41}_{-0.48} \pm 0.22) \times 10^{-6}$$
$$C_{\pi^0\pi^0} = -0.43 \pm 0.51^{+0.16}_{-0.17}$$

6.0σ Belle

Average

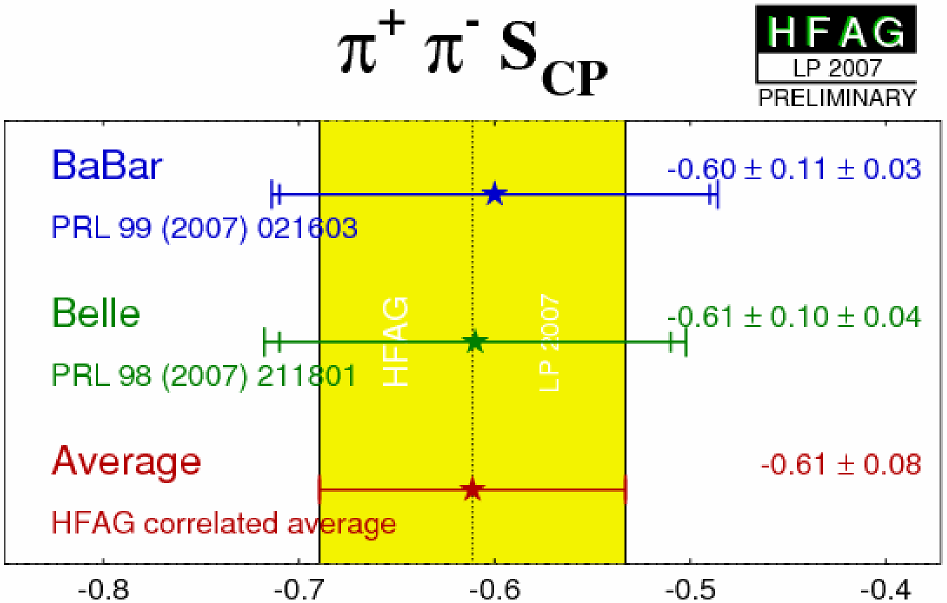
$$\frac{\Gamma(\overline{B} \to \pi^0\pi^0) - \Gamma(B \to \pi^0\pi^0)}{\Gamma(\overline{B} \to \pi^0\pi^0) + \Gamma(B \to \pi^0\pi^0)} = 0.28 \pm 0.39$$

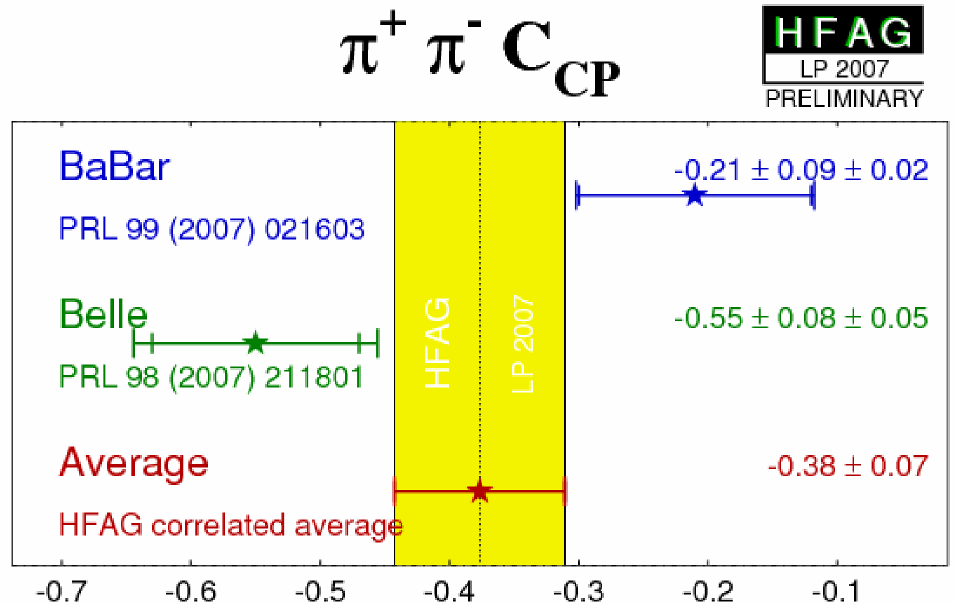
[BABAR, BELLE]

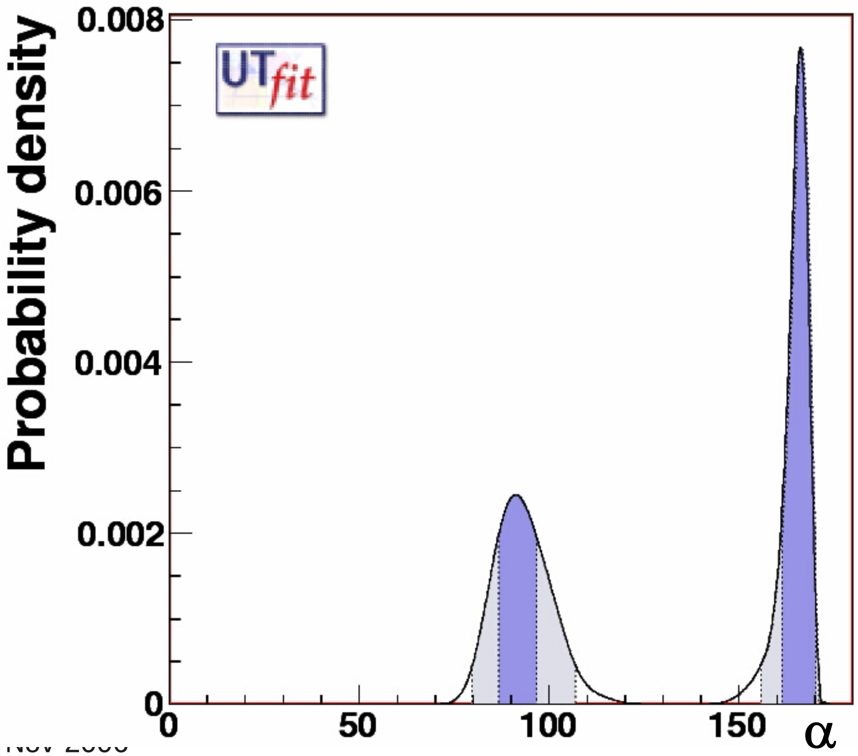
$$\mathcal{B}(B \to \pi^0\pi^0) = (1.51 \pm 0.28) \times 10^{-6}$$

Bad news is $C_{\pi\pi}$ is not precise enough and $B_{\pi 0 \pi 0}$ is too large for useful G-Q bound

Angle α From B→ $\pi^+ \pi^-$: Bottomline





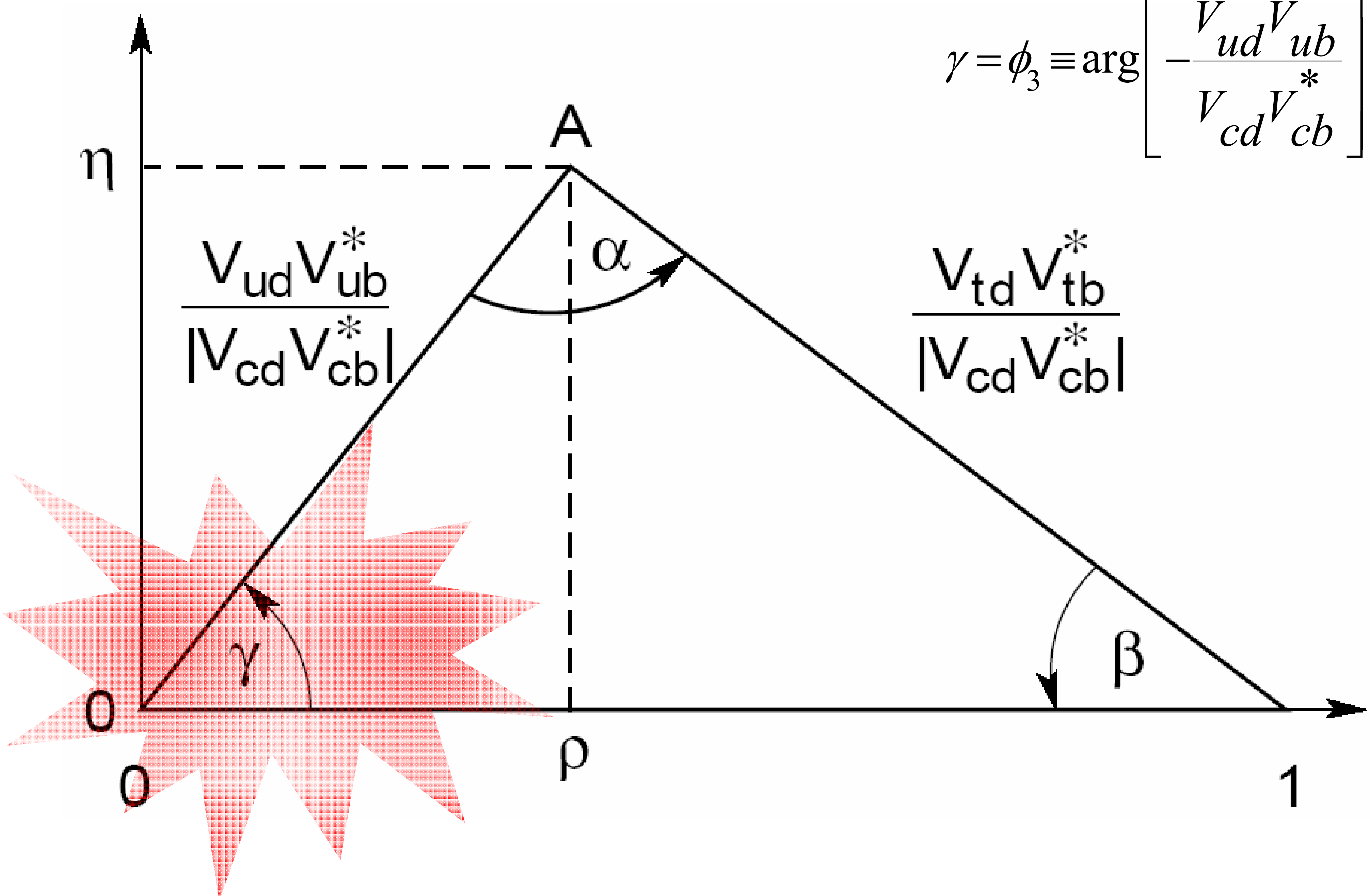


α = $[80,107]^o$ U $[156,171]^o$ @ 95% Prob.

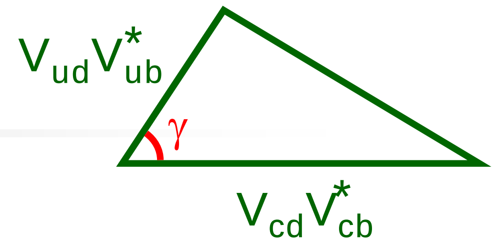
Standard Model Solution:

α =$(91 \pm 8)^o$ @ 68% Prob.)

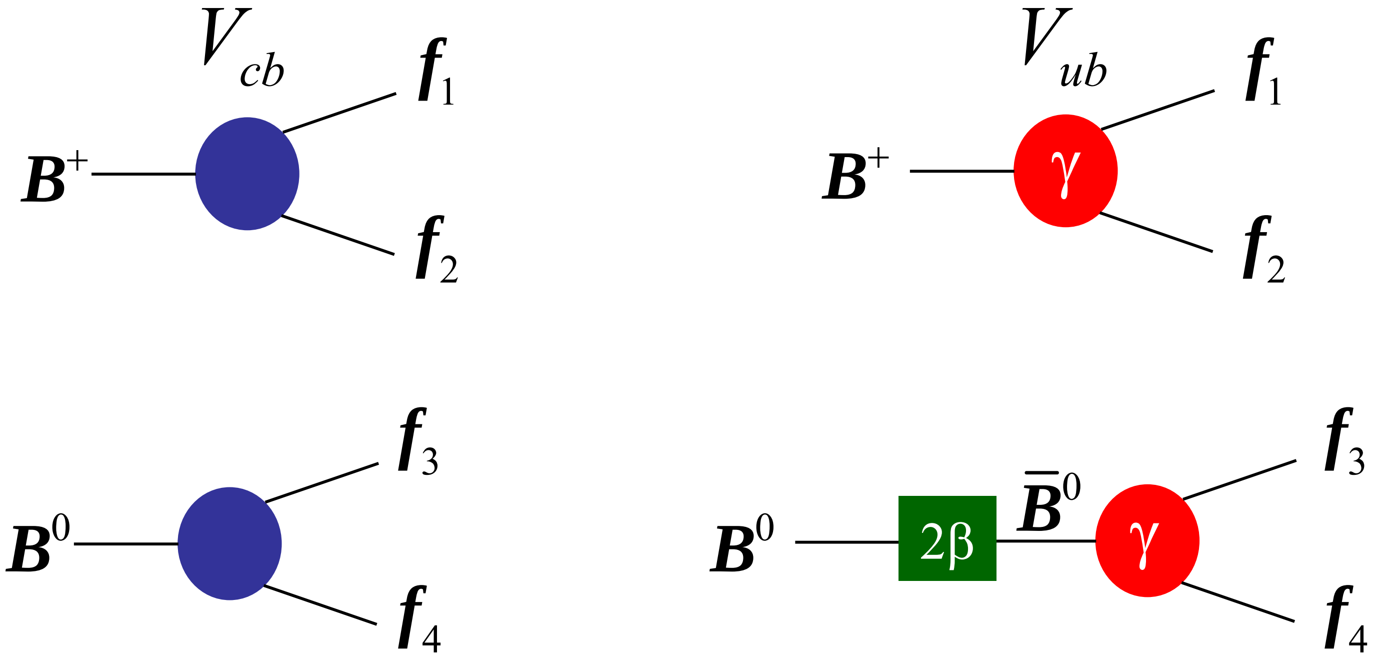
$$\gamma = \phi_3 \equiv \arg\left[-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right]$$



How To Measure γ ?



- Interference is the key to γ



$$A_{tot} = A_1 + A_2 e^{-i\gamma} e^{i\delta}$$

Strong phase

- Both charged and neutral B mesons can be used
  - Measure branching fractions for B^+
  - Time dependent studies for B^0

$$r_B = \frac{|A_2(b \to u)|}{|A_1(b \to c)|}$$

What to Watch Out For?

- Branching fractions of interesting B decays typically about 10^{-5} or smaller

- Every additional decay mode is important to increase statistics but...
  - Combining different decay modes not trivial

- Sensitivity to γ strongly depends on $r_B = \frac{|A_2(b \to u)|}{|A_1(b \to c)|}$
  - Small values of r_B make the measurement very difficult
  - Each decay mode has a different value of r_B

- Strong phase δ different for each final state
  - Combination of decay modes more complicated

- Experimentally need to determine: r_B, δ, and γ

Experimental Techniques to Measure γ

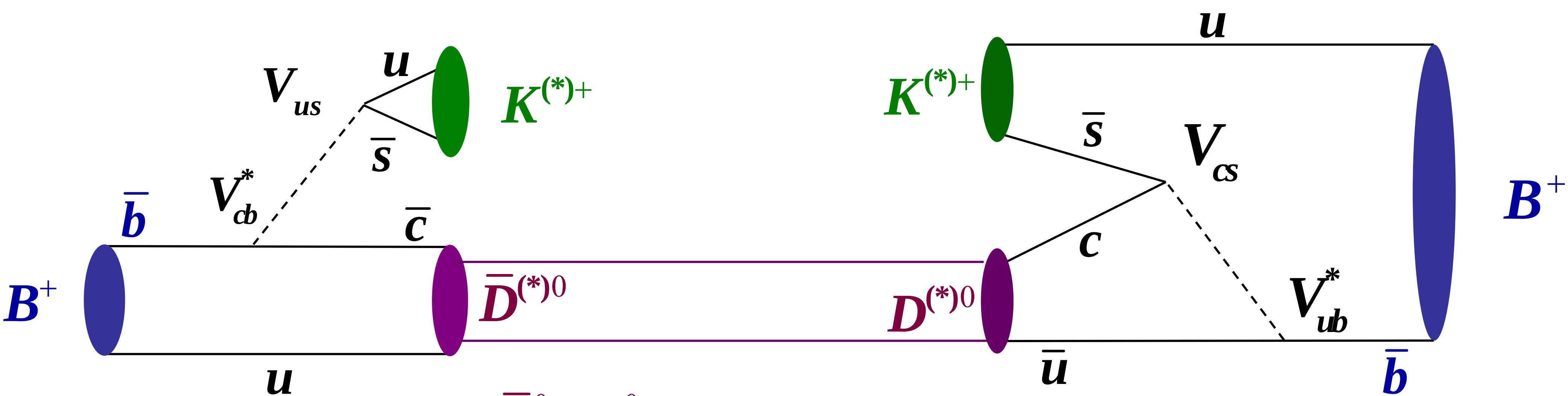
- Many papers about γ on the market

- Gronau-London-Wyler method with $B^- \to D^0 K^-$
  - CP eigenstates of D^0
- Atwood-Dunietz-Soni method with $B^- \to D^0 K^-$
  - Flavor eigenstates of D^0
- Dalitz Analysis of $B^- \to D^0 K^-$, $D^0 \to K_S \pi\pi$
- Time-dependent analysis $B^0 \to D^{(*)-} \pi/\rho$
- Search for decays $B^0 \to D^{(*)0} K^{(*)0}$

$B \to D^{(*)} K^{(*)}$ decays important ingredient for γ

- Other methods
  - Charmless B decays ($K\pi$)
  - Variations of GWL and ADS method

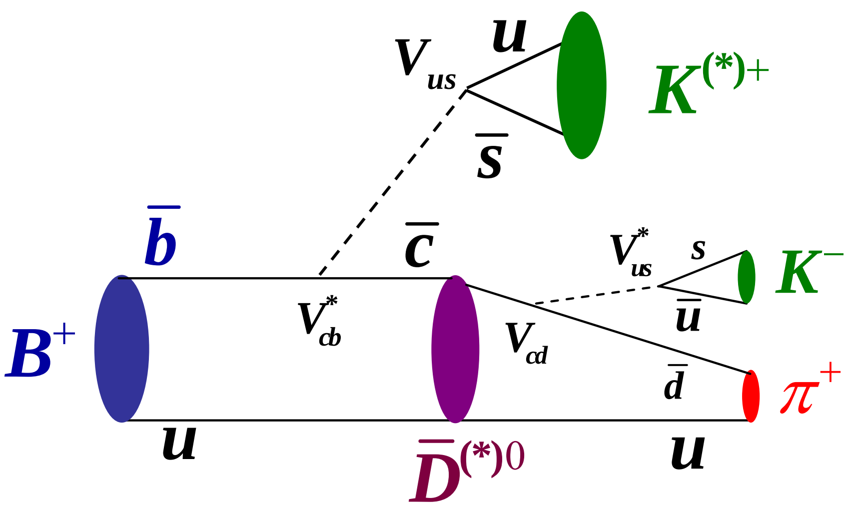
Gronau-London-Wyler Method with $B^+ \to D^0 K^+$



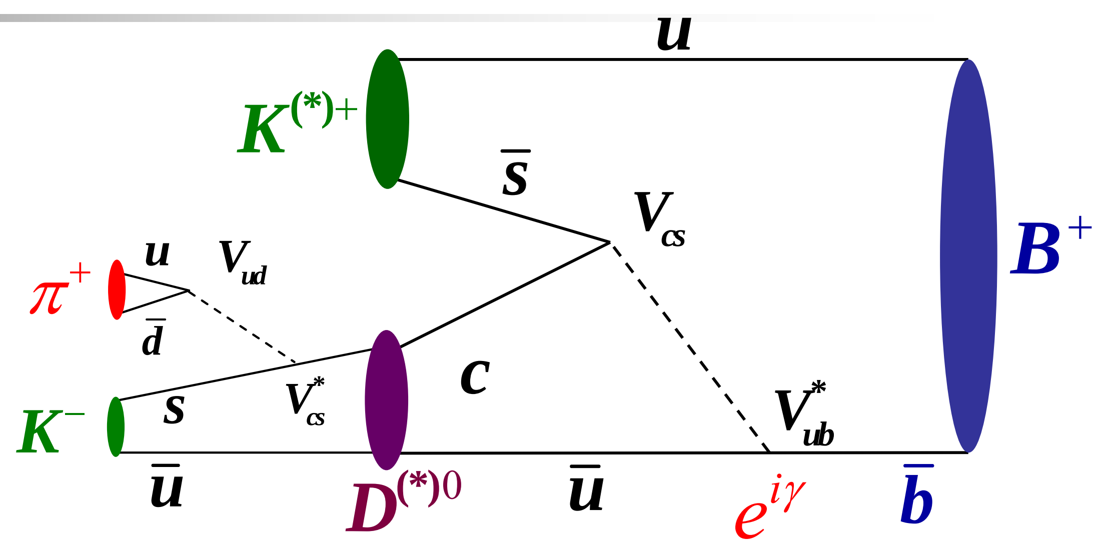
$$\bar{D}^0 / D^0 \to K^+K^-, \pi^+\pi^-$$

$$K_S\pi^0 / \phi / \eta / \eta'$$

Atwood-Dunietz-Soni Method in Pictures



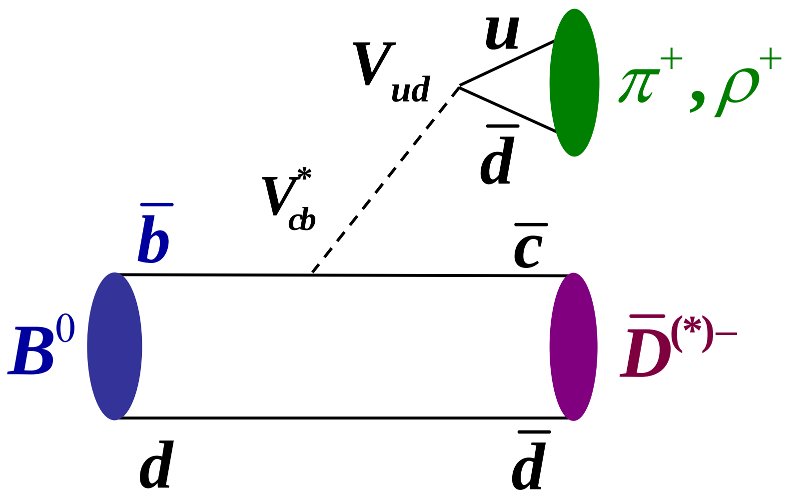
Favored b → c decay
Doubly Cabibbo suppressed D^0 decay

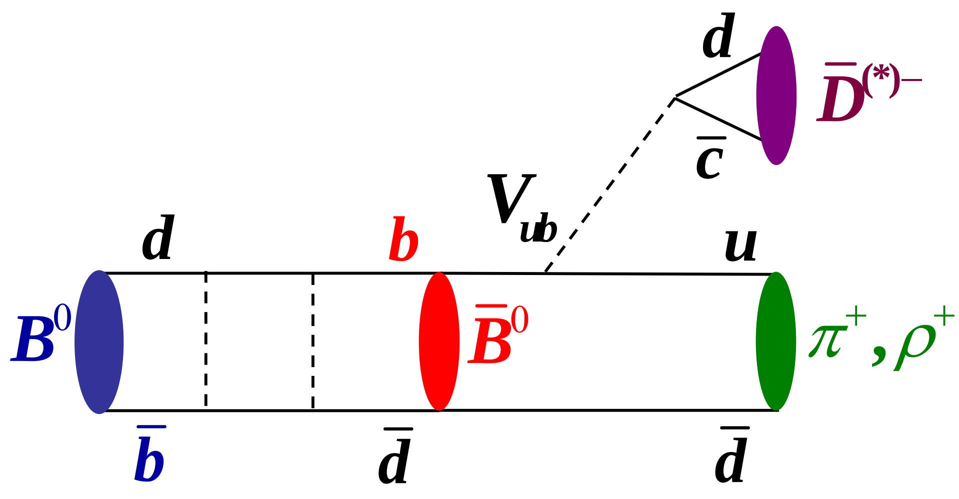


Suppressed b → u decay
Favored c → s decay

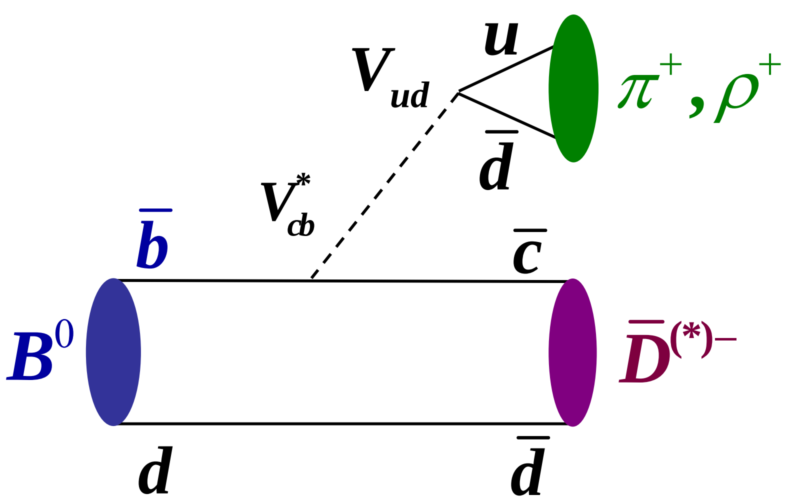
- Similar to GWL but replace CP eigenstates with flavor eigenstates of D^0
- Combine dominant b→c transition with doubly Cabibbo-suppressed D^0 decays
- Advantage: Both decay amplitudes are small but comparable $r_B = \frac{|A_2(b\to u)|}{|A_1(b\to c)|}$
  - hopefully large r_B
- Disadvantage: Effective BF($B^+ \to [K^- \pi^+]_D K^+$) ~ 10^{-7}

Time-Dependent Analysis and sin(2β+γ) with $B^0 \to D^{(*)}\pi$



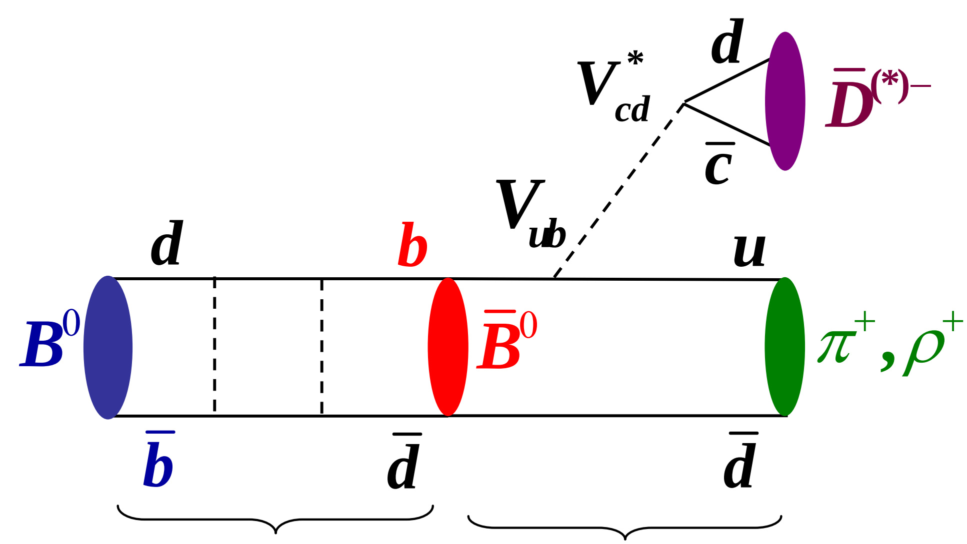


CP violation from interference of decay and mixing with $B^0 \rightarrow D^{(*)}\pi/\rho$



Favored b → c decay

$$V_{cb} V_{ud}^* = A$$



Mixing: $e^{i2\beta}$

Suppressed b → u decay

$$V_{ub} V_{cd}^* \, e^{i\delta} = r_B \, A \, e^{-i\gamma} e^{i\delta}$$

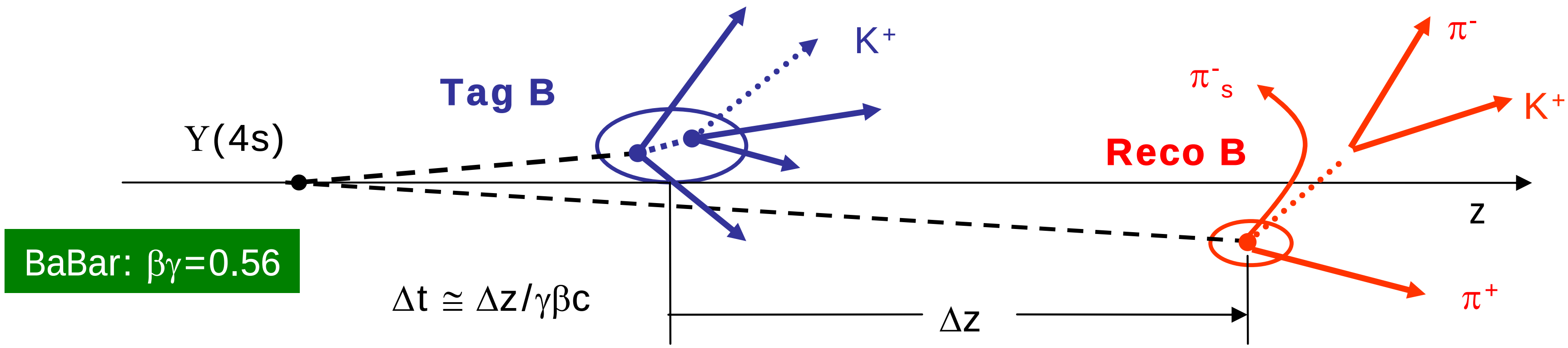
Determines the sensitivity of the method

CKM angle

Strong phase difference

- Advantage: Large branching fraction for favored decay (~3 x 10^{-3})
- Disadvantage: Small BR for suppressed decay (~10^{-6}) → Small CP violating amplitude!

Time-dependent Decay Time Distributions at Asymmetric e^+e^- Machines



BaBar: <|Δz|> ~ βγcτ: 260 μm   Average Δz resolution: 190 μm

$$f\left(B^0 \to D^{(*)-}\pi^+, \Delta t\right) = N e^{-\Gamma|\Delta t|}\left\{1 + C^{(*)}\cos\left(\Delta m_d \Delta t\right) + S^{(*)}\sin\left(\Delta m_d \Delta t\right)\right\}$$

$$f\left(\overline{B}^0 \to D^{(*)-}\pi^+, \Delta t\right) = N e^{-\Gamma|\Delta t|}\left\{1 - C^{(*)}\cos\left(\Delta m_d \Delta t\right) - S^{(*)}\sin\left(\Delta m_d \Delta t\right)\right\}$$

$$f\left(\overline{B}^0 \to D^{(*)+}\pi^-, \Delta t\right) = N e^{-\Gamma|\Delta t|}\left\{1 + C^{(*)}\cos\left(\Delta m_d \Delta t\right) - \overline{S}^{(*)}\sin\left(\Delta m_d \Delta t\right)\right\}$$

$$f\left(B^0 \to D^{(*)+}\pi^-, \Delta t\right) = N e^{-\Gamma|\Delta t|}\left\{1 - C^{(*)}\cos\left(\Delta m_d \Delta t\right) + \overline{S}^{(*)}\sin\left(\Delta m_d \Delta t\right)\right\}$$

Direct CP Violation

$$C^{(*)} = \frac{1 - r_{(*)}^2}{1 + r_{(*)}^2} \approx 1$$

Indirect CP Violation

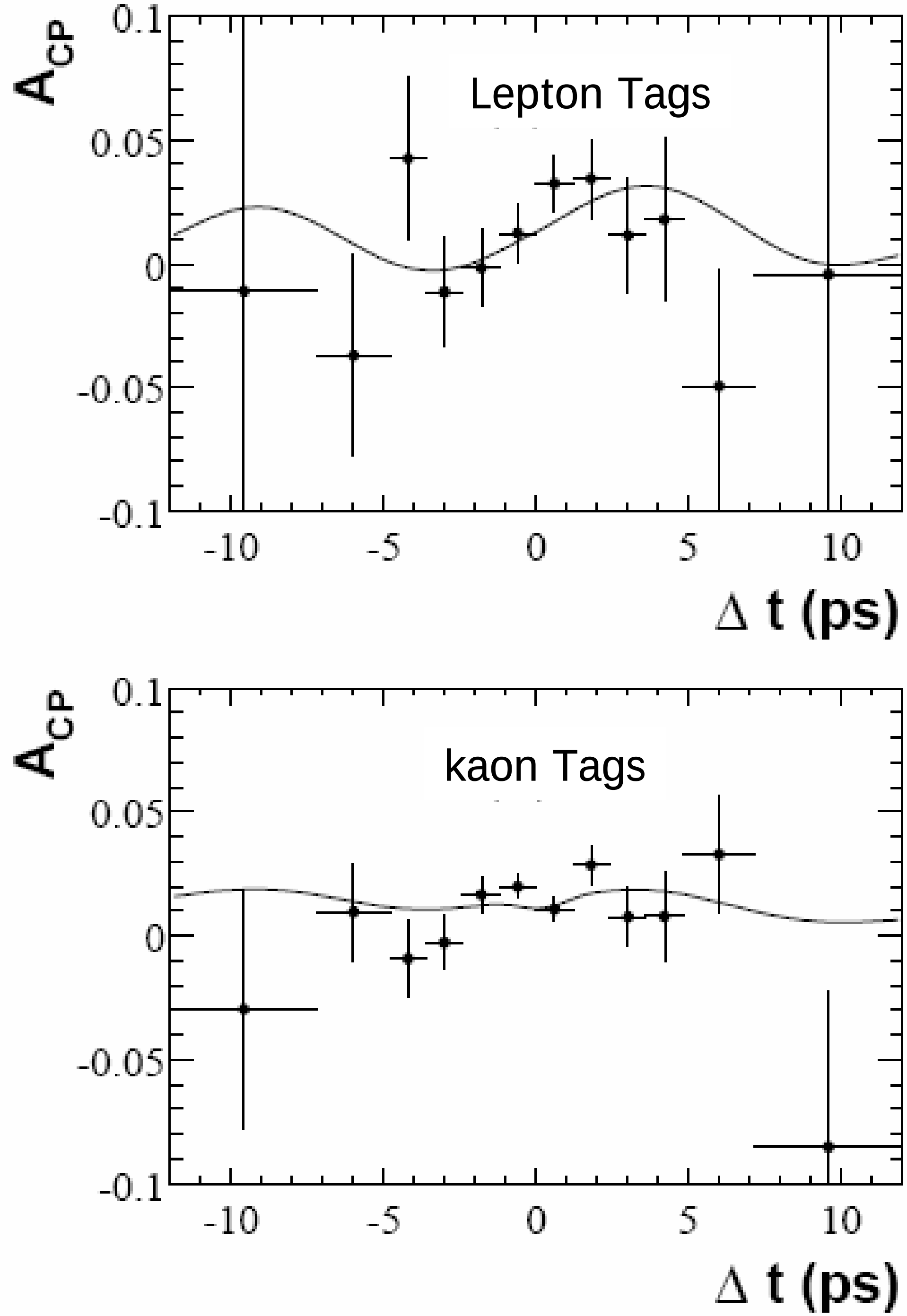
Sensitivity on sin(2β+γ) depends on value of r

$$\begin{cases} S^{(*)} = \dfrac{2r_{(*)}}{1 + r_{(*)}^2}\sin(2\beta + \gamma - \delta^{(*)}) \\ \overline{S}^{(*)} = \dfrac{2r_{(*)}}{1 + r_{(*)}^2}\sin(2\beta + \gamma + \delta^{(*)}) \end{cases}$$

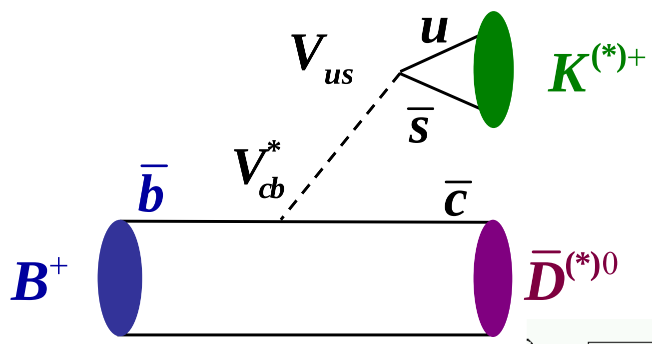
sin(2β+γ) Results in Pictures

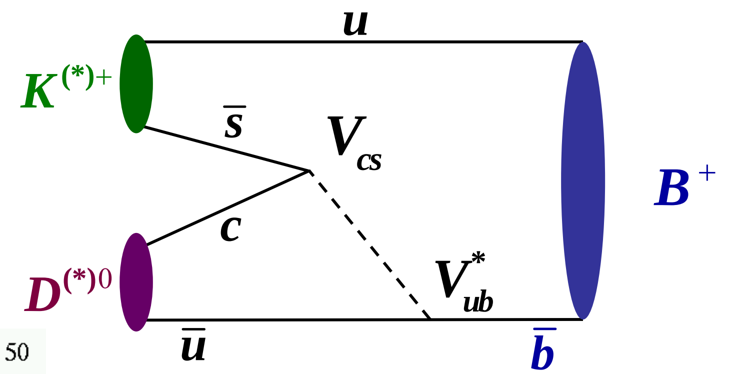
CP Asymmetry in Partially Reconstructed Sample

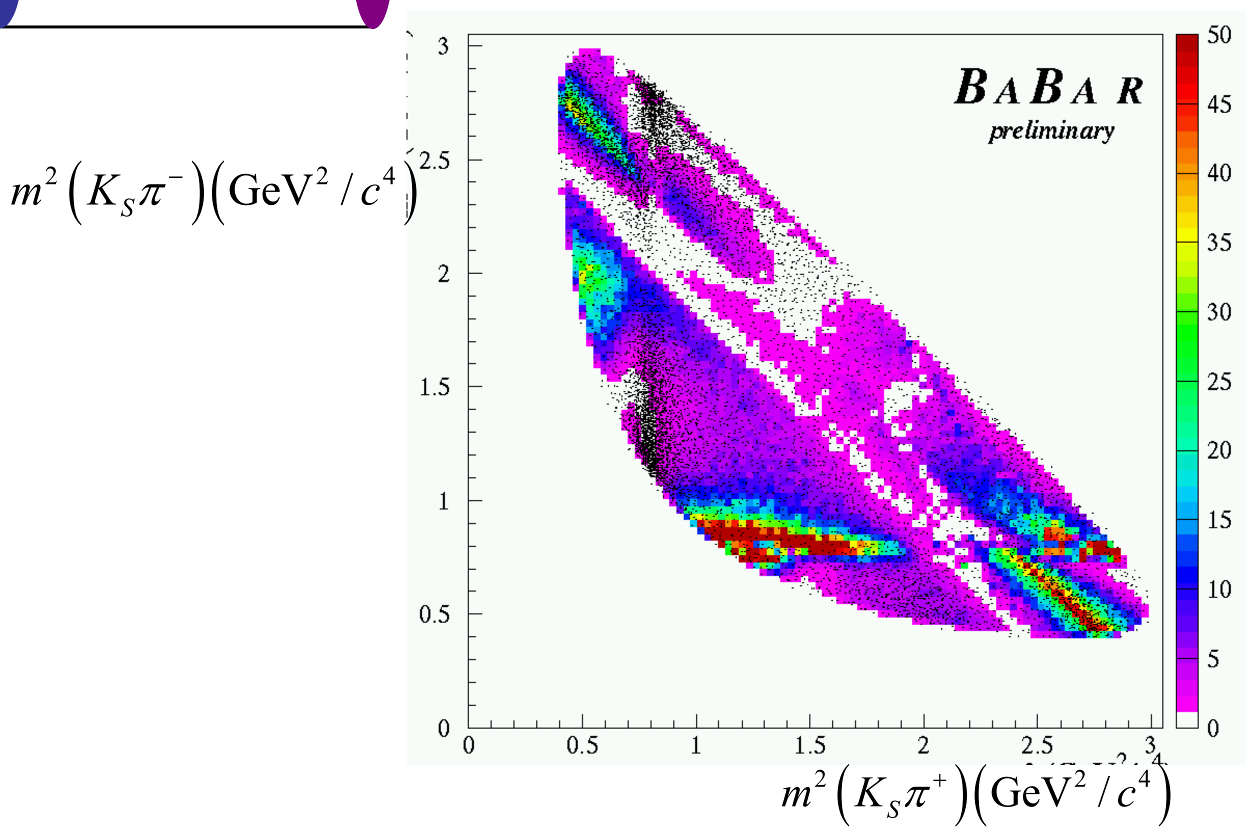
- No significant CP asymmetry so far with either method
- All measurements limited by statistics



Dalitz Analysis of $B^+ \to \bar{D}^0\left(K_S\pi^+\pi^-\right)K^+$

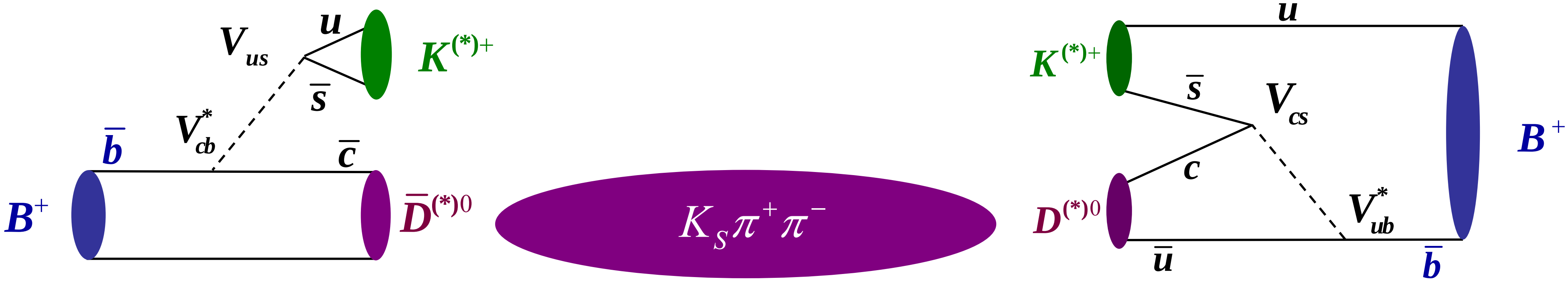






Interference in $B^- \to D^0[K_S\pi^+\pi^-]K^-$

- Possibly one of the cleanest methods to measure γ



$$A\left(B^+ \to [K_S\pi^+\pi^-]K^+\right) = A\left(B^+ \to \bar{D}^0 K^+\right) + r_B A\left(B^+ \to D^0 K^+\right)$$

$$A(B^+ \to [K_S\pi^+\pi^-]K^+) = \left|A(B^+ \to \bar{D}^0 K^+)\right|\left(f(m_+^2, m_-^2) + r_B e^{i(\gamma+\delta)} f(m_-^2, m_+^2)\right)$$

$$A(B^- \to [K_S\pi^+\pi^-]K^-) = \left|A(B^- \to D^0 K^-)\right|\left(f(m_-^2, m_+^2) + r_B e^{i(-\gamma+\delta)} f(m_+^2, m_-^2)\right)$$

- Measure $\gamma+\delta$ and $-\gamma+\delta$ from B^+ and B^- decay rates
  - Only 2-fold ambiguity in γ!
- Measure Dalitz structure $f\left(m_+^2, m_-^2\right) = A\left(\bar{D}^0 \to K_S\pi^+\pi^-\right)$ with high statistics sample $D^{*-} \to D^0[K_S\pi^+\pi^-]\pi^-$

$$m_-^2 = m^2(K_S^0\pi^-)$$
$$m_+^2 = m^2(K_S^0\pi^+)$$

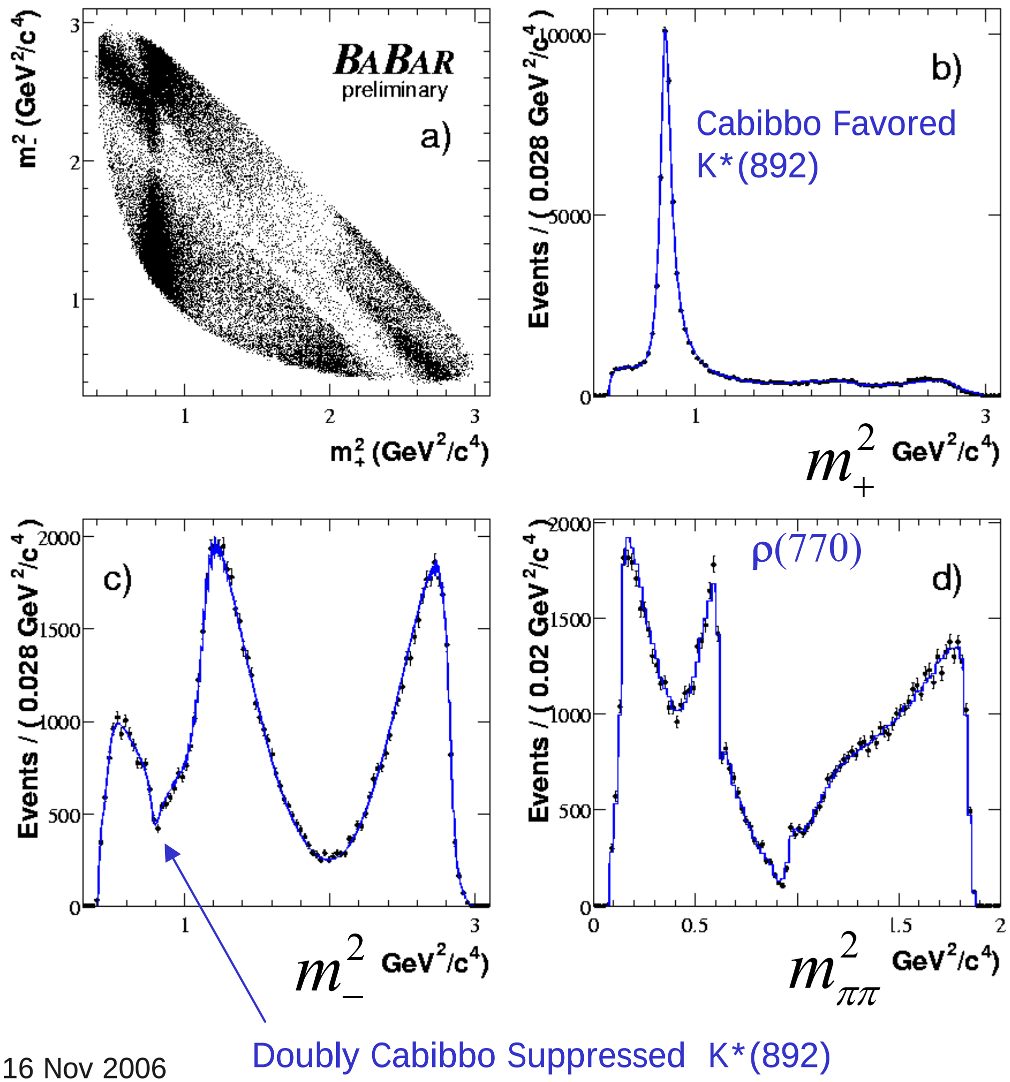
$D^0 \rightarrow K_s\pi^+\pi^-$ Dalitz Structure in $D^{*-} \rightarrow D^0\pi^-$

81k events with 97% purity (92 fb^{-1})

$$m_-^2 = M(K_S^0\pi^-)^2$$

$$m_+^2 = M(K_S^0\pi^+)^2$$

Isobar Model: sum of resonances
and 1 non-resonant component



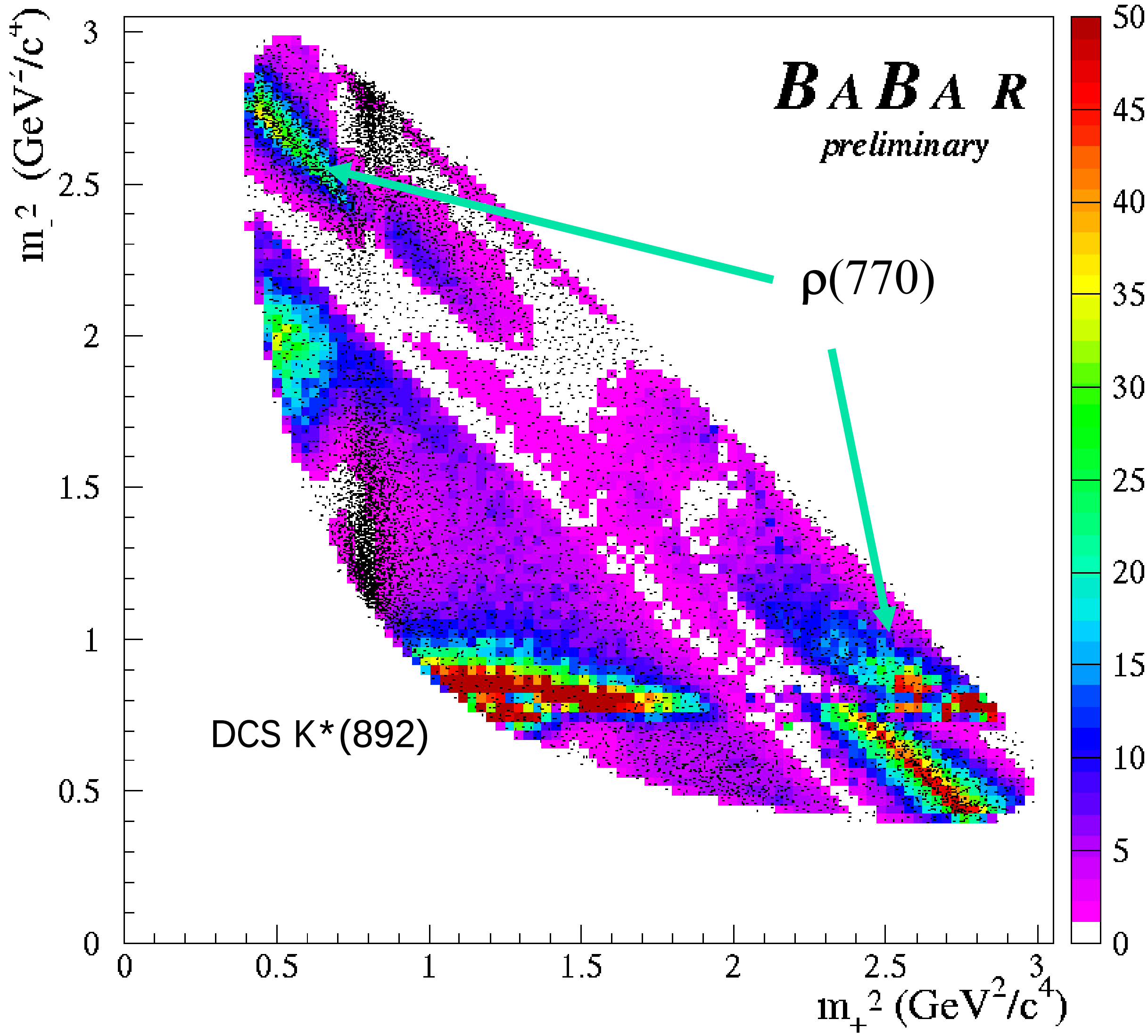
Resonance	Amplitude	Phase (deg)	Fit fraction
$K^*(892)^-$	1.781 ± 0.018	131.0 ± 0.82	0.586
$K_0^*(1430)^-$	2.447 ± 0.076	-8.3 ± 2.5	0.083
$K_2^*(1430)^-$	1.054 ± 0.056	-54.3 ± 2.6	0.027
$K^*(1410)^-$	0.515 ± 0.087	154 ± 20	0.004
$K^*(1680)^-$	0.89 ± 0.30	-139 ± 14	0.003
$K^*(892)^+$	0.1796 ± 0.0079	-44.1 ± 2.5	0.006
$K_0^*(1430)^+$	0.368 ± 0.071	-342 ± 8.5	0.002
$K_2^*(1430)^+$	0.075 ± 0.038	-104 ± 23	0.000
$\rho(770)$	1 (fixed)	0 (fixed)	0.224
$\omega(782)$	0.0391 ± 0.0016	115.3 ± 2.5	0.006
$f_0(980)$	0.4817 ± 0.012	-141.8 ± 2.2	0.061
$f_0(1370)$	2.25 ± 0.30	113.2 ± 3.7	0.032
$f_2(1270)$	0.922 ± 0.041	-21.3 ± 3.1	0.030
$\rho(1450)$	0.516 ± 0.092	38 ± 13	0.002
σ	1.358 ± 0.050	-177.9 ± 2.7	0.093
σ'	0.340 ± 0.026	153.0 ± 3.8	0.013
Non Resonant	3.53 ± 0.44	127.6 ± 6.4	0.073

No D-mixing
No CP violation in D decays

Host Spots for γ in the Dalitz Plot

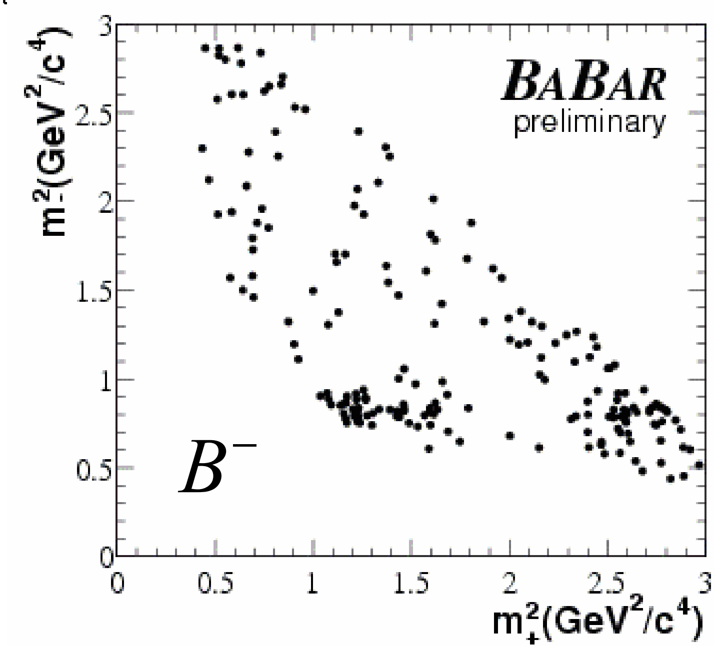
- Sensitivity to γ varies over the Dalitz plot
- Some resonances are better than others
- Perform analysis in each point of $(m_+.m_-)$

Example: $\gamma = 75$, $\delta = \pi$, $r_B=0.125$

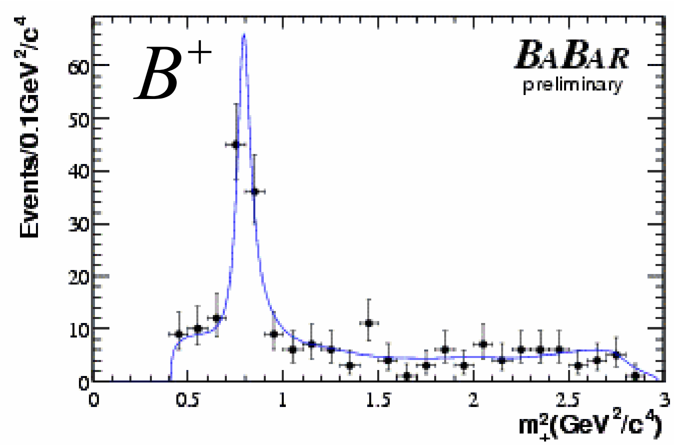


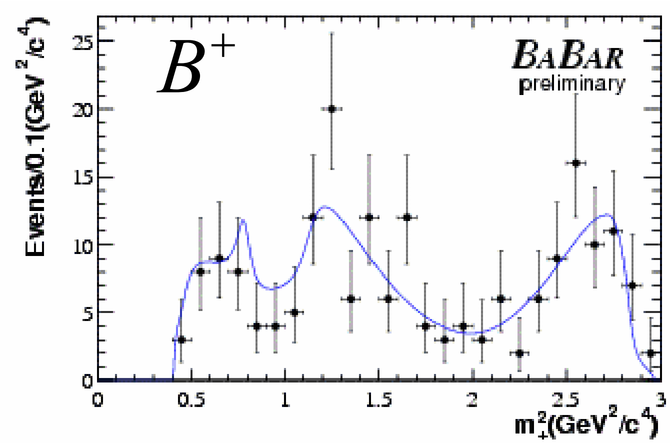
Dalitz Structure in $B^{\pm} \rightarrow [K_S\pi^+\pi^-]K^{\pm}$ Data

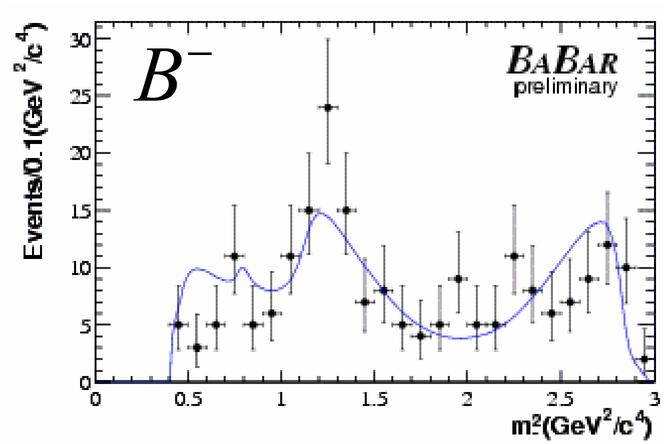
~260 events

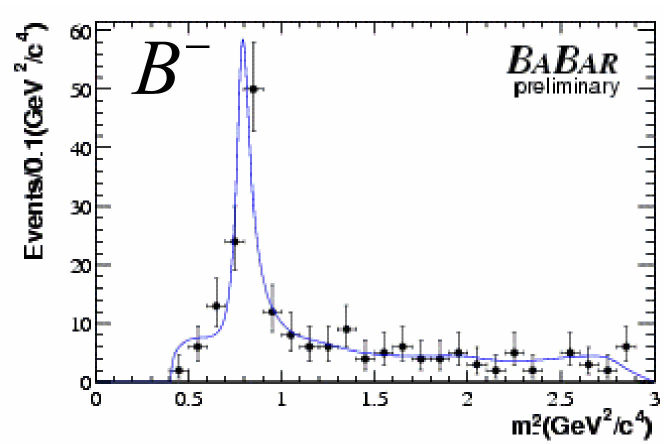


Dalitz Projection Plots
in signal region $m_{ES} > 5.27$ GeV/c^2









Constraints on γ and r_B with $B^- \to D^{(*)0}[K_S\pi^+\pi^-]\,K^-$

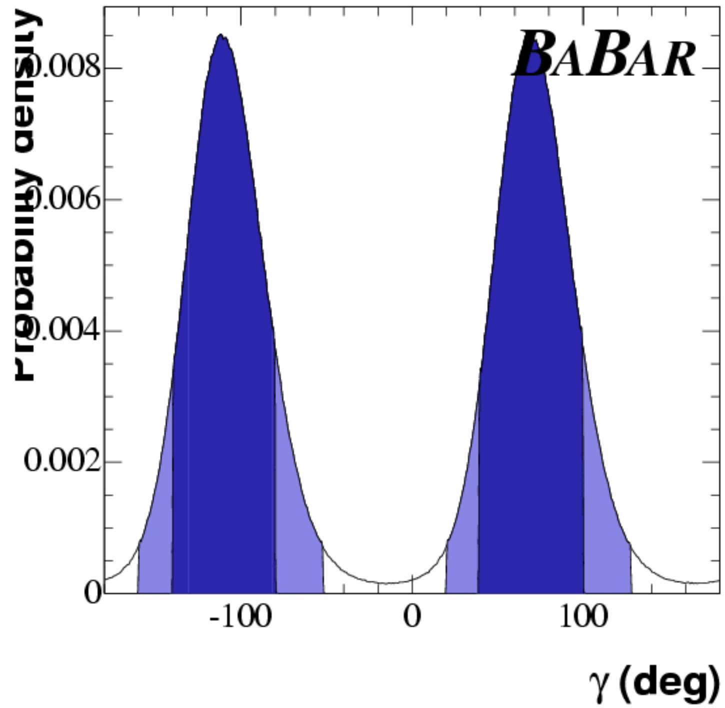
211 million BB

$$r_B = 0.118 \pm 0.079(stat) \pm 0.034(syst)^{+0.036}_{-0.034}(dalitz)$$

$$r_B^* = 0.169 \pm 0.096(stat)^{+0.030}_{-0.028}(syst)^{+0.029}_{-0.026}(dalitz)$$

$$\delta = 104^\circ \pm 45^\circ(stat)^{+17^\circ}_{-21^\circ}(syst)^{+16^\circ}_{-24^\circ}(dalitz)$$

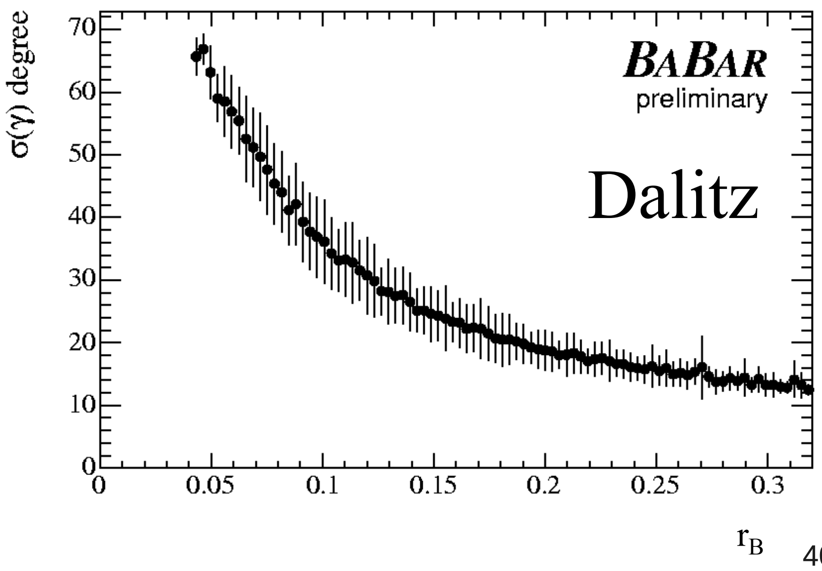
$$\delta^* = 296^\circ \pm 41^\circ(stat)^{+14^\circ}_{-12^\circ}(syst) \pm 15^\circ(dalitz)$$



$$\gamma = (70 \pm 31(stat)^{+12}_{-10}(syst)^{+14}_{-11}(dalitz))^o$$

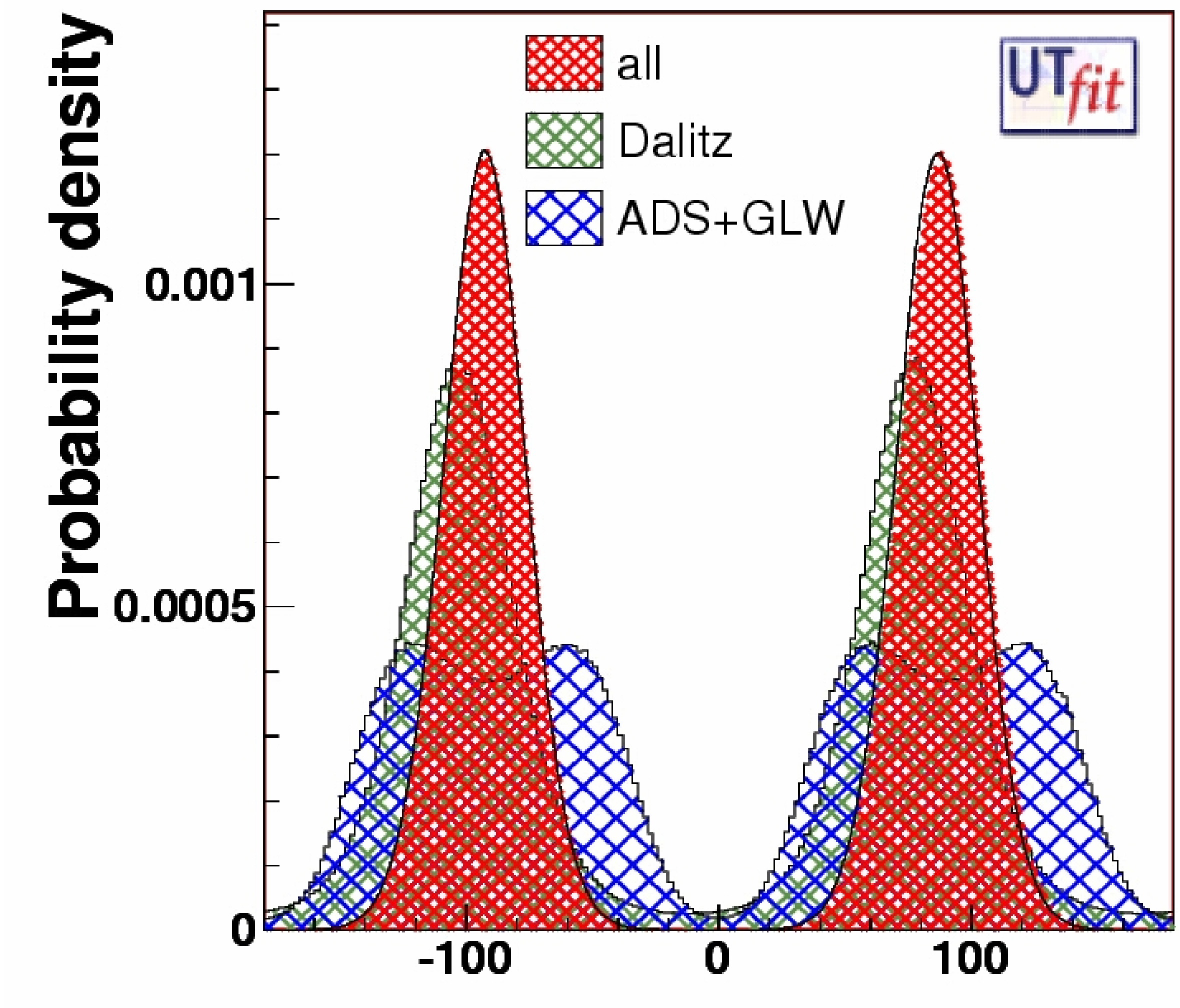
Dalitz structure of background
PDF shapes
Dalitz amplitudes and phases

Modeling of
Dalitz structure
for signal



Sensitivity to γ decreases significantly for small r_B

Sensitivity of GWL, ADS, and Dalitz to r_B

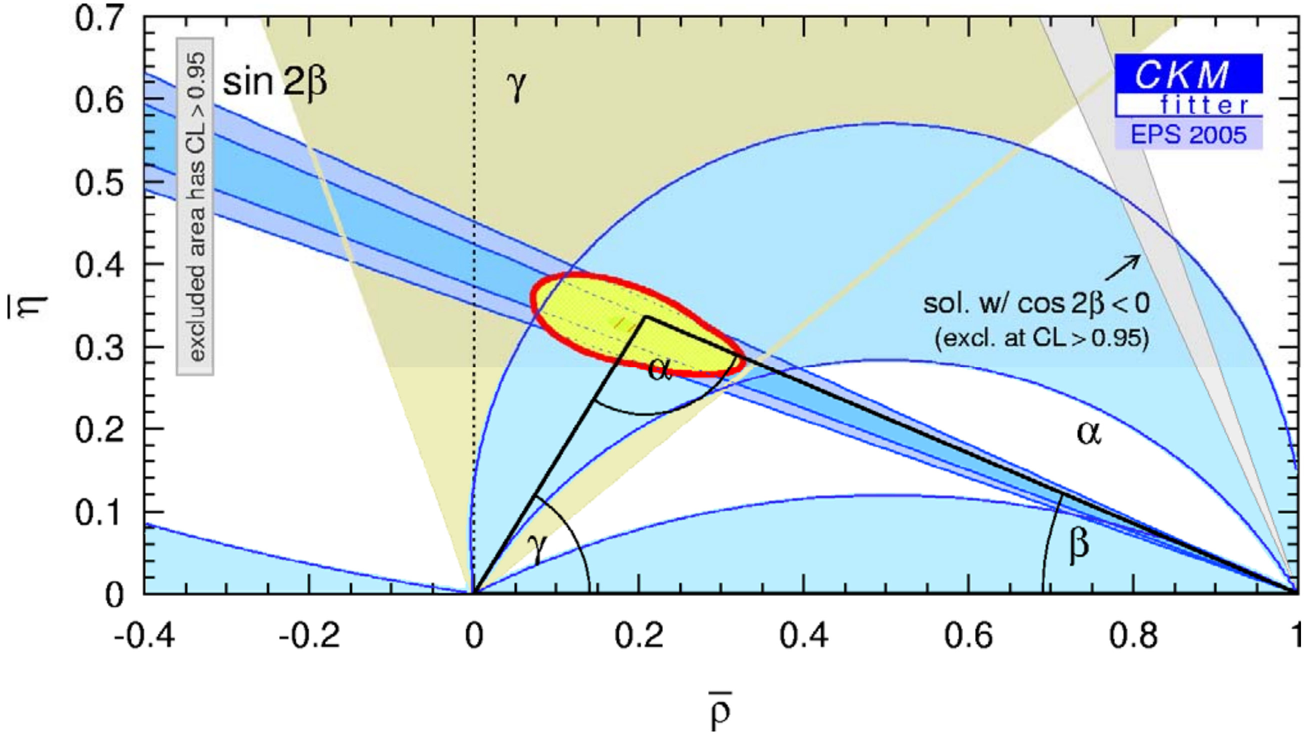


γ = 88 ± 16 ([41,123] @ 95% prob)
γ = -92 ± 16 ([-139,-57] @ 95% prob)

Dalitz Analysis Dominates Current Knowledge of γ

Most promising method for determination of γ in coming years

Constraints on Unitarity Triangle from Measurements of CKM Elements and CKM Angles



$$\mathbf{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

Determination of CKM Elements from Measurements

Measurement → Constraints on theoretical parameters

x_{exp}

Theoretical predictions

$y_{theo} = (A, \lambda, \rho, \eta, m_t \ldots)$

$x_{theo}(y_{model} = y_{theo}, y_{QCD})$

$y_{QCD} = (B_K, f_B, B_{Bd}, \ldots)$

Quarks confined in hadrons!

$$\text{“}\chi^2\text{”} = -2 \ln \mathcal{L}(y_{model})$$

$$\mathcal{L}(y_{model}) = \mathcal{L}_{exp}\,[\,x_{exp} - x_{theo}(y_{model})] \times \mathcal{L}_{theo}(y_{QCD})$$

Often Assumed to be Gaussian

« A part of theory error might come from Guesstimates »

Frequentist → Uniform likelihoods: “allowed *ranges*”

Bayesian → a-priori PDF

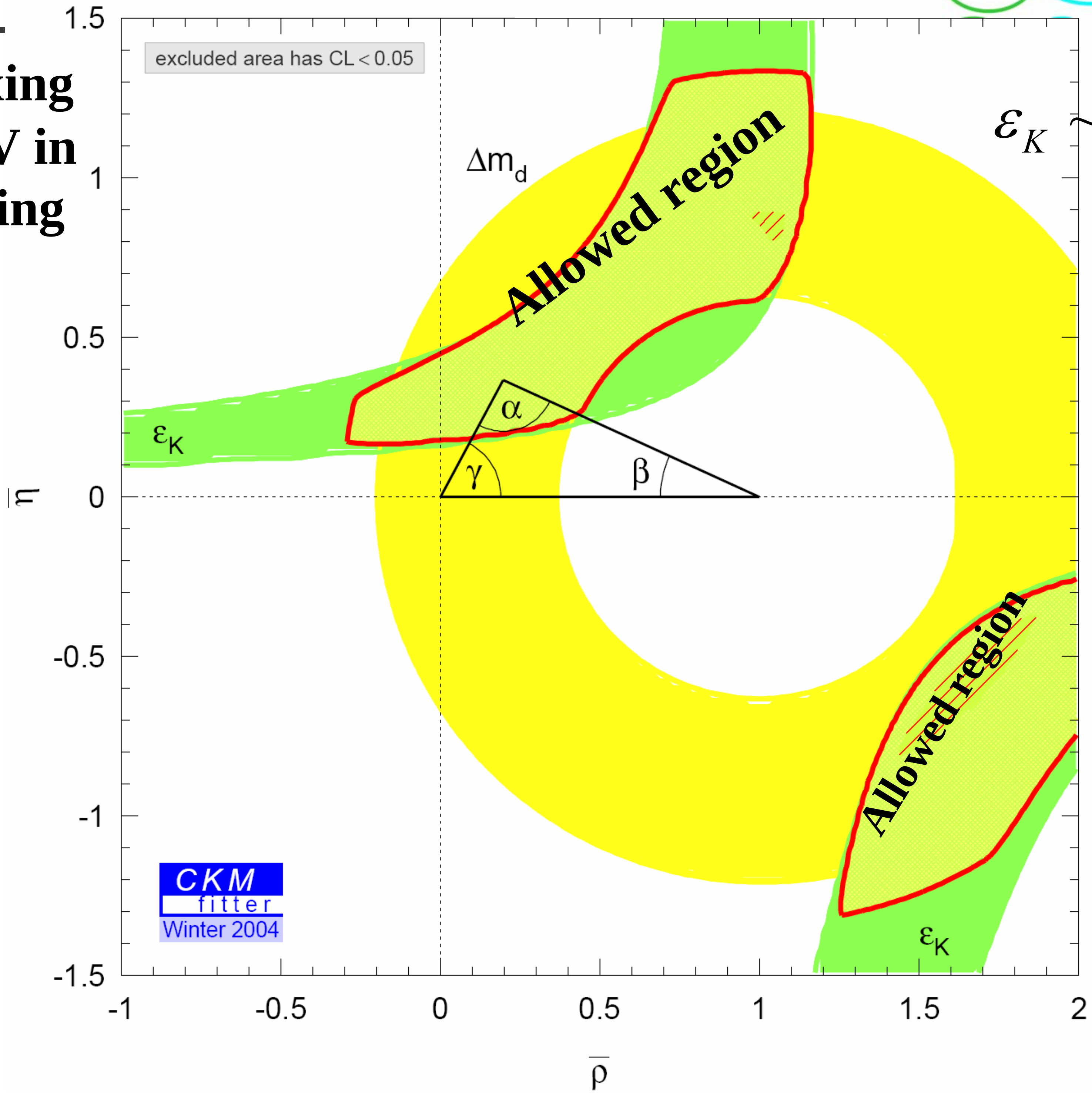
CKM Fitter
http://ckmfitter.in2p3.fr

UTfit
http://www.utfit.org

$$\Delta m_d \propto f_{B_d}^2 B_{B_d} |V_{tb}|^2 |V_{td}|^2 \propto f_{B_d}^2 B_{B_d} |V_{cb}|^2 \lambda^2 ((1-\bar{\rho})^2 + \bar{\eta}^2)$$

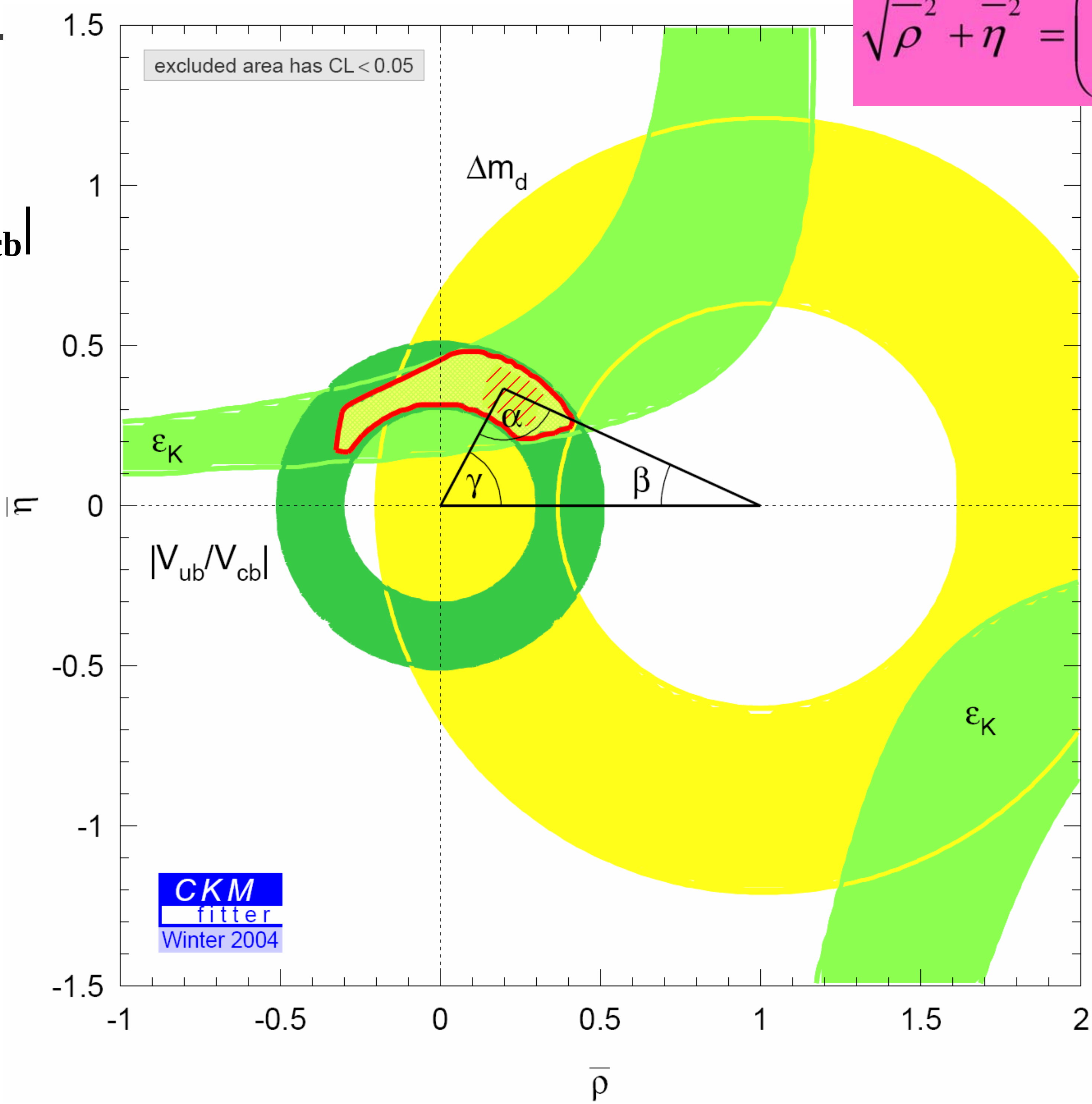
B_d mixing + CPV in K mixing (ε_K)

$$\varepsilon_K \sim \bar{\eta}(1-\bar{\rho})$$



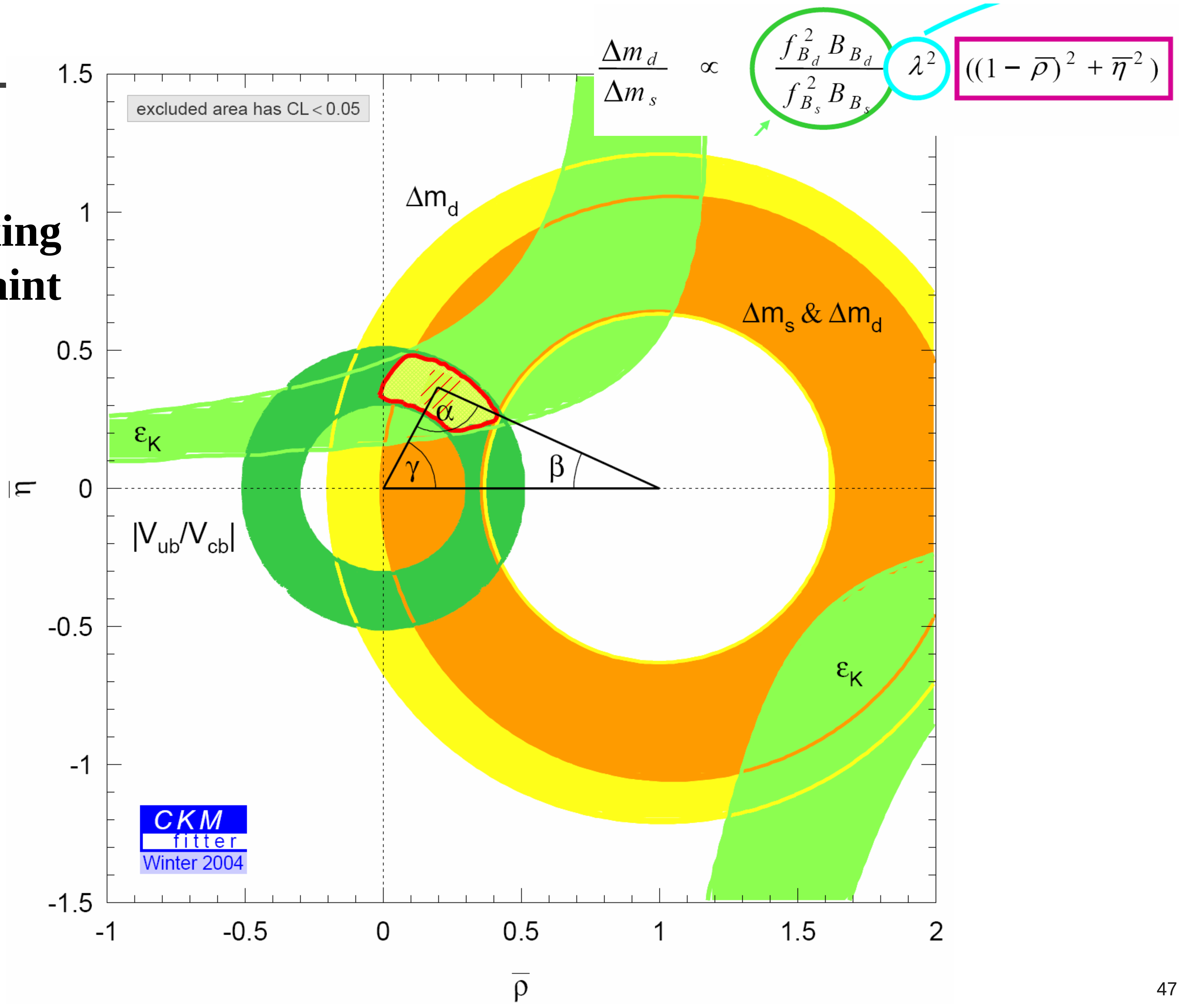
Add $|V_{ub}/ V_{cb}|$

$$\sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2}\right)\frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|$$

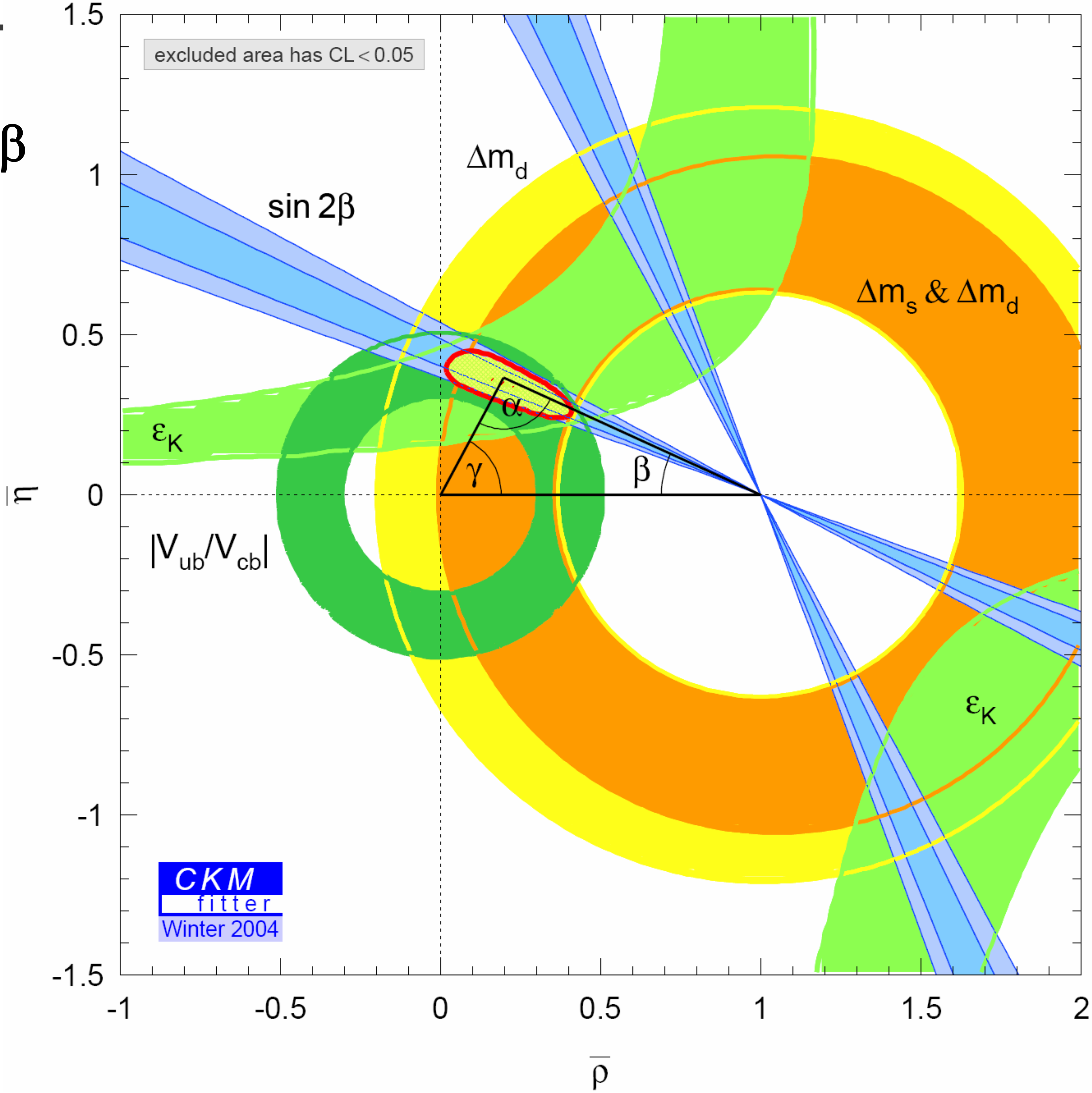


+

B_S Mixing constraint

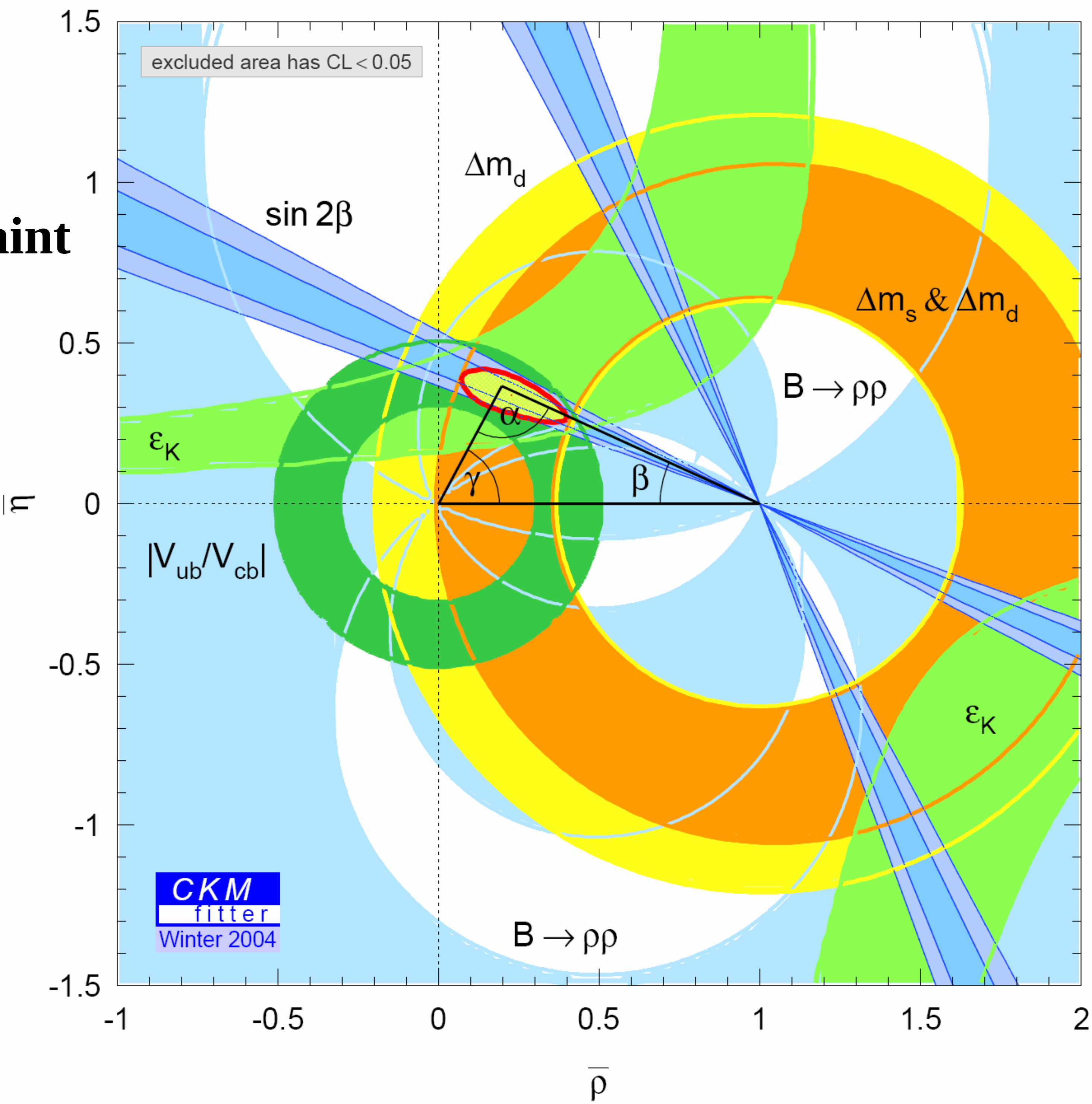


+ Sin2β



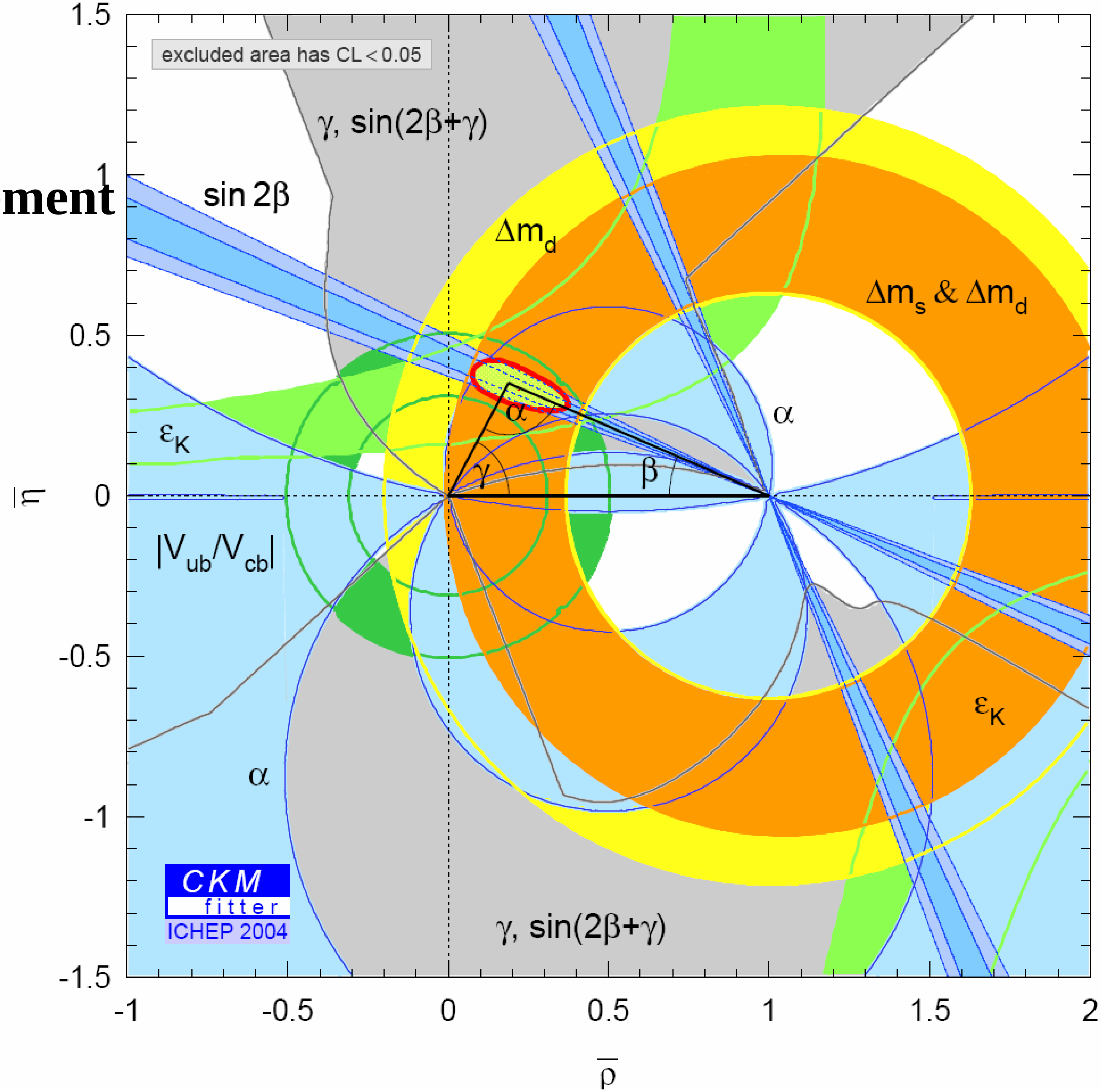
excluded area has CL < 0.05
Δm_d
sin 2β
Δm_s & Δm_d
ε_K
α
γ
β
|V_ub/V_cb|
ε_K
CKM fitter
Winter 2004
1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5
2
η̄
ρ̄

+ α constraint

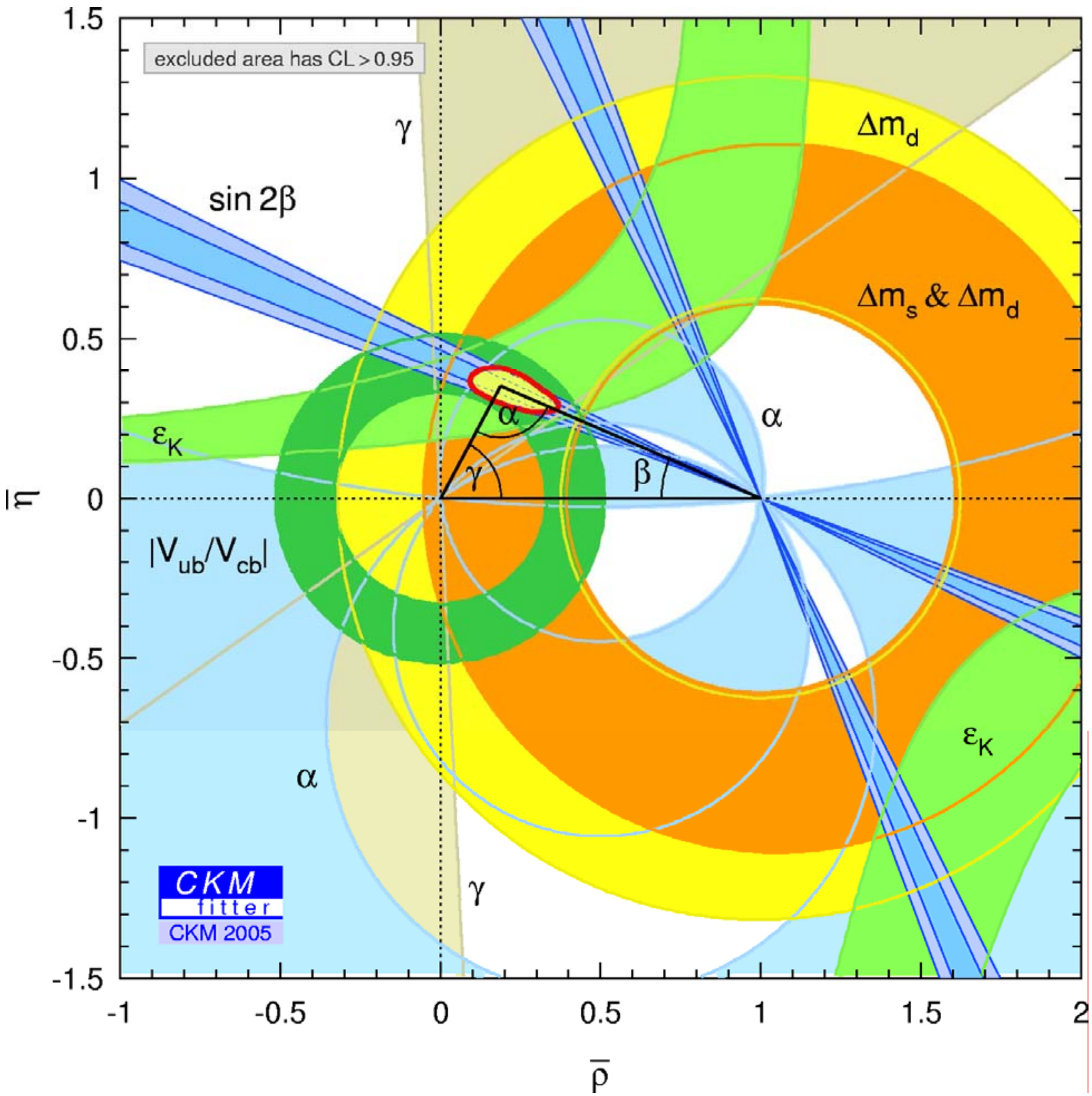


All measurements consistent, apex of (ρ,η) well defined

+ γ measurement

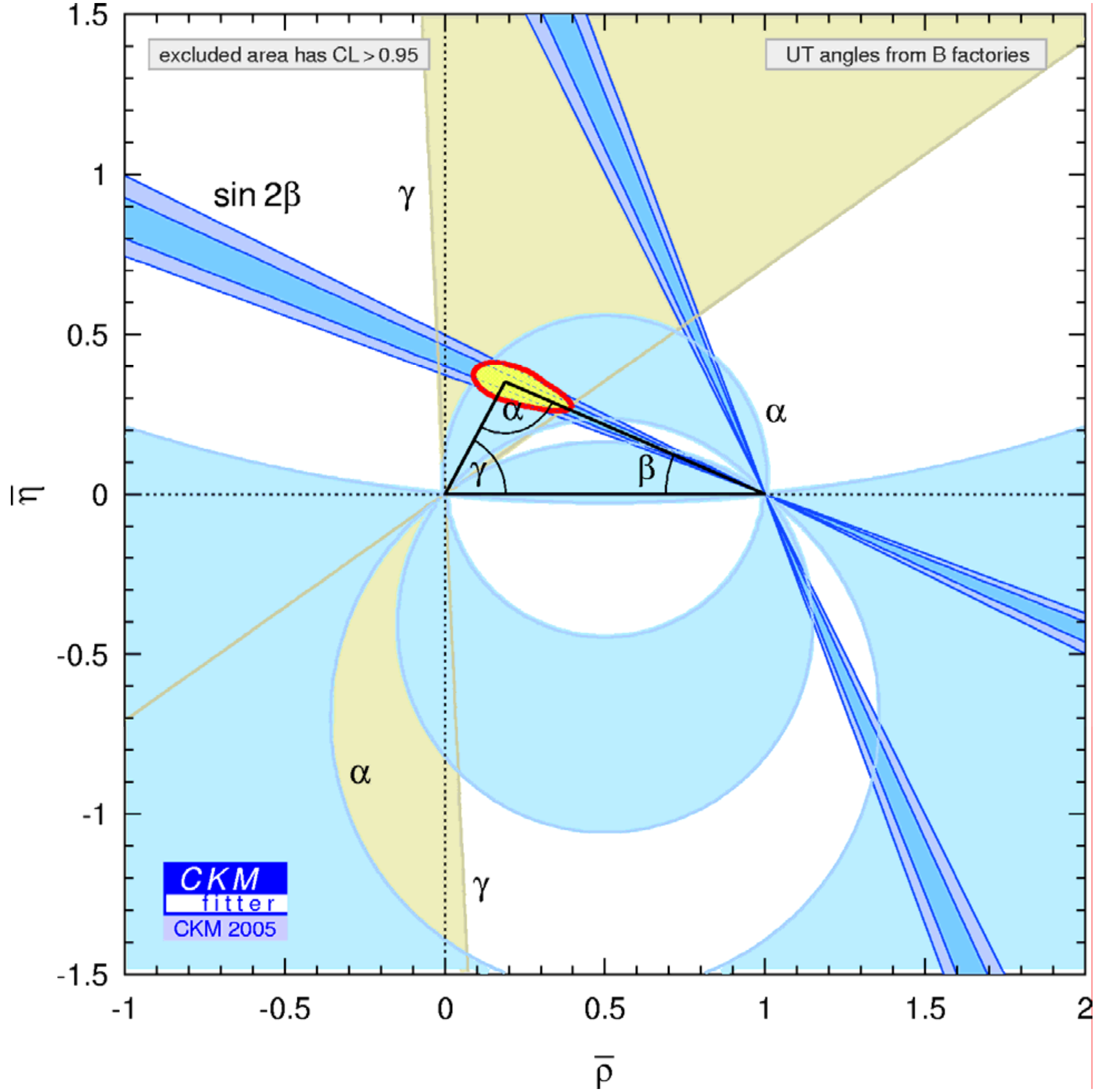


Unitarity Triangle after All Constraints



Includes constraints from all CKM-related measurements

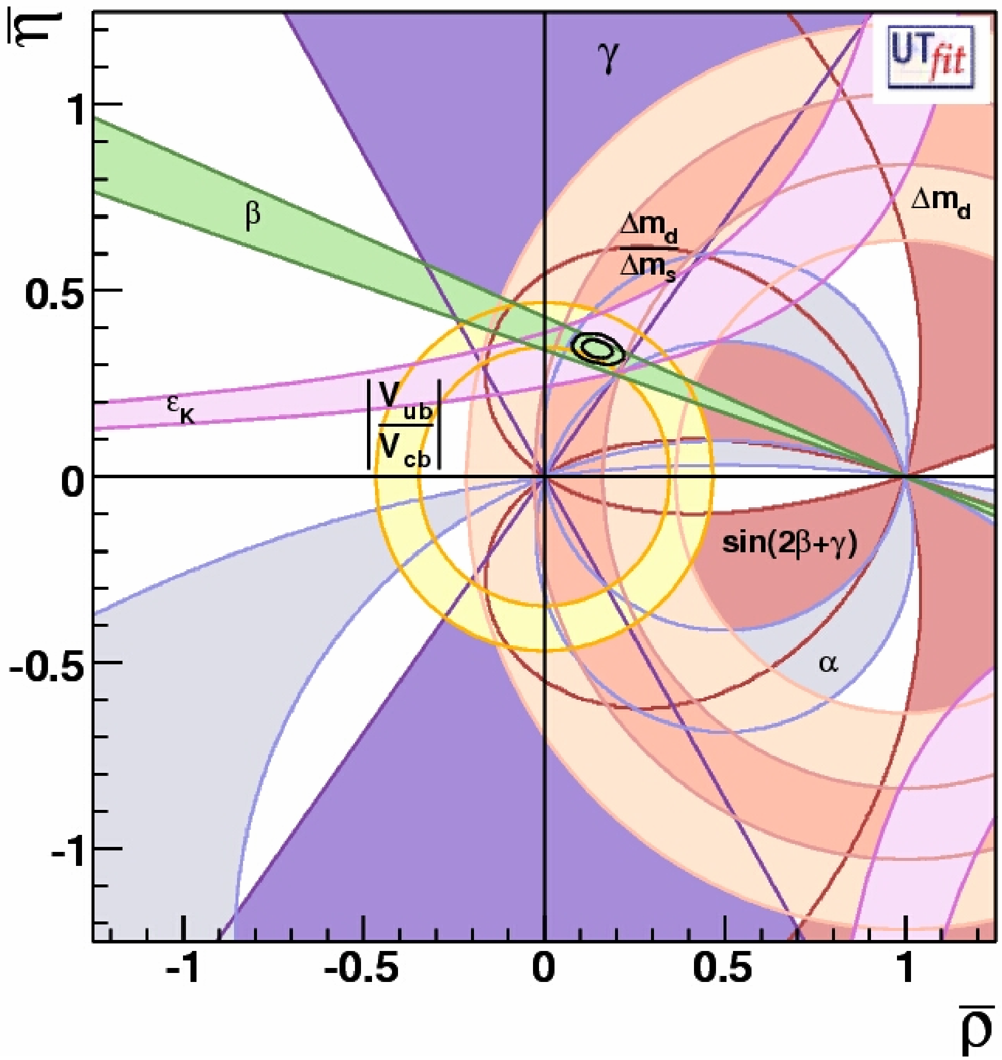
Impact of Angle Measurements on CKM Fit



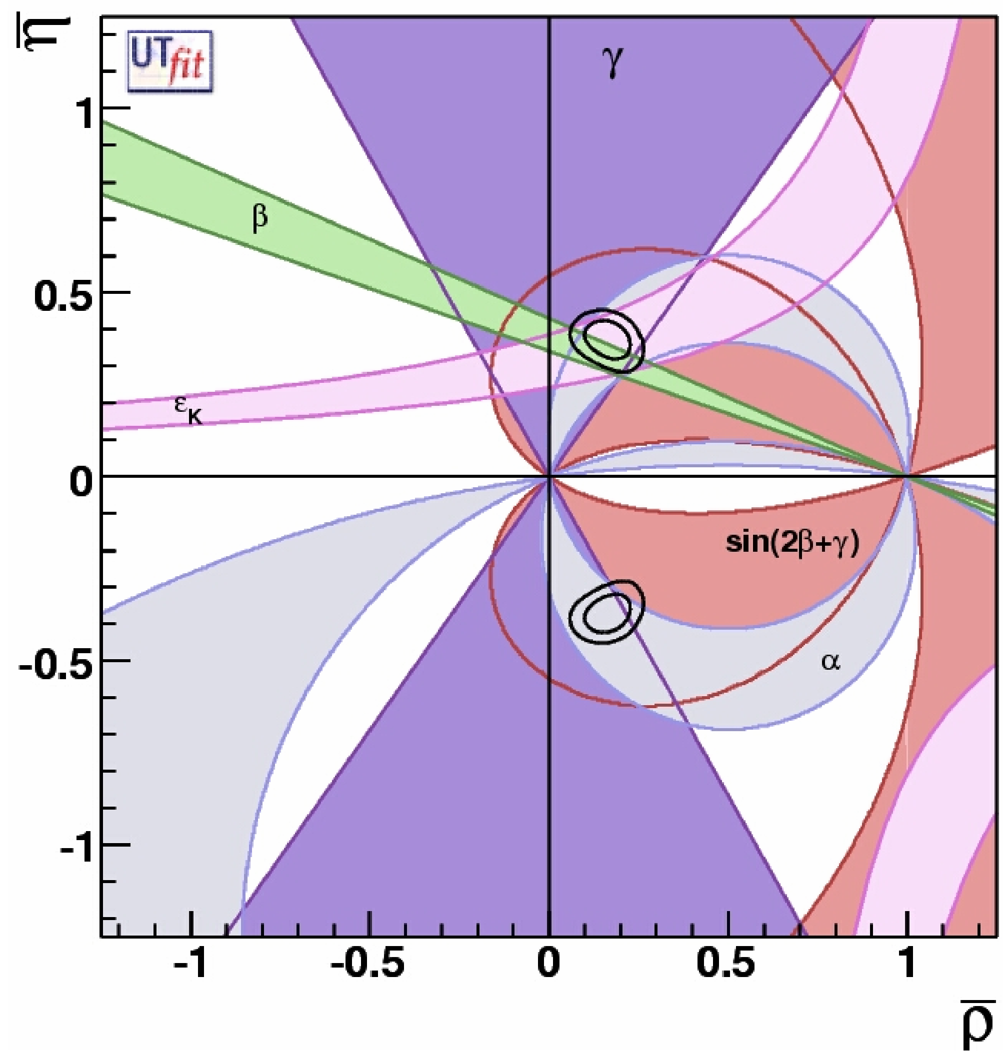
Measurements of angles dominate the constraint on the UT apex!

Status of UT in 2007

All constraints



Only angles from B and ε_K from Kaons



Future of CKM Physics

- BaBar & Belle are mature experiments and have a long term and a rich program for B physics (>2007)
  - Most CP asymmetry measurements are statistics limited
    - S-Penguins
    - α and γ measurements (multiple to be sure)
    - CPV in B mixing remains to be discovered
  - Rare decays such as Radiative and Electroweak are clean probe of NP
    - e.g. F-B Asymmetry in $b \to s\, l^+ l^-$
    - CPV in $B \to s\, \gamma$ etc

- Tevatron is accumulating large B samples:
  - CDF has finally provided measurement of B_s oscillation
  - They are the only current laboratory for studying B_s and Λ_b properties

- B Physics returns to Europe in with LHC-B !!
  - Will be an instrument for precision B physics
  - Precise exploration of CPV in B_S and B_d systems

- Super B Factory: studies started to evaluate possibility of very high luminosity B factory in near future