NEUTRON STAR OSCILLATIONS: NUMERICAL SIMULATIONS

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Plan of Talk

Overview of recent results on:

- Equilibrium models
- F-mode instability
- F-modes excited in NS-NS mergers

Numerical Method

Metric:

$$ds^2 = -e^{\gamma+\rho}dt^2 + e^{\gamma-\rho}r^2\sin^2\theta(d\phi - \omega dt)^2 + e^{2\alpha}(dr^2 + r^2 d\theta^2)$$

Field Equations:
integral form

$$\Delta[\rho e^{\gamma/2}] = S_\rho(r,\mu),$$

$$\left(\Delta + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\mu\frac{\partial}{\partial \mu}\right)\gamma e^{\gamma/2} = S_\gamma(r,\mu),$$

$$\left(\Delta + \frac{2}{r}\frac{\partial}{\partial r} - \frac{2}{r^2}\mu\frac{\partial}{\partial \mu}\right)\omega e^{(\gamma-2\rho)/2} = S_\omega(r,\mu),$$

$$\rho = -\frac{1}{4\pi}e^{-\gamma/2}\int_0^\infty dr'\int_{-1}^{1}d\mu'\int_0^{2\pi}d\phi' r'^2 S_\rho(r',\mu')\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,,$$

$$r\sin\theta\gamma = \frac{1}{2\pi}e^{-\gamma/2}\int_0^\infty dr'\int_0^{2\pi}d\theta' r'^2\sin\theta' S_\gamma(r',\theta')\log|\mathbf{r}-\mathbf{r}'|,$$

$$r\sin\theta\cos\phi\,\omega = -\frac{1}{4\pi}e^{(2\rho-\gamma)/2}\int_0^\infty dr'\int_0^\pi d\theta'\int_0^{2\pi}d\phi' r'^3\sin^2\theta'\cos\phi' S_\omega(r',\theta')\frac{1}{|\mathbf{r}-\mathbf{r}'|}\,.$$

RNS code, NS & Friedman

Komatsu, Eriguchi & Hachisu (1989) method

Cook, Shapiro & Teukolsky (1994)
compactified radial coordinate

$$\begin{aligned}
\alpha_{,\mu} = & -\nu_{,\mu} - \{(1-\mu^2)(1+rB^{-1}B_{,r})^2 + [\mu - (1-\mu^2)B^{-1}B_{,\mu}]^2\}^{-1} \\
& [\![\tfrac{1}{2}B^{-1}\{r^2 B_{,rr} - [(1-\mu^2)B_{,\mu}]_{,\mu} - 2\mu B_{,\mu}\}[-\mu + (1-\mu^2)B^{-1}B_{,\mu}] \\
& + rB^{-1}B_{,r}[\tfrac{1}{2}\mu + \mu r B^{-1}B_{,r} + \tfrac{1}{2}(1-\mu^2)B^{-1}B_{,\mu}] \\
& + \tfrac{3}{2}B^{-1}B_{,\mu}[-\mu^2 + \mu(1-\mu^2)B^{-1}B_{,\mu}] - (1-\mu^2)rB^{-1}B_{,\mu r}(1+rB^{-1}B_{,r}) \\
& - \mu r^2\nu_{,r}^2 - 2(1-\mu^2)r\nu_{,\mu}\nu_{,r} + \mu(1-\mu^2)\nu_{,\mu}^2 - 2(1-\mu^2)r^2B^{-1}B_{,r} \\
& \times \nu_{,\mu}\nu_{,r} + (1-\mu^2)B^{-1}B_{,\mu}[r^2\nu_{,r}^2 - (1-\mu^2)\nu_{,\mu}^2] + (1-\mu^2)B^2 e^{-4\nu} \\
& \times [\tfrac{1}{4}\mu r^4\omega_{,r}^2 + \tfrac{1}{2}(1-\mu^2)r^3\omega_{,\mu}\omega_{,r} - \tfrac{1}{4}\mu(1-\mu^2)r^2\omega_{,\mu}^2 + \tfrac{1}{2}(1-\mu^2) \\
& \times r^4B^{-1}B_{,r}\omega_{,\mu}\omega_{,r} - \tfrac{1}{4}(1-\mu^2)r^2B^{-1}B_{,\mu}[r^2\omega_{,r}^2 - (1-\mu^2)\omega_{,\mu}^2]]\}]\!],
\end{aligned}$$

Differential Rotation

For polytropic EOS the specific angular momentum measured by proper time of matter is

$$j \equiv u^t u_\phi = j(\Omega)$$

Rotation Law:

$$\Omega = \Omega_c - \frac{(\Omega-\omega) r^2 \sin^2\theta \, e^{-2\rho}}{A^2\left[1-(\Omega-\omega)^2 r^2 \sin^2\theta \, e^{-2\rho}\right]}$$

Dimensionless constant:

$$\hat{A} = A/r_e$$

Limits:

$$\hat{A}^{-1} \to \begin{cases} 0 & \text{uniform rotation} \\ \infty & j-\text{constant rotation law} \end{cases}$$

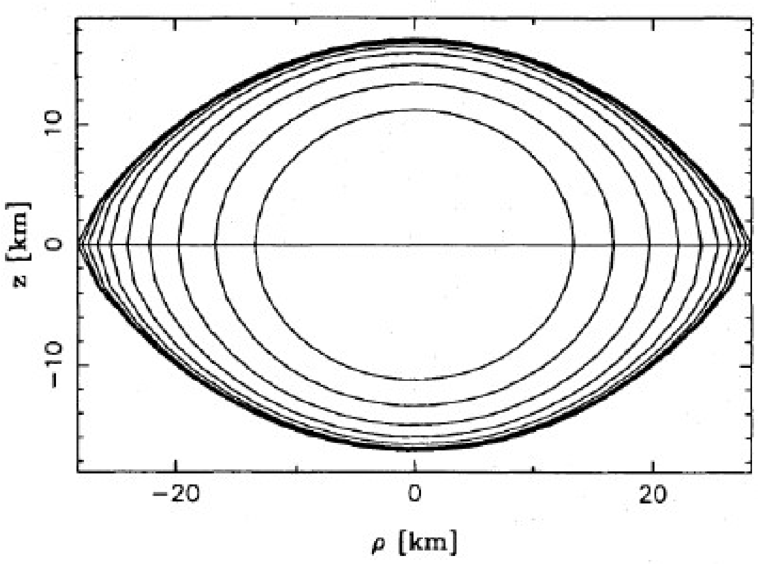
Specific angular momentum conserved during homologous collapse:

$$\tilde{j} \equiv u_\phi \left(\frac{\varepsilon + p}{\rho_0}\right)$$

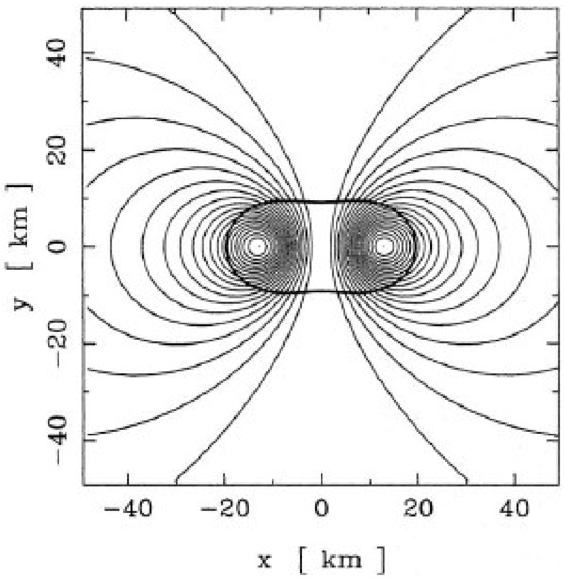
Satisfies Rayleigh criterion for local dynamical stability to axisymmetric perturbations:

$$\frac{d\tilde{j}}{d\Omega} < 0$$

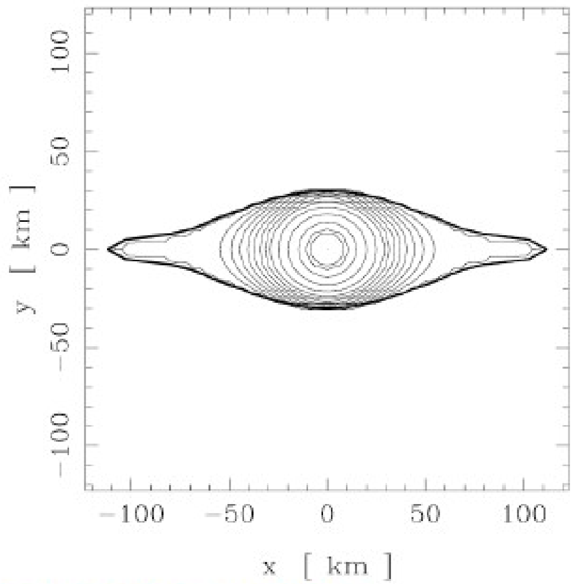
Examples of Equilibrium Models



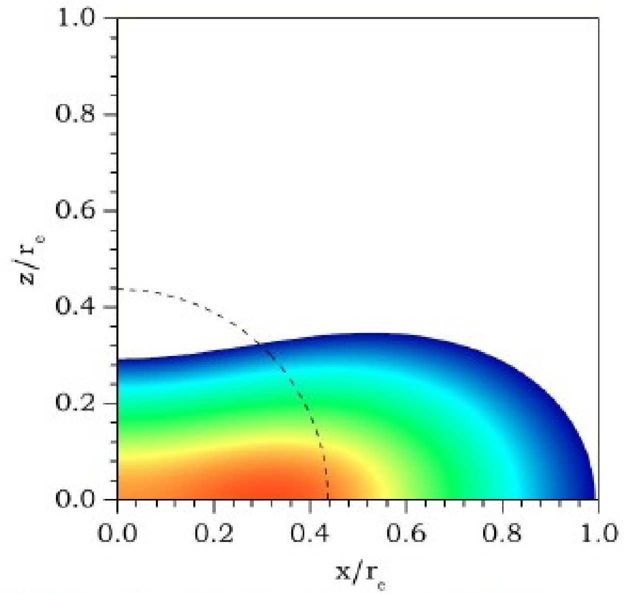
Bonazzola et al. 1993



Bocquet et al. 1995



Goussard et al. 1998

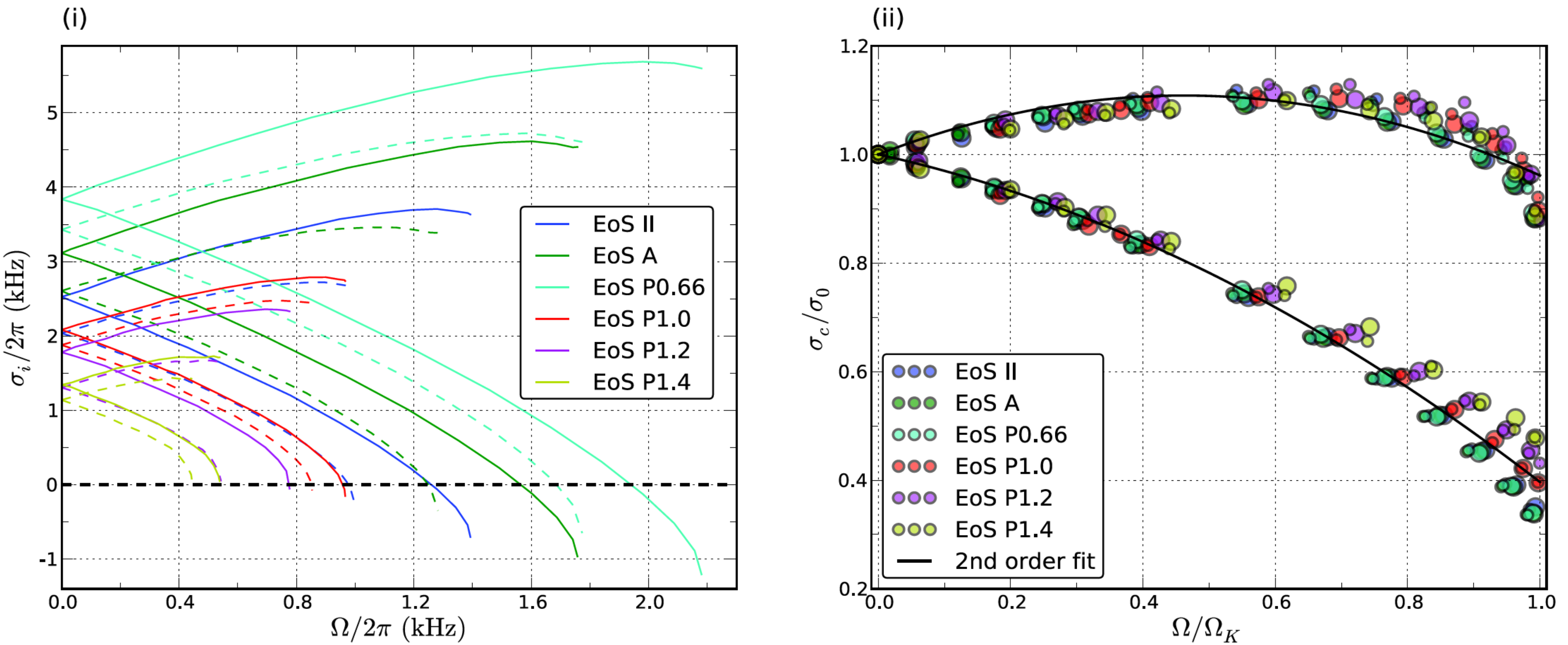


N. S., Apostolatos & Font, 2004

NS Oscillations

Gaertig, Kokkotas (2011)

Rapid rotation, Cowling approximation, $l=m=2$ f-mode frequency (linear time-evolution code)

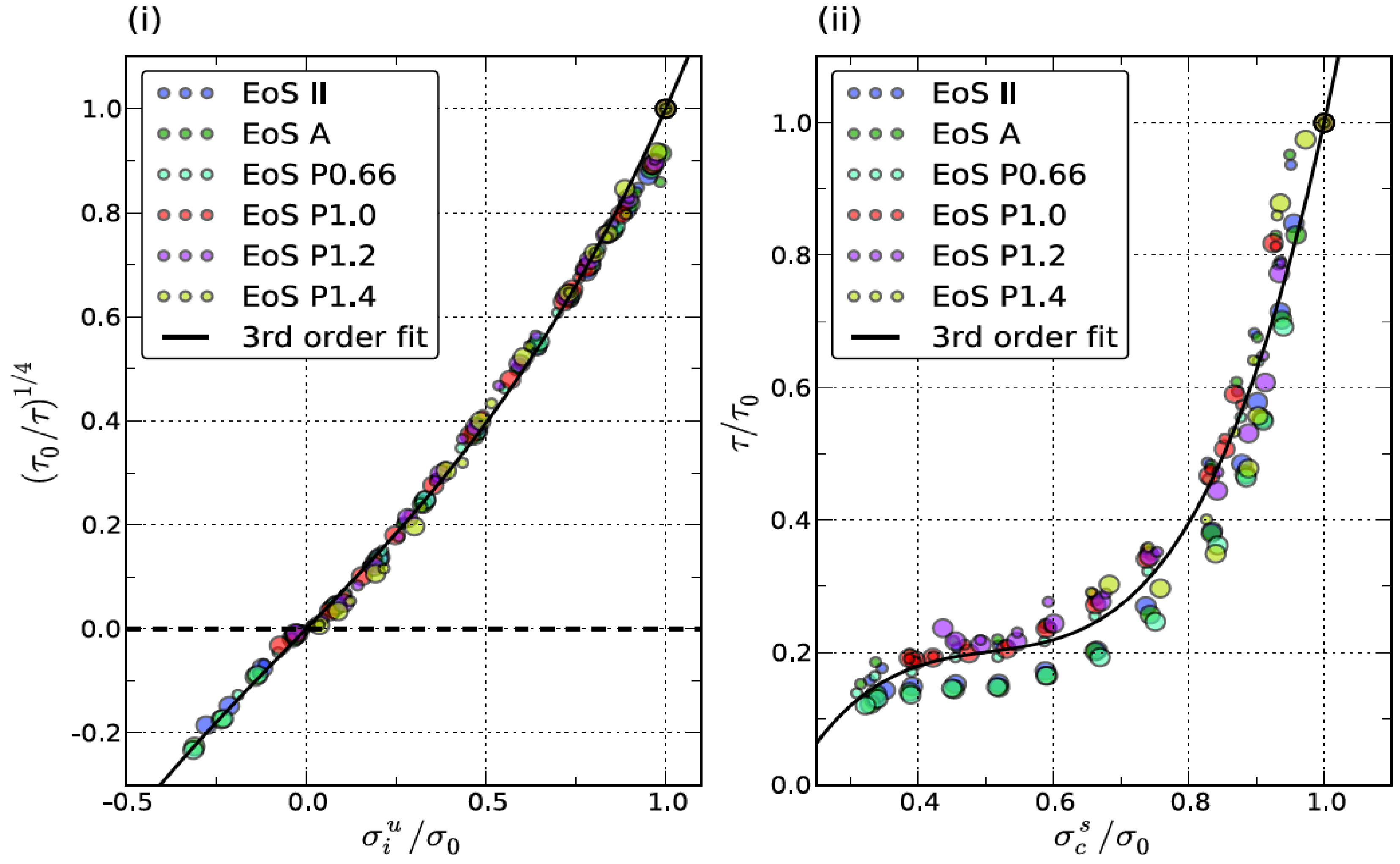


Inertial frame: large spread of frequencies.

Corotating frame: same rotational effect, independent of EOS!

NS Oscillations

Damping time:



Counter-rotating branch vs. frequency in inertial frame.

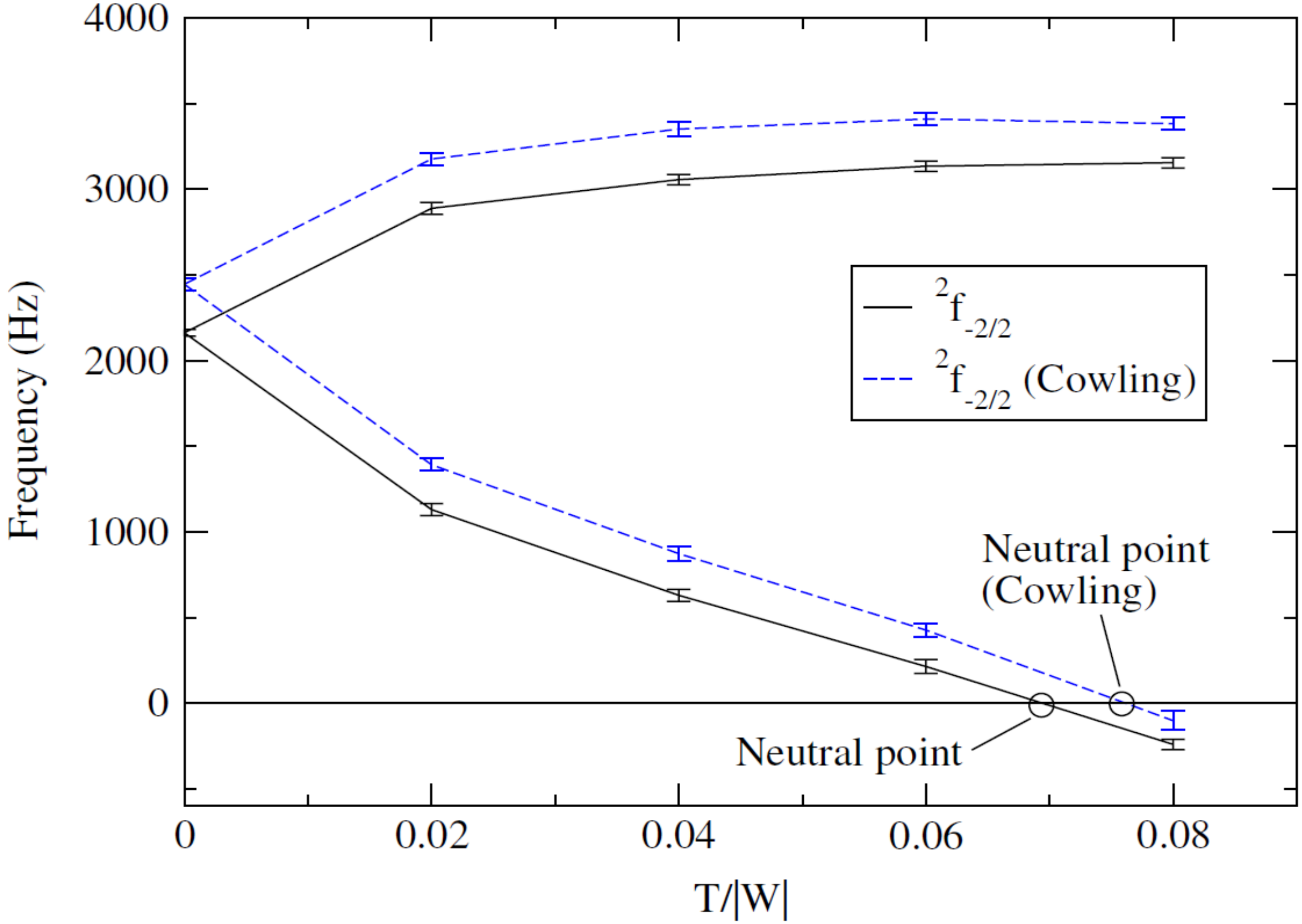
Corotating branch vs. frequency in corotating frame.

→ Empirical relations for GW asteroseismology.

Rotational Instabilities

Zink, Korobkin, Schnetter, NS (2011)

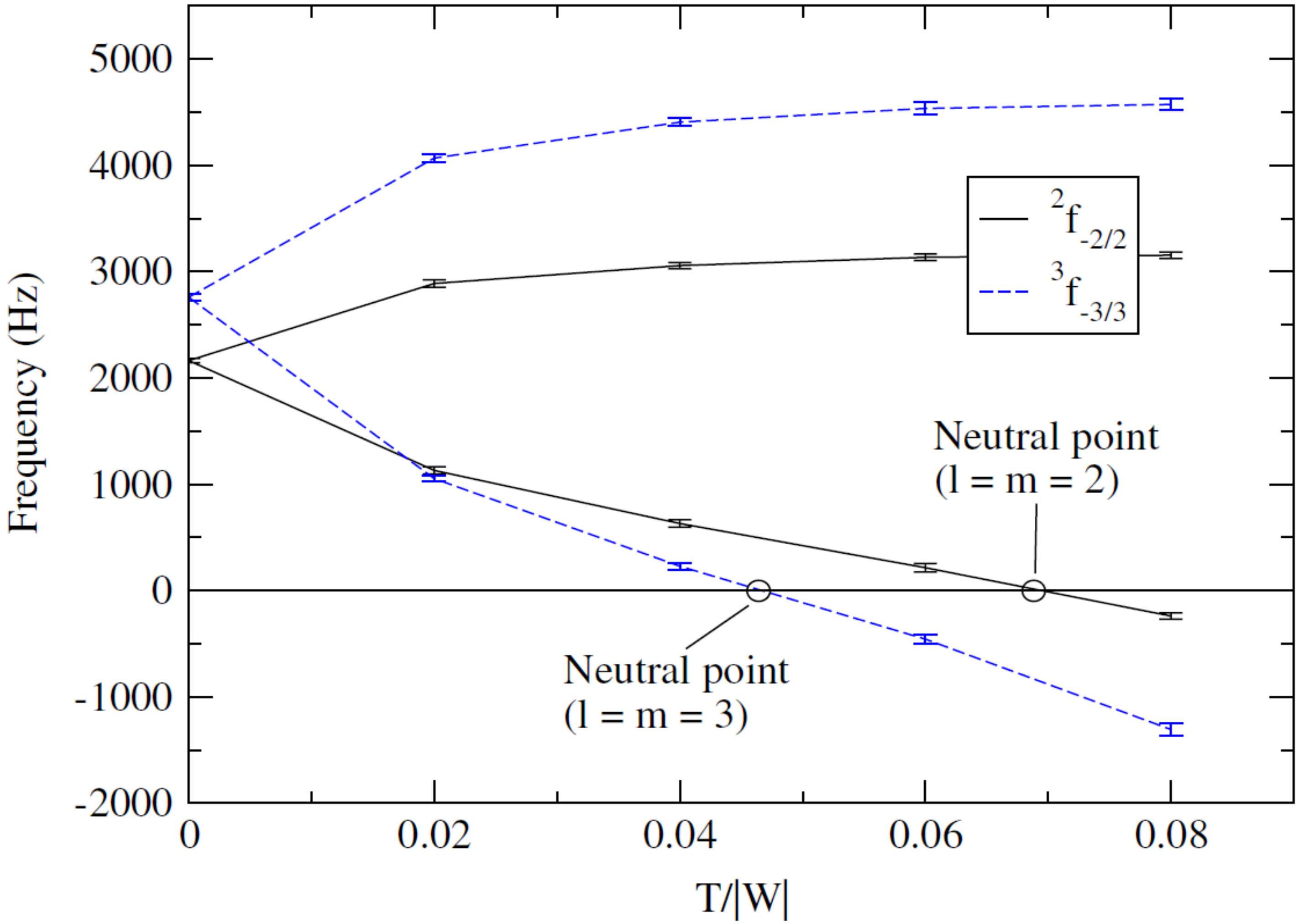
Rapid Rotation, full GR, *f*-modes
(3-D THOR code)



f-mode instability underestimated in Cowling approximation.
Neutral point near perturbative result by NS, Friedman (1998)

Rotational Instabilities

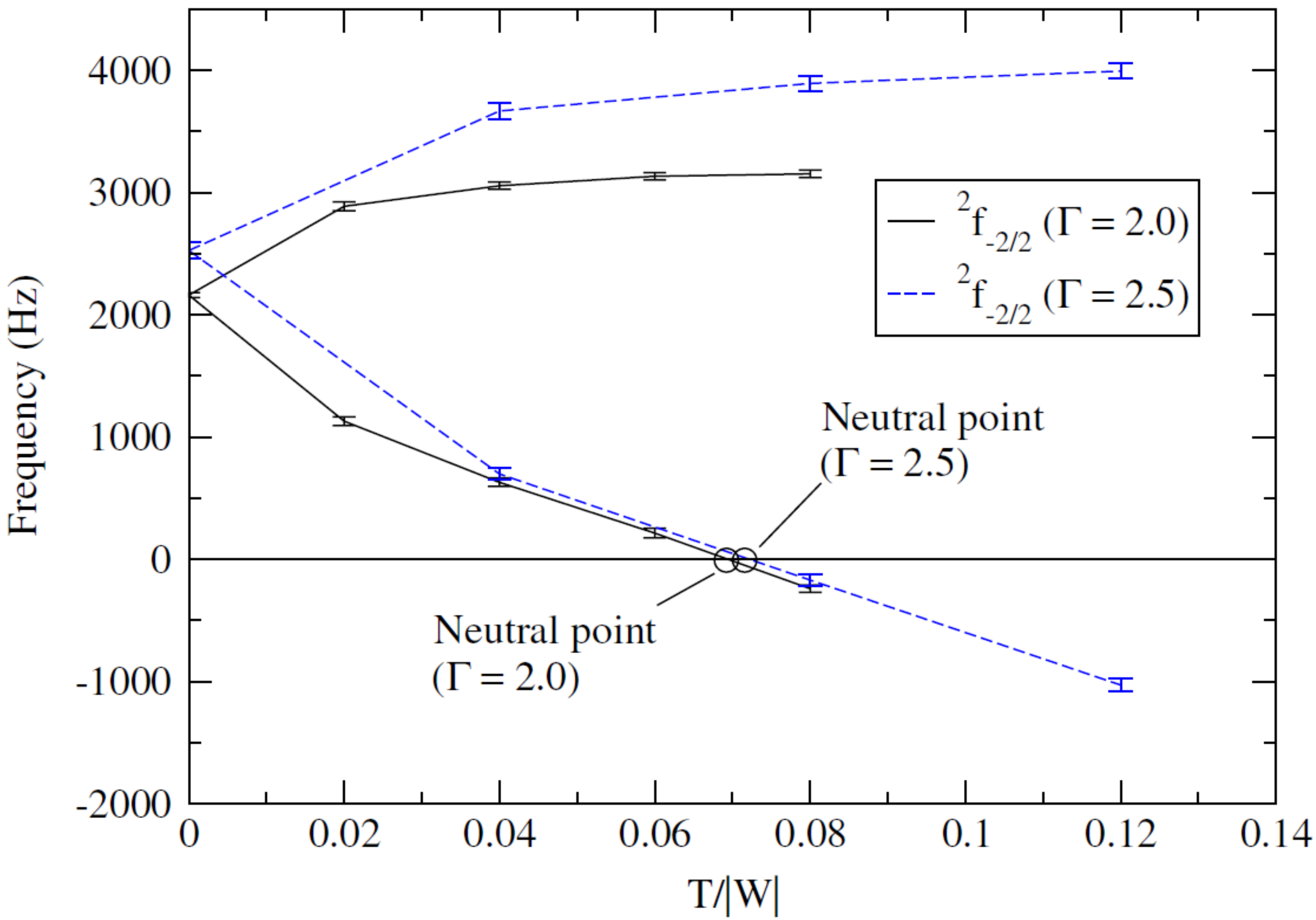
Comparison between l=m=2 and l=m=3 modes:



Larger instability window for $l=m=3$ (viscosity not taken into account, yet).

Rotational Instabilities

Comparison between $\Gamma = 2.0$ and $\Gamma = 2.5$ polytropes:



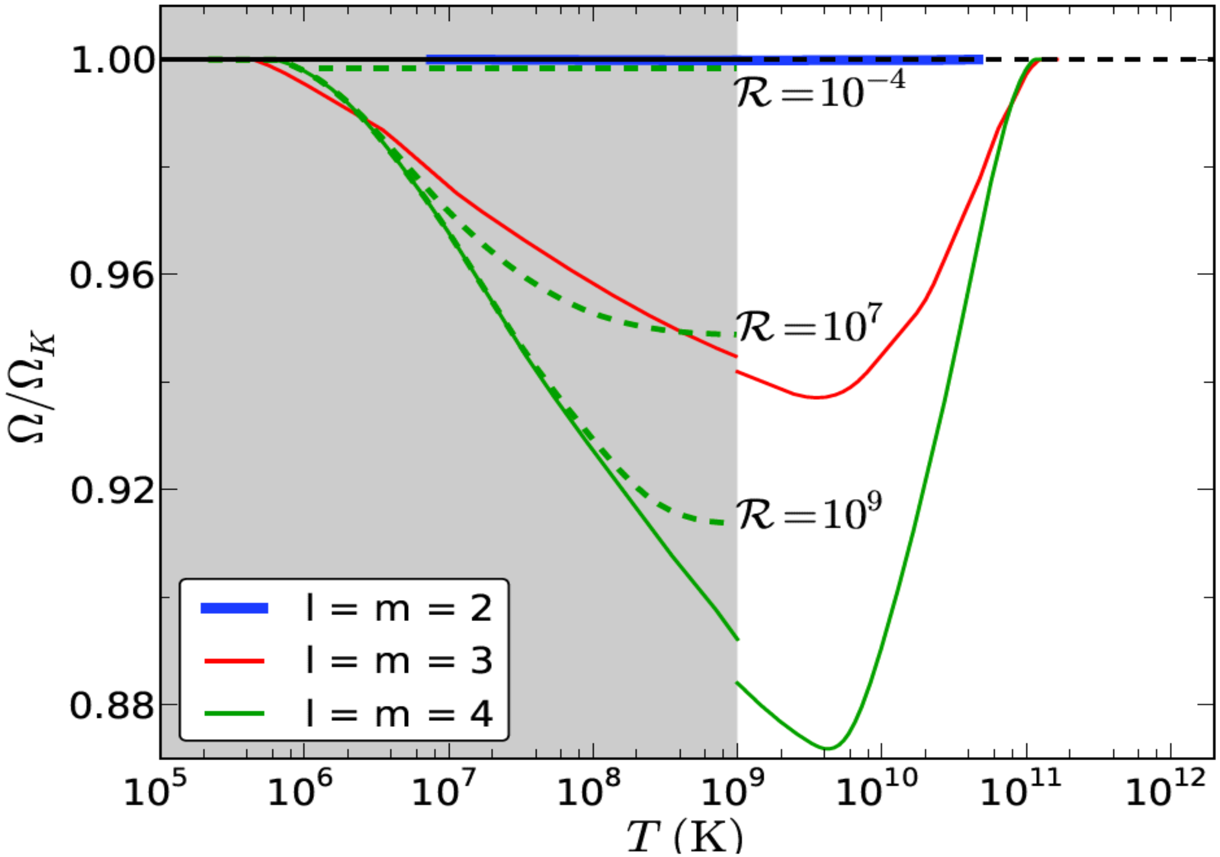
Similar neutral point, but larger instability window for stiffer EOS.

f-Mode Instability Window

Gaertig, Glampedakis, Kokkotas, Zink (2011)

Rapid rotation, Cowling approximation, <u>including dissipation</u>

(linear time-evolution code)



Shear vicosity (e^--e^- scattering)

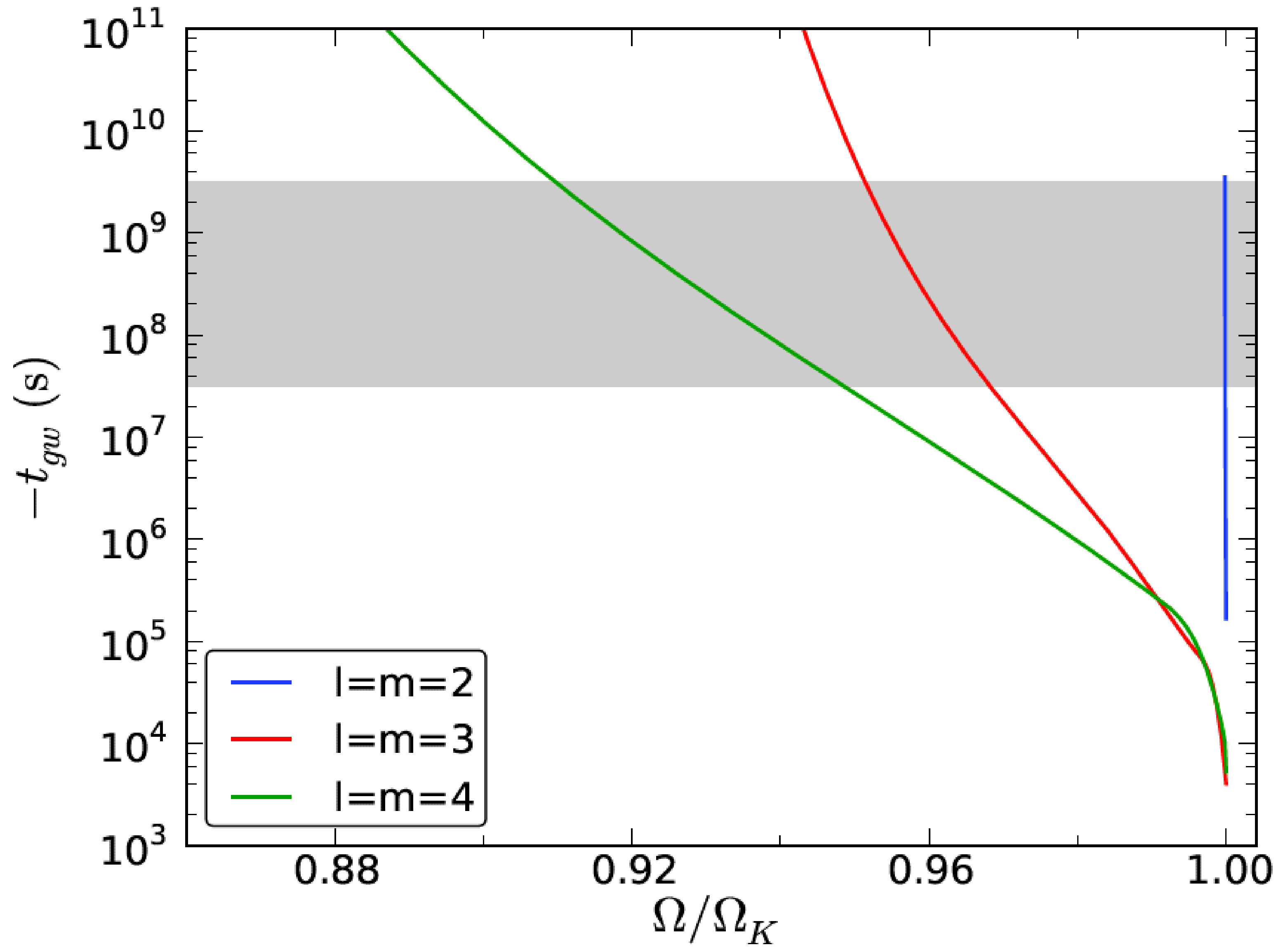
Mutual friction (e^- scattering off superfluid vortices)

Bulk vicosity

β-equilibrium, mURCA

f-Mode Instability Window

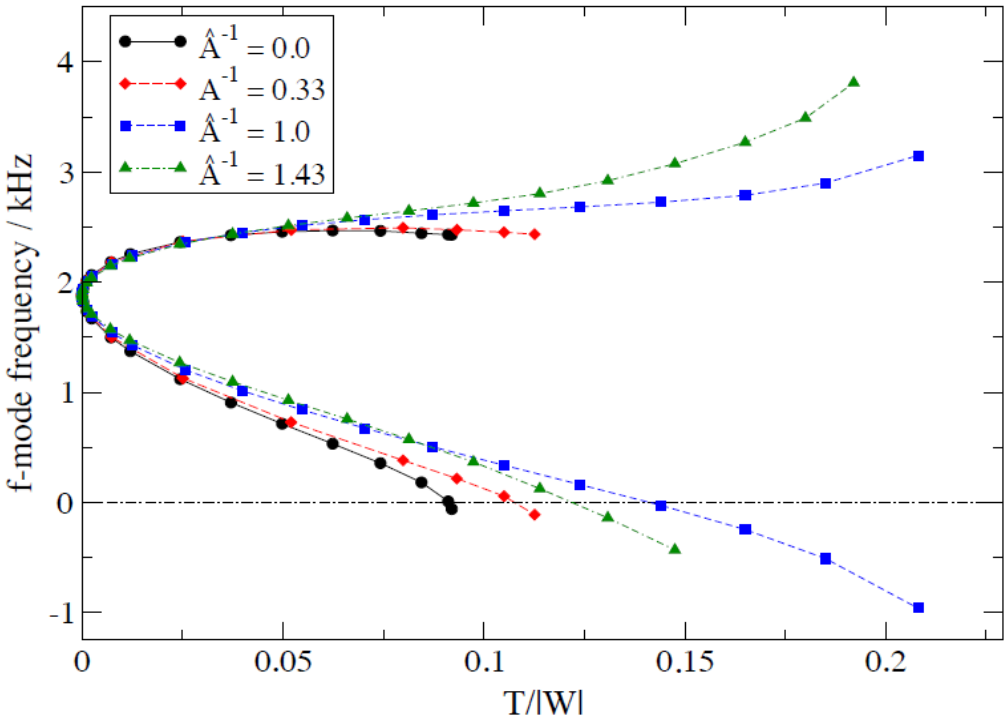
Instability growth times:

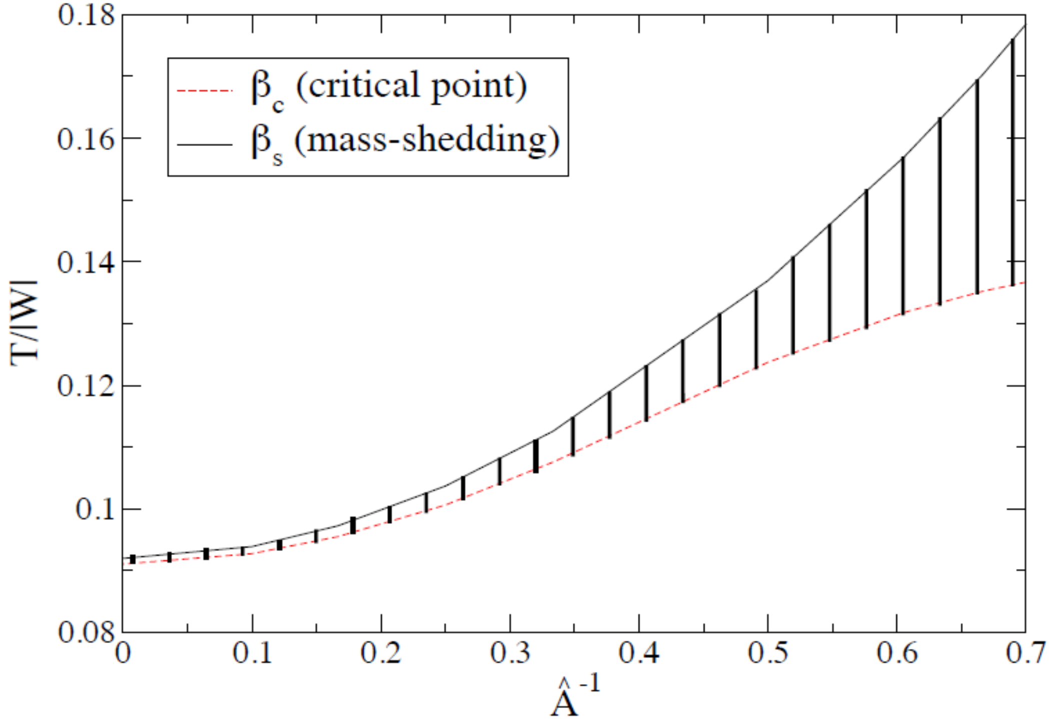


f-Mode Instability in Differentially Rotating Stars

Krueger, Gaertig, Kokkotas (2010)

Rapid <u>differential</u> rotation, Cowling approximation (linear time-evolution code)





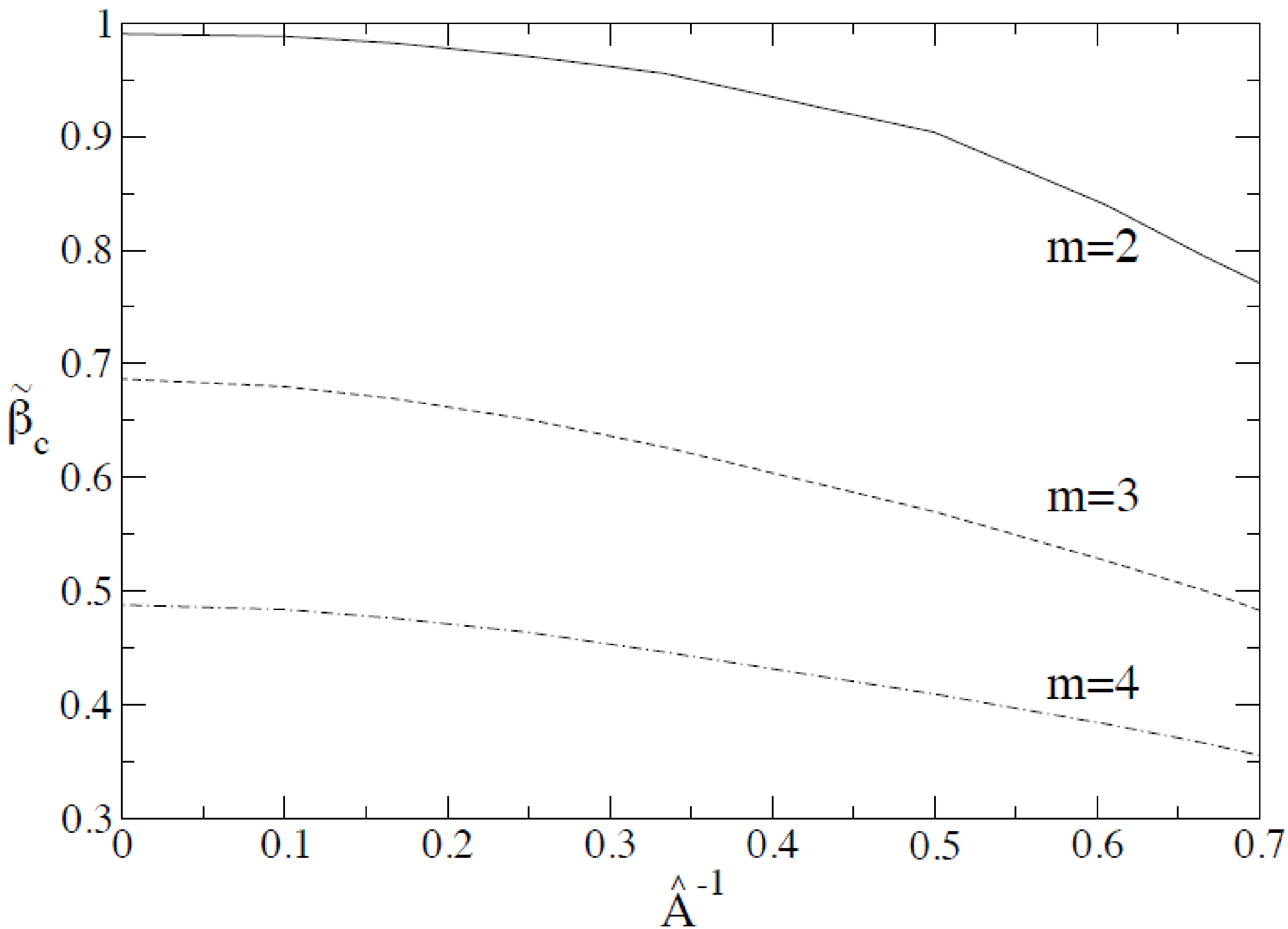
The critical absolute value of β=T/|W| for the instability increases.

At the same time, the value of β *at the mass-shedding limit* increases much more.

f-Mode Instability in Differentially Rotating Stars

Krueger, Gaertig, Kokkotas (2010)

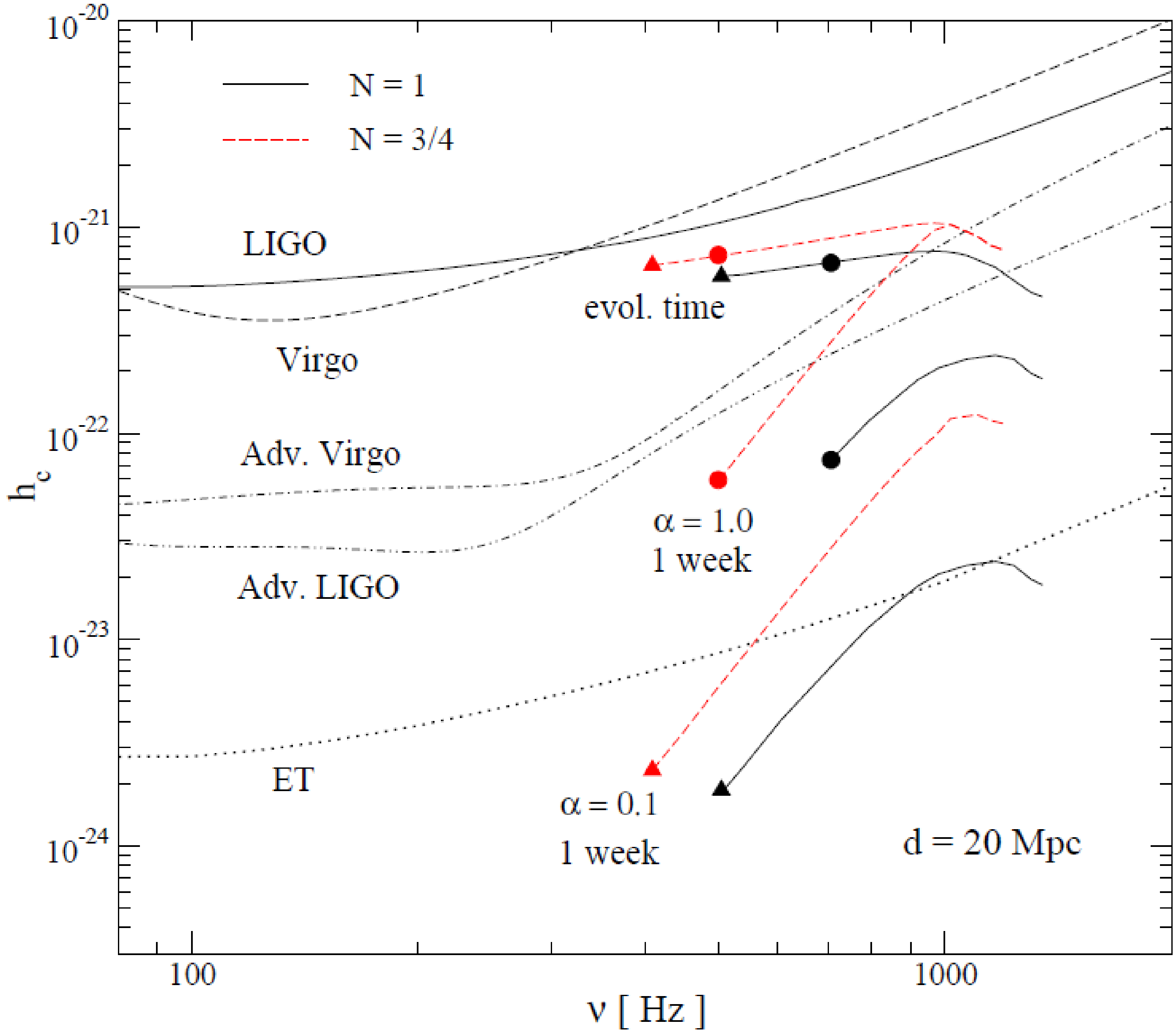
Overall, the *relative critical* β decreases with increasing degree of differential rotation.



Detectability of l=m=4 f-Modes

Passamonti, Glampedakis, (2010)

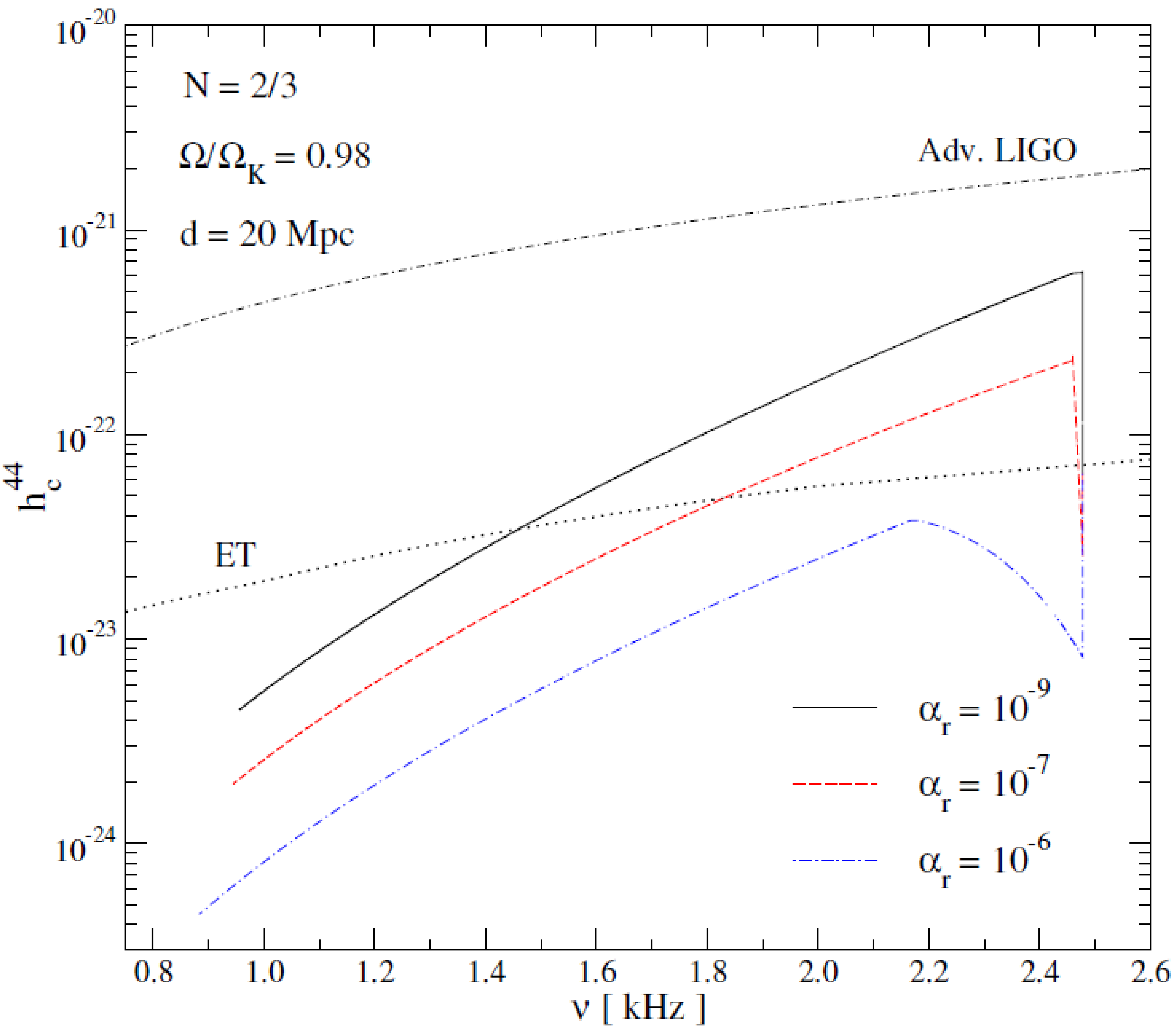
Detectability sensitive on: EOS, saturation amplitude, integration time.



l=m=4 *f*-Modes + l=m=2 *r*-Modes

Passamonti, Gaertig, Kokkotas (2013)

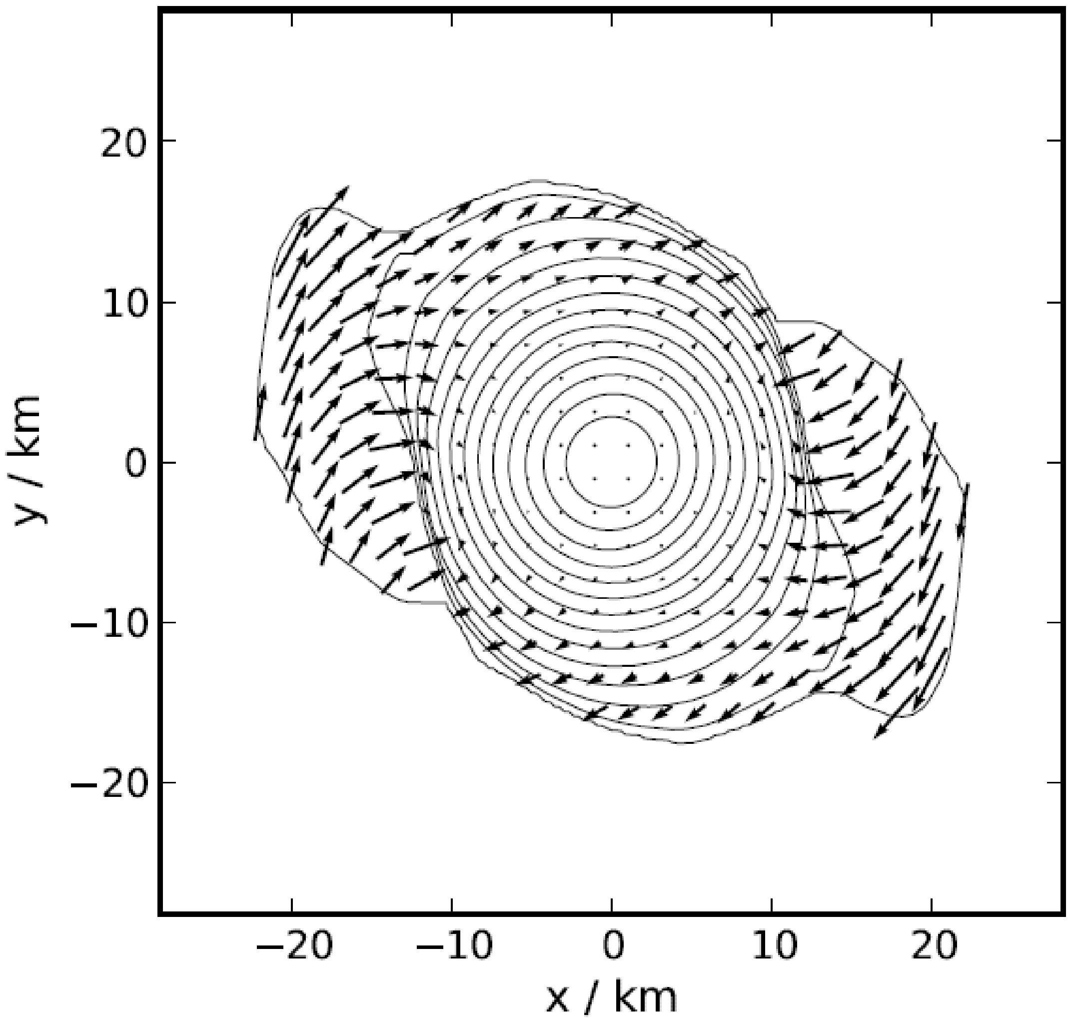
Assume 10^{-4} *f*-mode. Detectability sensitive on <u>*r*-mode amplitude !</u>



Nonlinear Saturation of l=m=2 *f*-Modes

Kastaun, Willburger, Kokkotas (2010)

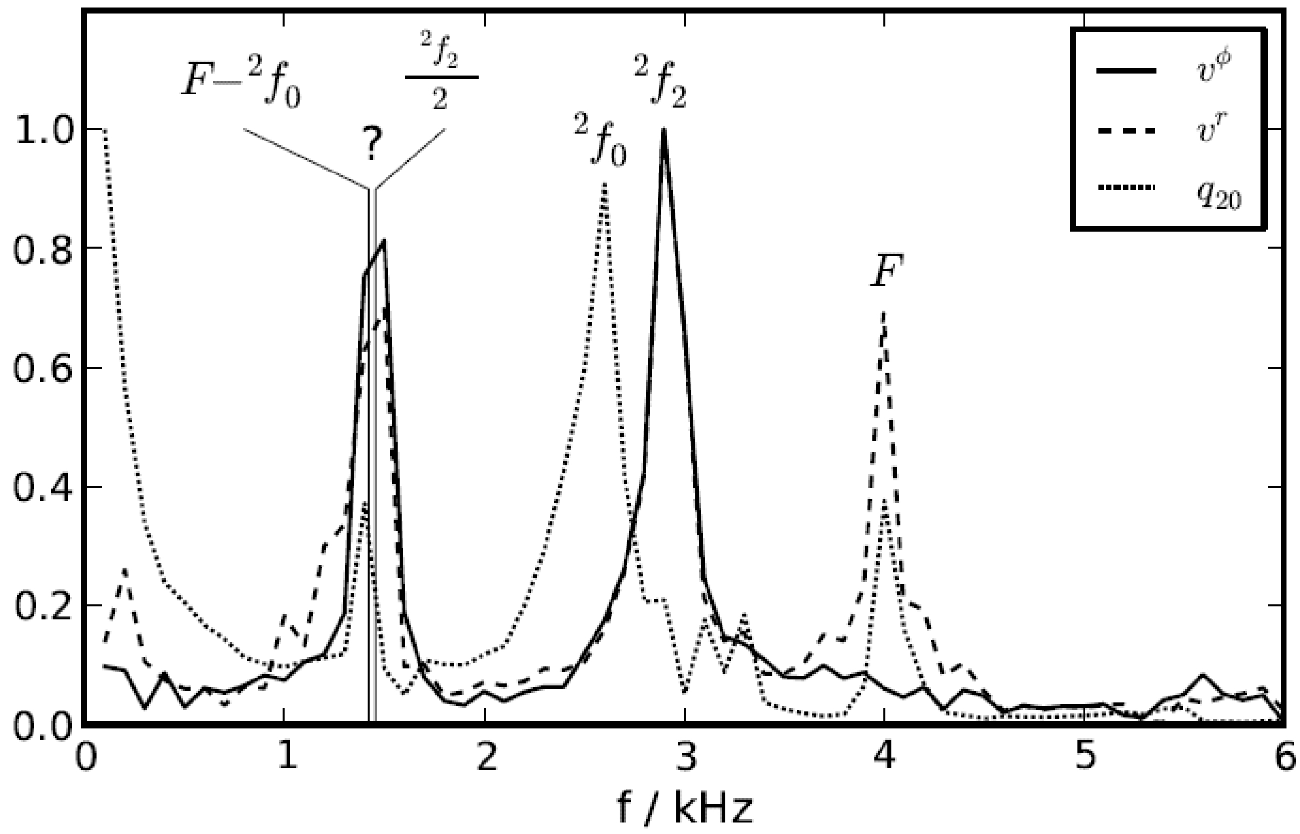
At amplitude of a few times 10^{-2} the *f*-mode is saturated by wave-breaking at the surface.



Nonlinear Coupling of f-Modes and Inertial Modes

Kastaun, Willburger, Kokkotas (2010)

At the same time, f-mode is coupled nonlinearly to an inertial mode of half the frequency.



Open Issues of f-Mode Instability

Which mode has the fastest growth time? l=m=2 or 4 ?

→ Need calculation of growth time in full GR and for tabulated EOSs.

Which is the nonlinear saturation amplitude of f-modes ?

→ Need high-resolution simulations to investigate coupling to inertial Modes.

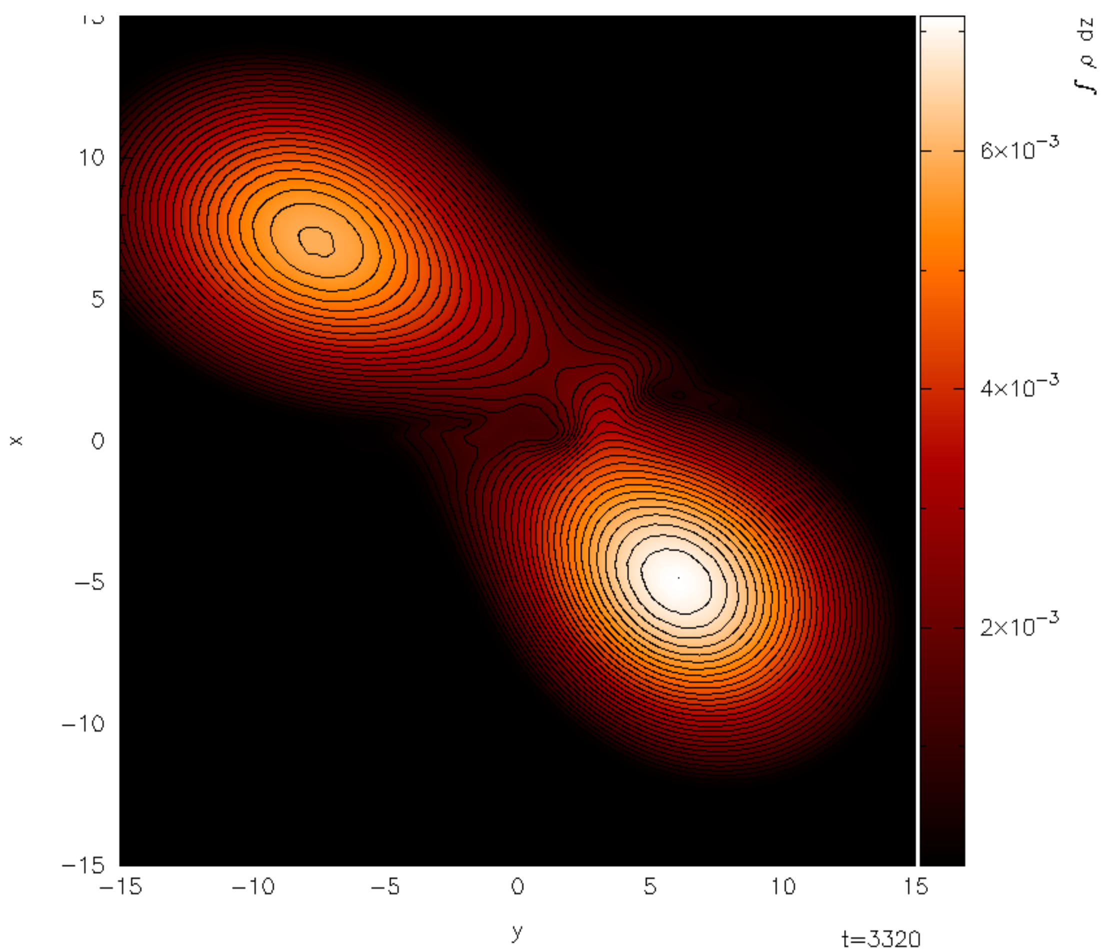
What is the effect of unstable r-modes on the f-mode instability ?

→ Need simulations in full GR with both instabilities present.

Mergers of Compact Object Binaries

NS, Bauswein, Zagkouris, Janka (2011)

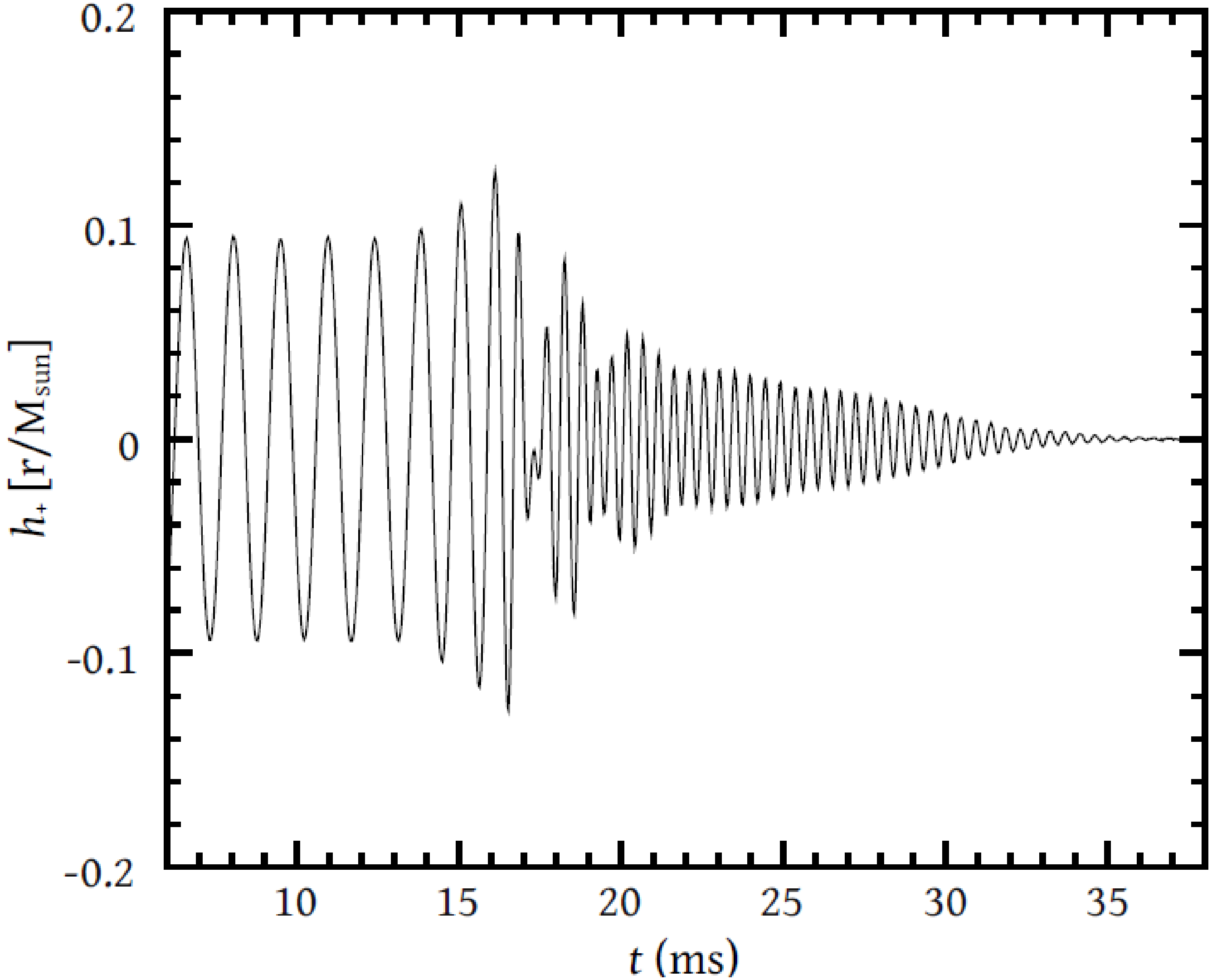
Merger of equal/unequal mass binaries with LS, Shen, MIT60 EOS.
(3-D GR CFC/SPH code)



Shen EOS: 1.2 Msun + 1.35 Msun

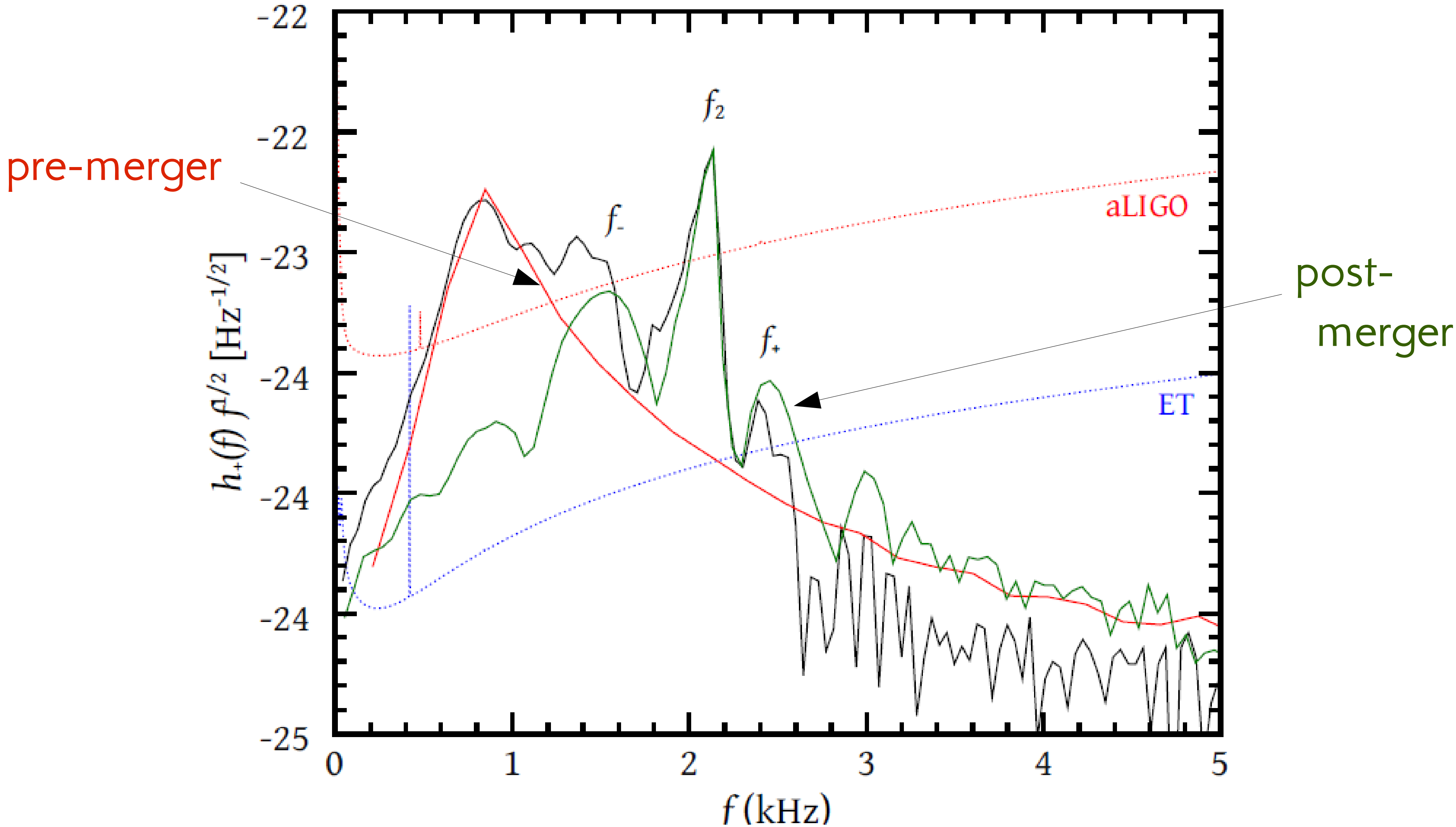
Mergers of Compact Object Binaries

Gravitational waves (via quadrupole formula)



Mergers of Compact Object Binaries

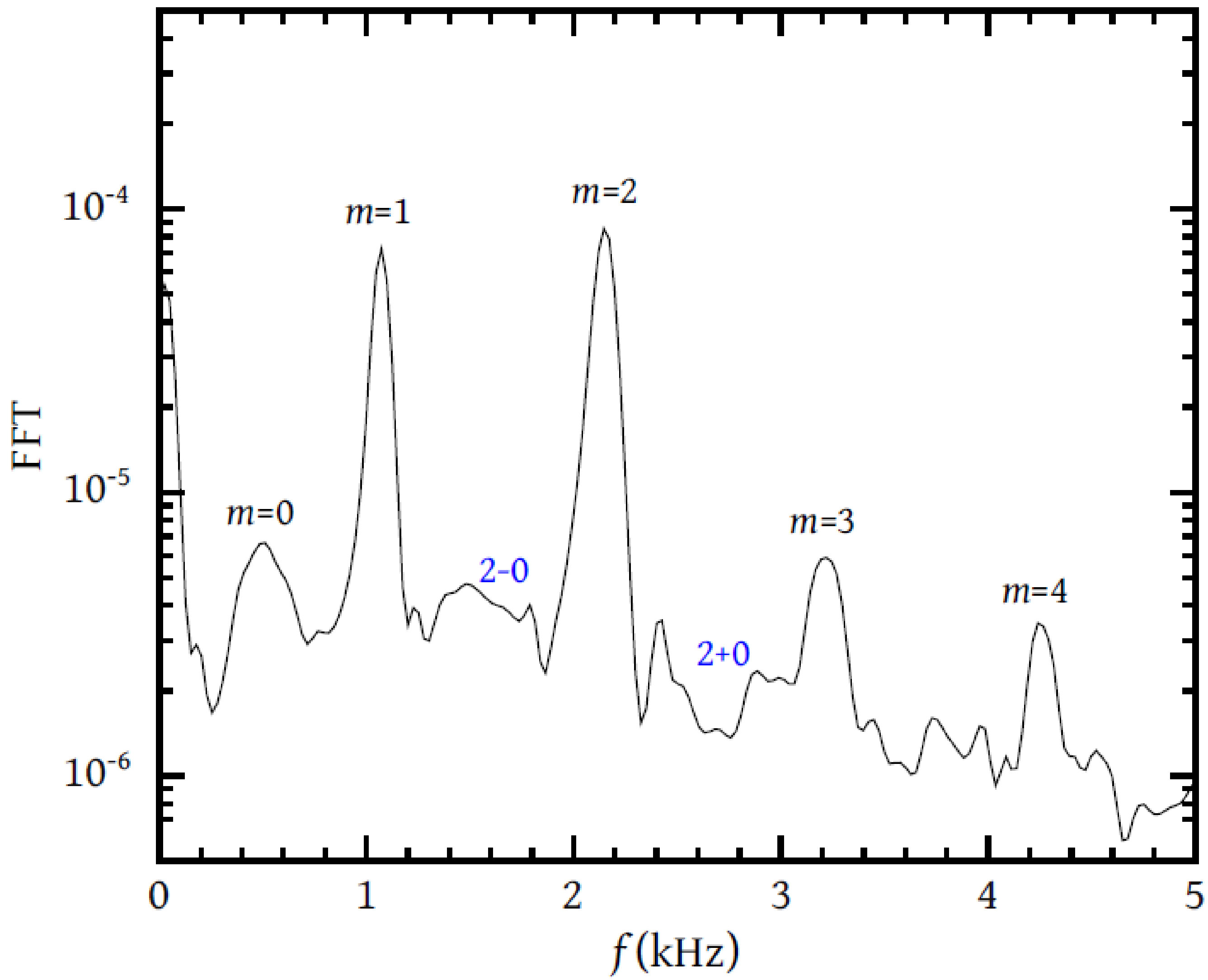
GW scaled power spectral density



Triplet of frequencies: f_-, f_2, f_+

Mergers of Compact Object Binaries

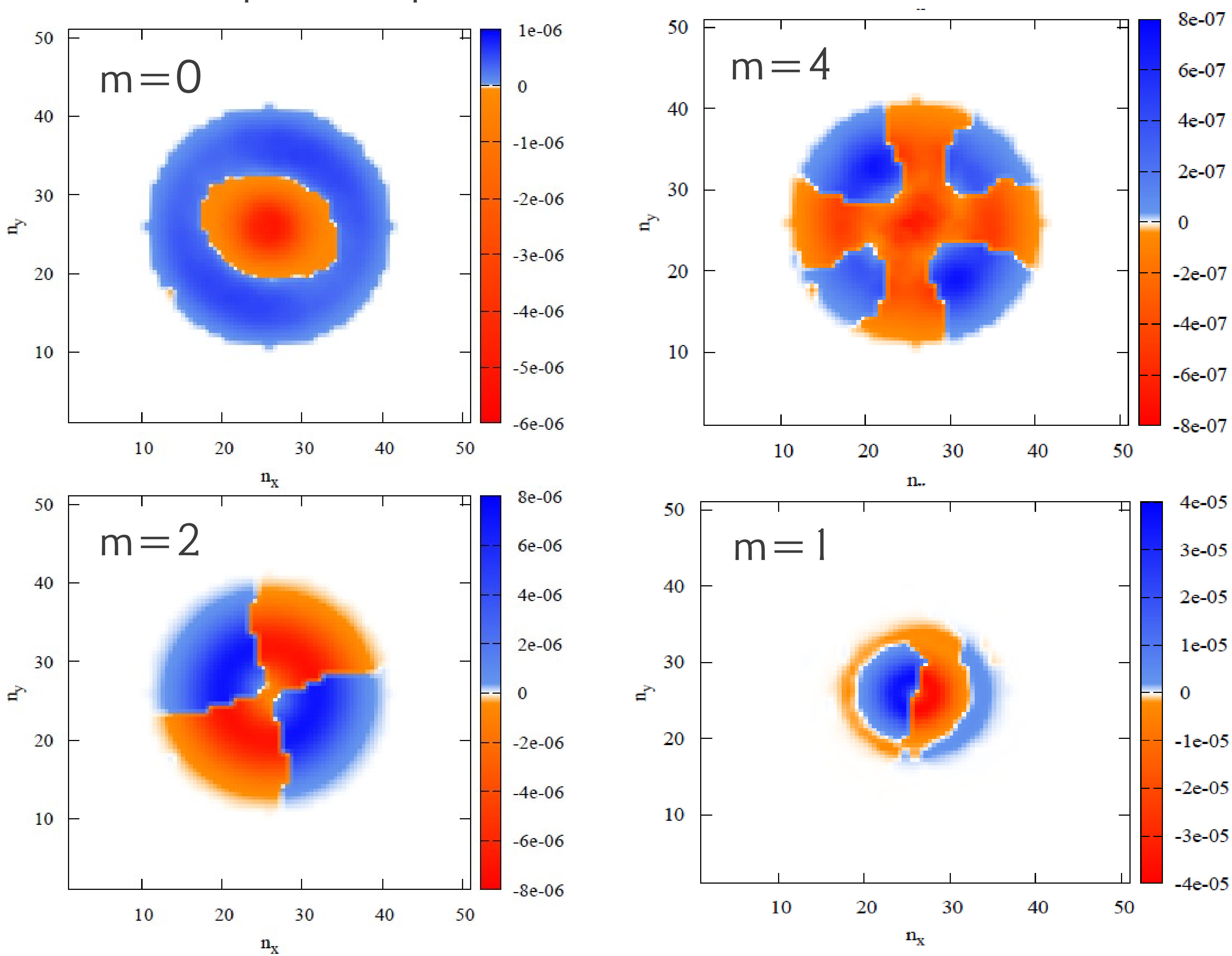
FFT of fluid variables:



Discrete mode frequencies!

Mergers of Compact Object Binaries

Eigenfunctions in equatorial plane



Mergers of Compact Object Binaries

Identification:

f_2 : $m=2$ mode excited after merger

f_- : $(m=2)$ - $(m=0)$ nonlinear combination frequency!

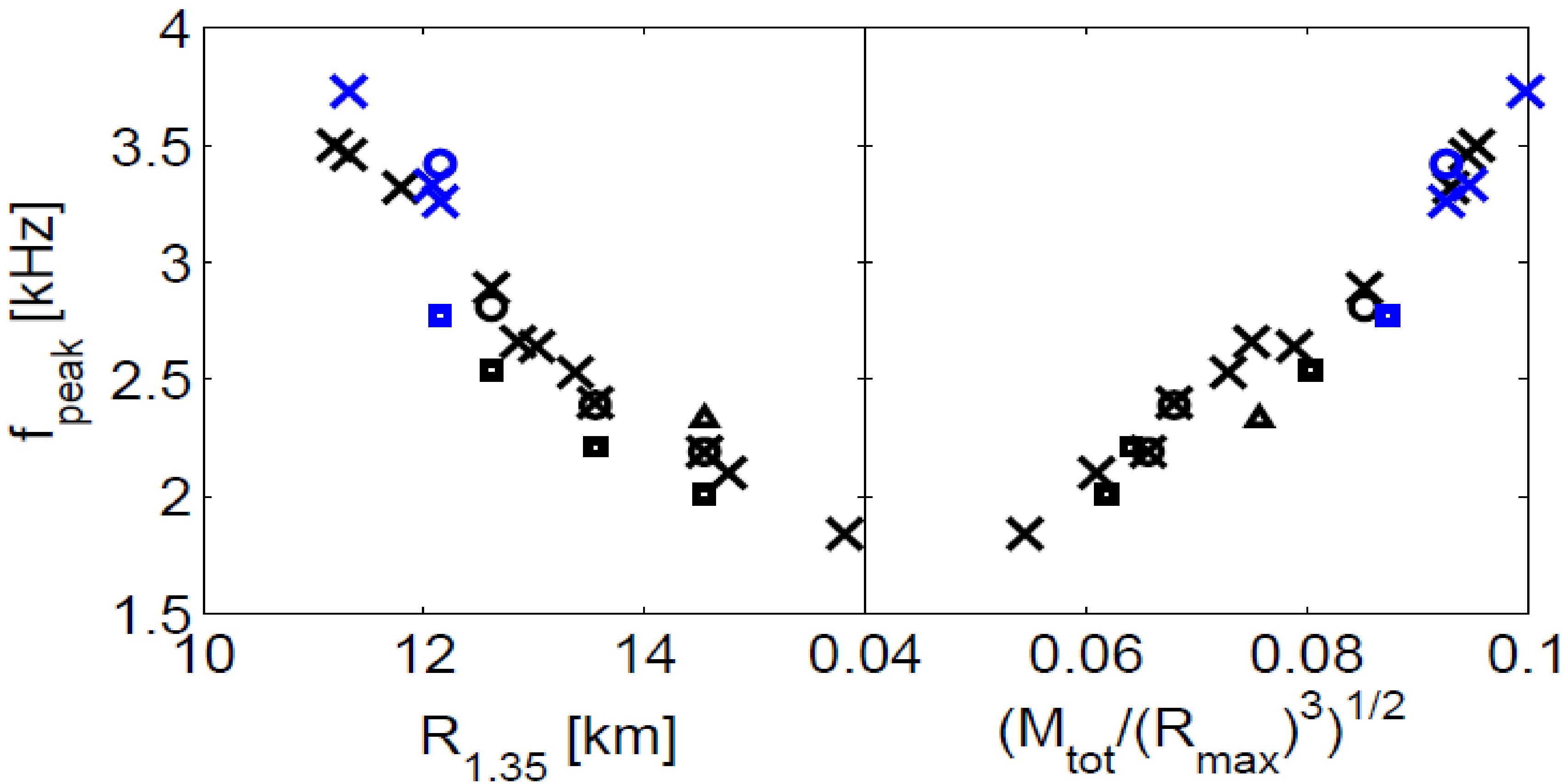
In case of detection: determine both m=0 and m=2 frequencies

GW gravitational-wave asteroseismology of binary mergers!

Mergers of Compact Object Binaries

Bauswein, Janka (2012)

Extracting EOS information from the post-merger signal:



THANK YOU