# The Standard Electroweak Theory and Beyond

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**Abstract**



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# 1 Introduction

These lectures on electroweak (EW) interactions start with a short summary of the Glashow–Weinberg–Salam theory and then cover in detail some main subjects of present interest in phenomenology.

The modern EW theory inherits the phenomenological successes of the $(V-A)\otimes(V-A)$ four-fermion low-energy description of weak interactions, and provides a well-defined and consistent theoretical framework including weak interactions and quantum electrodynamics in a unified picture.

As an introduction, we recall some salient physical features of the weak interactions. The weak interactions derive their name from their intensity. At low energy the strength of the effective four-fermion interaction of charged currents is determined by the Fermi coupling constant $G_F$. For example, the effective interaction for muon decay is given by

$$\mathcal{L}_{\rm eff} = (G_F/\sqrt{2})\,[\bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\mu]\,[\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e] \ , \qquad (1)$$

with [1] [1]

$$G_F = 1.16639(1)\times 10^{-5}\ \mathrm{GeV}^{-2}\ . \qquad (2)$$

In natural units $\hbar = c = 1$, $G_F$ has dimensions of $(\mathrm{mass})^{-2}$. As a result, the intensity of weak interactions at low energy is characterized by $G_F E^2$, where $E$ is the energy scale for a given process ($E \approx m_\mu$ for muon decay). Since

$$G_F E^2 = G_F m_p^2 (E/m_p)^2 \simeq 10^{-5}(E/m_p)^2\ , \qquad (3)$$

where $m_p$ is the proton mass, the weak interactions are indeed weak at low energies (energies of order $m_p$). Effective four fermion couplings for neutral current interactions have comparable intensity and energy behaviour. The quadratic increase with energy cannot continue for ever, because it would lead to a violation of unitarity. In fact, at large energies the propagator effects can no longer be neglected, and the current–current interaction is resolved into current–$W$ gauge boson vertices connected by a $W$ propagator. The strength of the weak interactions at high energies is then measured by $g_W$, the $W-\mu$–$\nu_\mu$ coupling, or, even better, by $\alpha_W = g_W^2/4\pi$ analogous to the fine-structure constant $\alpha$ of QED. In the standard EW theory, we have

$$\alpha_W = \sqrt{2}\ G_F\ m_W^2/\pi = \alpha/\sin^2\theta_W \cong 1/30\ . \qquad (4)$$

[1] For reasons of space, here only a few basic references are listed. Starting from those a more extended bibliography can easily be found.

That is, at high energies the weak interactions are no longer so weak.

The range $r_W$ of weak interactions is very short: it is only with the experimental discovery of the $W$ and $Z$ gauge bosons that it could be demonstrated that $r_W$ is non-vanishing. Now we know that

$$r_W = \hbar / m_W c \simeq 2.5 \times 10^{-16}\ \mathrm{cm}\ , \tag{5}$$

corresponding to $m_W \simeq 80$ GeV. This very large value for the $W$ (or the $Z$) mass makes a drastic difference, compared with the massless photon and the infinite range of the QED force. The direct experimental limit on the photon mass is [1] $m_\gamma < 2\ 10^{-16}\ eV$. Thus, on the one hand, there is very good evidence that the photon is massless. On the other hand, the weak bosons are very heavy. A unified theory of EW interactions has to face this striking difference.

Another apparent obstacle in the way of EW unification is the chiral structure of weak interactions: in the massless limit for fermions, only left-handed quarks and leptons (and right-handed antiquarks and antileptons) are coupled to $W$'s. This clearly implies parity and charge-conjugation violation in weak interactions.

The universality of weak interactions and the algebraic properties of the electromagnetic and weak currents [the conservation of vector currents (CVC), the partial conservation of axial currents (PCAC), the algebra of currents, etc.] have been crucial in pointing to a symmetric role of electromagnetism and weak interactions at a more fundamental level. The old Cabibbo universality for the weak charged current:

$$\begin{aligned} J_\alpha^{\mathrm{weak}} &= \bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\alpha (1-\gamma_5) e + \cos\theta_c\ \bar{u}\gamma_\alpha(1-\gamma_5)d + \\ & \sin\theta_c\ \bar{u}\gamma_\alpha(1-\gamma_5)s + \ldots\ , \end{aligned} \tag{6}$$

suitably extended, is naturally implied by the standard EW theory. In this theory the weak gauge bosons couple to all particles with couplings that are proportional to their weak charges, in the same way as the photon couples to all particles in proportion to their electric charges [in Eq. (6), $d' = \cos\theta_c\ d + \sin\theta_c\ s$ is the weak-isospin partner of $u$ in a doublet. The $(u, d')$ doublet has the same couplings as the $(\nu_e, \ell)$ and $(\nu_\mu, \mu)$ doublets].

Another crucial feature is that the charged weak interactions are the only known interactions that can change flavour: charged leptons into neutrinos or up-type quarks into down-type quarks. On the contrary, there are no flavour-changing neutral currents at tree level. This is a remarkable property of the weak neutral current, which is explained by the introduction of the Glashow-Iliopoulos-Maiani mechanism and has led to the successful prediction of charm.

The natural suppression of flavour-changing neutral currents, the separate conservation of $e, \mu$ and $\tau$ leptonic flavours, the mechanism of CP violation through the phase in the quark-mixing matrix, are all crucial features of the Standard Model. Many examples of new physics tend to break the selection rules of the standard theory. Thus the experimental study of rare flavour-changing transitions is an important window on possible new physics.

In the following sections we shall see how these properties of weak interactions fit into the standard EW theory.

# 2 Gauge Theories

In this section we summarize the definition and the structure of a gauge Yang–Mills theory. We will list here the general rules for constructing such a theory. Then in the next section these results will be applied to the EW theory.

Consider a Lagrangian density $\mathcal{L}[\phi, \partial_\mu \phi]$ which is invariant under a $D$ dimensional continuous group of transformations:

$$\phi' = U(\theta^A)\phi \qquad (A = 1, 2, ..., D) \ . \tag{7}$$

For $\theta^A$ infinitesimal, $U(\theta^A) = 1 + ig\sum_A \ \theta^A T^A$, where $T^A$ are the generators of the group $\Gamma$ of transformations (7) in the (in general reducible) representation of the fields $\phi$. Here we restrict ourselves to the case of internal symmetries, so that $T^A$ are matrices that are independent of the space–time coordinates. The generators $T^A$ are normalized in such a way that for the lowest dimensional non-trivial representation of the group $\Gamma$ (we use $t^A$ to denote the generators in this particular representation) we have

$$\mathrm{tr}(t^A t^B) = \frac{1}{2}\delta^{AB} \ . \tag{8}$$

The generators satisfy the commutation relations

$$[T^A, T^B] = iC_{ABC}T^C \ . \tag{9}$$

In the following, for each quantity $V^A$ we define

$$\mathbf{V} = \sum_A \ T^A V^A \ . \tag{10}$$

If we now make the parameters $\theta^A$ depend on the space–time coordinates $\theta^A = \theta^A(x_\mu)$, $\mathcal{L}[\phi, \partial_\mu \phi]$ is in general no longer invariant under the gauge transformations $U[\theta^A(x_\mu)]$, because of the derivative terms. Gauge invariance is recovered if the ordinary derivative is replaced by the covariant derivative:

$$D_\mu = \partial_\mu + ig\mathbf{V}_\mu \ , \tag{11}$$

where $V_\mu^A$ are a set of $D$ gauge fields (in one-to-one correspondence with the group generators) with the transformation law

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} - (1/ig)(\partial_\mu U)U^{-1} \ . \tag{12}$$

For constant $\theta^A$, $\mathbf{V}$ reduces to a tensor of the adjoint (or regular) representation of the group:

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} \simeq \mathbf{V}_\mu + ig[\theta, \mathbf{V}_\mu] \ , \tag{13}$$

which implies that

$$V_\mu^{\prime C} = V_\mu^C - gC_{ABC}\theta^A V_\mu^B \ , \tag{14}$$

where repeated indices are summed up.

As a consequence of Eqs. (11) and (12), $D_\mu \phi$ has the same transformation properties as $\phi$:

$$(D_\mu \phi)' = U(D_\mu \phi) \ . \tag{15}$$

Thus $\mathcal{L}[\phi, D_\mu \phi]$ is indeed invariant under gauge transformations. In order to construct a gauge-invariant kinetic energy term for the gauge fields $V^A$, we consider

$$[D_\mu, D_\nu]\phi = ig\{\partial_\mu \mathbf{V}_\nu - \partial_\nu \mathbf{V}_\mu + ig[\mathbf{V}_\mu, \mathbf{V}_\nu]\}\phi \equiv ig\mathbf{F}_{\mu\nu}\phi \ , \tag{16}$$

which is equivalent to

$$F_{\mu\nu}^A = \partial_\mu V_\nu^A - \partial_\nu V_\mu^A - gC_{ABC}V_\mu^B V_\nu^C \ . \tag{17}$$

From Eqs. (7), (15) and (16) it follows that the transformation properties of $F_{\mu\nu}^A$ are those of a tensor of the adjoint representation

$$\mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^{-1} \ . \tag{18}$$

The complete Yang–Mills Lagrangian, which is invariant under gauge transformations, can be written in the form

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}\sum_A F_{\mu\nu}^A F^{A\mu\nu} + \mathcal{L}[\phi, D_\mu \phi] \ . \tag{19}$$

For an Abelian theory, as for example QED, the gauge transformation reduces to $U[\theta(x)] = \exp[ieQ\theta(x)]$, where $Q$ is the charge generator. The associated gauge field (the photon), according to Eq. (12), transforms as

$$V'_\mu = V_\mu - \partial_\mu \theta(x) \; . \tag{20}$$

In this case, the $F_{\mu\nu}$ tensor is linear in the gauge field $V_\mu$ so that in the absence of matter fields the theory is free. On the other hand, in the non-Abelian case the $F^A_{\mu\nu}$ tensor contains both linear and quadratic terms in $V^A_\mu$, so that the theory is non-trivial even in the absence of matter fields.

# 3 The Standard Model of Electroweak Interactions

In this section, we summarize the structure of the standard EW Lagrangian and specify the couplings of $W^\pm$ and $Z$, the intermediate vector bosons.

For this discussion we split the Lagrangian into two parts by separating the Higgs boson couplings:

$$\mathcal{L} = \mathcal{L}_{\rm symm} + \mathcal{L}_{\rm Higgs} \; . \tag{21}$$

We start by specifying $\mathcal{L}_{\rm symm}$, which involves only gauge bosons and fermions:

$$\begin{aligned} \mathcal{L}_{\rm symm} \;\; = \;\; & -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}_L i\gamma^\mu D_\mu \psi_L \\ & + \bar{\psi}_R i\gamma^\mu D_\mu \psi_R \; . \end{aligned} \tag{22}$$

This is the Yang–Mills Lagrangian for the gauge group $SU(2) \otimes U(1)$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and} \quad F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\epsilon_{ABC}\; W^B_\mu W^C_\nu \tag{23}$$

are the gauge antisymmetric tensors constructed out of the gauge field $B_\mu$ associated with $U(1)$, and $W^A_\mu$ corresponding to the three $SU(2)$ generators; $\epsilon_{ABC}$ are the group structure constants [see Eqs. (9)] which, for $SU(2)$, coincide with the totally antisymmetric Levi-Civita tensor (recall the familiar angular momentum commutators). The normalization of the $SU(2)$ gauge coupling $g$ is therefore specified by Eq. (23).

The fermion fields are described through their left-hand and right-hand components:

$$\psi_{L,R} = [(1 \mp \gamma_5)/2]\psi, \quad \bar{\psi}_{L,R} = \bar{\psi}[(1 \pm \gamma_5)/2] \ , \tag{24}$$

with $\gamma_5$ and other Dirac matrices defined as in the book by Bjorken–Drell. In particular, $\gamma_5^2 = 1, \gamma_5^\dagger = \gamma_5$. Note that, as given in Eq. (24),

$$\bar{\psi}_L = \psi_L^\dagger \gamma_0 = \psi^\dagger[(1-\gamma_5)/2]\gamma_0 = \bar{\psi}[\gamma_0(1-\gamma_5)/2]\gamma_0 = \bar{\psi}[(1+\gamma_5)/2] \ .$$

The matrices $P_\pm = (1 \pm \gamma_5)/2$ are projectors. They satisfy the relations $P_\pm P_\pm = P_\pm, P_\pm P_\mp = 0, P_+ + P_- = 1$.

The sixteen linearly independent Dirac matrices can be divided into $\gamma_5$-even and $\gamma_5$-odd according to whether they commute or anticommute with $\gamma_5$. For the $\gamma_5$-even, we have

$$\bar{\psi}\Gamma_E\psi = \bar{\psi}_L\Gamma_E\psi_R + \bar{\psi}_R\Gamma_E\psi_L \qquad (\Gamma_E \equiv 1, i\gamma_5, \sigma_{\mu\nu}) \ , \tag{25}$$

whilst for the $\gamma_5$-odd,

$$\bar{\psi}\Gamma_O\psi = \bar{\psi}_L\Gamma_O\psi_L + \bar{\psi}_R\Gamma_O\psi_R \qquad (\Gamma_O \equiv \gamma_\mu, \gamma_\mu\gamma_5) \ . \tag{26}$$

In the Standard Model (SM) the left and right fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form $\bar{\psi}_L\psi_R$ + h.c.) are forbidden in the symmetric limit. In particular, all $\psi_R$ are singlets in the Minimal Standard Model (MSM). But for the moment, by $\psi_R$ we mean a column vector, including all fermions in the theory that span a generic reducible representation of $SU(2) \otimes U(1)$. The standard EW theory is a chiral theory, in the sense that $\psi_L$ and $\psi_R$ behave differently under the gauge group. In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing $\psi_L$ and $\psi_R$. Fermion masses will be introduced, together with $W^\pm$ and $Z$ masses, by the mechanism of symmetry breaking. The covariant derivatives $D_\mu\psi_{L,R}$ are explicitly given by

$$D_\mu\psi_{L,R} = \left[\partial_\mu + ig\sum_{A=1}^{3} \ t^A_{L,R}W^A_\mu + ig'\frac{1}{2}Y_{L,R}B_\mu\right]\psi_{L,R} \ , \tag{27}$$

where $t^A_{L,R}$ and $1/2Y_{L,R}$ are the $SU(2)$ and $U(1)$ generators, respectively, in the reducible representations $\psi_{L,R}$. The commutation relations of the $SU(2)$ generators are given by

$$[t^A_L, t^B_L] = i \ \epsilon_{ABC}t^C_L \quad \text{and} \quad [t^A_R, t^B_R] = i\epsilon_{ABC}t^C_R \ . \tag{28}$$

We use the normalization (8) [in the fundamental representation of $SU(2)$]. The electric charge generator $Q$ (in units of $e$, the positron charge) is given by

$$Q = t_L^3 + 1/2\ Y_L = t_R^3 + 1/2\ Y_R\ . \tag{29}$$

Note that the normalization of the $U(1)$ gauge coupling $g'$ in (27) is now specified as a consequence of (29).

All fermion couplings to the gauge bosons can be derived directly from Eqs. (22) and (27). The charged-current (CC) couplings are the simplest. From

$$\begin{aligned} g(t^1 W^1_\mu + t^2 W^2_\mu) &= g\left\{[(t^1 + it^2)/\sqrt{2}](W^1_\mu - iW^2_\mu)/\sqrt{2}] + \text{h.c.}\right\} \\ &= g\left\{[(t^+ W^-_\mu)/\sqrt{2}] + \text{h.c.}\right\}\ , \end{aligned} \tag{30}$$

where $t^\pm = t^1 \pm it^2$ and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$, we obtain the vertex

$$V_{\bar\psi\psi W} = g\bar\psi\gamma_\mu\left[(t^+_L/\sqrt{2})(1-\gamma_5)/2 + (t^+_R/\sqrt{2})(1+\gamma_5)/2\right]\psi W^-_\mu + \text{h.c.} \tag{31}$$

In the neutral-current (NC) sector, the photon $A_\mu$ and the mediator $Z_\mu$ of the weak NC are orthogonal and normalized linear combinations of $B_\mu$ and $W^3_\mu$:

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W^3_\mu\ , \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W\ W^3_\mu\ . \end{aligned} \tag{32}$$

Equations (32) define the weak mixing angle $\theta_W$. The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. Recalling Eq. (29) for the charge matrix $Q$, we immediately obtain

$$g\ \sin\theta_W = g'\cos\theta_W = e\ , \tag{33}$$

or equivalently,

$$\text{tg}\ \theta_W = g'/g \tag{34}$$

Once $\theta_W$ has been fixed by the photon couplings, it is a simple matter of algebra to derive the $Z$ couplings, with the result

$$\Gamma_{\bar\psi\psi Z} = g/(2\ \cos\theta_W)\bar\psi\gamma_\mu[t^3_L(1-\gamma_5) + t^3_R(1+\gamma_5) - 2Q\sin^2\theta_W]\psi Z^\mu\ , \tag{35}$$

where $\Gamma_{\bar\psi\psi Z}$ is a notation for the vertex. In the MSM, $t^3_R = 0$ and $t^3_L = \pm 1/2$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs. (31) and (35), we anticipate that large masses, as experimentally observed, are provided for $W^\pm$ and $Z$ by $\mathcal{L}_{\rm Higgs}$. For left–left CC couplings, when the momentum transfer squared can be neglected with respect to $m_W^2$ in the propagator of Born diagrams with single $W$ exchange, from Eq. (31) we can write

$$\mathcal{L}_{\rm eff}^{\rm CC} \simeq (g^2/8m_W^2)[\bar\psi\gamma_\mu(1-\gamma_5)t_L^+\psi][\bar\psi\gamma^\mu(1-\gamma_5)t_L^-\psi]\ . \tag{36}$$

By specializing further in the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of $g$ with the Fermi coupling constant $G_F$ measured from $\mu$ decay [see Eq. (2)]:

$$G_F/\sqrt{2} = g^2/8m_W^2\ . \tag{37}$$

By recalling that $g\ \sin\theta_W = e$, we can also cast this relation in the form

$$m_W = \mu_{\rm Born}/\sin\theta_W\ , \tag{38}$$

with

$$\mu_{\rm Born} = (\pi\alpha/\sqrt{2}G_F)^{1/2} \simeq 37.2802\ {\rm GeV}\ , \tag{39}$$

where $\alpha$ is the fine-structure constant of QED ($\alpha \equiv e^2/4\pi = 1/137.036$).

In the same way, for neutral currents we obtain in Born approximation from Eq. (35) the effective four-fermion interaction given by

$$\mathcal{L}_{\rm eff}^{\rm NC} \simeq \sqrt{2}\ G_F\rho_0\bar\psi\gamma_\mu[...]\psi\bar\psi\gamma^\mu[...]\psi\ , \tag{40}$$

where

$$[...] \equiv t_L^3(1-\gamma_5) + t_R^3(1+\gamma_5) - 2Q\sin^2\theta_W \tag{41}$$

and

$$\rho_0 = m_W^2/m_Z^2\ \cos^2\theta_W\ . \tag{42}$$

All couplings given in this section are obtained at tree level and are modified in higher orders of perturbation theory. In particular, the relations between $m_W$ and $\sin\theta_W$ [Eqs. (38) and (39)] and the observed values of $\rho$ ($\rho = \rho_0$ at tree level) in different NC processes, are altered by computable EW radiative corrections, as discussed in Section 6.

The gauge-boson self-interactions can be derived from the $F_{\mu\nu}$ term in $\mathcal{L}_{\rm symm}$, by using Eq. (32) and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$. Defining the three-gauge-boson vertex as in Fig. 1, we obtain ($V \equiv \gamma, Z$)

$$\Gamma_{W^-W^+V} = ig_{W^-W^+V}[g_{\mu\nu}(q-p)_\lambda + g_{\mu\lambda}(p-r)_\nu + g_{\nu\lambda}(r-q)_\mu]\ , \tag{43}$$

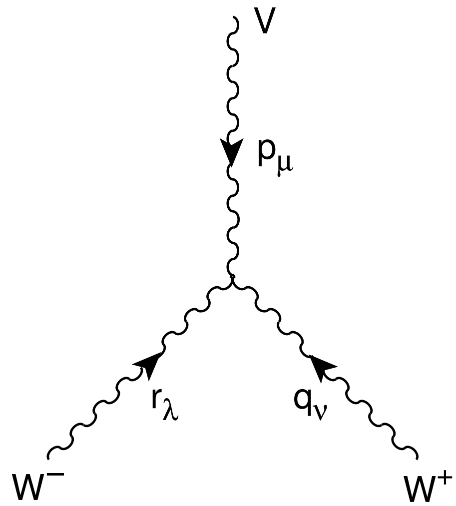


Figure 1: The three-gauge boson vertex: $V = \gamma, Z$

with

$$g_{W^-W^+\gamma} = g \ \sin\theta_W = e \quad \text{and} \quad g_{W^-W^+Z} = g \ \cos\theta_W \ . \tag{44}$$

This form of the triple gauge vertex is very special: in general, there could be departures from the above SM expression, even restricting us to $SU(2) \otimes U(1)$ gauge symmetric and C and P invariant couplings. In fact some small corrections are already induced by the radiative corrections. But, in principle, more important could be the modifications induced by some new physics effect. The experimental testing of the triple gauge vertices is presently underway at LEP2 and limits on departures from the SM couplings have also been obtained at the Tevatron and elsewhere.

We now turn to the Higgs sector of the EW Lagrangian. Here we simply review the formalism of the Higgs mechanism applied to the EW theory. In the next section we shall make a more general and detailed discussion of the physics of the EW symmetry breaking. The Higgs Lagrangian is specified by the gauge principle and the requirement of renormalizability to be

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) - \bar{\psi}_L\Gamma\psi_R\phi - \bar{\psi}_R\Gamma^\dagger\psi_L\phi^\dagger \ , \tag{45}$$

where $\phi$ is a column vector including all Higgs fields; it transforms as a reducible representation of the gauge group. The quantities $\Gamma$ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The potential $V(\phi^\dagger\phi)$, symmetric under $SU(2) \otimes U(1)$, contains, at most, quartic terms in $\phi$ so that the theory is renormalizable:

$$V(\phi^\dagger\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + \frac{1}{4}\lambda(\phi^\dagger\phi)^2 \tag{46}$$

As discussed in the next section, spontaneous symmetry breaking is induced if the minimum of V which is the classical analogue of the quantum mechanical vacuum state (both are the states of minimum energy) is obtained for non-vanishing $\phi$ values. Precisely, we denote the vacuum expectation value (VEV) of $\phi$, i.e. the position of the minimum, by $v$:

$$\langle 0|\phi(x)|0\rangle = v \neq 0 \ . \tag{47}$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by $v$:

$$M = \bar{\psi}_L \ \mathcal{M}\psi_R + \bar{\psi}_R\mathcal{M}^\dagger\psi_L \ , \tag{48}$$

with

$$\mathcal{M} = \Gamma \cdot v \ . \tag{49}$$

In the MSM, where all left fermions $\psi_L$ are doublets and all right fermions $\psi_R$ are singlets, only Higgs doublets can contribute to fermion masses. There are enough free couplings in $\Gamma$, so that one single complex Higgs doublet is indeed sufficient to generate the most general fermion mass matrix. It is important to observe that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, $\gamma_5$-free, and diagonal. In fact, we can make separate unitary transformations on $\psi_L$ and $\psi_R$ according to

$$\psi'_L = U\psi_L, \quad \psi'_R = V\psi_R \tag{50}$$

and consequently

$$\mathcal{M} \rightarrow \mathcal{M}' = U^\dagger\mathcal{M}V \ . \tag{51}$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\rm symm}$.

If only one Higgs doublet is present, the change of basis that makes $\mathcal{M}$ diagonal will at the same time diagonalize also the fermion–Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. This is not true, in general, when there are several Higgs doublets. But one Higgs doublet for each electric charge sector i.e. one doublet coupled only to $u$-type quarks, one doublet to $d$-type quarks, one doublet to charged leptons would also be all right, because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets in a given charge sector it is also possible to generate CP violation by complex phases in the Higgs couplings. In the presence of six quark flavours, this CP-violation mechanism is not necessary. In fact, at the moment, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomena.

We now consider the gauge-boson masses and their couplings to the Higgs. These effects are induced by the $(D_\mu\phi)^\dagger(D^\mu\phi)$ term in $\mathcal{L}_{\rm Higgs}$ [Eq. (45)], where

$$D_\mu\phi = \left[\partial_\mu + ig\sum_{A=1}^{3} t^A W_\mu^A + ig'(Y/2)B_\mu\right]\phi \ . \tag{52}$$

Here $t^A$ and $1/2Y$ are the $SU(2)\otimes U(1)$ generators in the reducible representation spanned by $\phi$. Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that the photon remains massless is equivalent to the condition that the vacuum is electrically neutral:

$$Q|v\rangle = (t^3 + \frac{1}{2}Y)|v\rangle = 0 \ . \tag{53}$$

The charged $W$ mass is given by the quadratic terms in the $W$ field arising from $\mathcal{L}_{\rm Higgs}$, when $\phi(x)$ is replaced by $v$. We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2|(t^+v/\sqrt{2})|^2 W_\mu^+ W^{-\mu} \ , \tag{54}$$

whilst for the $Z$ mass we get [recalling Eq. (32)]

$$\frac{1}{2}m_Z^2 Z_\mu Z^\mu = |[g\cos\theta_W t^3 - g'\sin\theta_W(Y/2)]v|^2 Z_\mu Z^\mu \ , \tag{55}$$

where the factor of 1/2 on the left-hand side is the correct normalization for the definition of the mass of a neutral field. By using Eq. (53), relating the action of $t^3$ and $1/2Y$ on the vacuum $v$, and Eqs. (34), we obtain

$$\frac{1}{2}m_Z^2 = (g\cos\theta_W + g'\sin\theta_W)^2|t^3v|^2 = (g^2/\cos^2\theta_W)|t^3v|^2 \ . \tag{56}$$

For Higgs doublets

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \quad v = \begin{pmatrix}0\\ v\end{pmatrix} \ , \tag{57}$$

we have

$$|t^+v|^2 = v^2, \quad |t^3v|^2 = 1/4v^2 \ , \tag{58}$$

so that

$$m_W^2 = 1/2g^2v^2, \quad m_Z^2 = 1/2g^2v^2/\cos^2\theta_W \ . \tag{59}$$

Note that by using Eq. (37) we obtain

$$v = 2^{-3/4} G_F^{-1/2} = 174.1 \text{ GeV} . \tag{60}$$

It is also evident that for Higgs doublets

$$\rho_0 = m_W^2 / m_Z^2 \cos^2\theta_W = 1 . \tag{61}$$

This relation is typical of one or more Higgs doublets and would be spoiled by the existence of Higgs triplets etc. In general,

$$\rho_0 = \sum_i ((t_i)^2 - (t_i^3)^2 + t_i) v_i^2 / \sum_i 2(t_i^3)^2 v_i^2 \tag{62}$$

for several Higgses with VEVs $v_i$, weak isospin $t_i$, and $z$-component $t_i^3$. These results are valid at the tree level and are modified by calculable EW radiative corrections, as discussed in Section 6.

In the minimal version of the SM only one Higgs doublet is present. Then the fermion–Higgs couplings are in proportion to the fermion masses. In fact, from the Yukawa couplings $g_{\phi\bar{f}f}(\bar{f}_L \phi f_R + h.c.)$, the mass $m_f$ is obtained by replacing $\phi$ by $v$, so that $m_f = g_{\phi\bar{f}f} v$. In the minimal SM three out of the four Hermitian fields are removed from the physical spectrum by the Higgs mechanism and become the longitudinal modes of $W^+, W^-$, and $Z$. The fourth neutral Higgs is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be around for each additional doublet.

The couplings of the physical Higgs $H$ to the gauge bosons can be simply obtained from $\mathcal{L}_{\text{Higgs}}$, by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \to \begin{pmatrix} 0 \\ v + (H/\sqrt{2}) \end{pmatrix} , \tag{63}$$

[so that $(D_\mu \phi)^\dagger (D^\mu \phi) = 1/2 (\partial_\mu H)^2 + ...$], with the result

$$\begin{aligned} \mathcal{L}[H, W, Z] &= g^2 (v/\sqrt{2}) W_\mu^+ W^{-\mu} H + (g^2/4) W_\mu^+ W^{-\mu} H^2 \\ &\quad + [(g^2 v Z_\mu Z^\mu)/(2\sqrt{2}\cos^2\theta_W)] H \\ &\quad + [g^2/(8\cos^2\theta_W)] Z_\mu Z^\mu H^2 . \end{aligned} \tag{64}$$

In the minimal SM the Higgs mass $m_H^2 \sim \lambda v^2$ is of order of the weak scale v. We will discuss in sect.9 the direct experimental limit on $m_H$ from

LEP, which is $m_H \gtrsim 113\ GeV$. We shall also see in sect.9 , that, if there is no physics beyond the SM up to a large scale $\Lambda$, then, on theoretical grounds, $m_H$ can only be within a narrow range between 135 and 180 GeV. But the interval is enlarged if there is new physics nearby. Also the lower limit depends critically on the assumption of only one doublet. The dominant decay mode of the Higgs is in the $b\bar{b}$ channel below the WW threshold, while the $W^+W^-$ channel is dominant for sufficiently large $m_H$. The width is small below the WW threshold, not exceeding a few MeV, but increases steeply beyond the threshold, reaching the asymptotic value of $\Gamma \sim 1/2 m_H^3$ at large $m_H$, where all energies are in TeV.

# 4 The Higgs Mechanism

The gauge symmetry of the Standard Model was difficult to discover because it is well hidden in nature. The only observed gauge boson that is massless is the photon. The gluons are presumed massless but are unobservable because of confinement, and the $W$ and $Z$ weak bosons carry a heavy mass. Actually a major difficulty in unifying weak and electromagnetic interactions was the fact that e.m. interactions have infinite range ($m_\gamma = 0$), whilst the weak forces have a very short range, owing to $m_{W,Z} \neq 0$.

The solution of this problem is in the concept of spontaneous symmetry breaking, which was borrowed from statistical mechanics.

Consider a ferromagnet at zero magnetic field in the Landau–Ginzburg approximation. The free energy in terms of the temperature $T$ and the magnetization $\mathbf{M}$ can be written as

$$F(\mathbf{M}, T) \simeq F_0(T) + 1/2\ \mu^2(T)\mathbf{M}^2 + 1/4\ \lambda(T)(\mathbf{M}^2)^2 + ... \ . \tag{65}$$

This is an expansion which is valid at small magnetization. The neglect of terms of higher order in $\vec{M}^2$ is the analogue in this context of the renormalizability criterion. Also, $\lambda(T) > 0$ is assumed for stability; $F$ is invariant under rotations, i.e. all directions of $\mathbf{M}$ in space are equivalent. The minimum condition for $F$ reads

$$\partial F/\partial M = 0, \quad [\mu^2(T) + \lambda(T)\mathbf{M}^2]\mathbf{M} = 0\ . \tag{66}$$

There are two cases. If $\mu^2 > 0$, then the only solution is $\mathbf{M} = 0$, there is no magnetization, and the rotation symmetry is respected. If $\mu^2 < 0$, then another solution appears, which is

$$|\mathbf{M}_0|^2 = -\mu^2/\lambda\ . \tag{67}$$

The direction chosen by the vector $\mathbf{M}_0$ is a breaking of the rotation symmetry. The critical temperature $T_{\rm crit}$ is where $\mu^2(T)$ changes sign:

$$\mu^2(T_{\rm crit}) = 0 \ . \tag{68}$$

It is simple to realize that the Goldstone theorem holds. It states that when spontaneous symmetry breaking takes place, there is always a zero-mass mode in the spectrum. In a classical context this can be proven as follows. Consider a Lagrangian

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi) \tag{69}$$

symmetric under the infinitesimal transformations

$$\phi \to \phi' = \phi + \delta\phi, \quad \delta\phi_i = i\delta\theta t_{ij}\phi_j \ . \tag{70}$$

The minimum condition on $V$ that identifies the equilibrium position (or the ground state in quantum language) is

$$(\partial V/\partial\phi_i)(\phi_i = \phi_i^0) = 0 \ . \tag{71}$$

The symmetry of $V$ implies that

$$\delta V = (\partial V/\partial\phi_i)\delta\phi_i = i\delta\theta(\partial V/\partial\phi_i)t_{ij}\phi_j = 0 \ . \tag{72}$$

By taking a second derivative at the minimum $\phi_i = \phi_i^0$ of the previous equation, we obtain

$$\partial^2 V/\partial\phi_k\partial\phi_i(\phi_i = \phi_i^0)t_{ij}\phi_i^0 + \frac{\partial V}{\partial\phi_i}(\phi_i = \phi_i^0)t_{ik} = 0 \ . \tag{73}$$

The second term vanishes owing to the minimum condition, Eq. (71). We then find

$$\partial^2 V/\partial\phi_k\partial\phi_i \ (\phi_i = \phi_i^0)t_{ij}\phi_j^0 = 0 \ . \tag{74}$$

The second derivatives $M^2_{ki} = (\partial^2 V/\partial\phi_k\partial\phi_i)(\phi_i = \phi_i^0)$ define the squared mass matrix. Thus the above equation in matrix notation can be read as

$$M^2 t\phi^0 = 0 \ , \tag{75}$$

which shows that if the vector $(t\phi^0)$ is non-vanishing, i.e. there is some generator that shifts the ground state into some other state with the same energy, then $t\phi^0$ is an eigenstate of the squared mass matrix with zero eigenvalue. Therefore, a massless mode is associated with each broken generator.

When spontaneous symmetry breaking takes place in a gauge theory, the massless Goldstone mode exists, but it is unphysical and disappears from the spectrum. It becomes, in fact, the third helicity state of a gauge boson that takes mass. This is the Higgs mechanism. Consider, for example, the simplest Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\ F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2\phi^*\phi - (\lambda/4)(\phi^*\phi)^2\ . \tag{76}$$

Note the 'wrong' sign in front of the mass term for the scalar field $\phi$, which is necessary for the spontaneous symmetry breaking to take place. The above Lagrangian is invariant under the $U(1)$ gauge symmetry

$$A_\mu \to A'_\mu = A_\mu - (1/e)\partial_\mu\theta(x), \quad \phi \to \phi' = \phi\ \exp[i\theta(x)]\ . \tag{77}$$

Let $\phi^0 = v \neq 0$, with $v$ real, be the ground state that minimizes the potential and induces the spontaneous symmetry breaking. Making use of gauge invariance, we can make the change of variables

$$\begin{aligned} &\phi(x) \to (1/\sqrt{2})[\rho(x) + v]\ \exp[i\zeta(x)/v]\ , \\ &A_\mu(x) \to A_\mu - (1/ev)\partial_\mu\zeta(x). \end{aligned} \tag{78}$$

Then $\rho = 0$ is the position of the minimum, and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho vA_\mu^2 + \mathcal{L}(\rho)\ . \tag{79}$$

The field $\zeta(x)$, which corresponds to the would-be Goldstone boson, disappears, whilst the mass term $\frac{1}{2}e^2v^2A_\mu^2$ for $A_\mu$ is now present; $\rho$ is the massive Higgs particle.

The Higgs mechanism is realized in well-known physical situations. For a superconductor in the Landau–Ginzburg approximation the free energy can be written as

$$F = F_0 + \frac{1}{2}\mathbf{B}^2 + |(\nabla - 2ie\mathbf{A})\phi|^2/4m - \alpha|\phi|^2 + \beta|\phi|^4\ . \tag{80}$$

Here $\mathbf{B}$ is the magnetic field, $|\phi|^2$ is the Cooper pair ($e^-e^-$) density, $2e$ and $2m$ are the charge and mass of the Cooper pair. The 'wrong' sign of $\alpha$ leads to $\phi \neq 0$ at the minimum. This is precisely the non-relativistic analogue of the Higgs model of the previous example. The Higgs mechanism implies the absence of propagation of massless phonons (states with dispersion relation $\omega = kv$ with constant $v$). Also the mass term for $\mathbf{A}$ is manifested by the exponential decrease of $\mathbf{B}$ inside the superconductor (Meissner effect).

# 5 The CKM Matrix

Weak charged currents are the only tree level interactions in the SM that change flavour: by emission of a W an up-type quark is turned into a down-type quark, or a $\nu_l$ neutrino is turned into a $l^-$ charged lepton (all fermions are letf-handed). If we start from an up quark that is a mass eigenstate, emission of a W turns it into a down-type quark state d' (the weak isospin partner of u) that in general is not a mass eigenstate. In general, the mass eigenstates and the weak eigenstates do not coincide and a unitary transformation connects the two sets:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{81}$$

V is the Cabibbo-Kobayashi-Maskawa matrix. Thus in terms of mass eigenstates the charged weak current of quarks is of the form:

$$J^+_\mu \propto \bar{u}\gamma_\mu(1-\gamma_5)t^+Vd \tag{82}$$

Since V is unitary (i.e. $VV^\dagger = V^\dagger V = 1$) and commutes with $T^2$, $T_3$ and Q (because all d-type quarks have the same isospin and charge) the neutral current couplings are diagonal both in the primed and unprimed basis (if the Z down-type quark current is abbreviated as $\bar{d}'\Gamma d'$ then by changing basis we get $\bar{d}V^\dagger\Gamma V d$ and V and $\Gamma$ commute because, as seen from eq.(41), $\Gamma$ is made of Dirac matrices and $T_3$ and Q generator matrices). It follows that $\bar{d}'\Gamma d' = \bar{d}\Gamma d$. This is the GIM mechanism that ensures natural flavour conservation of the neutral current couplings at the tree level.

For N generations of quarks, V is a NxN unitary matrix that depends on $N^2$ real numbers ($N^2$ complex entries with $N^2$ unitarity constraints). However, the $2N$ phases of up- and down-type quarks are not observable. Note that an overall phase drops away from the expression of the current in eq.(82), so that only $2N-1$ phases can affect V. In total, V depends on $N^2 - 2N + 1 = (N-1)^2$ real physical parameters. A similar counting gives $N(N-1)/2$ as the number of independent parameters in an orthogonal NxN matrix. This implies that in V we have $N(N-1)/2$ mixing angles and $(N-1)^2 - N(N-1)/2$ phases: for $N = 2$ one mixing angle (the Cabibbo angle) and no phase, for $N = 3$ three angles and one phase etc.

Given the experimental near diagonal structure of V a convenient parametrisation is the one proposed by Maiani. One starts by the definition:

$$|d'\rangle = c_{13}|d_C\rangle + s_{13}e^{-i\phi}|b\rangle \tag{83}$$

where $c_{13} \equiv cos\theta_{13}$, $s_{13} \equiv sin\theta_{13}$ (analogous shorthand notations will be used in the following), $d_C$ is the Cabibbo down quark and $\theta_{12} \equiv \theta_C$ is the Cabibbo angle (experimentally $s_{12} \equiv \lambda \sim 0.22$).

$$|d_C\rangle = c_{12}|d\rangle + s_{12}|s\rangle \tag{84}$$

Note that in a four quark model the Cabibbo angle fixes both the ratio of couplings $(u \to d)/(\nu_e \to e)$ and the ratio of $(u \to d)/(u \to s)$. In a six quark model one has to choose which to keep as a definition of the Cabibbo angle. Here the second definition is taken and, in fact the $u \to d$ coupling is given by $V_{ud} = c_{13}c_{12}$ so that it is no longer specified by $\theta_{12}$ only. Also note that we can certainly fix the phases of u, d, s so that a real coefficient appears in front of $d_C$ in eq.(83). We now choose a basis of two orthonormal vectors, both orthogonal to $|d'\rangle$:

$$|s_C\rangle = -s_{12}|d\rangle + c_{12}|s\rangle, \qquad |v\rangle = -s_{13}e^{i\phi}|d_C\rangle + c_{13}|b\rangle \tag{85}$$

Here $|s_C\rangle$ is the Cabibbo s quark. Clearly s' and b' must be othonormal superpositions of the above base vectors defined in terms of an angle $\theta_{23}$:

$$|s'\rangle = c_{23}|s_C\rangle + s_{23}|v\rangle, \qquad |b'\rangle = -s_{23}|s_C\rangle + c_{23}|v\rangle \tag{86}$$

The general expression of $V_{ij}$ can be obtained from the above equations. But a considerable notational simplification is gained if one takes into account that from experiment we know that $s_{12} \equiv \lambda$, $s_{23} \sim o(\lambda^2)$ and $s_{13} \sim o(\lambda^3)$ are increasingly small and of the indicated orders of magnitude. Thus, following Wolfenstein one can set:

$$s_{12} \equiv \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{-i\phi} = A\lambda^3(\rho - i\eta) \tag{87}$$

As a result, by neglecting terms of higher order in $\lambda$ one can write down:

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \sim \begin{bmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{bmatrix}. \tag{88}$$

Indicative values of the CKM parameters as obtained from experiment are (a survey of the current status of the CKM parameters can be found in ref.[1]):

$$\begin{aligned} \lambda &= 0.2196 \pm 0.0023 \\ A &= 0.83 \pm 0.04 \\ \sqrt{\rho^2 + \eta^2} = 0.4 \pm 0.1; \qquad \eta &\sim 0.3 \pm 0.1 \end{aligned} \tag{89}$$

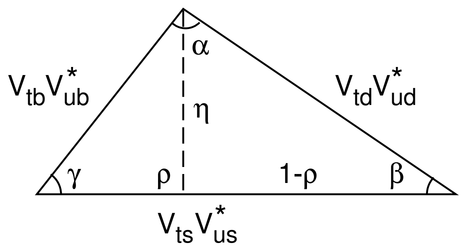


Figure 2: The Bjorken triangle corresponding to eq.(90):

In the SM the non vanishing of the $\eta$ parameter is the only source of CP violation. Unitarity of the CKM matrix V implies relations of the form $\sum_a V_{ba}V_{ca}^* = \delta_{bc}$. In most cases these relations do not imply particularly instructive constraints on the Wolfenstein parameters. But when the three terms in the sum are of comparable magnitude we get interesting information. The three numbers which must add to zero form a closed triangle in the complex plane, with sides of comparable length. This is the case for the t-u triangle (Bjorken triangle) shown in fig.2:

$$V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* = 0 \tag{90}$$

All terms are of order $\lambda^3$. For $\eta$=0 the triangle would flatten down to vanishing area. In fact the area of the triangle, J of order $J \sim \eta A^2\lambda^6$, is the Jarlskog invariant (its value is independent of the parametrization). In the SM all CP violating observables must be proportional to J, hence to the area of the triangle or to $\eta$. The most direct and solid evidence for J non vanishing is obtained from the measurement of $\epsilon$ in K decay. Additional direct evidence is being obtained from the measurement of $\sin 2\beta$ in B decay.

We have only discussed flavour mixing for quarks. But, clearly, if neutrino masses exist, as indicated by neutrino oscillations (see section 8.2.3), then a similar mixing matrix must also be introduced in the leptonic sector.

# 6 Renormalisation and Higher Order Corrections

The Higgs mechanism gives masses to the Z, the $W^\pm$ and to fermions while the Lagrangian density is still symmetric. In particular the gauge Ward identities and the conservation of the gauge currents are preserved. The validity of these relations is an essential ingredient for renormalisability. For example the massive gauge boson propagator would have a bad ultraviolet behaviour:

$$W_{\mu\nu} = \frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}}{q^2 - m_W^2} \tag{91}$$

But if the propagator is sandwiched between conserved currents $J_\mu$ the bad terms in $q_\mu q_\nu$ give a vanishing contribution because $q_\mu J^\mu = 0$ and the high energy behaviour is like for a scalar particle and compatible with renormalisation.

The fundamental theorem that in general a gauge theory with spontaneous symmetry breaking and the Higgs mechanism is renormalisable was proven by 't Hooft. For a chiral theory like the SM an additional complication arises from the existence of chiral anomalies. But this problem is avoided in the SM because the quantum numbers of the quarks and leptons in each generation imply a remarkable (and apparently miracoulous) cancellation of the anomaly, as originally observed by Bouchiat, Iliopoulos and Meyer. In quantum field theory one encounters an anomaly when a symmetry of the classical lagrangian is broken by the process of quantisation, regularisation and renormalisation of the theory. For example, in massless QCD there is no mass scale in the classical lagrangian. Thus one would predict that dimensionless quantities in processes with only one large energy scale Q cannot depend on Q and must be constants. As well known this naive statement is false. The process of regularisation and renormalisation necessarily introduces an energy scale which is essentially the scale where renormalised quantities are defined. For example the renormalised coupling must be defined from the vertices at some scale. This scale $\mu$ cannot be zero because of infrared divergences. The scale $\mu$ destroys scale invariance because dimensionless quantities can now depend on $Q/\mu$. The famous $\Lambda_{QCD}$ parameter is a tradeoff of $\mu$ and leads to scale invariance breaking. Of direct relevance for the EW theory is the Adler-Bell-Jackiw chiral anomaly. The classical lagrangian of a theory with massless fermions is invariant under a U(1) chiral transformations $\psi\prime = e^{i\gamma_5\theta}\psi$. The associated axial Noether current is conserved at the classical level. But, at the quantum level, chiral symmetry is broken due to the ABJ anomaly and the current is not conserved. The chiral breaking is introduced by a clash between chiral symmetry, gauge invariance and the regularisation procedure. The anomaly is generated by triangular fermion loops with one axial and two vector vertices (fig.3). For neutral currents (Z and $\gamma$) the axial coupling is proportional to the 3rd component of weak isospin $t_3$, while vector couplings are proportional to a linear combination of $t_3$ and the electric charge Q. Thus in order for the chiral anomaly to vanish all traces of the form $tr\{t_3 QQ\}$,

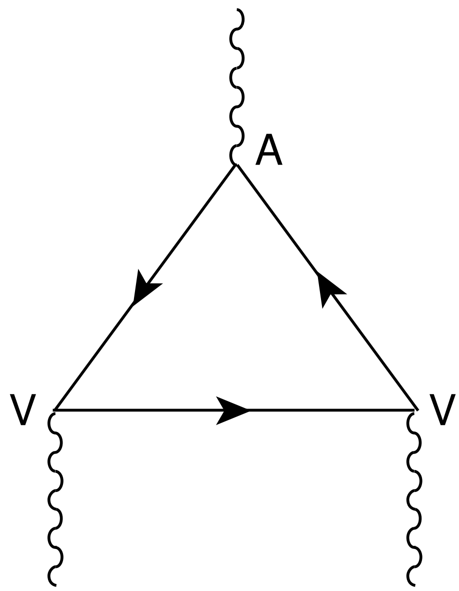


Figure 3: Triangle diagram that generates the ABJ anomaly.

$tr\{t_3 t_3 Q\}$, $tr\{t_3 t_3 t_3\}$ (and also $tr\{t_+ t_- t_3\}$ when charged currents are also included) must vanish, where the trace is extended over all fermions in the theory that can circulate in the loop. Now all these traces happen to vanish for each fermion family separately. For example take $tr\{t_3 QQ\}$. In one family there are, with $t_3 = +1/2$, three colours of up quarks with charge $Q = +2/3$ and one neutrino with $Q = 0$ and, with $t_3 = -1/2$, three colours of down quarks with charge $Q = -1/3$ and one $l^-$ with $Q = -1$. Thus we obtain $tr\{t_3 QQ\} = 1/2\ 3\ 4/9 - 1/2\ 3\ 1/9 - 1/2\ 1 = 0$. This impressive cancellation suggests an interplay among weak isospin, charge and colour quantum numbers which appears as a miracle from the point of view of the low energy theory but is more understandable from the point of view of the high energy theory. For example in GUTs there are similar relations where charge quantisation and colour are related: in the 5 of SU(5) we have the content $(d, d, d, e^+, \bar{\nu})$ and the charge generator has a vanishing trace in each SU(5) representation (the condition of unit determinant, represented by the letter S in the SU(5) group name, translates into zero trace for the generators). Thus the charge of d quarks is -1/3 of the positron charge because there are three colours.

Since the SM theory is renormalisable higher order perturbative corrections can be reliably computed. Radiative corrections are very important for precision EW tests. The SM inherits all successes of the old V-A theory of charged currents and of QED. Modern tests focus on neutral current processes, the W mass and the measurement of triple gauge vertices. For Z physics and the W mass the state of the art computation of radiative corrections include the complete one loop diagrams and selected dominant two loop corrections. In addition some resummation techniques are also implemented, like Dyson resummation of vacuum polarisation functions and important renormalisation group improvements for large QED and QCD logarithms. We now discuss in more detail sets of large radiative corrections which are particularly significant [2].

A set of important quantitative contributions to the radiative corrections arise from large logarithms [e.g. terms of the form $(\alpha/\pi \ \ln\ (m_Z/m_{f_\ell}))^n$ where $f_\ell$ is a light fermion]. The sequences of leading and close-to-leading logarithms are fixed by well-known and consolidated techniques ($\beta$ functions, anomalous dimensions, penguin-like diagrams, etc.). For example, large logarithms dominate the running of $\alpha$ from $m_e$, the electron mass, up to $m_Z$. Similarly large logarithms of the form $[\alpha/\pi \ \ln\ (m_Z/\mu)]^n$ also enter, for example, in the relation between $\sin^2\theta_W$ at the scales $m_Z$ (LEP, SLC) and $\mu$ (e.g. the scale of low-energy neutral-current experiments). Also, large logs from initial state radiation dramatically distort the line shape of the Z resonance as observed at LEP1 and SLC and must be accurately taken into account in the measure of the Z mass and total width.

For example, a considerable amount of work has deservedly been devoted to the theoretical study of the $Z$ line-shape. The experimental accuracy on $m_Z$ obtained at LEP1 is $\delta m_Z = \pm 2.1$ MeV. This small error was obtained by a precise calibration of the LEP energy scale achieved by taking advantage of the transverse polarization of the beams and implementing a sophisticated resonant spin depolarization method. Similarly, a measurement of the total width to an accuracy $\delta\Gamma = \pm 2.4$ MeV has been achieved. The prediction of the Z line-shape in the SM to such an accuracy has posed a formidable challenge to theory, which has been successfully met. For the inclusive process $e^+e^- \to f\bar{f}X$, with $f \neq e$ (for simplicity, we leave Bhabha scattering aside) and $X$ including $\gamma$'s and gluons, the physical cross-section can be written in the form of a convolution [2]:

$$\sigma(s) = \int_{z_0}^{1} dz\ \hat{\sigma}(zs) G(z,s)\ , \tag{92}$$

where $\hat{\sigma}$ is the reduced cross-section, and $G(z,s)$ is the radiator function that describes the effect of initial-state radiation; $\hat{\sigma}$ includes the purely weak corrections, the effect of final-state radiation (of both $\gamma$'s and gluons), and also non-factorizable terms (initial- and final-state radiation interferences, boxes, etc.) which, being small, can be treated in lowest order and effectively absorbed in a modified $\hat{\sigma}$. The radiator $G(z,s)$ has an expansion of the form

$$G(z,s) = \delta(1-z) + \alpha/\pi(a_{11}L + a_{10}) + (\alpha/\pi)^2(a_{22}L^2 + a_{11}L + a_{20}) + ... + (\alpha/\pi)^n \sum_{i=0}^{n} a_{ni}L^i \ , \qquad (93)$$

where $L = \ln s/m_e^2 \simeq 24.2$ for $\sqrt{s} \simeq m_Z$. All first- and second-order terms are known exactly. The sequence of leading and next-to-leading logs can be exponentiated (closely following the formalism of structure functions in QCD). For $m_Z \approx 91$ GeV, the convolution displaces the peak by +110 MeV, and reduces it by a factor of about 0.74. The exponentiation is important in that it amounts to a shift of about 14 MeV in the peak position.

Among the one loop EW radiative corrections, a very remarkable class of contributions are those terms that increase quadratically with the top mass. The sensitivity of radiative corrections to $m_t$ arises from the existence of these terms. The quadratic dependence on $m_t$ (and on other possible widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD, the running of $\alpha$ and $\alpha_s$ at a scale $Q$ is not affected by heavy quarks with mass $M \gg Q$. According to an intuitive decoupling theorem, diagrams with heavy virtual particles of mass $M$ can be ignored at $Q \ll M$ provided that the couplings do not grow with $M$ and that the theory with no heavy particles is still renormalizable. In the spontaneously broken EW gauge theories both requirements are violated. First, one important difference with respect to unbroken gauge theories is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). Second the theory without the top quark is no more renormalisable because the gauge symmetry is broken because the doublet (t,b) would not be complete (also the chiral anomaly would not be completely cancelled). With the observed value of $m_t$ the quantitative importance of the terms of order $G_F m_t^2/4\pi^2\sqrt{2}$ is substancial but not dominant (they are enhanced by a factor $m_t^2/m_W^2 \sim 5$ with respect to ordinary terms). Both the large logarithms and the $G_F m_t^2$ terms have a simple structure and are to a large extent universal, i.e. common to a wide class of processes. In particular the $G_F m_t^2$ terms appear in vacuum polarization diagrams which are universal and in the $Z \to b\bar{b}$ vertex which is not (this vertex is connected with the top quark which runs in the loop, while other types of heavy particles could in principle also contribute to vacuum polarization diagrams). Their study is important for an understanding of the pattern of radiative corrections. One can also derive approximate formulae (e.g. improved Born approximations), which can be useful in cases where a limited precision may be adequate. More in general, another very important consequence of non decoupling is that precision tests of the electroweak theory may be sensitive to new physics even if the new particles are too heavy for their direct production.

While radiative corrections are quite sensitive to the top mass, they are unfortunately much less dependent on the Higgs mass. If they were sufficiently sensitive by now we would precisely know the mass of the Higgs. But the dependence of one loop diagrams on $m_H$ is only logarithmic: $\sim G_F m_W^2 \log(m_H^2/m_W^2)$. Quadratic terms $\sim G_F^2 m_H^2$ only appear at two loops and are too small to be important. The difference with the top case is that the difference $m_t^2 - m_b^2$ is a direct breaking of the gauge symmetry that already affects the one loop corrections, while the Higgs couplings are "custodial" SU(2) symmetric in lowest order.

The basic tree level relations:

$$\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}}, \qquad g^2 \sin^2\theta_W = e^2 = 4\pi\alpha \tag{94}$$

can be combined into

$$\sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_F m_W^2} \tag{95}$$

A different definition of $\sin^2\theta_W$ is from the gauge boson masses:

$$\frac{m_W^2}{m_Z^2\cos^2\theta_W} = \rho_0 = 1 \quad \Longrightarrow \quad \sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \tag{96}$$

where $\rho_0 = 1$ assuming that there are only Higgs doublets. The last two relations can be put into the convenient form

$$(1 - \frac{m_W^2}{m_Z^2})\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F m_Z^2} \tag{97}$$

These relations are modified by radiative corrections:

$$\begin{aligned} (1 - \frac{m_W^2}{m_Z^2})\frac{m_W^2}{m_Z^2} &= \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2}\frac{1}{1-\Delta r_W} \\ \frac{m_W^2}{m_Z^2\cos^2\theta_W} &= 1 + \rho_m \end{aligned} \tag{98}$$

In the first relation the replacement of $\alpha$ with the running coupling at the Z mass $\alpha(m_Z)$ makes $\Delta r_W$ completely determined by the purely weak corrections. This relation defines $\Delta r_W$ unambigously, once the meaning of $\alpha(m_Z)$

is specified. On the contrary, in the second relation $\Delta\rho_m$ depends on the definition of $\sin^2\theta_W$ beyond the tree level. For LEP physics $\sin^2\theta_W$ is usually defined from the $Z \to \mu^+\mu^-$ effective vertex. At the tree level we have:

$$Z \to f^+f^- = \frac{g}{2\cos\theta_W}\bar{f}\gamma_\mu(g_V^f - g_A^f\gamma_5)f \tag{99}$$

with $g_A^{f2} = 1/4$ and $g_V^f/g_A^f = 1 - 4|Q_f|\sin^2\theta_W$. Beyond the tree level a corrected vertex can be written down in the same form of eq.(99) in terms of modified effective couplings. Then $\sin^2\theta_W \equiv \sin^2\theta_{eff}$ is in general defined through the muon vertex:

$$\begin{aligned} g_V^\mu/g_A^\mu &= 1 - 4\sin^2\theta_{eff} \\ \sin^2\theta_{eff} &= (1+\Delta k)s_0^2, \qquad s_0^2c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \\ g_A^{\mu 2} &= \frac{1}{4}(1+\Delta\rho) \end{aligned} \tag{100}$$

Actually, since in the SM lepton universality is only broken by masses and is in agreement with experiment within the present accuracy, in practice the muon channel is replaced with the average over charged leptons.

We end this discussion by writing a symbolic equation that summarises the status of what has been computed up to now for the radiative corrections $\Delta r_W$, $\Delta\rho$ and $\Delta k$:

$$\Delta r_W, \Delta\rho, \Delta k = g^2\frac{m_t^2}{m_W^2}(1+\alpha_s+\alpha_s^2) + g^2(1+\alpha_s + \sim \alpha_s^2) + g^4\frac{m_t^4}{m_W^4} + g^4\frac{m_t^2}{m_W^2} + ... \tag{101}$$

The meaning of this relation is that the one loop terms of order $g^2$ are completely known, together with their first order QCD corrections (the second order QCD corrections are only estimated for the $g^2$ terms not enhanced by $m_t^2/m_W^2$), and the terms of order $g^4$ enhanced by the ratios $m_t^4/m_W^4$ or $m_t^2/m_W^2$ are also known.

In recent years new powerful tests of the SM have been performed mainly at LEP but also at SLC and at the Tevatron. The running of LEP1 was terminated in 1995 and close-to-final results of the data analysis are now available. The SLC is also finished. The experiments at the Z resonance have enormously improved the accuracy in the electroweak neutral current sector. The top quark has been at last found at the Tevatron and the errors on $m_Z$ and $\sin^2\theta_{eff}$ went down by two and one orders of magnitude respectively since the start of LEP in 1989. The LEP2 programme is almost completed by now. The validity of the SM has been confirmed to a level that we can say was unexpected at the beginning. In the present data there is no significant evidence for departures from the SM, no convincing hint of new physics. The impressive success of the SM poses strong limitations on the possible forms of new physics. Favoured are models of the Higgs sector and of new physics that preserve the SM structure and only very delicately improve it, as is the case for fundamental Higgs(es) and Supersymmetry. Disfavoured are models with a nearby strong non perturbative regime that almost inevitably would affect the radiative corrections, as for composite Higgs(es) or technicolor and its variants.

# 7 Why we do Believe in the SM: Precision Tests

## 7.1 Precision Electroweak Data and the Standard Model

The relevant electro-weak data together with their SM values are presented in table 1 [3]. The SM predictions correspond to a fit of all the available data (including the directly measured values of $m_t$ and $m_W$) in terms of $m_t$, $m_H$ and $\alpha_s(m_Z)$, described later in sect., table 4.

Other important derived quantities are, for example, $N_\nu$ the number of light neutrinos, obtained from the invisible width: $N_\nu = 2.9835(83)$, which is $2\sigma$ below 3 and indicates that only three fermion generations exist with $m_\nu < 45\ GeV$. This is one of the most important results of LEP. Other important quantities are the leptonic width $\Gamma_l$, averaged over e, $\mu$ and $\tau$: $\Gamma_l = 83.959(89) MeV$ and the hadronic width $\Gamma_h = 1743.9(2.0) MeV$.

For indicative purposes, in table the "pulls" are also shown, defined as: pull = (data point- fit value)/(error on data point). At a glance we see that the agreement with the SM is quite good. The distribution of the pulls is statistically normal. The presence of a few $\sim 2\sigma$ deviations is what is to be expected. For example, the atomic parity violation in Cs, a low energy experiment, shows a $2.5\sigma$ deviation. While there could be new physics terms that only sizeably contribute to this channel (a specific contact term, a Z' unmixed with the Z), the apparent deviation may simply be due to the difficulty of the measurement and the complicacies of the Cesium wavefunction[2]. One unpleasant feature of the data is the difference between the values of $\sin^2\theta_{eff}$ measured at LEP and at SLC. The value of $\sin^2\theta_{eff}$ is obtained from a set of combined asymmetries. From asymmetries one derives the ratio $x = g^l_V/g^l_A$ of the vector and axial vector couplings of the Z, averaged over the charged leptons. In turn $\sin^2\theta_{eff}$ is defined by $x = 1 - 4\sin^2\theta_{eff}$. SLD obtains x from the single measurement of $A_{LR}$, the left-right asymmetry, which requires longitudinally polarized beams. The LEP average, $\sin^2\theta_{eff} = 0.23192(23)$, differs by $2.2\sigma$ from the SLD value $\sin^2\theta_{eff} = 0.23099(26)$. The most precise individual measurement at LEP is from $A_b^{FB}$: the combined LEP error on this quantity is comparable to the SLD error, but the two values are $2.7\sigma$'s away. It is difficult to find a simple explanation for the SLD-LEP discrepancy on $\sin^2\theta_{eff}$. In the following we will tentatively use the official average

$$\sin^2\theta_{eff} = 0.23151 \pm 0.00017 \qquad (102)$$

obtained by a simple combination of the LEP-SLC data. However, one could be more conservative and enlarge the error because of the larger dispersion.

For the analysis of electroweak data in the SM one starts from the input parameters: some of them, $\alpha$, $G_F$ and $m_Z$, are very well measured, some other ones, $m_{f_{light}}$, $m_t$ and $\alpha_s(m_Z)$ are only approximately determined while $m_H$ is largely unknown. With respect to $m_t$ the situation has much improved since the CDF/D0 direct measurement of the top quark mass. From the input parameters one computes the radiative corrections to a sufficient precision to match the experimental capabilities. Then one compares the theoretical predictions and the data for the numerous observables which have been measured, checks the consistency of the theory and derives constraints on $m_t$, $\alpha_s(m_Z)$ and hopefully also on $m_H$.

Some comments on the least known of the input parameters are now in order.The only practically relevant terms where precise values of the light quark masses, $m_{f_{light}}$, are needed are those related to the hadronic contribution to the photon vacuum polarization diagrams, that detemine $\alpha(m_Z)$. This correction is of order 6%, much larger than the accuracy of a few per mille of the precision tests. Fortunately, one can use the actual data to in principle solve the related ambiguity. But we shall see that the left over uncertainty is still one of the main sources of theoretical error. As is well

[2] In a very recent paper [4] new terms from the Breit interaction in the atomic-structure calculation are shown to bring the discrepancy down to the $1\sigma$ level. So this problem is probably resolved.

Table 1: Data on precision electroweak test

| Quantity | Data (March 2000) | Pull |
|---|---|---|
| $m_Z$ (GeV) | 91.1871(21) | 0.1 |
| $\Gamma_Z$ (GeV) | 2.4944(24) | $-0.6$ |
| $\sigma_h$ (nb) | 41.544(37) | 1.7 |
| $R_h$ | 20.768(24) | 1.2 |
| $R_b$ | 0.21642(73) | 0.85 |
| $R_c$ | 0.1674(38) | $-1.3$ |
| $A_{FB}^l$ | 0.01701(95) | 0.8 |
| $A_\tau$ | 0.1425(44) | $-1.2$ |
| $A_e$ | 0.1483(51) | 0.1 |
| $A_{FB}^b$ | 0.0988(20) | $-2.3$ |
| $A_{FB}^c$ | 0.0692(37) | $-1.3$ |
| $A_b$ (SLD direct) | 0.911(25) | $-1.0$ |
| $A_c$ (SLD direct) | 0.630(26) | $-1.5$ |
| $\sin^2\theta_{eff}$(LEP-combined) | 0.23192(23) | 2.1 |
| $A_{LR} \to \sin^2\theta_{eff}$ | 0.23099(26) | $-1.9$ |
| $m_W$ (GeV) (LEP2+p$\bar{p}$) | 80.419(38) | 0.1 |
| $1 - \frac{m_W^2}{m_Z^2}$ ($\nu$N) | 0.2255(21) | 1.2 |
| $Q_W$ (Atomic PV in Cs) | -72.06(44) | 2.5 |
| $m_t$ (GeV) | 174.3(5.1) | 0.1 |

known [2], the QED running coupling is given by:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \tag{103}$$

$$\Delta\alpha(s) = \Pi(s) = \Pi_\gamma(0) - \mathrm{Re}\Pi_\gamma(s) \tag{104}$$

where $\Pi(s)$ is proportional to the sum of all 1-particle irreducible vacuum polarization diagrams. In perturbation theory $\Delta\alpha(s)$ is given by:

$$\Delta\alpha(s) = \frac{\alpha}{3\pi}\sum_f Q_f^2 N_{Cf}\left(\log\frac{2}{m_f^2} - \frac{5}{3}\right) \tag{105}$$

where $N_{Cf} = 3$ for quarks and 1 for leptons. However, the perturbative formula is only reliable for leptons, not for quarks (because of the unknown values of the effective quark masses). Separating the leptonic, the light quark and the top quark contributions to $\Delta\alpha(s)$ we have:

$$\Delta\alpha(s) = \Delta\alpha(s)_1 + \Delta\alpha(s)_h + \Delta\alpha(s)_t \tag{106}$$

with:

$$\Delta\alpha(s)_1 = 0.0331421\ ; \quad \Delta\alpha(s)_t = \frac{\alpha}{3\pi}\,\frac{4}{15}\,\frac{m_Z^2}{m_t^2} = -0.000061 \tag{107}$$

Note that in QED there is decoupling so that the top quark contribution approaches zero in the large $m_t$ limit. For $\Delta\alpha(s)_h$ one can use eq.(104) and the Cauchy theorem to obtain the representation:

$$\Delta\alpha(m_Z^2)_h = -\frac{\alpha m_Z^2}{3\pi}\mathrm{Re}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}\,\frac{R(s)}{s - m_Z^2 - i\epsilon} \tag{108}$$

where $R(s)$ is the familiar ratio of the hadronic to the pointlike $\ell^+\ell^-$ cross-section from photon exchange in $e^+e^-$ annihilation. At $s$ large one can use the perturbative expansion for $R(s)$ while at small $s$ one can use the actual data. In recent years there has been a lot of activity on this subject [3]. A conservative value, directly obtained from the data, is given by

$$\alpha(m_Z)^{-1} = 128.90 \pm 0.09 \qquad [\Delta\alpha(m_Z^2)_h = 0.02804 \pm 0.00064] \tag{109}$$

As I said, for the derivation of this result the QCD theoretical prediction is actually used for large values of s where the data do not exist. But the sensitivity of the dispersive integral to this region is strongly suppressed, so that no important model dependence is introduced. More recently some analyses have appeared where one studied by how much the error on $\alpha_s(m_Z)$

is reduced by using the QCD prediction down to $\sqrt{s} = m_\tau$, with the possible exception of the regions around the charm and beauty thresholds. These attempts were motivated by the apparent success of QCD predictions in $\tau$ decays, despite the low $\tau$ mass (note however that the relevant currents are V-A in $\tau$ decay but V in the present case). One finds that the central value is not much changed while the error in eq.(109) is reduced but, of course, at the price of more model dependence. For example, one quoted value is:

$$\alpha(m_Z)^{-1} = 128.913 \pm 0.035 \qquad [\Delta\alpha(m_Z^2)_h = 0.027782 \pm 0.000254] \tag{110}$$

The data from BES and Daphne are expected to somewhat improve the accuracy.

As for the strong coupling $\alpha_s(m_Z)$ the world average central value is by now quite stable. The error is going down because the dispersion among the different measurements is much smaller in the most recent set of data. The error on the final average is taken by all authors between $\pm 0.003$ and $\pm 0.005$ depending on how conservative one is. In the following our reference value will be

$$\alpha_s(m_Z) = 0.119 \pm 0.003 \tag{111}$$

Finally a few words on the current status of the direct measurement of $m_t$. The present combined CDF/D0 result is

$$m_t = 174.3 \pm 5.1 \ GeV \tag{112}$$

The error is so small by now that one is approaching a level where a more careful investigation of the effects of colour rearrangement on the determination of $m_t$ will be needed. One wants to determine the top quark mass, defined as the invariant mass of its decay products (i.e. b+W+ gluons + $\gamma$'s). However, due to the need of colour rearrangement, the top quark and its decay products cannot be really isolated from the rest of the event. Some smearing of the mass distribution is induced by this colour crosstalk which involves the decay products of the top, those of the antitop and also the fragments of the incoming (anti)protons. A reliable quantitative computation of the smearing effect on the $m_t$ determination is difficult because of the importance of non perturbative effects. An induced error of the order of 1 GeV on $m_t$ could reasonably be expected. So this problem is still not urgent.

In order to appreciate the relative importance of the different sources of theoretical error for precision tests of the SM, we report in table 2 a comparison for the most relevant observables. What is important to stress is that the ambiguity from $m_t$, once by far the largest one, is by now smaller

Table 2: Errors from different sources: $\Delta^{exp}_{now}$ is the present experimental error; $\Delta\alpha^{-1}$ is the impact of $\Delta\alpha^{-1} = \pm 0.09$; $\Delta_{th}$ is the estimated theoretical error from higher orders; $\Delta m_t$ is from $\Delta m_t = \pm 6$GeV; $\Delta m_H$ is from $\Delta m_H =$ 60–1000 GeV; $\Delta\alpha_s$ corresponds to $\Delta\alpha_s = \pm 0.003$. The epsilon parameters are defined in sect.7.2.

| Parameter | $\Delta^{exp}_{now}$ | $\Delta\alpha^{-1}$ | $\Delta_{th}$ | $\Delta m_t$ | $\Delta m_H$ | $\Delta\alpha_s$ |
|---|---|---|---|---|---|---|
| $\Gamma_Z$ (MeV) | $\pm 2.4$ | $\pm 0.7$ | $\pm 0.8$ | $\pm 1.4$ | $\pm 4.6$ | $\pm 1.7$ |
| $\sigma_h$ (pb) | 37 | 1 | 4.3 | 3.3 | 4 | 17 |
| $R_h \cdot 10^3$ | 24 | 4.3 | 5 | 2 | 13.5 | 20 |
| $\Gamma_l$ (keV) | 89 | 11 | 15 | 55 | 120 | 3.5 |
| $A^l_{FB} \cdot 10^4$ | 9.5 | 4.2 | 1.3 | 3.3 | 13 | 0.18 |
| $\sin^2\theta \cdot 10^4$ | 1.7 | 2.3 | 0.8 | 1.9 | 7.5 | 0.1 |
| $m_W$ (MeV) | 38 | 12 | 9 | 37 | 100 | 2.2 |
| $R_b \cdot 10^4$ | 7.3 | 0.1 | 1 | 2.1 | 0.25 | 0 |
| $\epsilon_1 \cdot 10^3$ | 1.1 | | $\sim$0.1 | | | 0.2 |
| $\epsilon_3 \cdot 10^3$ | 1.0 | 0.5 | $\sim$0.1 | | | 0.12 |
| $\epsilon_b \cdot 10^3$ | 1.8 | | $\sim$0.1 | | | 1 |

than the error from $m_H$. We also see from table 2 that the error from $\Delta\alpha(m_Z)$ is expecially important for $\sin^2\theta_{eff}$ and, to a lesser extent, is also sizeable for $\Gamma_Z$ and $\epsilon_3$.

An important recent advance in the theory of radiative corrections is the calculation of the $o(g^4 m_t^2/m_W^2)$ terms in $\sin^2\theta_{eff}$, $m_W$ and, more recently in $\delta\rho$ [3]. The result implies a small but visible correction to the predicted values but expecially a seizable decrease of the ambiguity from scheme dependence (a typical effect of truncation). These calculations are now implemented in the fitting codes used in the analysis of LEP data. The fitted value of the Higgs mass is lowered by about 30 $GeV$ due to this effect.

We now discuss fitting the data in the SM. As the mass of the top quark is now rather precisely known from CDF and D0 one must distinguish two different types of fits. In one type one wants to answer the question: is $m_t$ from radiative corrections in agreement with the direct measurement at the Tevatron? Similarly how does $m_W$ inferred from radiative corrections compare with the direct measurements at the Tevatron and LEP2? For answering these interesting but somewhat limited questions, one must clearly exclude the measurements of $m_t$ and $m_W$ from the input set of data. Fitting all other data in terms of $m_t$, $m_H$ and $\alpha_s(m_Z)$ one finds the results shown in the second column of table 3 [3]. The extracted value of $m_t$ is in good

Table 3: Standard Model fits of electroweak data.

| Parameter | LEP(incl.$m_W$) | All but $m_W$, $m_t$ | All Data |
|---|---|---|---|
| $m_t$ (GeV) | 172+14 − 11 | 167+11 − 8 | $173.2 \pm 4.5$ |
| $m_H$ (GeV) | 134+268 − 81 | 55+84 − 27 | 77+69 − 39 |
| $log[m_H(GeV)]$ | 2.13+0.48 − 0.40 | 1.74+0.40 − 0.30 | 1.88+0.28 − 0.30 |
| $\alpha_s(m_Z)$ | $0.120 \pm 0.003$ | $0.118 \pm 0.003$ | $0.118 \pm 0.003$ |
| $\chi^2/dof$ | 11/9 | 21/12 | 23/15 |

agreement with the direct measurement.In fact, as shown in the table 3, from all the electroweak data except the direct production results on $m_t$ and $m_W$, one finds $m_t = 167 \pm_8^{11}\ GeV$. There is a strong correlation between $m_t$ and $m_H$. In a more general type of fit, e.g. for determining the overall consistency of the SM or to evaluate the best present estimate for some quantity, say $m_W$, one should of course not ignore the existing direct determinations of $m_t$ and $m_W$. Then, from all the available data, by fitting $m_t$, $m_H$ and $\alpha_s(m_Z)$ one finds the values shown in the last column of table 3.

This is the fit also referred to in table 1. The corresponding fitted values of $\sin^2\theta_{eff}$ and $m_W$ are:

$$\sin^2\theta_{eff} = 0.23150 \pm 0.00016; \qquad m_W = 80.385 \pm 0.022\ GeV \tag{113}$$

The fitted value of $\sin^2\theta_{eff}$ is practically identical to the LEP+SLD average. The error of 22 MeV on $m_W$ clearly sets up a goal for the direct measurement of $m_W$ at LEP2, the Tevatron and the LHC.

The main lesson of the precision tests of the standard electroweak theory can be summarised as follows. It has been checked that the couplings of quark and leptons to the weak gauge bosons $W^\pm$ and $Z$ are indeed precisely those prescribed by the gauge symmetry. The accuracy of a few 0.1% for these tests implies that, not only the tree level, but also the structure of quantum corrections has been verified. To a lesser accuracy the triple gauge vertices $\gamma W^+W^-$ and $ZW^+W^-$ have also been found in agreement with the specific prediction, at the tree level, of the $SU(2)\bigotimes U(1)$ gauge theory. This means that it has been verified that the gauge symmetry is indeed unbroken in the vertices of the theory: the currents are indeed conserved. Yet there is obvious evidence that the symmetry is otherwise badly broken in the masses. In fact the $SU(2)\bigotimes U(1)$ gauge symmetry forbids masses for all the particles that have been sofar observed: quarks, leptons and gauge bosons. But of all these particles only the photon and the gluons are massless ( protected by the

$SU(3) \bigotimes U(1)_Q$ unbroken colour-electric charge gauge symmetry), all other are massive (probably also the neutrinos). Thus the currents are conserved but the spectrum of particle states is not symmetric. This is the definition of spontaneous symmetry breaking. The practical implementation of spontaneous symmetry breaking in a gauge theory is via the Higgs mechanism. In the minimal SM one single fundamental scalar Higgs isospin doublet is introduced and its vacuum expectation value v breaks the symmetry. All masses are proportional to v, although the Yukawa couplings that multiply v in the expression for the masses of quarks and leptons are distributed over a wide range. The Higgs sector is still very much untested. The Higgs particle has not been found but its mass can well be heavier than the present direct lower limit $m_H \gtrsim 113\ GeV$ from LEP2 [3]. One knew from the beginning that the Higgs search would be difficult: being coupled in proportion to masses one has first to produce heavy particles and then try to detect the Higgs (itself heavy) in their couplings. What has been tested is the relation $m_W^2 = m_Z^2 \cos^2\theta_W$, modified by computable radiative corrections. This relation means that the effective Higgs (be it fundamental or composite) is indeed a weak isospin doublet.

We have seen that quantum corrections depend only logaritmically on $m_H$. In spite of this small sensitivity, the data are precise enough that one obtains a quantitative indication of the mass range: [3] $\log_{10} m_H(GeV) = 1.88^{+0.28}_{-0.30}$ (or $m_H = 77^{+69}_{-39}$). This result on the Higgs mass is particularly remarkable. The value of $\log_{10} m_H(GeV)$ is right on top of the small window between $\sim 2$ and $\sim 3$ which is allowed by the direct limit, on the one side, and the theoretical upper limit on the Higgs mass in the minimal SM (see later), $m_H \lesssim 600 - 800\ GeV$, on the other side. If one had found a central value like $\gtrsim 4$ the model would have been discarded. Thus the whole picture of a perturbative theory with a fundamental Higgs is well supported by the data on radiative corrections. It is important that there is a clear indication for a particularly light Higgs. This is quite encouraging for the ongoing search for the Higgs particle. More in general, if the Higgs couplings are removed from the lagrangian the resulting theory is non renormalisable. A cutoff $\Lambda$ must be introduced. In the quantum corrections $\log m_H$ is then replaced by $\log \Lambda$ plus a constant. The precise determination of the associated finite terms would be lost (that is, the value of the mass in the denominator in the argument of the logarithm). Thus the fact that, from experiment, one finds $\log m_H \sim 2$ is a strong argument in favour of the specific form of the Higgs mechanism as in the SM. A heavy Higgs would need some unfortunate conspiracy: the

[3] this combined limit was presented by the LEP collaborations at the 2000 summer conferences

finite terms should accidentally compensate for the heavy Higgs in the few key parameters of the radiative corrections (mainly $\epsilon_1$ and $\epsilon_3$). Or additional new physics, for example in the form of effective contact terms added to the minimal SM lagrangian, should accidentally do the compensation, which again needs some sort of conspiracy.

## 7.2 A More General Analysis of Electroweak Data

We now discuss an update of the epsilon analysis [5] which is a method to look at the data in a more general context than the SM. This is important to put constraints on extensions of the SM. The starting point is to isolate from the data that part which is due to the purely weak radiative corrections. In fact the epsilon variables are defined in such a way that they are zero in the approximation when only effects from the SM at the tree level plus pure QED and pure QCD corrections are taken into account. This very simple version of improved Born approximation is a good first approximation according to the data and is independent of $m_t$ and $m_H$. In fact the whole $m_t$ and $m_H$ dependence arises from weak loop corrections and therefore is only contained in the epsilon variables. Thus the epsilons are extracted from the data without need of specifying $m_t$ and $m_H$. But their predicted value in the SM or in any extension of it depend on $m_t$ and $m_H$. This is to be compared with the competitor method based on the S, T, U variables. The latter cannot be obtained from the data without specifying $m_t$ and $m_H$ because they are defined as deviations from the complete SM prediction for specified $m_t$ and $m_H$. Of course there are very many variables that vanish if pure weak loop corrections are neglected, at least one for each relevant observable. Thus for a useful definition we choose a set of representative observables that are used to parametrize those hot spots of the radiative corrections where new physics effects are most likely to show up. These sensitive weak correction terms include vacuum polarization diagrams which being potentially quadratically divergent are likely to contain possible non decoupling effects (like the quadratic top quark mass dependence in the SM). There are three independent vacuum polarization contributions. In the same spirit, one must add the $Z \to b\bar{b}$ vertex which also includes a large top mass dependence. Thus altogether we consider four defining observables: one asymmetry, for example $A^l_{FB}$, (as representative of the set of measurements that lead to the determination of $\sin^2\theta_{eff}$), one width (the leptonic width $\Gamma_l$ is particularly suitable because it is practically independent of $\alpha_s$), $m_W$ and $R_b$. Here lepton universality has been taken for granted, because the data show that it is verified within the present accuracy. The four variables, $\epsilon_1$, $\epsilon_2$, $\epsilon_3$ and $\epsilon_b$ are defined in correspondence with the set of observables $A_l^{FB}$, $\Gamma_l$, $m_W$, and $R_b$. The definition is so chosen that the quadratic top mass dependence is only present in $\epsilon_1$ and $\epsilon_b$, while the $m_t$ dependence of $\epsilon_2$ and $\epsilon_3$ is logarithmic. The definition of $\epsilon_1$ and $\epsilon_3$ is specified in terms of $A_l^{FB}$ and $\Gamma_l$ only. Then adding $m_W$ or $R_b$ one obtains $\epsilon_2$ or $\epsilon_b$. We now specify the relevant definitions in detail.

We start from the basic observables $m_W/m_Z$, $\Gamma_l$ and $A_l^{FB}$ and $\Gamma_b$. From these four quantities one can isolate the corresponding dynamically significant corrections $\Delta r_W$, $\Delta\rho$, $\Delta k$ and $\epsilon_b$, which contain the small effects one is trying to disentangle and are defined in the following. First we introduce $\Delta r_W$ as obtained from $m_W/m_Z$ by the relation:

$$(1-\frac{m_W^2}{m_Z^2})\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2(1-\Delta r_W)} \tag{114}$$

Here $\alpha(m_Z) = \alpha/(1-\Delta\alpha)$ is fixed to the central value $1/128.90$ so that the effect of the running of $\alpha$ due to known physics is extracted from $1-\Delta r = (1-\Delta\alpha)(1-\Delta r_W)$. In fact, the error on $1/\alpha(m_Z)$, as given in eq.(109) would then affect $\Delta r_W$. In order to define $\Delta\rho$ and $\Delta k$ we consider the effective vector and axial-vector couplings $g_V$ and $g_A$ of the on-shell Z to charged leptons, given by the formulae:

$$\begin{aligned}\Gamma_l &= \frac{G_F m_Z^3}{6\pi\sqrt{2}}(g_V^2+g_A^2)(1+\frac{3\alpha}{4\pi}),\\ A_l^{FB}(\sqrt{s} = m_Z) &= \frac{3g_V^2 g_A^2}{(g_V^2+g_A^2)^2} = \frac{3x^2}{(1+x^2)^2}.\end{aligned} \tag{115}$$

Note that $\Gamma_l$ stands for the inclusive partial width $\Gamma(Z\to l\bar{l}+\text{photons})$. We stress the following points. First, we have extracted from $(g_V^2+g_A^2)$ the factor $(1+3\alpha/4\pi)$ which is induced in $\Gamma_l$ from final state radiation. Second, by the asymmetry at the peak in eq.(115) we mean the quantity which is commonly referred to by the LEP experiments (denoted as $A_{FB}^0$ in ref.[3]), which is corrected for all QED effects, including initial and final state radiation and also for the effect of the imaginary part of the $\gamma$ vacuum polarization diagram. In terms of $g_A$ and $x = g_V/g_A$, the quantities $\Delta\rho$ and $\Delta k$ are given by:

$$\begin{aligned}g_A &= -\frac{\sqrt{\rho}}{2} \sim -\frac{1}{2}(1+\frac{\Delta\rho}{2}),\\ x = \frac{g_V}{g_A} &= 1-4\sin^2\theta_{eff} = 1-4(1+\Delta k)s_0^2.\end{aligned} \tag{116}$$

Here $s_0^2$ is $\sin^2\theta_{eff}$ before non pure-QED corrections, given by:

$$s_0^2 c_0^2 \;=\; \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \tag{117}$$

with $c_0^2 \;=\; 1 - s_0^2$ ($s_0^2 = 0.231095$ for $m_Z \;=\; 91.188\ GeV$).

We now define $\epsilon_b$ from $\Gamma_b$, the inclusive partial width for $Z \to b\bar{b}$ according to the relation

$$\Gamma_b \;=\; \frac{G_F m_Z^3}{6\pi\sqrt{2}}\beta(\frac{3-\beta^2}{2}g_{bV}^2 \;+\; \beta^2 g_{bA}^2)N_C R_{QCD}(1+\frac{\alpha}{12\pi}) \tag{118}$$

where $N_C = 3$ is the number of colours, $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$, with $m_b = 4.7$ GeV, $R_{QCD}$ is the QCD correction factor given by

$$R_{QCD} \;=\; 1 \;+\; 1.2a \;-\; 1.1a^2 \;-\; 13a^3\;; \quad a \;=\; \frac{\alpha_s(m_Z)}{\pi} \tag{119}$$

and $g_{bV}$ and $g_{bA}$ are specified as follows

$$\begin{aligned} g_{bA} \;&=\; -\frac{1}{2}(1+\frac{\Delta\rho}{2})(1+\epsilon_b), \\ \frac{g_{bV}}{g_{bA}} \;&=\; \frac{1 - 4/3\sin^2\theta_{eff} + \epsilon_b}{1+\epsilon_b}. \end{aligned} \tag{120}$$

This is clearly not the most general deviation from the SM in the $Z \to b\bar{b}$ but $\epsilon_b$ is closely related to the quantity $-Re(\delta_{b-vertex})$ where the large $m_t$ corrections are located in the SM.

As is well known, in the SM the quantities $\Delta r_W$, $\Delta\rho$, $\Delta k$ and $\epsilon_b$, for sufficiently large $m_t$, are all dominated by quadratic terms in $m_t$ of order $G_F m_t^2$. As new physics can more easily be disentangled if not masked by large conventional $m_t$ effects, it is convenient to keep $\Delta\rho$ and $\epsilon_b$ while trading $\Delta r_W$ and $\Delta k$ for two quantities with no contributions of order $G_F m_t^2$. We thus introduce the following linear combinations:

$$\begin{aligned} \epsilon_1 \;&=\; \Delta\rho, \\ \epsilon_2 \;&=\; c_0^2\Delta\rho \;+\; \frac{s_0^2\Delta r_W}{c_0^2 - s_0^2} \;-\; 2s_0^2\Delta k, \\ \epsilon_3 \;&=\; c_0^2\Delta\rho \;+\; (c_0^2 - s_0^2)\Delta k. \end{aligned} \tag{121}$$

The quantities $\epsilon_2$ and $\epsilon_3$ no longer contain terms of order $G_F m_t^2$ but only logarithmic terms in $m_t$. The leading terms for large Higgs mass, which are logarithmic, are contained in $\epsilon_1$ and $\epsilon_3$. In the Standard Model one has the following "large" asymptotic contributions:

$$
\begin{aligned}
\epsilon_1 &= \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} - \frac{3G_F m_W^2}{4\pi^2\sqrt{2}}\tan^2\theta_W \ln\frac{m_H}{m_Z} + ...., \\
\epsilon_2 &= -\frac{G_F m_W^2}{2\pi^2\sqrt{2}}\ln\frac{m_t}{m_Z} + ...., \\
\epsilon_3 &= \frac{G_F m_W^2}{12\pi^2\sqrt{2}}\ln\frac{m_H}{m_Z} - \frac{G_F m_W^2}{6\pi^2\sqrt{2}}\ln\frac{m_t}{m_Z} ...., \\
\epsilon_b &= -\frac{G_F m_t^2}{4\pi^2\sqrt{2}} + ....
\end{aligned} \tag{122}
$$

The relations between the basic observables and the epsilons can be linearised, leading to the approximate formulae

$$
\begin{aligned}
\frac{m_W^2}{m_Z^2} &= \frac{m_W^2}{m_Z^2}|_B(1 + 1.43\epsilon_1 - 1.00\epsilon_2 - 0.86\epsilon_3), \\
\Gamma_l &= \Gamma_l|_B(1 + 1.20\epsilon_1 - 0.26\epsilon_3), \\
A_l^{FB} &= A_l^{FB}|_B(1 + 34.72\epsilon_1 - 45.15\epsilon_3), \\
\Gamma_b &= \Gamma_b|_B(1 + 1.42\epsilon_1 - 0.54\epsilon_3 + 2.29\epsilon_b).
\end{aligned} \tag{123}
$$

The Born approximations, as defined above, depend on $\alpha_s(m_Z)$ and also on $\alpha(m_Z)$. Defining

$$
\delta\alpha_s = \frac{\alpha_s(m_Z) - 0.119}{\pi}; \quad \delta\alpha = \frac{\alpha(m_Z) - \frac{1}{128.90}}{\alpha}, \tag{124}
$$

we have

$$
\begin{aligned}
\frac{m_W^2}{m_Z^2}|_B &= 0.768905(1 - 0.40\delta\alpha), \\
\Gamma_l|_B &= 83.563(1 - 0.19\delta\alpha)\mathrm{MeV}, \\
A_l^{FB}|_B &= 0.01696(1 - 34\delta\alpha), \\
\Gamma_b|_B &= 379.8(1 + 1.0\delta\alpha_s - 0.42\delta\alpha).
\end{aligned} \tag{125}
$$

Note that the dependence on $\delta\alpha_s$ for $\Gamma_b|_B$, shown in eq.(125), is not simply the one loop result for $m_b = 0$ but a combined effective shift which takes into account both finite mass effects and the contribution of the known higher order terms.

The important property of the epsilons is that, in the Standard Model, for all observables at the Z pole, the whole dependence on $m_t$ (and $m_H$) arising

Table 4: Values of the epsilons in the SM as functions of $m_t$ and $m_H$ as obtained from recent versions of ZFITTER and TOPAZ0. These values (in $10^{-3}$ units) are obtained for $\alpha_s(m_Z) = 0.119$, $\alpha(m_Z) = 1/128.90$, but the theoretical predictions are essentially independent of $\alpha_s(m_Z)$ and $\alpha(m_Z)$

| $m_t$ (GeV) | $\epsilon_1$ $m_H$ (GeV) = | | | $\epsilon_2$ $m_H$ (GeV) = | | | $\epsilon_3$ $m_H$ (GeV) = | | | $\epsilon_b$ All $m_H$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | 70 | 300 | 1000 | 70 | 300 | 1000 | 70 | 300 | 1000 | |
| 150 | 3.55 | 2.86 | 1.72 | −6.85 | −6.46 | −5.95 | 4.98 | 6.22 | 6.81 | −4.50 |
| 160 | 4.37 | 3.66 | 2.50 | −7.12 | −6.72 | −6.20 | 4.96 | 6.18 | 6.75 | −5.31 |
| 170 | 5.26 | 4.52 | 3.32 | −7.43 | −7.01 | −6.49 | 4.94 | 6.14 | 6.69 | −6.17 |
| 180 | 6.19 | 5.42 | 4.18 | −7.77 | −7.35 | −6.82 | 4.91 | 6.09 | 6.61 | −7.08 |
| 190 | 7.18 | 6.35 | 5.09 | −8.15 | −7.75 | −7.20 | 4.89 | 6.03 | 6.52 | −8.03 |
| 200 | 8.22 | 7.34 | 6.04 | −8.59 | −8.18 | −7.63 | 4.87 | 5.97 | 6.43 | −9.01 |

from one-loop diagrams only enters through the epsilons. The same is actually true, at the relevant level of precision, for all higher order $m_t$-dependent corrections. Actually, the only residual $m_t$ dependence of the various observables not included in the epsilons is in the terms of order $\alpha_s^2(m_Z)$ in the pure QCD correction factors to the hadronic widths. But this one is quantitatively irrelevant, especially in view of the errors connected to the uncertainty on the value of $\alpha_s(m_Z)$. The theoretical values of the epsilons in the SM from state of the art radiative corrections are given in table 4. It is important to remark that the theoretical values of the epsilons in the SM, as given in table 4, are not affected, at the percent level or so, by reasonable variations of $\alpha_s(m_Z)$ and/or $\alpha(m_Z)$ around their central values. By our definitions, in fact, no terms of order $\alpha_s^n(m_Z)$ or $\alpha \ln m_Z/m$ contribute to the epsilons. In terms of the epsilons, the following expressions hold, within the SM, for the various precision observables

$$\begin{aligned}
\Gamma_T &= \Gamma_{T0}(1 + 1.35\epsilon_1 - 0.46\epsilon_3 + 0.35\epsilon_b), \\
R &= R_0(1 + 0.28\epsilon_1 - 0.36\epsilon_3 + 0.50\epsilon_b), \\
\sigma_h &= \sigma_{h0}(1 - 0.03\epsilon_1 + 0.04\epsilon_3 - 0.20\epsilon_b), \\
x &= x_0(1 + 17.6\epsilon_1 - 22.9\epsilon_3), \\
R_b &= R_{b0}(1 - 0.06\epsilon_1 + 0.07\epsilon_3 + 1.79\epsilon_b). \qquad (126)
\end{aligned}$$

where x=$g_V/g_A$ as obtained from $A_l^{FB}$ . The quantities in eqs.(123),(126) are clearly not independent and the redundant information is reported for convenience. By comparison with the computed radiative corrections we

Table 5: Experimental values of the epsilons in the SM from different sets of data. These values (in $10^{-3}$ units) are obtained for $\alpha_s(m_Z) = 0.119 \pm 0.003$, $\alpha(m_Z)^{-1} = 128.913 \pm 0.035$, the corresponding uncertainties being included in the quoted errors

| $\epsilon\ \ 10^3$ | Only def. quantities | All asymmetries | All High Energy | All Data |
|---|---|---|---|---|
| $\epsilon_1\ 10^3$ | $4.1 \pm 1.2$ | $4.3 \pm 1.2$ | $3.9 \pm 1.1$ | $3.2 \pm 1.1$ |
| $\epsilon_2\ 10^3$ | $-8.35 \pm 1.6$ | $-9.0 \pm 1.4$ | $-9.3 \pm 1.5$ | $-9.7 \pm 1.5$ |
| $\epsilon_3\ 10^3$ | $3.4 \pm 1.8$ | $4.5 \pm 1.1$ | $4.2 \pm 1.0$ | $3.5 \pm 1.0$ |
| $\epsilon_b\ 10^3$ | $-3.7 \pm 1.9$ | $-3.8 \pm 1.9$ | $-4.4 \pm 1.8$ | $-4.0 \pm 1.8$ |

obtain

$$\begin{aligned}
\Gamma_{T0} &= 2489.46(1 + 0.73\delta\alpha_s - 0.35\delta\alpha)\ MeV, \\
R_0 &= 20.8228(1 + 1.05\delta\alpha_s - 0.28\delta\alpha), \\
\sigma_{h0} &= 41.420(1 - 0.41\delta\alpha_s + 0.03\delta\alpha)\ nb, \\
x_0 &= 0.075619 - 1.32\delta\alpha, \\
R_{b0} &= 0.2182355.
\end{aligned} \tag{127}$$

Note that the quantities in eqs.(127) should not be confused, at least in principle, with the corresponding Born approximations, due to small "non universal" electroweak corrections. In practice, at the relevant level of approximation, the difference between the two corresponding quantities is in any case significantly smaller than the present experimental error.

In principle, any four observables could have been picked up as defining variables. In practice we choose those that have a more clear physical significance and are more effective in the determination of the epsilons. In fact, since $\Gamma_b$ is actually measured by $R_b$ (which is nearly insensitive to $\alpha_s$), it is preferable to use directly $R_b$ itself as defining variable, as we shall do hereafter. In practice, since the value in eq.(127) is practically indistinguishable from the Born approximation of $R_b$, this determines no change in any of the equations given above but simply requires the corresponding replacement among the defining relations of the epsilons.

The values of the epsilons as obtained from the defining variables $m_W$, $\Gamma_l$, $A_l^{FB}$ and $R_b$ are shown in the first column of table 5.

To proceed further and include other measured observables in the analy-

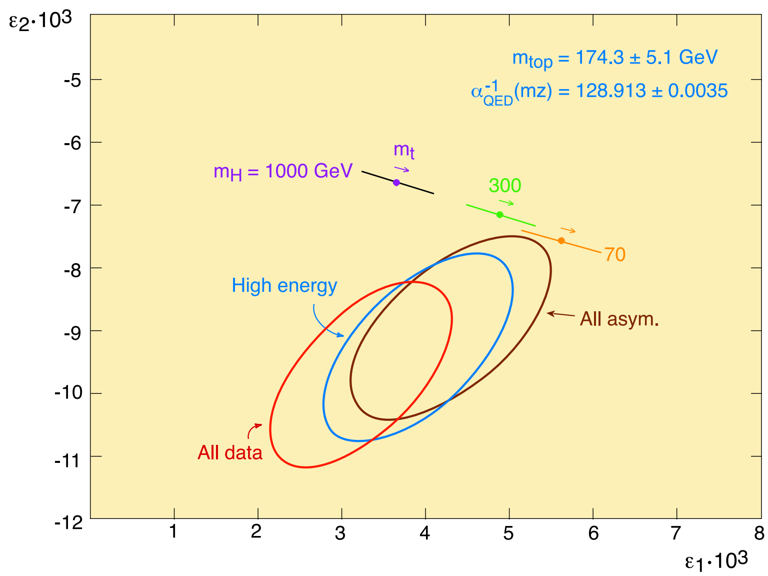


Figure 4: Data vs theory in the $\epsilon_2$-$\epsilon_1$ plane. The origin point corresponds to the "Born" approximation obtained from the SM at tree level plus pure QED and pure QCD corrections. The predictions of the full SM are shown for $m_H = 70$, 300 and 1000 GeV and $m_t = 174.3 \pm 5.5\ GeV$ (a segment for each $m_H$ with the arrow showing the direction of $m_t$ increasing from $-1\sigma$ to $+1\sigma$). The three $1-\sigma$ ellipses (38% probability contours) are obtained from a) "All Asymm." :$\Gamma_l$, $m_W$ and $\sin^2\theta_{eff}$ as obtained from the combined asymmetries (the value in eq. (102)); b) "All High En.": the same as in a) plus all the hadronic variables at the Z; c) "All Data": the same as in b) plus the low energy data

sis we need to make some dynamical assumptions. The minimum amount of model dependence is introduced by including other purely leptonic quantities at the Z pole such as $A_\tau$, $A_e$ (measured from the angular dependence of the $\tau$ polarization) and $A_{LR}$ (measured by SLD). For this step, one is simply assuming that the different leptonic asymmetries are equivalent measurements of $\sin^2\theta_{eff}$. We add, as usual, the measure of $A_b^{FB}$ because this observable is dominantly sensitive to the leptonic vertex. We then use the combined value of $\sin^2\theta_{eff}$ obtained from the whole set of asymmetries measured at LEP and SLC given in eq.(8). At this stage the best values of the epsilons are shown in the second column of table 5. In figs. 4-7 we report the $1\sigma$ ellipses in the indicated $\epsilon_i$-$\epsilon_j$ planes that correspond to this set of input data.

All observables measured on the Z peak at LEP can be included in the analysis provided that we assume that all deviations from the SM are only

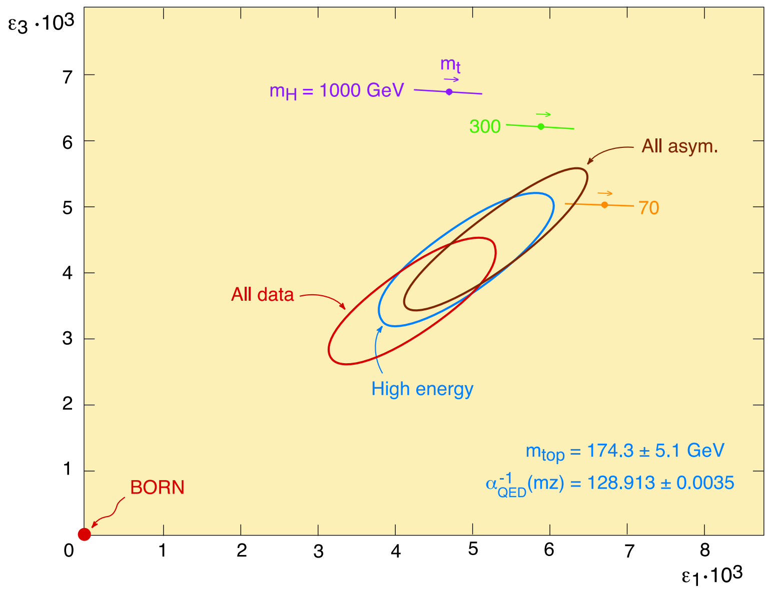


Figure 5: Data vs theory in the $\epsilon_3 - \epsilon_1$ plane (notations as in fig. 4)

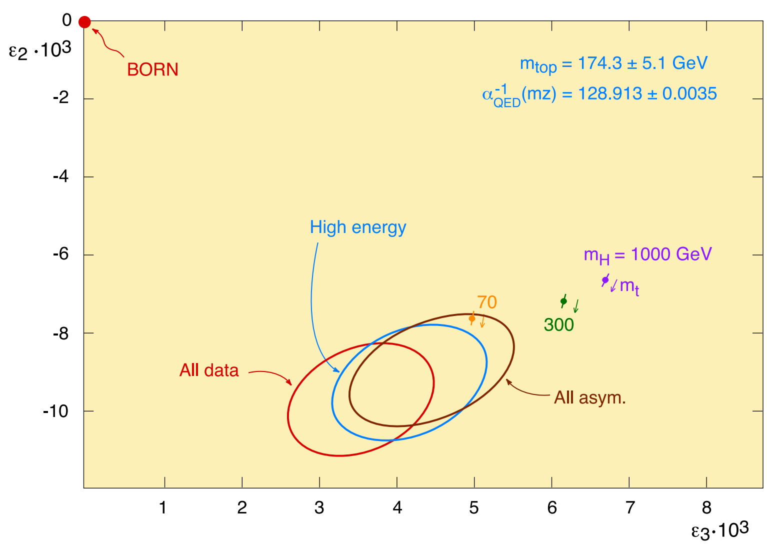


Figure 6: Data vs Theory in the $\epsilon_2 - \epsilon_3$ plane (notations as in fig. 4)

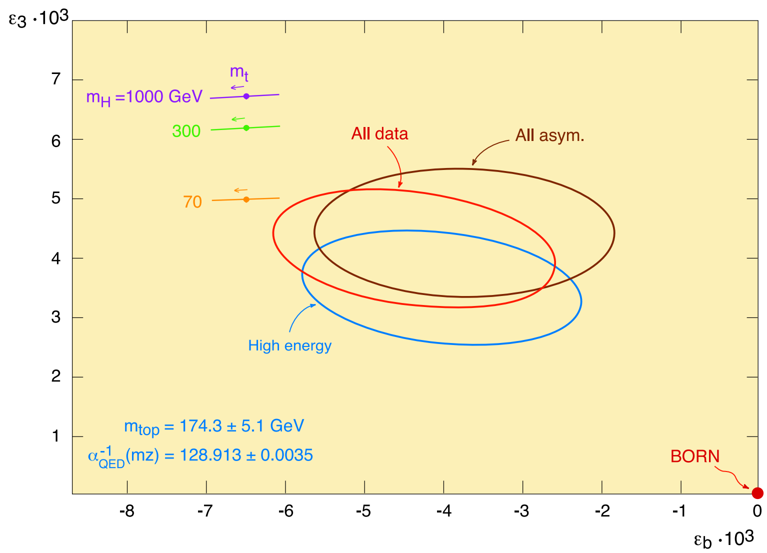


Figure 7: Data vs theory in the $\epsilon_b - \epsilon_3$ plane (notations as in fig. 4)

contained in vacuum polarization diagrams (without demanding a truncation of the $q^2$ dependence of the corresponding functions) and/or the $Z \to b\bar{b}$ vertex. From a global fit of the data on $m_W$, $\Gamma_T$, $R_h$, $\sigma_h$, $R_b$ and $\sin^2\theta_{eff}$ (for LEP data, we have taken the correlation matrix for $\Gamma_T$, $R_h$ and $\sigma_h$ given by the LEP experiments [3], while we have considered the additional information on $R_b$ and $\sin^2\theta_{eff}$ as independent) we obtain the values shown in the third column of table 6. The comparison of theory and experiment at this stage is also shown in figs. 4-7.

To include in our analysis lower energy observables as well, a stronger hypothesis needs to be made: vacuum polarization diagrams are allowed to vary from the SM only in their constant and first derivative terms in a $q^2$ expansion. In such a case, one can, for example, add to the analysis the ratio $R_\nu$ of neutral to charged current processes in deep inelastic neutrino scattering on nuclei, the "weak charge" $Q_W$ measured in atomic parity violation experiments on Cs and the measurement of $g_V/g_A$ from $\nu_\mu e$ scattering. In this way one obtains the global fit given in the fourth column of table 5 and shown in figs. 4-7. In fig. 8 we see the ellipse in the $\epsilon_1$-$\epsilon_3$ plane that is obtained from the low energy data by themselves, in comparison with the results from high energy data. We clearly see the effect of 2.5$\sigma$ deviation from the SM fit of the measured parity violation in atomic physics. It can be shown that the data on neutrino scattering fix the slope of the ellipse major axis, which is in agreement with the high energy data. The atomic parity violation fix the center of the ellipse, which is instead displaced.

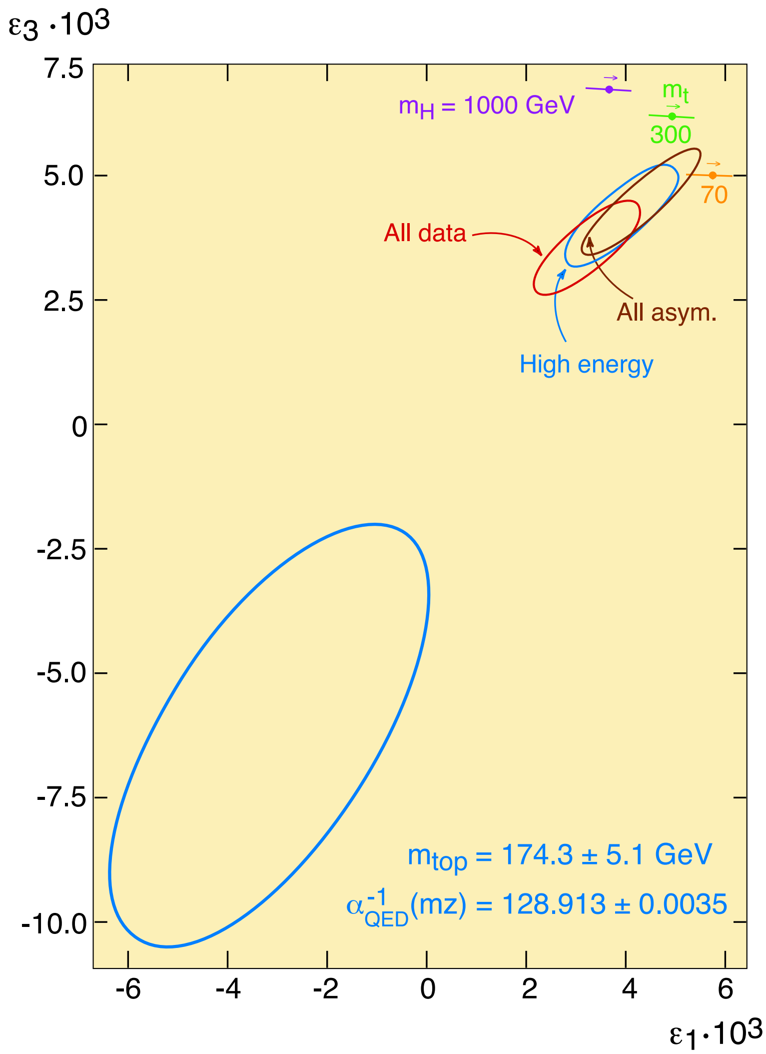


Figure 8: Data vs theory in the $\epsilon_3 - \epsilon_1$ plane (notations as in fig. 4). The ellipse from the low energy data only is compared with that from high energy data and with the SM predictions.

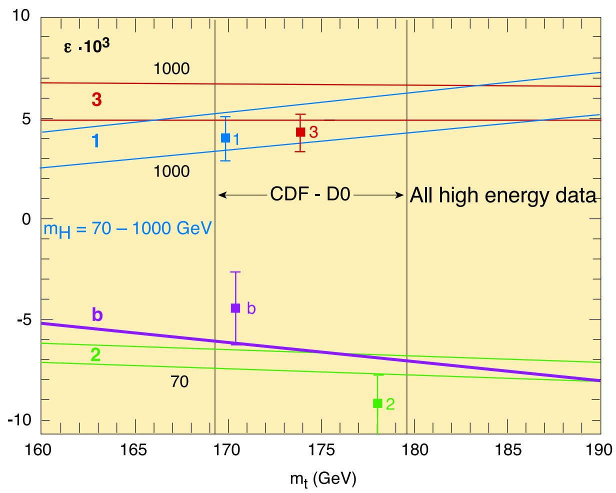


Figure 9: The bands (labeled by the $\epsilon$ index) are the predicted values of the epsilons in the SM as functions of $m_t$ for $m_H = 70 - 1000$ GeV (the $m_H$ value corresponding to one edge of the band is indicated). The CDF/D0 experimental 1-$\sigma$ range of $m_t$ is shown. The experimental results for the epsilons from all data are displayed (from the last column of table 5). The position of the data on the $m_t$ axis has been arbitrarily chosen and has no particular meaning.

The best values of the $\epsilon$'s from all the data are given in the last column of table 5.

Note that the ambiguity on the value of $\delta\alpha^{-1}(m_Z) = \pm 0.035$ (or $\pm 0.09$) corresponds to an uncertainty on $\epsilon_3$ (the other epsilons are not much affected) given by $\Delta\epsilon_3\ 10^3 = \pm 0.25$ (or $\pm 0.6$). Thus the theoretical error is still confortably less than the experimental error. In fig.9 we present a summary of the experimental values of the epsilons as compared to the SM predictions as functions of $m_t$ and $m_H$, which shows agreement within $1\sigma$, but the central value of $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are all a little bit low, while the central value of $\epsilon_b$ is shifted upward with respect to the SM as a consequence of the still imperfect matching of $R_b$.

A number of interesting features are clearly visible from figs.5-11. First, the good agreement with the SM and the evidence for weak corrections, measured by the distance of the data from the improved Born approximation point (based on tree level SM plus pure QED or QCD corrections). There is by now a solid evidence for departures from the improved Born approximation where all the epsilons vanish. In other words a clear evidence for the pure weak radiative corrections has been obtained and one is sensitive to the various components of these radiative corrections. For example, some authors have studied the sensitivity of the data to a particularly interesting subset of the weak radiative corrections, i.e. the purely bosonic part. These terms arise from virtual exchange of gauge bosons and Higgses. The result is that indeed the measurements are sufficiently precise to require the presence of these contributions in order to fit the data. Second, the general results of the SM fits are reobtained from a different perspective. We see the preference for light Higgs manifested by the tendency for $\epsilon_3$ to be rather on the low side. Since $\epsilon_3$ is practically independent of $m_t$, its low value demands $m_H$ small. If the Higgs is light then the preferred value of $m_t$ is slightly lower than the Tevatron result (which in the epsilon analysis is not included among the input data). This is because also the value of $\epsilon_1 \equiv \delta\rho$, which is determined by the widths, in particular by the leptonic width, is somewhat low. In fact $\epsilon_1$ increases with $m_t$ and, at fixed $m_t$, decreases with $m_H$, so that for small $m_H$ the low central value of $\epsilon_1$ pushes $m_t$ down. Note that also the central value of $\epsilon_2$ is on the low side, because the experimental value of $m_W$ is a little bit too large. Finally, we see that adding the hadronic quantities or the low energy observables hardly makes a difference in the $\epsilon_i$-$\epsilon_j$ plots with respect to the case with only the leptonic variables being included (the ellipse denoted by "All Asymm."). But, for example for the $\epsilon_1$-$\epsilon_3$ plot, while the leptonic ellipse contains the same information as one could obtain from a $\sin^2\theta_{eff}$ vs $\Gamma_l$ plot, the content of the other two ellipses is much larger because it shows that the hadronic as well as the low energy quantities match the leptonic variables without need of any new physics. Note that the experimental values of $\epsilon_1$ and $\epsilon_3$ when the hadronic quantities are included also depend on the input value of $\alpha_s$ specified in table 5.

The good agreement of the fitted epsilon values with the SM impose strong constraints on possible forms of new physics. Consider, for example, new quarks or leptons. Mass splitted multiplets contribute to $\Delta\epsilon_1$, in analogy to the t-b quark doublet. Recall that $\Delta\epsilon_1 \sim +9.5\ 10^{-3}$ for the t-b doublet, which is about 10 $\sigma$'s in terms of the present error. Even mass degenerate multiplets are strongly constrained. They contribute to $\Delta\epsilon_3$ according to

$$\Delta\epsilon_3 \sim N_C \frac{G_F m_W^2}{8\pi^2\sqrt{2}} \frac{4}{3}(T_{3L} - T_{3R})^2 \tag{128}$$

For example a new left-handed quark doublet, degenerate in mass, would contribute $\Delta\epsilon_3 \sim +1.3\ 10^{-3}$, that is more than one $\sigma$, but in the wrong direction, in the sense that the experimental value of $\epsilon_3$ favours a displacement, if any, with negative sign. Only vector fermions ($T_{3L} = T_{3R}$) are not constrained. In particular, naive technicolour models [7], that introduce several new technifermions, are strongly disfavoured because they tend to produce large corrections with the wrong sign to $\epsilon_1$, $\epsilon_3$ and also to $\epsilon_b$.

# 8 Why we do not Believe in the SM

## 8.1 Conceptual Problems

Given the striking success of the SM why are we not satisfied with that theory? Why not just find the Higgs particle, for completeness, and declare that particle physics is closed? The main reason is that there are strong conceptual indications for physics beyond the SM.

It is considered highly unplausible that the origin of the electro-weak symmetry breaking can be explained by the standard Higgs mechanism, without accompanying new phenomena. New physics should be manifest at energies in the TeV domain. This conclusion follows fron an extrapolation of the SM at very high energies. The computed behaviour of the $SU(3) \otimes SU(2) \otimes U(1)$ couplings with energy clearly points towards the unification of the electro-weak and strong forces (Grand Unified Theories: GUT's) at scales of energy $M_{GUT} \sim 10^{14} - 10^{16}\ GeV$ which are close to the scale of quantum gravity, $M_{Pl} \sim 10^{19}\ GeV$ [6]. One can also imagine a unified theory of all interactions also including gravity (at present superstrings provide the best attempt at such a theory). Thus GUT's and the realm of quantum gravity set a very distant energy horizon that modern particle theory cannot anymore ignore. Can the SM without new physics be valid up to such large energies? This appears unlikely because the structure of the SM could not naturally explain the relative smallness of the weak scale of mass, set by the Higgs mechanism at $\mu \sim 1/\sqrt{G_F} \sim 250\ GeV$ with $G_F$ being the Fermi coupling constant. This so-called hierarchy problem is related to the presence of fundamental scalar fields in the theory with quadratic mass divergences and no protective extra symmetry at $\mu = 0$. For fermion masses, first, the divergences are logaritmic and, second, they are forbidden by the $SU(2) \bigotimes U(1)$ gauge symmetry plus the fact that at $m = 0$ an additional symmetry, i.e. chiral symmetry, is restored. Here, when talking of divergences we are not worried of actual infinities. The theory is renormalisable and finite once the dependence on the cut off is absorbed in a redefinition of masses and couplings. Rather the hierarchy problem is one of naturalness. If we consider the cut off as a manifestation of new physics that will modify the theory at large energy scales, then it is relevant to look at the dependence of physical quantities on the cut off and to demand that no unexplained enormously accurate cancellations arise.

According to the above argument the observed value of $\mu \sim 250\ GeV$ is indicative of the existence of new physics nearby. There are two main possibilities. Either there exist fundamental scalar Higgses but the theory is stabilised by supersymmetry, the boson-fermion symmetry that would downgrade the bosonic degree of divergence from quadratic to logarithmic. For approximate supersymmetry the cut off is replaced by the splitting between the normal particles and their supersymmetric partners. Then naturalness demands that this splitting (times the size of the weak gauge coupling) is of the order of the weak scale of mass, i.e. the separation within supermultiplets should be of the order of no more than a few TeV. In this case the masses of most supersymmetric partners of the known particles, a very large managerie of states, would fall, at least in part, in the discovery reach of the LHC. There are consistent, fully formulated field theories constructed on the basis of this idea, the simplest one being the MSSM [8]. As already mentioned, all normal observed states are those whose masses are forbidden in the limit of exact $SU(2)\otimes U(1)$. Instead for all SUSY partners the masses are allowed in that limit. Thus when supersymmetry is broken in the TeV range but $SU(2)\otimes U(1)$ is intact only s-partners take mass while all normal particles remain massless. Only at the lower weak scale the masses of ordinary particles are generated. Thus a simple criterium exists to understand the difference between particles and s-particles.

The other main avenue is compositeness of some sort. The Higgs boson is not elementary but either a bound state of fermions or a condensate, due to a new strong force, much stronger than the usual strong interactions, responsible for the attraction. A plethora of new "hadrons", bound by the new strong force would exist in the LHC range. A serious problem for this idea is that nobody sofar has been able to build up a realistic model along these lines, but that could eventually be explained by a lack of ingenuity on the theorists side. The most appealing examples are technicolor theories [7]. These models were inspired by the breaking of chiral symmetry in massless QCD induced by quark condensates. In the case of the electroweak breaking new heavy techniquarks must be introduced and the scale analogous to $\Lambda_{QCD}$ must be about three orders of magnitude larger. The presence of such a large force relatively nearby has a strong tendency to clash with the results of the electroweak precision tests. New versions have been developed to overcome the negative response of the data, but models are far from offering a realistic picture.

Are there other ways to solve the hierarchy problem? Recently an exotic way was proposed [9]. The idea is that perhaps the scale of gravity is only apparently so large. It has been shown that it is in principle possible to bring down the scale of gravity in the multi TeV energy range. This can happen if one assumes the existence of extra space dimensions with sufficiently large compactification radius, with the graviton propagating in all dimensions, while ordinary gauge interactions are trapped on a four dimensional wall. The corresponding modification of gravity at submillimetric distances is compatible with existing limits. The vicinity of the decompactification scale can manifest itself in high energy processes at $e^+e^-$ and hadron colliders where gravitons can be produced and appear as missing energy. This very speculative scenario is certainly interesting especially as a stimulus to look for specific signals. But does not appear as particularly plausible because some large compactification scale have to be ad hoc introduced and large ratios of scales still remain (e.g. the scale where gravity changes behaviour and the weak scale and largely different compactification scales like the depth of the wall and the radius of the bulk). In addition all the positive hints we have in favour of the ordinary picture of GUTs from coupling unification, neutrino masses, dark matter and so on would be emptied. Finally early time cosmology should be rewritten.

The hierarchy problem is certainly not the only conceptual problem of the SM. There are many more: the proliferation of parameters, the mysterious pattern of fermion masses and so on. But while most of these problems can be postponed to the final theory that will take over at very large energies, of order $M_{GUT}$ or $M_{Pl}$, the hierarchy problem arises from the unstability of the low energy theory and requires a solution at relatively low energies.

A supersymmetric extension of the SM provides a way out which is well defined, computable and that preserves all virtues of the SM. The necessary SUSY breaking can be introduced through soft terms that do not spoil the good convergence properties of the theory. Precisely those terms arise from supergravity when it is spontaneoulsly broken in a hidden sector. This is the case in the Minimal Supersymmetric Standard Model (MSSM) [8]. In this most traditional approach SUSY is broken in a hidden sector and the scale of SUSY breaking is very large of order $\Lambda \sim \sqrt{G_F^{-1/2} M_P}$ where $M_P$ is the Planck mass. But since the hidden sector only communicates with the visible sector through gravitational interactions the splitting of the SUSY multiplets is much smaller, in the TeV energy domain, and the Goldstino is practically decoupled. But alternative mechanisms of SUSY breaking are also being considered [9]. In one alternative scenario the (not so much) hidden sector is connected to the visible one by ordinary gauge interactions. As these are much stronger than the gravitational interactions, $\Lambda$ can be much smaller, as low as 10-100 TeV. It follows that the Goldstino is very light in these models (with mass of order or below 1 eV typically) and is the lightest, stable SUSY particle, but its couplings are observably large. The radiative decay of the lightest neutralino into the Goldstino leads to detectable photons. The signature of photons comes out naturally in this SUSY breaking pattern: with respect to the MSSM, in the gauge mediated model there are typically more photons and less missing energy. The main appeal of gauge mediated models is a better protection against flavour changing neutral currents. In the gravitational version even if we accept that gravity leads to degenerate scalar masses at a scale near $M_P$ the running of the masses down to the weak scale can generate mixing induced by the large masses of the third generation fermions [9]. More recently it has been pointed out that there are pure gravity contributions to soft masses that arise from gravity theory anomalies [9]. In the assumption that these terms are dominant the associated spectrum and phenomenology has been studied. In this case gaugino masses are proportional to gauge coupling beta functions, so that the gluino is much heavier than the electroweak gauginos, and the wino is most often the lightest SUSY particle.

The MSSM [8] is a completely specified, consistent and computable theory. There are too many parameters to attempt a direct fit of the electroweak precision data to the most general framework. But we can consider two significant limiting cases: the "heavy" and the "light" MSSM.

The "heavy" limit corresponds to all s-particles being sufficiently massive, still within the limits of a natural explanation of the weak scale of mass. In this limit a very important result holds: for what concerns the precision electroweak tests, the MSSM predictions tend to reproduce the results of the SM with a light Higgs, say $m_H \sim 100$ GeV. So if the masses of SUSY partners are pushed at sufficiently large values the same quality of fit as for the SM is guaranteed.

In the "light" MSSM option some of the superpartners have a relatively small mass, close to their experimental lower bounds. In this case the pattern of radiative corrections may sizeably deviate from that of the SM.. The potentially largest effects occur in vacuum polarization amplitudes and/or the $Z \to b\bar{b}$ vertex. Since no sign of deviations from the SM is seen in the data and no light SUSY partners have been found at LEP2 or at the Tevatron, the "light" case can no more be that light.

According to the prevailing view at present, the large scale structure of particle physics consists of a unified theory at $M \approx M_{GUT} \sim M_P$ and a low energy effective theory valid at and above the weak scale of energy. The lagrangian density of the low energy effective theory , after integrating out all very heavy degrees of freedom, consists of a set of operators of dimension non larger than 4, that correspond to the renormalisable part, plus a set of higher dimension, non renormalisable, operators. Schematically, we have:

$$\mathcal{L} = \mu^2\phi^2 + m\bar{\psi}\psi + g\bar{\psi}i\not{D}\psi + \lambda\phi^4 + ...... + \frac{\lambda_5}{M}\bar{\psi}\psi\phi\phi + \frac{\lambda_6}{M^2}\bar{\psi}\psi\bar{\psi}\psi + .... \quad (129)$$

Indicatively, we have shown a number of typical terms of dimension 2 (boson masses), 3 (fermion masses), 4 (renormalisable interactions) plus examples of operators of higher dimension, 5 and 6. Due to the very large scale of energy where the really fundamental theory applies, the conditions on the low energy effective theory are severe. First, the dimension $\leq 4$ part must be renormalisable. This is a minimum requirement in order to have a closed, consistent and predictive description of the dynamics after the presence of the very high cut off has been hidden inside renormalised masses and couplings. But this is not enough because the dependence of masses and couplings from the cut off must be reasonable in order to avoid the necessity of immense fine tuning. For this to be true additional conditions must be satisfied. The coupling in front of each operator, in absence of specific reasons, should be proportional to the large cut off $M$ raised to the power d fixed by dimensions. For example, $\mu^2$ should be proportional to $M^2$. In the SM there is no symmetry reason why this should not be the case. So boson masses, like the W and Z masses, should be of order M. This the hierarchy problem. In supersymmetric extensions of the SM $\mu^2$ is instead of order the mass splittings of SUSY multiplets, because in the limit of exact SUSY symmetry there are no quadratic divergences (in presence of boson-fermion symmetry the stronger bosonic divergences must disappear, in order that bosonic and fermionic divergences can both be logaritmic). For fermions $m$ is not of order $M$ but of order $v \log M$ because the divergences in the fermionic sector are always at most logaritmic. Also, chiral symmetry ensures that if you start from zero masses the quantum corrections to $m$ must vanish. Once supersymmetry or some other stabilising mechanism is introduced, the renormalisable part of the lagrangian is sufficiently insensitive to the presence of the very large cut off $M$. The additional non renormalisable terms are suppressed by powers of $M$. At energies of order $v$, the electro-weak scale, their effects are proportional to $(v/M)^d$, $d = 1, 2, ...$, hence very small.

## 8.2 Hints from Experiment

### 8.2.1 Unification of Couplings

At present the most direct phenomenological evidence in favour of supersymmetry is obtained from the unification of couplings in GUTs. Precise LEP data on $\alpha_s(m_Z)$ and $\sin^2\theta_W$ confirm what was already known with less accuracy: standard one-scale GUTs fail in predicting $\sin^2\theta_W$ given $\alpha_s(m_Z)$ (and $\alpha(m_Z)$) while SUSY GUTs [6] are in agreement with the present, very precise, experimental results. According to a recent analysis, if one starts from the known values of $\sin^2\theta_W$ and $\alpha(m_Z)$, one finds for $\alpha_s(m_Z)$ the results:

$$\alpha_s(m_Z) = 0.073 \pm 0.002 \qquad \text{(Standard GUTs)}$$
$$\alpha_s(m_Z) = 0.129 \pm 0.010 \qquad \text{(SUSY GUTs)} \tag{130}$$

to be compared with the world average experimental value $\alpha_s(m_Z)$ =0.119(3).

### 8.2.2 Dark Matter

There is solid astrophysical and cosmological evidence [10], that most of the matter in the universe does not emit electromagnetic radiation, hence is "dark". Some of the dark matter must be baryonic but most of it must be non baryonic. Non baryonic dark matter can be cold or hot. Cold means non relativistic at freeze out, while hot is relativistic. There is general consensus that most of the non baryonic dark matter must be cold dark matter. A couple of years ago the most likely composition was quoted to be around 80% cold and 20% hot. At present it appears that the need of a sizeable hot dark matter component is more uncertain. In the last few years great progress has been made in the experimental determination of fundamental cosmological parameters. The Hubble constant has been measured, also using the Hubble telescope, ($H_0 = 65 \pm 8 km\ s{-1}\ Mpc^{-1}$). There is growing experimental evidence (for example, from the supernovae distribtion vs redshift) of the presence of a cosmological constant component in $\Omega = \Omega_m + \Omega_\Lambda$. Here $\Omega$ is the total matter-energy density in units of the critical density, $\Omega_m$ is the matter component (dominated by non baryonic cold dark matter) and $\Omega_\Lambda$ is the cosmological component. $\Omega_m$ is extimated reliably, for example from the mass distribution at large distances, measured by gravitational lensing, which gives $\Omega_m \approx 0.35$. Inflationary theories strongly favour $\Omega = 1$ which is consistent with present data (in particular the beautiful new data on the position of the first acoustic peak by Boomerang and Maxima). At present, still within large uncertainties, the approximate composition is indicated to be $\Omega_m \sim 0.35$ and $\Omega_\Lambda \sim 0.65$ (baryonic dark matter from big bang nucleosynthesis gives $\Omega_b \sim 0.05$).

The implications for particle physics is that certainly there must exist a source of cold dark matter. By far the most appealing candidate is the neutralino, the lowest supersymmetric particle, in general a superposition of photino, Z-ino and higgsinos. This is stable in supersymmetric models with R parity conservation, which are the most standard variety for this class of models (including the MSSM). A neutralino with mass of order 100 GeV would fit perfectly as a cold dark matter candidate. Another common candidate for cold dark matter is the axion, the elusive particle associated to a possible solution of the strong CP problem along the line of a spontaneously broken Peccei-Quinn symmetry. To my knowledge and taste this option is less plausible than the neutralino. One favours supersymmetry for very diverse conceptual and phenomenological reasons, as described in the previous sections, so that neutralinos are sort of standard by now. For hot dark matter, the self imposing candidates would be neutrinos. If we demand a density fraction $\Omega_\nu \sim 0.1$ from neutrinos, the maximum which is allowed by observations,then it turns out that the sum of stable neutrino masses should be around 5 eV.

### 8.2.3 Neutrino Masses

Recent data [11] from Superkamiokande have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition the solar neutrino deficit, observed by several experiments, is also probably an indication of a different sort of neutrino oscillations. Results from the laboratory experiment by the LSND collaboration, not confirmed by KARMEN, can also be considered as a possible indication of yet another type of neutrino oscillation. Neutrino oscillations imply neutrino masses. The extreme smallness of neutrino masses in comparison with quark and charged lepton masses indicate a different nature of neutrino masses, linked to lepton number violation and the Majorana nature of neutrinos. Thus neutrino masses provide a window on the very large energy scale where lepton number is violated and on GUTs. The new experimental evidence on neutrino masses could also give an important feedback on the problem of quark and charged lepton masses, as all these masses are possibly related in GUTs. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \rightarrow \nu_\tau$ is particularly interesting. Perhaps also solar neutrinos may occur with large mixing angle. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings are very interesting because a first guess was in favour of small mixings in the neutrino sector in analogy to what is observed for quarks. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses.

From a strict minimal standard model point of view neutrino masses could vanish if no right handed neutrinos existed (no Dirac mass) and lepton number was conserved (no Majorana mass). In GUTs both these assumptions are violated. The right handed neutrino is required in all unifying groups larger than SU(5). In SO(10) the 16 fermion fields in each family, including the right handed neutrino, exactly fit into the 16 dimensional representation of this group. This is really telling us that there is something in SO(10)! The SU(5) alternative in terms of $\bar{5} + 10$, without a right handed neutrino, is certainly less elegant. The breaking of $|B - L|$, B and L is also a generic feature of GUTs. In fact, the see-saw mechanism [12] explains the smallness of neutrino masses in terms of the large mass scale where $|B - L|$ and L are violated. Thus, neutrino masses, as would be proton decay, are important as a probe into the physics at the GUT scale.

Oscillations only determine squared mass differences and not masses. If in addition to solar and atmospheric neutrino oscillations also the LSND evidence will be confirmed, then one would need to add a fourth sterile neutrino (i.e. without weak interactions, to avoid the LEP veto against additional light weakly interacting neutrinos). This is because oscillation frequencies determine squared mass differences and with three masses there are only two independent differences. However, sterile neutrinos are at present disfavoured both from atmospheric and solar neutrino oscillation observations. Thus in the following we assume that the LSND evidence for a third oscillation frequency will disappear and we restrict to three neutrinos. In terms of our labelling of masses the two frequencies are given by $\Delta_{sun} \propto m_2^2 - m_1^2$ and $\Delta_{atm} \propto m_3^2 - m_{1,2}^2$.

Neutrino oscillations only determine differences of squared masses and not the absolute mass scale. the case of three almost perfectly degenerate neutrinos is the only one that could in principle accomodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. According to our previous discussion, the common mass should be around 1-2 eV. The solar frequency ($\Delta m^2_{sun} \sim 10^{-5} - 10^{-10}\ eV^2$, depending on which solution is finally established) could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting ($\Delta m^2_{atm} \sim 3\ 10^{-3}\ eV^2$). A strong constraint arises in the degenerate case from neutrinoless double beta decay which requires that the ee entry of $m_\nu$ must obey $|(m_\nu)_{11}| \leq 0.2 - 0.5$ eV [12]. It has been observed that this bound can only be satisfied if double maximal mixing is realized, i.e. if also solar neutrino oscillations occur with nearly maximal mixing. We have mentioned that it is not at all clear at the moment that a hot dark matter component is really needed [10]. However the only reason to consider the fully degenerate solution is that it is compatible with hot dark matter. Note that for degenerate masses with $m \sim 1 - 2$ eV we need a relative splitting $\Delta m/m \sim \Delta m^2_{atm}/2m^2 \sim 10^{-3}$ and a much smaller one for solar neutrinos. It is not simple to imagine a natural mechanism compatible with unification and the see-saw mechanism to arrange such a precise near symmetry.

If neutrino masses are smaller than for cosmological relevance, we can have the hierarchies $|m_3| >> |m_{2,1}|$ or $|m_1| \sim |m_2| >> |m_3|$. We prefer the first case, because for quarks and leptons one mass eigenvalue, the third generation one, is largely dominant. Thus the dominance of $m_3$ for neutrinos corresponds to what we observe for the other fermions. In this case, $m_3$ is determined by the atmospheric neutrino oscillation frequency to be around $m_3 \sim 0.05\,eV$. By the see-saw mechanism $m_3$ is related to some large mass M, by $m_3 \sim m^2/M$. If we identify m with either the Higgs vacuum expectation value or the top mass (which are of the same order), as suggested for third generation neutrinos by GUTs in simple SO(10) models, then M turns out to be around $M \sim 10^{15}\ GeV$, which is consistent with the connection with GUTs.

A lot of attention [12] is being devoted to the problem of a natural explanation of the observed nearly maximal mixing angle for atmospheric neutrino oscillations and possibly also for solar neutrino oscillations, if explained by vacuum oscillations. Large mixing angles are somewhat unexpected because the observed quark mixings are small and the quark, charged lepton and neutrino mass matrices are to some extent related in GUT's. There must be some special interplay between the neutrino Dirac and Majorana matrices in the see-saw mechanism in order to generate maximal mixing. It is hoped that looking for a natural explanation of large neutrino mixings can lead us to decripting some interesting message on the physics at the GUT scale.

### 8.2.4 Baryogenesis

Baryogenesis is interesting because it could occur at the weak scale [13] but not in the SM. For baryogenesis one needs the three famous Sakharov conditions: B violation, CP violation and no termal equilibrium. In principle these conditions could be verified in the SM. B is violated by instantons when kT is of the order of the weak scale (but B-L is conserved). CP is violated by the CKM phase and sufficiently marked out of equilibrium conditions could be realised during the electroweak phase transition. So the conditions for baryogenesis at the weak scale in the SM appear superficially to be present. However, a more quantitative analysis [13], shows that baryogenesis is not possible in the SM because there is not enough CP violation and the phase transition is not sufficiently strong first order, unless $m_H < 80\ GeV$, which is by now completely excluded by LEP. However, it is interesting that baryogenesis at the weak scale is not yet excluded in SUSY extensions of the SM. In particular, in the MSSM there are additional sources of CP violations and the bound on $m_H$ is modified by a sufficient amount by the presence of scalars with large couplings to the Higgs sector, typically the s-top. What is required is that $m_h \sim 80 - 110\ GeV$, a s-top not heavier than the top quark and, preferentially, a small $\tan\beta$. This possibility has become more and more marginal with the progress of the LEP2 running.

If baryogenesis at the weak scale is excluded by the data it can occur at or just below the GUT scale, after inflation. But only that part with $|B-L| > 0$ would survive and not be erased at the weak scale by instanton effects. Thus baryogenesis at $kT \sim 10^{12} - 10^{15}\ GeV$ needs B-L violation at some stage like for $m_\nu$, if neutrinos are Majorana particles. The two effects could be related if baryogenesis arises from leptogenesis then converted into baryogenesis by instantons [14]. Recent results on neutrino masses are compatible with this elegant possibility. Thus the case of baryogenesis through leptogenesis has been boosted by the recent results on neutrinos.

# 9 Status of the Search for the Higgs and for New Physics

The LEP2 programme has started in the second part of 1995. The the total center of mass energy was gradually increased up to 208 $GeV$. The main goals of LEP2 are the search for the Higgs and for new particles, the measurement of $m_W$ and the investigation of the triple gauge vertices $WWZ$ and $WW\gamma$

[15].

Concerning the Higgs, the present limits (summer '00) obtained by the LEP collaborations , are, for the SM Higgs, $m_H \gtrsim 113 GeV$ and for the lightest MSSM Higgs, $m_h \gtrsim 90 GeV$. To understand the significance of these limits we recall the theoretical bounds on the Higgs mass.

It is well known that in the SM with only one Higgs doublet a lower limit on $m_H$ can be derived from the requirement of vacuum stability. This criterium is equivalent to demand that the coupling $\lambda$ of the quartic term $\lambda(\phi \dagger \phi)^2$ does not become negative while running from the weak scale up to the scale $\Lambda$. The initial value of $\lambda$ at the weak scale increases with $m_H^2$, while the derivative, for $m_H$ near the limit, is dominated, for a not too heavy Higgs, by the top quark term which is large and negative. The value of the limit is a function of $m_t$ and of the energy scale $\Lambda$ where the model breaks down and new physics appears. If one requires that $\lambda$ remains positive up to $\Lambda = 10^{15}$–$10^{19}$ GeV, then the resulting bound on $m_H$ in the SM with only one Higgs doublet is given by:

$$m_H > 135 + 2.1\,[m_t - 174.3] - 4.5\,\frac{\alpha_s(m_Z) - 0.119}{0.006}\ . \qquad (131)$$

It follows that he discovery of a SM-like Higgs particle at LEP2, or $m_H \lesssim 115\ GeV$, would imply that the SM breaks down at a scale $\Lambda$ of the order of $\lesssim 100\ TeV$. Note, however, that the lower bound is invalidated if more than one single Higgs doublet exists: for more doublets the limit applies to some average mass and not to the lightest Higgs particle.

Similarly an upper bound on $m_H$ (with mild dependence on $m_t$) is obtained from the requirement that up to the scale $\Lambda$ no Landau pole appears. The upper limit on the Higgs mass in the SM is important to guarantee the success of the LHC as an accelerator designed to solve the Higgs problem. In fact, for large Higgs masses, the initial value of $\lambda$ is large and the derivative of $\lambda$ is positive, because the positive $\lambda$ term (the $\lambda\phi^4$ theory is not asymptotically free!) overwhelms the top Yukawa negative contribution. As a consequence the coupling $\lambda$ tends to infinity (the Landau pole) at some finite scale. The upper limit on $m_H$ has been studied not only in perturbation theory but also using lattice simulations of the Higgs sector in the region near the pole which is non perturbative. For $m_t \sim 175\ GeV$ one finds $m_H \lesssim 180\ GeV$ for $\Lambda \sim M_{GUT} - M_{Pl}$ and $m_H \lesssim 0.5 - 0.8\ TeV$ for $\Lambda \sim 1\ TeV$. Thus, in conclusion [16], if the SM holds up to $\Lambda \sim M_{GUT}$ or $M_{Pl}$, then, for $m_t \sim 174$ GeV, only a small range of values for $m_H$ is allowed, $130 < m_H < \ \sim 200$ GeV.

A particularly important example of theory where the lower bound is violated, is the MSSM, which we now discuss. As is well known [8], in the MSSM there are two Higgs doublets, which implies three neutral physical Higgs particles and a pair of charged Higgses. The lightest neutral Higgs, called $h$, should be lighter than $m_Z$ at tree-level approximation. However, radiative corrections increase the $h$ mass by a term proportional to $m_t^4$ and logarithmically dependent on the stop mass. Once the radiative corrections are taken into account the $h$ mass still remains rather small: for $m_t = 174\ GeV$ one finds the limit $m_h \lesssim 130\ GeV$ (valid for all values of $tg\beta$ and saturated at large $tg\beta$). Actually one can well expect that $m_h$ is sizeably below the bound if $tg\beta$ is small, $tg\beta = v_{up}/v_{down} < 10$). LEP2 is progressively excluding a part of the small $\tan\beta$ domain. If no Higgs is found at LEP the domain $\tan\beta \lesssim 2 - 8$ will be excluded, depending on the value of other MSSM parameters. By now most of the discovery potential of LEP2 for supersymmetric particles has been deployed. For example, the limit on the chargino mass was about 45 $GeV$ after LEP1 and is now about $m_{\chi^+} \lesssim 103\ GeV$, apart from exceptional regions of the MSSM parameter space. The lightest neutralino mass limit is around $m_{\chi^0} \lesssim 40\ GeV$. The region of the MSSM parameter space that has been by now excluded by LEP is a very important one. The low $tg\beta$ solution was appealing in many respects. Some more constrained forms of the model, like the supergravity version, where degenerate scalar masses and gaugino masses are assumed at the GUT scale, are by now disfavoured. With no discovery of the Higgs and SUSY at LEP the case for the MSSM certainly becomes less natural, and even less natural become the gauge mediated models.

An important competitor of CERN is the Tevatron collider. In 2001 the Tevatron will start RunII with the purpose of collecting a few $fb^{-1}$ of integrated luminosity at 2 $TeV$. The competition is especially on the search of new particles and the Higgs, but also on $m_W$ and the triple gauge vertices. For example, for supersymmetry, LEP2 was strong on Higgses, charginos, neutralinos and sleptons while the Tevatron is superior for gluinos and squarks, . There are plans for RunIII to start in $\gtrsim 2004$ with the purpose of collecting of the order 5 $fb^{-1}$ of integrated luminosity per year. If so the Tevatron could also hope to find the Higgs before the LHC if the Higgs mass is close to the LEP2 range.

## 10 Conclusion

Today in particle physics we follow a double approach: from above and from below. From above there are, on the theory side, quantum gravity (that is superstrings), GUT theories and cosmological scenarios. On the experimental side there are underground experiments (e.g. searches for neutrino oscillations and proton decay), cosmic ray observations, space experiments (like COBE, Boomerang, Maxima etc), cosmological observations and so on. From below, the main objectives of theory and experiment are the search of the Higgs and of signals of particles beyond the Standard Model (typically supersymmetric particles). Another important direction of research is aimed at the exploration of the flavour problem: study of CP violation and rare decays. The general expectation is that new physics is close by and that should be found very soon if not for the complexity of the necessary experimental technology that makes the involved time scale painfully long.

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